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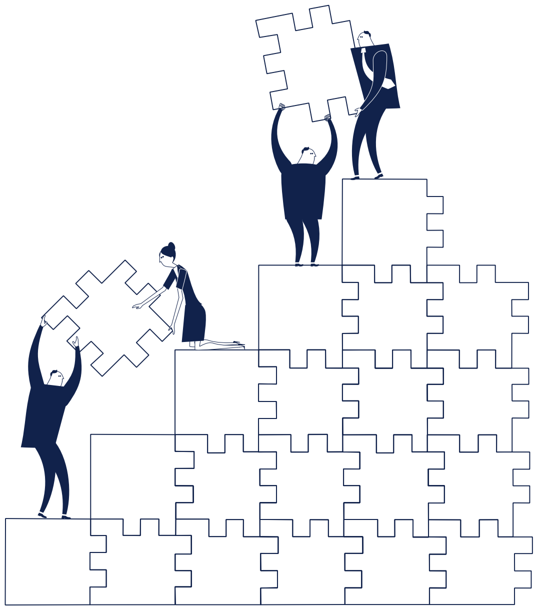

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Capital Flows, Default, and Renegotiation in a Small Open Economy[1]

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Abstract

The post-2008 period focused attention on "twin-crises". Banking crises may lead to sovereign crises where fiscal vulnerabilities are exacerbated by the extension of support for the banking system. We develop a model that describes private sector generated capital inflow that is used to finance investment and consumption expenditure. In the event of an economic contraction, the (convex) haircut on outstanding debt is negotiated, or bargained, centrally by the sovereign. Two results arise: the volume of debt and haircut rate are inefficient. In this setting the accumulation of capital achieves two goals. First, it generates sufficient optimism about future income to allow the debt market to function. Second, and counter-intuitively, it increases expected haircuts by raising the value of the outside option of complete default. These competing forces characterize the optimal balanced-budget macroprudential policy targeting capital investment.

Keywords: open economy, capital flows, debt, default, renegotiation

JEL Codes: *F34, G15, G18*

1 Introduction

Global banking-financial systems have become increasingly integrated since the 1980s. This has resulted in debt-denominated capital flows driving private sector credit growth. Although such flows have promoted growth and investment, they also exacerbated vulnerabilities in domestic systems that required sovereign support. The literature on the "twin-crises" of banking and sovereign has emphasized the interconnected nature of banking-financial sector and fiscal vulnerabilities.[1] Indeed such crises may originate from either sector: in Iceland they began in the banking sector whereas in Greece they originated in the sovereign. In this paper we focus on crises originating from private-sector generated external debt that ultimately required sovereign support.

In practice governments may "bail-out" banking-financial institutions by transferring their indebtedness onto the sovereign balance sheet.[2] In our model the sovereign negotiates with foreign lenders over the outstanding national stock of external debt. This process results in a "hair-cut", or partial default on what is to be repaid, that is then relayed to private sector borrowers. Our framework allows us to examine the role of centralized bargaining abstracting from fiscal considerations. Private sector debtors anticipate future hair-cuts on their debt before accumulating external debt. The pecuniary externality arising from privately accumulated debt renders the level of debt, and consequently the rate of default, inefficiently low. If the bargaining power of the sovereign is particularly low, private sector debt may collapse as

[1] Balteanu and Erce (2014) documents systematic differences between 'single' and 'twin' crises and examine the feedback loop between fiscal and financial distress. Reinhart and Rogoff (2013) show that banking crises cause severe contractions in fiscal revenues, raising government debt by about 86% in the years following the crisis.

[2] In the period following 2008 governments of advanced economies issued guarantees of bank debt, initiated renegotiations with bank creditors, undertook bank bailouts and nationalizations and sought assistance from big international lenders (e.g. the 'troika') to finance bank recapitalizations. An important lesson we can draw from these episodes: when faced with looming banking crises, governments attempt to mitigate losses of the financial system by taking charge over the fate of bank debt. For this reason sovereign spreads are affected by risks in the banking system. Acharya et al. (2014) examine the period of 2007-2009 associated with bank bailouts in Europe and find that the bailouts of banks by European governments triggered a hike in sovereign bond spreads. Reinhart and Rogoff (2011a) document that banking crises usually either precede or coincide with sovereign debt crises, and find that banking crises help explain sovereign defaults. Arellano and Kocherlakota (2014) arrive at a similar conclusion, and stipulate that domestic financial crises are typically associated with large transfers from the sovereigns to the private sector. The interconnection between bank and sovereign spreads is best examined by following the unraveling of the Irish crisis, see Appendix A.

required repayments in economic contractions are prohibitive. More surprisingly, if the bargaining power of the sovereign is *too high* it may also collapse. This is because lenders rationally expect higher hair-cuts and demand higher interest rates on flows to the point that gains-to-trade are exhausted. As we focus on private-sector flows, our model provides for explicit consideration of macro-prudential policies. We show that policies that reduce the relative price of capital (or, equivalently, provide incentives for investment) counter the pecuniary externality, stimulate flows and improve efficiency. Such policies may even generate flows where otherwise, due to pessimistic expectations of repayment, capital flows may collapse.

Models of national defaults typically incorporate losses associated with default events. In some models defaults are followed by output losses (e.g. Arellano 2008, Aguiar and Gopinath 2006, Asonuma and Trebesch 2016), in other models (welfare) losses are concurrent (e.g. Peiris and Tsomocos (2015), Walsh (2015a), Walsh (2015b)). Our model abstracts from these considerations and instead focuses on losses that occur *ex ante* and are driven by expectations of future defaults.[3] These expectations influence current capital investment and alter the path of capital stock. The majority of models on national defaults omit these effects because they either features endowment economies, or consider centralized versions of their models in which a planner accounts for the effect her current decisions would have on future welfare in equilibrium. By contrast, our framework gives rise to pecuniary externalities originating from the decentralized nature of borrowing and capital investment decisions.[4]

We develop a small open economy model with capital and incomplete markets that incorporates 1) private sector cross border flows (modeled as decentralized borrowing) and 2) renegotiation of private debt handled by the government (modeled as Nash Bargaining).

Our model yields endogenous partial defaults that occur in 'bad times', following unfavorable output shocks. In this context default can be thought of as an insurance against economic slowdowns.[5] We argue, however, that decentralized nature of the economy and the bargaining process introduce inefficiencies that impair consumption smoothing. First, we show that the haircut resulting from renegotiation is 'too low' and that higher haircuts expected ex ante could be pareto-improving. Second, we show that higher

[3] We also abstract from the link between defaults and currency devaluations examined in Na et al. (2014) and Schreger and Du (2014).

[4] Examples of other models of decentralized borrowing and centralized default are Kim and Zhang (2012) and Na et al. (2014).

[5] Our model shares this feature with the prominent studies of the field targeting developing economies, such as Arellano (2008), Aguiar and Gopinath (2006), Yue (2010).

bargaining power of the borrower does not fully resolve this problem: when bargaining power is 'too high', borrowing ex ante ceases to be possible. Finally, we investigate the role of macro-prudential policy and show that a subsidy on capital investment financed by a lump-sum tax can alleviate both problems: it can raise expected haircut on debt in 'bad times' and make borrowing sustainable ex ante, even when absent the subsidy investors do not lend.

This setup allows us to study properties of an economy displaying constrained suboptimality, as in Geanakoplos and Polemarchakis (1986), in which there is room for pareto-improving macroprudential policies. Our study contributes to the discussion in Dubey et al. (2005a), Peiris and Tsomocos (2015) and Walsh (2015b), who examine the interaction between market incompleteness, default rates and equilibrium allocations. We develop the ideas of Jeske (2006), Wright (2006) and Kim and Zhang (2012) who highlight importance of and channels through which decentralized debt affects capital flows and default.

Our finding that raising capital through capital subsidy can improve welfare echoes the result of Peiris and Vardoulakis (2013), that higher savings ex ante reduce expected default rates. The intuition behind their result is that current savings raise future consumption and reduce the marginal utility gain from defaulting. In our model this mechanism operates through capital: higher capital today makes repayment sustainable tomorrow. An important distinction is that in our model optimal capital subsidy results in *more* default, not less. This happens because of bargaining. Raising capital requires borrowing more, but larger debt in the future makes full default more attractive compared to partial restructuring. This improves households' stance during renegotiation: when home country has little to gain from renegotiating with the creditors, it repays less.

Recent empirical findings of Trebesch and Zabel (2016) suggest that the process of debt renegotiation (and the degree of debtor coerciveness) plays an important role in defining economic dynamics after defaults. Our framework shares some key elements with Yue (2010) who introduces Nash bargaining as a means of determining default decisions and recovery rates.[6] Although we follow Yue (2010) in the way we model bargaining between risk-neutral lenders and the government, in our model borrowing decisions are made by atomistic households that do not internalize the effect their current choices will have on future bargaining outcome. We show that in this setup lower ex-

[6]Other examples of models with defaults that feature Nash Bargaining protocol are Asonuma and Trebesch (2016), where the borrower may also choose to initiate a pre-emptive restructuring, and Arellano and Bai (2014), in which two borrowing countries simultaneously renegotiate with a common lender.

pected equilibrium default rates hinder potential for consumption smoothing across states.[7]

In contrast to the literature on sovereign default along the lines of Eaton and Gersovitz (1981) (e.g. Yue 2010, Arellano 2008, Aguiar and Gopinath 2006, Kim and Zhang 2012) that describe endowment economies, in our model capital and investment play a central role. Capital serves as a form of inter-period commitment that ensures repayment in the future and allows for positive borrowing ex ante. We show that if capital is subsidized it can deliver higher average levels of debt to GDP in equilibrium, because such a subsidy ensures that positive borrowing is always sustainable.

Our argument concerning optimal default rates is connected to that of D'Erasmo and Mendoza (2016), who also examine an economy with incomplete markets and stipulate that default can be chosen optimally to promote consumption smoothing. Unlike our paper, their work features defaults on both domestic and foreign creditors, and examines how defaults can serve to optimally redistribute resources across heterogeneous agents.

We also contribute to literature that models default costs as non-pecuniary losses developed by Shubik and Wilson. (1977) and Dubey et al. (2005b) and applied in Tsomocos (2003), Goodhart et al. (2005), Goodhart et al. (2006), De Walque et al. (2010) and Goodhart et al. (2016). We show that the allocation obtained in a model with bargaining can be replicated in a model with properly specified non-pecuniary loss. Within this literature our model shares many features with Peiris and Tsomocos (2015), who set up a two period large open international economy model with incomplete markets and default, and Walsh (2015a) and Walsh (2015b), that examine a small open dynamic incomplete markets economy.

In section 2 we introduce our baseline 3 period model, where in the first period representative households make decisions about capital and borrowing, while in the second period after uncertainty is realized they may choose to renegotiate the amount of repayment. If renegotiation occurs, the default rate is chosen to solve the Nash bargaining problem. In subsection 2.2 we show that this economy is equivalent to a decentralised economy where households decide on their individual rates of default by evaluating the (non-pecuniary) costs and benefits of defaulting. As this equivalent economy fits into standard general equilibrium methodology, we then use it to derive equilibrium properties of our baseline model. In subsection 3.1 we show that bargaining mechanism may prevent consumption smoothing. We calibrate our

[7]Another important difference between our setup and that of Yue (2010) is that in our model the economy does not experience output losses or market exclusion in an equilibrium with orderly renegotiation. We believe, however, that these features would not affect our qualitative results.

model to capture features of European debt crisis following 2008 and compare equilibria in a model with bargaining and a model where default rates accommodate full consumption smoothing. In subsection 3.2 we prove that when bargaining power is too high, borrowing becomes infeasible ex ante. In section 4 we set up a centralized version of our model and show that a planner can resolve this problem by manipulating the amount of capital investment. In section 5 we go back to a decentralized economy and demonstrate that a government can replicate main features of the centralized solution by introducing a subsidy on capital investment.

1.1 Defaults and default costs

Empirical studies show that defaults lead to capital market exclusion, a decrease in FDI flows (see Fuentes and Saravia 2010), a reduction in trade between borrowing and lending countries (see Rose 2005). Furthermore, Reinhart and Rogoff (2011b) provide evidence linking sovereign defaults and banking crises, while Arteta and Hale (2008) show that defaults are associated with a reduction of foreign credit to the private sector. Thus, empirical evidence suggests that defaults are typically accompanied by economic slowdowns. Theoretical studies interpret this evidence twofold: as indicative of 1) defaults *being triggered* by economic downturn and 2) defaults *causing* further output losses. These assumptions give rise to a tight negative correlation between defaults and output. In practice, as noted by Panizza et al. (2009), it is hard to disentangle the cause and the effect. For example, it may be that fears of looming default trigger distress in the banking sector which deepens the recession; the recession in turn exacerbates sovereign default crisis, and by the time government defaults, the losses associated with default has already occurred.This reasoning is reinforced by the findings of Yeyati and Panizza (2011) who use quarterly data—higher frequency than commonly used—to show that output losses typically precede default events, not vice versa.

Furthermore, Tomz and Wright (2007) argue that the evidence for defaults occurring in slowdowns is weak, based on dataset covering the period 1820-2004 and 175 debtor countries. In their sample, only 62% of the 169 defaults began when output was below trend. Moreover, when the authors look at all years in which countries were in default, they find that about 44% of those years coincided with output being above trend, which again casts doubt on the notion of defaults causing output declines. Overall, defaults coincided with output being 1.6 percentage points below trend on average. Even if this can be attributed to default causing output loss, this cost is still lower than 2% output loss assumed by Aguiar and Gopinath (2006) and

Yue (2010). To sum up, the evidence of defaults causing (and being cased by) economic downturn is not as clear cut as the distinction adopted by the theoretical literature.

These considerations suggest that default is not just insurance against bad output shocks and, according to Tomz and Wright (2007), this poses a question of why countries do not default enough in 'bad' times. Our methodology can provide a partial explanation for this puzzle. First, to generate positive borrowing and default, our model does not have to rely on equilibrium default losses: in our baseline framework output losses only occur if the country refuses to renegotiate and exercises its outside option, defaulting by full amount and entering permanent autarky. The losses associated with outside option are not observed in equilibrium—therefore, our framework is flexible enough to allow for a less tight negative correlation between default and output. Second, by introducing bargaining we allow for an extra variable governing the timing of default events—the borrower's bargaining power. Variation in this parameter can potentially explain, why sometimes countries choose to continue repayment even though the economy is in bad shape, while other times the default ensues even when economy is doing well. Third, in our model decentralized borrowing helps explain why default is an imperfect mean for insuring against bad output shocks. Unlike in Yue (2010), in our model households that borrow do not fully internalize the effect of their decisions on the bargaining outcome, default probabilities and the bond price. We show that introducing such centralized borrowing would improve consumption smoothing.

2 Model

In this section we present a model that features decentralized borrowing by private agents and collective default. When we refer to partial default or default rates, we mean haircuts that are a successful outcome of bargaining where debtors agree to repay a portion of outstanding debt. When we refer to full or complete default, we mean the breakdown of renegotiation resulting in creditors receiving nothing. In subsection 2.1 we consider a setup along the lines of Yue (2010), in which the borrowers bargain with risk-neutral lenders over default rates, and the outcome is determined via a Nash Bargaining Solution. In subsection 2.2 we construct an alternative model, in which default has no direct material costs, but instead leads to non-pecuniary losses in household utility. We specify the non-pecuniary losses in a way that allows replicating the allocation in the bargaining model of subsection 2.1. As the setup with non-pecuniary losses proves more convenient for equilibrium and welfare analysis, we use it in the subsequent sections to further investigate the properties of our economy, and refer to it as the λ-equilibrium.

2.1 Nash Bargaining Framework

We consider a model of two countries, Home and Foreign, and three time periods, $t = \{0, 1, 2\}$. Home is inhabited by a unit measure of identical households, h, and a production technology (Firm). Households supply labour inelastically at the beginning of every period, before production of final output occurs; households receive competitive wage w. Firms then transform capital and labour into final output using a constant returns to scale technology; they then distribute any profits to Home households. Firms are endowed with a domestic productive technology A. Furthermore, the firms may raise their productivity by the share Z, that has to be prepaid by borrowing from abroad in the intraperiod capital market. When such borrowing is available, the total factor productivity is $A(1 + Z)$. Access to the intraperiod capital market is predicated on Home households having the ability to access foreign capital markets.

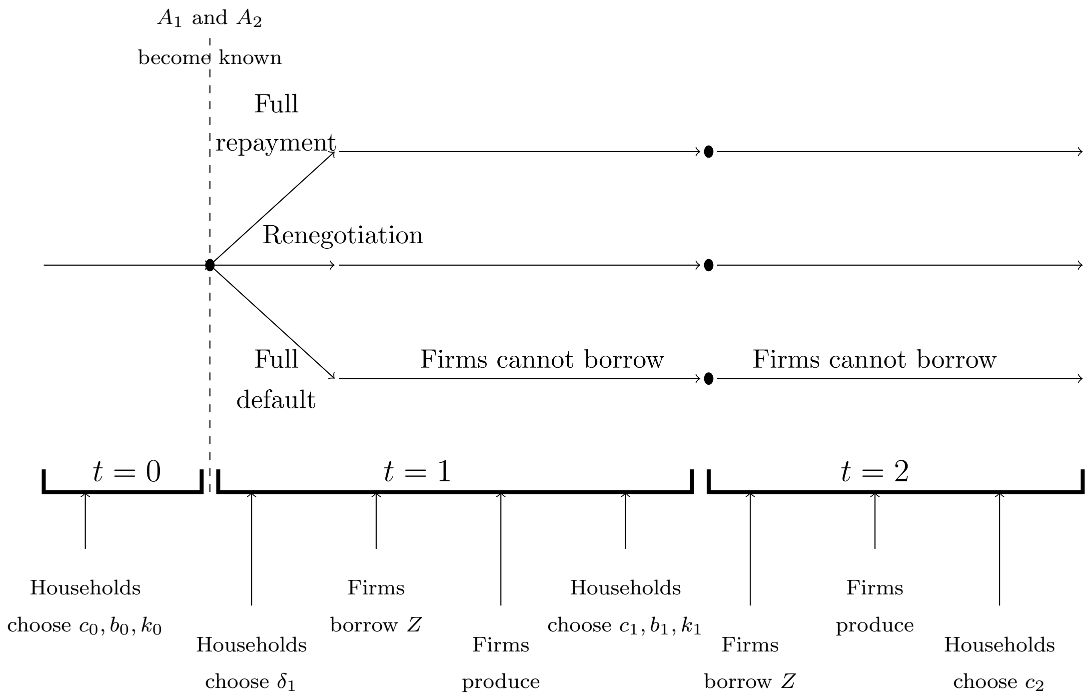
Productivity in period 1 is uncertain. At the beginning of period 1, one of two states is realized, $s = \{H, L\}$. In the last period there is no further uncertainty. Thus, all uncertainty is realized in period 1. In total there are 5 date events, 1 in the first period, 2 in the second period and 2 in the last period.

Households enter date 0 having rented k_{-1} units of capital, on which they receive the gross return R_0. They supply a unit of labour l_0 to firms and receive competitive wage w_0. In addition to the total rental income from factors of production, households can borrow from the Foreign country an amount b_0 to be repaid in the second period at gross interest rate I_0. We follow Yue (2010) in assuming that foreign lenders are risk-neutral.

In period 1, households not wishing to honour the full debt due ($b_0 I_0$) may send a request to the government to bargain with lenders on a reduced final repayment. The government then initiates renegotiation, taking the total requests of households as a given, and acting in their interests. The outcome of this debt renegotiation—the default rates $\delta_1(s)$—is determined via the Nash Bargaining solution. When bargaining is concluded, the default rates are reported to households who decide whether to accept, in which case they repay the agreed rate, or not, in which case they default fully but incur additional costs. Specifically, households that renege cannot access the foreign debt market in period 1.

The timing of events is depicted on Figure 1. Bargaining occurs before any production decisions, therefore, households that do not agree with the aggregate bargaining outcome can only rent their second period capital to firms that are also denied the ability to access the capital markets. Thus the pecuniary costs of not accepting renegotiation are two-fold: financial autarky and lower final period total factor productivity.[8] Default does not occur in the last period as there is no uncertainty, so we ignore it. We restrict our attention to rational expectations equilibria.

Figure 1: Timeline



2.1.1 Firms

Firm productivity depends on two components: a part A that is domestic and depends on the realization of the state of the world, and a fixed part Δ that has to be refinanced each period with external debt. Firm's profit in each period is:

$$\Pi = A(1+Z)K^{\alpha}L^{1-\alpha} - RK - wL - \kappa Z, \tag{1}$$

[8]Formally speaking there are three possibilities: full default, in which case there would be nothing lent *ex ante*; partial repayment, in which case the default rate is priced into the interest rate; and full repayment. As only the latter two are possible in equilibrium, we ignore the case in which a country enters financial autarky in period 1.

where κ is a cost of refinancing Z. Define output as $Y = A(1+Z)K^{\alpha}L^{1-\alpha}$ Optimality implies:

$$R = \alpha \frac{Y}{K}, \tag{2}$$

$$w = (1-\alpha)\frac{Y}{L}. \tag{3}$$

2.1.2 Home Households

Households are identical; their lifetime utility is:

$$u(c_0) + \sum_{s=L,H} \pi(s) \sum_{\tau=1}^{2} \beta^{\tau} u(c_{\tau}(s)), \tag{4}$$

where $\pi(s) > 0$ is the probability of the state s occurring, and $\sum_{s=1}^{S} \pi(s) = 1$. The budget constraints are:

$$\begin{aligned} c_0 + k_0 &= w_0 l_0 + R_0 k_{-1} + b_0, & (5)\\ c_1(s) + k_1(s) + (1-\delta_1(s))b_0 I_0 &= w_1(s)l_1(s) + R_1(s)k_0 + b_1(s), & (6)\\ c_2(s) + b_1(s)I_1(s) &= w_2(s)l_2(s) + R_2(s)k_1(s), & (7) \end{aligned}$$

where $c_t(s)$, $k_t(s)$, $w_t(s)$, $R_t(s)$ are consumption, capital, wage and return on capital in period t, state s; b_0 is the amount borrowed in period 0; I_0 is the gross interest on household borrowing due in period 1, and $\delta_1(s)$ is the rate of default; $b_1(s)$ is borrowing in period 1 state s, and $I_1(s)$ is the gross interest to be paid next period. Labour is supplied inelastically $l_0 = l_1(s) = l_2(s) = 1$, $\forall s \in \{H, L\}$.

First order conditions that solve households' problem are:

$$\frac{\partial u(c_0)}{\partial c_0} = \beta I_0 \sum_{s=1}^{2} \pi(s)(1-\delta_1(s)) \frac{\partial u(c_1(s))}{\partial c_1(s)}, \tag{8}$$

$$\frac{\partial u(c_0)}{\partial c_0} = \beta \sum_{s=1}^{2} \pi(s) R_1(s) \frac{\partial u(c_1(s))}{\partial c_1(s)}, \tag{9}$$

$$\frac{\partial u(c_1(s))}{\partial c_1(s)} = \beta I_1(s) \frac{\partial u(c_2(s))}{\partial c_2(s)}, \tag{10}$$

$$R_2(s) = I_1(s). \tag{11}$$

2.1.3 Foreign Households

Foreign country is populated by identical households with lifetime utility:

$$c_0^f + \sum_{s=1}^{2} \pi(s) \sum_{\tau=1}^{2} \beta^\tau c_\tau^f(s) \tag{12}$$

Their budget constraints are:

$$\begin{aligned} c_0^f + b_0^f + s_0^f &= y_0^f & (13)\\ c_1^f(s) + s_1^f(s) &= y_1^f(s) + I_0^w s_0 + (1-\delta_1(s)) I_0 b_0^f & (14)\\ c_2^f(s) &= y_2^f(s) + I_1^w(s) s_2^f(s) + I_1(s) b_2^f(s) & (15) \end{aligned}$$

Foreign bonds, s^f, are riskless. They are traded at an exogenous world interest rate I_t^w. Utility maximization demands:

$$\begin{aligned} 1 &= \beta I_0 \sum_{s=1}^{2} \pi(s)(1-\delta_1(s)), & (16)\\ 1 &= \beta I_1^w(s), & (17)\\ 1 &= \beta I_1^w(s). & (18) \end{aligned}$$

Additionally, no arbitrage implies

$$I_1(s) = I_1^w(s). \tag{19}$$

2.1.4 Nash Bargaining and Equilibrium

Our Nash Bargaing setup follows closely that of Yue (2010) featuring bargaining with foreign risk-neutral lenders over debt redemption. The Nash Bargaining problem determines the default rate outcome by evaluating welfare surpluses gained by each party by participating in bargaining. Formally, for each $s \in \{H, L\}$:

$$\begin{aligned} \delta_1(s) \quad &= \quad \begin{cases} \arg\max_{\delta \in [0,1]} \left[(\Delta_1^h(s))^\theta ((\Delta_1^f(s))^{1-\theta} \right] & \text{if } \Delta_1^h(s) > 0, \\ 1, \ \text{else.} \end{cases} & (20)\\ s.t. \quad & \quad \Delta_1^h(s) \geq 0, \\ & \quad \Delta_1^f(s) \geq 0. \end{aligned}$$

The functions $\Delta_1^h(s)$ and $\Delta_1^f(s)$ give welfare gains from participating in renegotiation, for Home and Foreign. These surpluses are assigned weights that represent relative bargaining power of each party, with θ corresponding to the bargaining power of the borrower, and $(1-\theta)$ to the bargaining power of the lender. For each party, the surplus $\Delta_1(s)$ is given by the difference between the value function of repayment, and the value function that would arise if the agent decided to exit renegotiation (invoking full default). Specifically, for Home (the borrower):

$$\Delta_1^h(s) = \{VR\}_1^h(s) - \{VA\}_1^h(s), \tag{21}$$

where $\{VR\}_1^h(s)$ gives value function under renegotiation:

$$\{VR\}_1^h(s) = u(c_1(s)) + \beta u(c_2(s)), \tag{22}$$

and $\{VA\}_1^h(s)$ gives value function under financial autarky:

$$\{VA\}_1^h(s) = u(c_1(s,a)) + \beta u(c_2(s,a)). \tag{23}$$

Similarly, for the Foreign (the lender):

$$\Delta_1^f(s) = \{VR\}_1^f(s) - \{VA\}_1^f(s), \tag{24}$$

where $\{VR\}_1^f(s)$ is foreign households' value of renegotiation:

$$\{VR\}_1^f(s) = c_1^f(s) + \beta c_2^f(s), \tag{25}$$

and $\{VA\}_1^f(s)$ is their value of autarky:

$$\{VA\}_1^f(s) = c_1^f(s,a) + \beta c_2^f(s,a). \tag{26}$$

In our model, autarky means the absence of financial trade between Home and Foreign. As such, it has different implications for the two countries. Foreign country has access to unlimited risk-free borrowing, and can still trade with the rest of the world if it decides not to trade with Home. By contrast, Home can only borrow on the international financial market through Foreign. Thus, for Home the absence of trade has two effects: *1)* it impairs productivity of domestic firms, as they cannot borrow Z to up the return on capital, and *2)* it prevents households from borrowing to smooth consumption.

Exiting renegotiation for a foreign households means receiving 0 from Home. As the foreign households are risk-neutral, their surplus from renegotiation simply equals the amount they gain by agreeing to renegotiate:

$$\Delta_1^f(s) = (1-\delta_1(s))I_0 b_0^f. \tag{27}$$

For Home, consumption choices under renegotiation can be obtained by maximizing $\{VR\}_1^h(s)$ with respect to period 1 and 2 budget constraints

(6),(7), and plugging in equilibrium wage (3) and interest rates (2),(18). Furthermore, assuming CRRA utility, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, we can directly solve for $c_1(s)$ and $c_2(s)$:

$$
\begin{aligned}
c_1(s) &= \frac{I_1^w}{I_1^w + 1}[A_1(s)(1+Z)k_0^\alpha - (1-\delta_1(s))b_0 I_0 + \\
&\quad + \frac{1-\alpha}{\alpha}\left[\frac{\alpha A_2(s)(1+Z)}{I_1^w}\right]^{\frac{1}{1-\alpha}}], && (28)\\
c_2(s) &= c_1(s). && (29)
\end{aligned}
$$

An increase in the default rate $\delta_1(s)$ or capital k_0 raises consumption in both periods, and raises $\{VR\}_1^h(s)$.

When Home enters autarky, the country is no longer able to borrow between periods 1 and 2. In that case the households would solve:

$$
\begin{aligned}
\{VA\}_1^h(s) &= u(c_1(s,a)) + \beta u(c_2(s,a)) \to \max && (30)\\
s.t. \quad & c_1(s) + k_1(s) = R_1(s,a)k_0 + w_1(s,a), && (31)\\
& c_2(s) = R_2(s,a)k_1(s,a) + w_2(s,a). && (32)
\end{aligned}
$$

Solving this problem and plugging autarky consumption choices into (30) gives the welfare under autarky as function of state and period 1 level of capital, k_0.

An interior solution to the Nash Bargaining problem requires the following expression to have a zero:

$$
\frac{\partial u(c_1(s))}{\partial c_1(s)} - \frac{1-\theta}{\theta}\frac{\Delta_1^h(s)}{\Delta_1^f(s)}. \tag{33}
$$

A solution to this exists if $\Delta_1^h(s), \Delta_1^f(s) > 0$. Substituting surpluses of Home and Foreign, we obtain an equation that determines default rate for cases when bargaining yields partial default:

$$
u'(c_1(s))b_0 I_0 = \frac{1-\theta}{\theta}\frac{\{VR\}_1^h(s) - \{VA\}_1^h(s)}{1-\delta_1(s)}. \tag{34}
$$

We can now define equilibrium in our economy.

Definition 1: Decentralized equilibrium with bargaining *A set* $\{\delta_1(s), I_0, I_0^w, I_1^w, I_1(s), R_t(s), w_t(s), c_t(s), k_t(s), b_t(s),\ c_t^f(s), s_t^f(s), b_t^f(s)\}$ *is a competitive equilibrium with rational expectations if:*

1. *Given allocation and interest rates, each $\delta_1(s)$ solves Nash Bargaining Problem (20);*

2. *Given all $\delta_1(s)$, the interest rate on risky bonds satisfies (16);*

3. *Risk-free interest rates satisfy foreign household's first order conditions (17) and (18);*

4. *Given interest rates and default rates, $c_t(s), k_t(s), b_t(s)$ satisfy Home household's budget constraints (5)-(7) and first order conditions (8)-(11);*

5. *Given interest rates and default rates, $c_t^f(s), s_t^f(s), b_t^f(s)$ satisfy Foreign budget constraints (13)-(15);*

6. *The return on Home capital, $R_t(s)$, and Home wage, $w_t(s)$, satisfy firms' first order conditions (2) and (3);*

7. *No arbitrage for risk-free interest rates holds: $I_1(s) = I_1^w(s)$;*

8. *Asset markets clear at $b_t^f(s) = b_t(s)$*

In the next subsection we construct an equivalent economy (in terms of allocation), in which debtors incur a non-pecuniary punishment proportional to default rates.

2.2 Non-Pecuniary costs

In subsection 2.1 we described a model in which full default on the debt led to productivity losses and financial autarky. This section presents an alternative model, in which default is associated with utility losses rather than direct material costs. Particularly, we assume that a default rate $\delta_1(s)$ is associated with a loss of utility $\delta_1(s)\lambda(s)$, and the borrower's expected utility reads:

$$u(c_0) + \sum_{s=1}^{S} \pi(s) \left\{ \sum_{\tau=1}^{2} \beta^\tau u(c_\tau(s)) - \beta\lambda(s) \cdot \max\{\delta_1(s), 0\} \right\} \tag{35}$$

We specify utility loss $\lambda(s)$ in a way that allows matching equilibrium allocation of subsection 2.1:

$$\lambda(s) = \frac{1-\theta}{\theta} \frac{\{\bar{VR}\}_1^h(s) - \{\bar{VA}\}_1^h(s)}{1-\bar{\delta}_1(s)}, \tag{36}$$

where $\{\bar{VR}\}_1^h(s)$, $\{\bar{VA}\}_1^h(s)$ are *aggregate* values of repayment and autarky, and $\bar{\delta}_1(s)$ is the aggregate default rate. According to this definition, the non-pecuniary cost is an increasing function of $0 \leq \delta_1(s) \leq 1$, and at $\delta_1(s) = 1$ we have $\lambda(s) = +\infty$. Therefore, provided that $\{\bar{VR}\}_1^h(s) - \{\bar{VA}\}_1^h(s) > 0$, full default with autarky remains an out of equilibrium outcome.

The budget and resource constraints of this economy are identical to those defined in subsection 2.1. First order conditions for firms, households and foreign lenders mirror those stated before with one exception: in the household problem, there is now one additional optimality condition per state governing the choice of the default rate $\delta_1(s)$. As the households do not internalize the effect their decisions have on aggregate values and on $\lambda(s)$, household's optimization over $\delta_1(s)$ simply requires:

$$\begin{aligned} u'(c_1(s))b_0I_0 &= \lambda(s) \equiv \\ &\equiv \frac{1-\theta}{\theta}\frac{\{\bar{VR}\}_1^h(s) - \{\bar{VA}\}_1^h(s)}{1-\bar{\delta}_1(s)}, \end{aligned} \tag{37}$$

that is, the marginal gain from an increase in the default rate must match the marginal loss. In equilibrium all aggregate values match their individual choice counterparts—therefore, the condition in the non-pecuniary economy that determines the (interior) default rate is identical to that of the Nash Bargaining model (see eq. 34).

This alternative formulation of the problem proves intuitive when discussing partial-equilibrium outcomes, because it allows to address the problem by comparing marginal gains and losses of a default decision. For example, if, given current allocation, the interest rates and a $\hat{\delta}_1(s)$, a household observes $u'(c_1(s))b_0I_0 > \lambda(s)$, this means that from the household's viewpoint the marginal gain from raising the default rate is higher than the marginal cost, thus, the default rate $\hat{\delta}_1(s)$ is too small. A full repayment occurs when, given $\delta_1 = 0$, a household observes $u'(c_1(s))b_0I_0 - \lambda(s) \leq 0$, that is, the marginal gain from defaulting is lower or equal to the marginal cost at the $\delta_1 = 0$ corner solution.

3 Properties of the decentralized equilibrium

In this section we examine the relationship between model parameters and the properties of the equilibrium outlined above. In subsection 3.1 we show that default rates that allow for consumption smoothing exceed those generated in equilibrium of the model with bargaining. In subsection 3.2 we investigate whether this problem can be mitigated by raising the bargaining power of the borrowers. We conclude that this is not always the case: when bargaining power becomes 'to high', borrowing ceases to be possible ex ante.

3.1 Optimal default

In this section we construct an economy, in which default rates in both states are exogenously given; we call the equilibrium associated with this model a δ-equilibrium. We show that for this economy if there exists a pair of default rates that ensures full consumption smoothing, then a default rate in high state must be higher than that in the low state. We contrast the allocation under optimal default rates with that generated in economy with bargaining, and show numerically that the latter is associated with lower Home household expected welfare.[9]

3.1.1 δ-equilibrium

Consider a reduced version of the economy set up in subsection 2.1, in which there is no bargaining but instead the default rates $\delta_1(L)$ and $\delta_1(H)$ are exogenously given. In this economy budget constraints and first order conditions of Home and Foreign households and firms are the same as in subsection 2.1. The following definition of equilibrium applies.

Definition 2: Decentralized δ-equilibrium *Given $\bar{\delta}_1(L)$ and $\bar{\delta}_1(H)$, a set $\{I_0, I_0^w, I_1^w, I_1(s), R_t(s), w_t(s), c_t(s), k_t(s), b_t(s),\ c_t^f(s), s_t^f(s), b_t^f(s)\}$ is a competitive δ-equilibrium with rational expectations if:*

1. *Given all $\delta_1(s)$, the interest rate on risky bonds satisfies (16);*
2. *Risk-free interest rates satisfy foreign household's first order conditions (17) and a(18);*
3. *Given interest rates and default rates, $c_t(s), k_t(s), b_t(s)$ satisfy Home household's budget constraints (5)-(7) and first order conditions (8)-(11);*
4. *Given interest rates and default rates, $c_t^f(s), s_t^f(s), b_t^f(s)$ satisfy Foreign budget constraints (13)-(15);*
5. *The return on Home capital, $R_t(s)$, and Home wage, $w_t(s)$, satisfy firms' first order conditions (2) and (3);*

[9]The expected welfare of Foreign households remains unchanged regardless of the model, as those households are risk-neutral and the interest rate on risky bonds reflects expected default rates.

6. *No arbitrage for risk-free interest rates holds:* $I_1(s) = I_1^w(s)$*;*

7. *Asset markets clear at* $b_t^f(s) = b_t(s)$

We now show that for the case of risk-neutral lenders for each pair $0 \leq \delta_1^*(L), \delta_1^*(H) < 1$ there exists an infinite number of pairs $\delta_1^{**}(L), \delta_1^{**}(H)$ that give rise to the same allocation in δ-equilibrium.

Proposition 1 *Consider a pair of default rates* $0 \leq \delta_1^*(L), \delta_1^*(H) < 1$*. There exists an infinite number of pairs of state-specific default rates that replicate* δ*-equilibrium allocation associated with* $0 \leq \delta_1^*(L), \delta_1^*(H) < 1$*.*
Proof. Because investors are risk-neutral, in δ-equilibrium the interest rate on bonds is pinned down by the default rates $0 \leq \delta_1^*(L), \delta_1^*(H) < 1$ and does not depend on the allocation:

$$I_0^* = \frac{I^w}{\sum_{s=L,H} \pi(s)(1 - \delta_1^*(s))}.$$

Notice that the budget constraints of the household problem (5), (6) only include the interest rate I_0 in conjunction with a repayment rate in one of the state $1 - \delta(s)$. Thus, the model can be rewritten in terms of products $I_0(1 - \delta_1(s))$, and the δ-equilibrium allocation c^*, k^*, b^* can be expressed as a function of exogenous products $(1 - \delta_1^*(s))I_0^*$. Rewrite the products as:

$$\begin{aligned}
(1 - \delta_1^*(L))I_0^* &= \frac{I_1^w}{\pi(L) + (1 - \pi(L))\frac{1-\delta_1^*(H)}{1-\delta_1^*(L)}}, \\
(1 - \delta_1^*(H))I_0^* &= \frac{I_1^w}{\pi(L)\frac{1-\delta_1^*(L)}{1-\delta_1^*(H)} + (1 - \pi(L))}.
\end{aligned}$$

Notice that the right-hand sides of both equations are functions of $\frac{1-\delta_1^*(L)}{1-\delta_1^*(H)}$. As allocation is a function of products $(1 - \delta_1^*(s))I_0^*$, we can alternatively express the allocation as a function of $\frac{1-\delta_1^*(L)}{1-\delta_1^*(H)}$, rather than $\delta_1^*(L)$ and $\delta_1^*(H)$ separately. This implies that the allocation will remain the same for all pairs $\delta_1^{**}(L), \delta_1^{**}(H)$, such that:

$$\frac{1 - \delta_1^*(L)}{1 - \delta_1^*(H)} = \frac{1 - \delta_1^{**}(L)}{1 - \delta_1^{**}(H)}. \tag{38}$$

□

Proposition 1 implies that without loss of generality we can set the smaller default rate to 0: any pair $0 < \delta_1^*(L), \delta_1^*(H) < 1$ has one counterpart

$\delta_1^{**}(L), \delta_1^{**}(H)$ where one of the default rates is zero, but the allocation is exactly the same.

Corollary 1.1 *A δ-equilibrium allocation associated with default rates that satisfy $0 < \delta_1^*(s) < \delta_1^*(-s) < 1$ can be replicated under $\delta_1(s) = 0$ and $\delta_1(-s) = 1 - \frac{1-\delta_1^*(-s)}{1-\delta_1^*(s)}$.*

To improve consumption smoothing the default rate in the low state has to be higher than the default rate in the high state. Intuitively, as the households cannot choose negative capital stock, their income from capital and labor is always positively correlated with the level of productivity A_t. To mitigate fluctuations in consumption, a household would have to default more when income is low.

Proposition 2. *Suppose there exist default rates $\delta_1^*(L), \delta_1^*(H)$ that ensure full consumption smoothing in δ-equilibrium with positive borrowing. Then $\delta_1^*(L) > \delta_1^*(H)$.*

Proof. Denote the δ-equilibrium allocation corresponding to $\delta_1^*(L), \delta_1^*(H)$ as c^*, b_0^*, k_0^*, and the associated interest rate I_0^*. If full consumption smoothing is feasible, then $c_1^*(L) = c_1^*(H)$ and (from 28) default rates must be such that:

$$\begin{aligned}\delta_1^*(L) - \delta_1^*(H) &= \frac{(1+Z)k_0^*\left[A_1(H) - A_1(L)\right] +}{b_0^* I_0^*} + \\ &\quad + \frac{\frac{1-\alpha}{\alpha}\left(\frac{\alpha(1+Z)}{I_1^w}\right)^{\frac{1}{1-\alpha}}\left[A_2(H)^{\frac{1}{1-\alpha}} - A_2(L)^{\frac{1}{1-\alpha}}\right]}{b_0^* I_0^*}.\end{aligned}$$

For positive borrowing, the right-hand side is positive, therefore, $\delta_1^*(L) > \delta_1^*(H)$. By **Corollary 1.1**, under default rates $\delta_1(L) = 1 - \frac{1-\delta_1^*(L)}{1-\delta_1^*(H)}$ and $\delta_1(H) = 0$, δ-equilibrium allocation exactly matches c^*, b_0^*, k_0^*. □

Therefore, a default schedule that promotes consumption smoothing should allow for more default in a state with bad productivity shock. In the next subsections we calibrate our model and solve numerically for the optimal default rate in δ-equilibrium. We contrast the result with the equilibrium that arises in a decentralized model with bargaining discussed in subsection 2.1.

3.1.2 Calibration

To capture key aspects of the European debt crisis that followed financial collapse of 2007-2008, we use statistics that characterize Greek economy of the corresponding period. We set the risk-free interest rate to 2% which compares to 1.916% observed for 10-year German bond yields (average for

January 2010 to October 2014). We follow Angelopoulos et al. (2010) and set the share of capital income to 0.34. The standard deviation of productivity shock is set to 1% matching 1% obtained by Bi,Traum (2012) after performing bayesian estimation of their model on Greek data. We set the productivity loss from autarky to 40%, and we calibrate the bargaining power to let the model produce equilibrium with partial default in one state ($\theta = 0.255$).[10] As in Yue (2010), we set relative risk aversion in Home households CRRA utility to $\sigma = 2$.

Given these parameters, we compute equilibrium in the decentralized model with bargaining. The model produces partial default in low state with a default rate at 11.2%. The interest rate on bonds in our model is 8.1% (compared to 12.81% observed from January 2010 to January 2015 for Greek 10-year bonds).[11] The debt-to-GDP ratio in the default state is 65%.

The upper plot on Figure 2 depicts expected welfare in δ-equilibrium, $E_0 \sum_{t=0,1,2} \beta^t u(c_t)$, as function of the default rate in low state. The welfare-maximizing default rate is around 29%. As shown in the right column of Table 1, this default rates allows for full consumption smoothing across states in δ-equilibrium, as $c_0 = c_1(L) = c_1(H) = c_2$. [12]

Although 29% default in low state guarantees maximum expected welfare, this default rate is not feasible in a model with bargaining. Figure 2 depicts the difference between marginal gains and costs from defaulting in low and high states corresponding to the λ-equilibrium.[13] In subsection 2.2 we established that in equilibrium with bargaining a given state s is associated with partial default if the marginal gain from defaulting exactly equals the marginal cost λ, whereas negative difference between marginal gain and

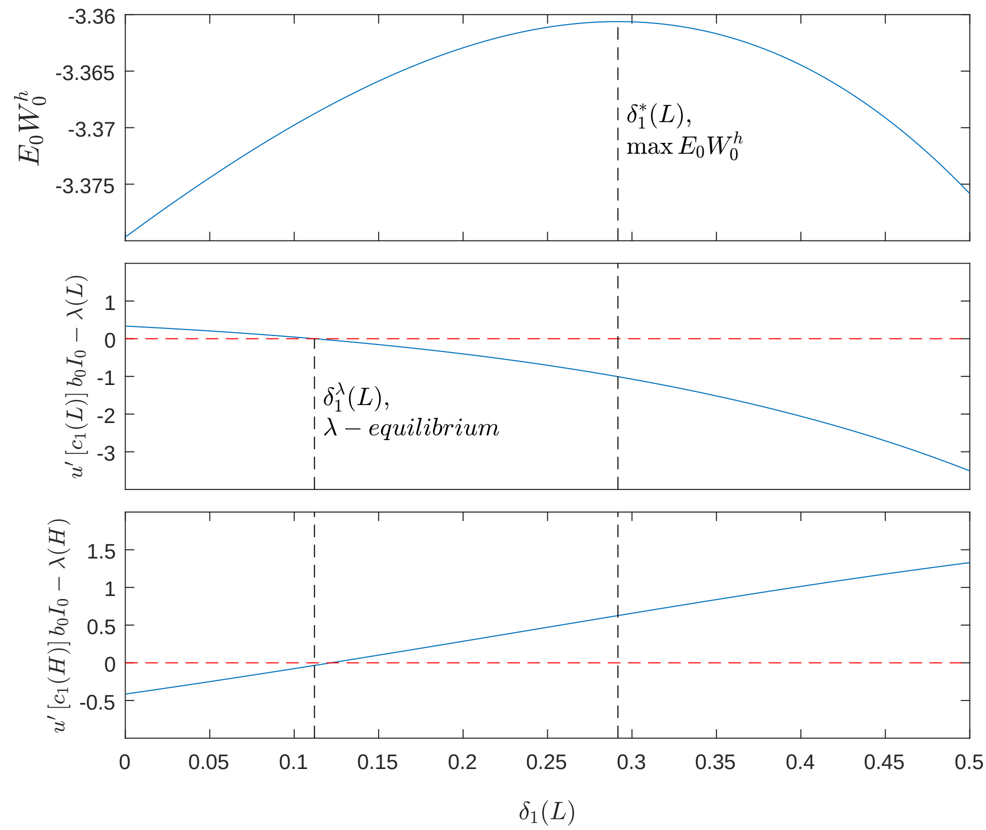
[10] Productivity loss of 40% compares with estimates of GDP costs that would have arisen if Greece exited Eurozone, published at the time of the crisis. For instance, a UBS (2011) study claims that for Greece exiting and abandoning the euro would have meant a 60% currency devalutaion, a 50% decline in the volume of trade and a loss of 60% of deposits, amounting up to 50% GDP loss.

[11] Taken from St Louis FRED database Long-Term Government Bond Yields: 10-year: Main (Including Benchmark).

[12] In light of **Proposition 1** our result means that there also exists an infinite number of pairs $\delta_1(L), \delta_1(H) > 0$ that replicate this allocation, but because of **Proposition 2** we know that for all those pairs $\delta_1(L) > \delta_1(H)$, and we can therefore limit our attention to the case where we normalize default rate in high state to 0.

[13] As shown in subsection 2.2, under our assumptions the model with non-pecuniary costs (the λ-equilibrium) generates the same allocation as the model with bargaining. Here we find it more convenient to discuss equilibrium results in terms of marginal gains and marginal losses from defaulting, using concepts and notation introduced in subsection 2.2.

Figure 2: Decentralized equilibrium with bargaining vs. optimal δ-equilibrium



Notes: the figure depicts values associated with δ-equilibrium depending on $\delta_1(L)$ and assuming $\delta_1(H) = 0$. The graphs plot expected welfare and differences between the left- and the right-hand sides of (34) for the two states. Vertical dashed lines mark equbrium $\delta_1(L)$ in decentralized model, and an optimal $\delta_1(L)$ in δ-equilibrium.

loss is associated with full repayment:

$$\begin{cases} u'(c_1(s))I_0 b_0 - \lambda(s) = 0 & for \quad \delta(s) > 0, \\ u'(c_1(s))I_0 b_0 - \lambda(s) < 0 & for \quad \delta(s) = 0. \end{cases} \tag{39}$$

For a default rate of 29% to be supported as equilibrium with bargaining it must yield $c_1(L))I_0 b_0 = \lambda(L)$ and $c_1(H))I_0 b_0 \leq \lambda(H)$ in δ-equilibrium. But on Figure 2 this is not the case: in low state marginal gain is lower than marginal loss suggesting that a household would benefit from defaulting less, while in high state it is the reverse. The λ-equilibrium (and the equilibrium

Table 1: Decentralized equilibrium with bargaining vs. optimal δ-equilibrium

Variable	Decentralized	$\max E_0 W$ δ-equilibrium
Welfare home	-3.369	-3.361
b_0	0.9218	0.9220
δ_1^L	0.112	0.292
I_0	1.081	1.195
k_0	0.542	0.541
c_0	0.874	0.875
$c_1(L)$	0.823	0.875
$c_1(H)$	0.928	0.875

Notes: The table compares equilibrium allocations for the baseline decentralized model and the optimal δ-equilibrium.

with bargaining) are obtained at a much lower default rate of 11.2%, at which marginal gains and costs in high and low states comply with the prerequisits discussed above.

Left column of Table 1 documents that in the decentralized equilibrium with bargaining consumption differs across time and states, as $c_0 \neq c_1(L) \neq c_1(H)$, while expected welfare is lower than in the optimal δ-equilibrium. Therefore, bargaining results in an inefficiency that impairs consumption smoothing, as borrowers do not default 'enough'.

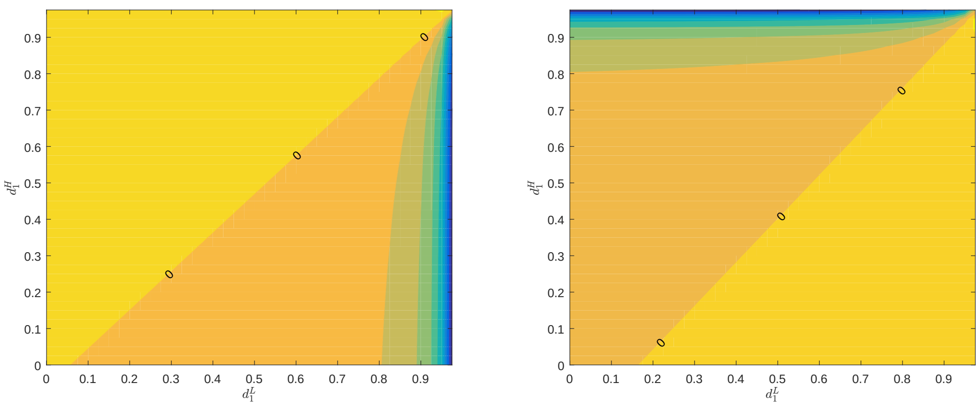
3.2 Decentralized equilibrium and the bargaining power

In previous section we concluded that in presence of bargaining consumption smoothing is imperfect, and that higher default in low state leads to a welfare improvement for the borrower. In fact, with risk-neutral creditors we can refine this statement by claiming that more default is pareto-improving, as higher default rates have no effect on the expected welfare for Foreign country because, as the default rate rises, so does the risk-neutral interest on bonds. The remainder of the paper seeks refinements to the model setup that could bring the equilibrium outcomes closer to those observed under optimal default rates.

In this section we ask, whether the inefficiency associated with bargaining could be solved by giving the Home borrower higher bargaining power. Intuitively, more bargaining power for the borrower implies higher haircut, which should be pareto-improving. However, in this subsection we show that this logic breaks down when the bargaining power becomes 'too high', as it leads to full default in both states and renders borrowing infeasible *ex ante.*

Figure 3: Marginal gains minus losses from defaulting

(a) Low state (b) High state



Notes: the figures present heat plots for the difference between the left- and the right-hand sides of (34) for the two states, obtained for δ-equilibrium allocations. For this plot we use $\theta = 0.23$. Warmer colors (lighter in grayscale) indicate higher values, cooler colors (darker in grayscale) indicate lower values.

Consider δ-equilibrium in which default rates are exogenous. Figure 3 depicts differences between marginal gains and losses from defaulting in low and high states, for all possible combinations of $\delta_1(L)$ and $\delta_1(H)$. By **Proposition 1**, there is a continuum of pairs of $\delta_1(L), \delta_1(H)$ that result in zero difference between gains and costs—on Figure 3 these combinations are depicted by straight lines labeled '0'.[14] The difference between gains and losses from defaulting in a given state is higher, when default rate in the alternative state is substantial. The intuition behind this result is as follows. Given default rate in state s, when default in states s^{-1} is high, equilibrium on the financial market implies higher compensation for the lenders, I_0, raising the gain from defaulting in state s.

Conversely, higher default in state s means lower difference between gains and losses from defaulting in state s. This result comes from the definition of $\lambda(s)$ we adopted to make it reflect the underlying bargaining process: $\lambda(s) = \frac{1-\theta}{\theta} \frac{\{\bar{VR}\}_1^h(s) - \{\bar{VA}\}_1^h(s)}{1-\delta_1(s)}$. Higher $\delta(s)$ exponentially increases the punishment $\lambda(s)$ as the denominator approaches 0.

In the full model with bargaining, an equilibrium with positive borrowing

[14]The lines are defined by $\delta_1(L) = \delta_1^*(s) + (1-\delta_1^*(s))\delta_1(H)$, where $\delta_1^*(s)$ is an intersection between each line and the axis $\delta_1(H) = 0$, see **Proposition 1**.

exists if there exists a pair $\delta_1(L), \delta_1(H)$ such that the associated δ-equilibrium allocation satisfies conditions given in (39). Because of the way (39) is specified, there are four qualitatively different possibilities regarding $\delta_1(L), \delta_1(H)$:

(i) $0 < \delta_1(L) \leq 1$ and $\delta_1(H) = 0$.
In this case the associated δ-equilibrium allocation must satisfy:

$$\begin{cases} u'(c_1(L))I_0 b_0 - \lambda(L) = 0, \\ u'(c_1(H))I_0 b_0 - \lambda(H) \leq 0. \end{cases}$$

(ii) $\delta_1(L) = 0$ and $0 < \delta_1(H) \leq 1$.
This requires δ-equilibrium to satisfy:

$$\begin{cases} u'(c_1(L))I_0 b_0 - \lambda(L) \leq 0, \\ u'(c_1(H))I_0 b_0 - \lambda(H) = 0. \end{cases}$$

(iii) $\delta_1(L) = 0$ and $\delta_1(H) = 0$.
Then the δ-equilibrium must satisfy:

$$\begin{cases} u'(c_1(L))I_0 b_0 - \lambda(L) \leq 0, \\ u'(c_1(H))I_0 b_0 - \lambda(H) \leq 0. \end{cases}$$

(iv) $0 < \delta_1(L) \leq 1$ and $0 < \delta_1(H) \leq 1$.
This is feasible if δ-equilibrium is such that:

$$\begin{cases} u'(c_1(L))I_0 b_0 - \lambda(L) = 0, \\ u'(c_1(H))I_0 b_0 - \lambda(H) = 0. \end{cases}$$

In our numerical example, the only equilibrium with bargaining is achieved at $\delta_1(L) > 0$ and $\delta_1(H) = 0$, which falls into category (i). To see this, consider Figure 3a, specifically the pair of default rates at the intersection between the inclined line that gives zero marginal gains minus losses, and the horizontal axis ($\delta_1(L) \approx 0.07$ and $\delta_1(H) = 0$). For this pair, in the low state the marginal gain from defaulting equals the marginal loss. Furthermore, in high state the marginal gain from defaulting is negative (see Figure 3b). Therefore, conditions outlined by (i) hold.

By contrast, there does not exist a pair of default rates $\delta_1(L) = 0$ and $0 < \delta_1(H) \leq 1$ that would satisfy (ii). For some $0 < \delta_1(H) \leq 1$ to be supported as equilibrium with bargaining, the associated marginal gain from defaulting in high state must equal the marginal loss. But according to

Figure 3b, for all values of $\delta_1(H)$ lying on the vertical axis marginal losses from defaulting are higher than marginal gains. Thus, there are no default rates for which conditions in (ii) would be satisfied.

An equilibrium with full repayment in both states is not feasible either. Under case (iii), marginal gain must be lower or equal to marginal loss in both states of δ-equilibrium associated with $(0, 0)$. But according to Figure 3a, this is not the case for the low state—the conditions specified in (iii) are violated.

Finally, an equilibrium with partial default in both state is not attainable, because the lines at which marginal gains equal marginal losses depicted on Figure 3 (a) and (b) do not intersect at $\delta_1(L), \delta_1(H) < 1$, therefore, conditions of (iv) do not hold.

This last result applies more generally and holds true for almost all combinations of model parameters.

Proposition 3 *In general there does not exist a pair $\delta_1(L), \delta_1(H) \in (0, 1)$ such that corresponding marginal gains from defaulting equal marginal losses in both states. Therefore, in the model with bargaining partial default in both state is not possible in equilibrium.*

Proof. By **Proposition 1**, combinations of default rates associated with zero marginal gains in low and high state are given by

$$\delta_1(L) = \delta_1^* + (1 - \delta_1^*)\delta_1(H), \tag{40}$$

$$\delta_1(L) = \delta_1^{**} + (1 - \delta_1^{**})\delta_1(H), \tag{41}$$

where δ_1^* is such that $u'(c_1(L))b_0 I_0 = \lambda(L)$ under $\delta_1(L) = \delta_1^*, \delta_1(H) = 0$ (i.e. corresponds to intersection between inclined zero line on Figure 3a and horizontal axis), and δ_1^{**} is such that $u'(c_1(H))b_0 I_0 = \lambda(H)$ under $\delta_1(L) = \delta_1^{**}, \delta_1(H) = 0$ (same but for Figure 3b).

Writing these two equations as a system and solving in terms of $\delta_1(L), \delta_1(H)$ yields:

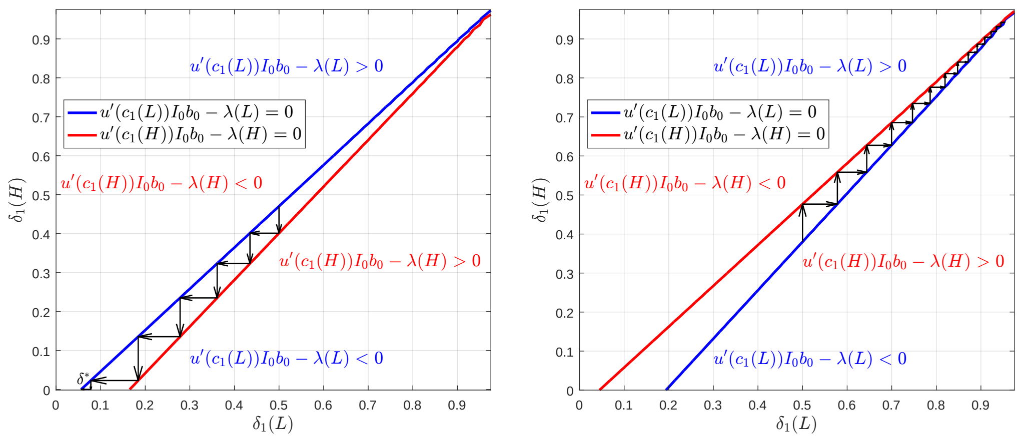
$$\delta_1(L) = 1,$$
$$\delta_1(H) = 1,$$

given $\delta_1^* \neq \delta_1^{**}$. As these two lines do not intersect at default rates below 1, there are no combinations of default rates smaller than 1 for which marginal gains equal losses in both states. $\square$

Figure 4a plots together the two inclined zero lines depicted on Figure 3(a) and (b), and illustrates how in the economy with bargaining agents adjusts their expectations about default rates until equilibrium is reached. For example, at $\delta_1(L) = 0.5$ and $\delta_1(H) = 0.47$ the marginal gain from defaulting

Figure 4: Marginal gains and losses: equilibrium

(a) $\theta = 0.23$ (b) $\theta = 0.30$



Notes: the figures plot zero lines for the difference between the left- and the right-hand sides of (34) for the two states, obtained for δ-equilibrium allocations. The graphs differ by the assumption made about borrowers' bargaining power.

in low state equals the marginal cost, whereas in high state the loss exceeds the gain. That means that expected default rate in high state is 'too high', and $\delta_1(H)$ falls to 0.4, where in high state the marginal loss from defaulting equals the gain. But at this new combination of default rates the default rate in low state turns out to be 'too high', as the associated loss from defaulting is higher than the gain, causing expected $\delta_1(L)$ to reduce to 0.43, etc. This process continues until the equilibrium default rates $\delta_1(L) = 0.07$ and $\delta_1(H) = 0$ are reached.

Now let us compare this economy with the one in which the borrower has higher bargaining power ($\theta = 0.30$), Figure 4b. As before, zero level lines of marginal gains minus losses do not intersect, indicating there is no equilibrium with partial defaults in both state. But unlike on Figure 4a, we now observe that the zero level line for default in high state is above that corresponding to the low state. Let us again consider the intersection between the horizontal axis, $\delta_1(H) = 0$, and the line $u'(c_1(L))I_0b_0 - \lambda(L) = 0$, given by $\delta_1(L) = 0.2, \delta_1(H) = 0$. For this combination of default rates the marginal gain from defaulting in high state exceeds the marginal loss, meaning that $\delta_1(L) = 0.2, \delta_1(H) = 0$ cannot be supported as equilibrium with bargaining. In fact, for every combination of $\delta_1(L), \delta_1(H) < 1$ marginal gain from defaulting more in one of the states (or in both) is higher than the loss.

On Figure 4b we show that in such a setup, given any starting pair of default rates agents' expectations will adjust toward higher default rates in both states, and at the limit both expected default rates will approach 1. But if agents rationally expect full default in both states, then borrowing ex ante would not be possible. Therefore, when the borrowers' bargaining power equals $\theta = 0.30$, there is no equilibrium with positive borrowing. This result can be generalized.

Proposition 4 *Consider a pair $\hat{\delta}_1(L), \hat{\delta}_1(H) < 1$ that yields δ-equilibrium with positive borrowing. For each such pair $\hat{\delta}_1(L), \hat{\delta}_1(H) < 1$ there exists*

1. *a $\bar{\theta}$ such that if $\theta \geq \bar{\theta}$, then in δ-equilibrium originated by $\hat{\delta}_1(L), \hat{\delta}_1(H)$ marginal gains from defaulting exceed marginal losses in both states.*
2. *a $\underline{\theta}$ such that if $\theta \leq \underline{\theta}$, then in δ-equilibrium originated by $\hat{\delta}_1(L), \hat{\delta}_1(H)$ marginal losses from defaulting exceed gains in both states.*

Proof. Consider a pair $0 \leq \hat{\delta}_1(L), \hat{\delta}_1(H) < 1$. Marginal gains from defaulting will be positive in both states if:

$$u'(c_1(L))I_0 b_0 \quad > \quad \lambda(L) \equiv \frac{1-\theta}{\theta} \frac{\{VR\}_1^h(L) - \{VA\}_1^h(L)}{1-\hat{\delta}_1(L)} \tag{42}$$

$$u'(c_1(H))I_0 b_0 \quad > \quad \lambda(H) \equiv \frac{1-\theta}{\theta} \frac{\{VR\}_1^h(H) - \{VA\}_1^h(H)}{1-\hat{\delta}_1(H)} \tag{43}$$

The values of $\{VR\}_1^h(s)$, $\{VA\}_1^h(s)$, b_0, I_0 and $c_1(s)$ are pinned down by the δ-equilibrium allocation associated with $\hat{\delta}_1(L), \hat{\delta}_1(H)$. The right-hand sides of both (42) and (43) depend negatively on the borrower's bargaining power, θ: higher θ decreases the punishment $\lambda(s)$ in each state. Furthermore, for $\theta = 1$ both $\lambda(L)$ and $\lambda(H)$ equal zero. At the same time, by assumption $u'(.) > 0$, $b_0 > 0$ (equilibrium with positive borrowing), and $I_0 > 0$ (as $\hat{\delta}_1(L), \hat{\delta}_1(H) < 1$ by assumption). Therefore, (42) and (43) hold for $\theta = 1$. Since $\lambda(L)$ and $\lambda(H)$ depend on θ monotonically, there exists an $\epsilon > 0$ such that (42) and (43) hold as strict inequalities for $\theta = 1 - \epsilon$. Thus, there exists a $\bar{\theta}$ such that for any $\theta > \bar{\theta}$ inequalities (42) and (43) hold.

Analogously, we observe that given the δ-equilibrium allocation, for each $\lambda(s)$ we have: $\lim_{\theta \to 0} \lambda(s) = \infty$. Since the left-hand sides of (42) and (43) are finite, we can always find a 'low enough' θ that will ensure negative difference between marginal gains and losses in both states. □

By **Proposition 4**, for any δ-equilibrium with positive borrowing there exists a 'low enough' value of the borrower's bargaining power that will ensure that losses from defaulting exceed gains. This implies that it is always possible to find a bargaining power such that the equilibrium with bargaining will result in full repayment in both state.

Corollary 4.1 *Suppose that a δ-equilibrium originated by $\delta_1(L) = \delta_1(H) = 0$ is associated with positive borrowing. Then, there exists a $\underline{\theta}$ such that for all $\theta < \underline{\theta}$ there exists an equilibrium in the model with bargaining that delivers full repayment in both states.*

Proof. By **Proposition 3**, for $\delta_1(L) = \delta_1(H) = 0$ there exists a $\underline{\theta}$ such that for all $\theta < \underline{\theta}$ in the associated δ-equilibrium the marginal gains from defaulting are lower than costs in both states. This means that for $\theta < \underline{\theta}$ the δ-equilibrium originated by $\delta_1(L) = \delta_1(H) = 0$ is supported as equilibrium with bargaining. □

By contrast, when the borrower's bargaining power becomes 'too high', it destroys the households ability to borrow ex ante.

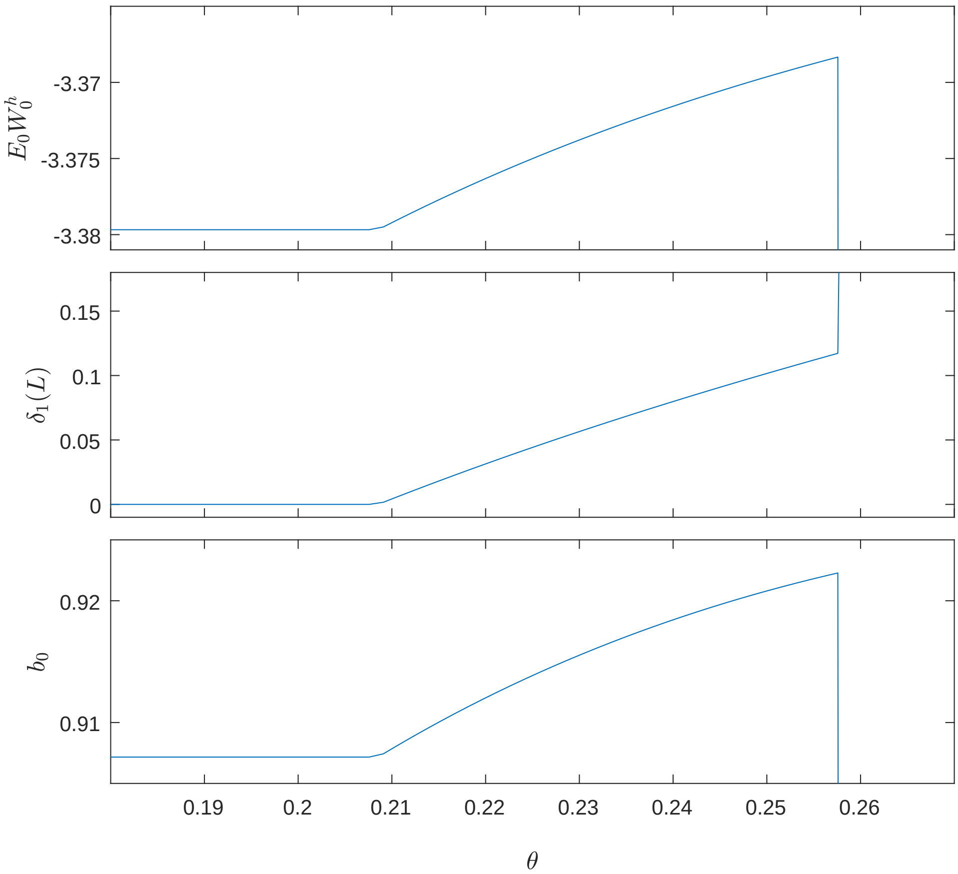
Corollary 4.2 *For any set of model parameters there exists a $\bar{\theta}$ such that if $\theta > \bar{\theta}$, then in the model with bargaining there is no equilibrium with positive borrowing.*

Proof. Consider a set D of all pairs $0 \leq (\hat{\delta}_1(L), \hat{\delta}_1(H)) < 1$ such that $\forall(\delta_1(L), \delta_1(H)) \in D$ households choose positive borrowing in δ-equilibrium. By **Proposition 4**, there exists $\bar{\theta}$ such that if $\theta > \bar{\theta}$, then none of the pairs $(\hat{\delta}_1(L), \hat{\delta}_1(H))$ in D are supported as equilibrium with bargaining. Since D contains all combinations of default rates that yield positive borrowing, with $\theta > \bar{\theta}$ there are no $(\hat{\delta}_1(L), \hat{\delta}_1(H))$ that yield positive borrowing in equilibrium with bargaining. □

Therefore, an increase in the borrowers' bargaining power may compromise their ability to borrow. On Figure 5 we plot equilibrium values of Home households expected utility, default rate and the level of borrowing that arise in the model with bargaining, depending on the Home's bargaining power. When the borrowers' bargaining power is low, they are forced to fully repay the debt in both states, and borrowing is risk-free. When the bargaining power increases beyond 0.21, the default rate in the low state rises. At higher $\delta_1(L)$ households wish to borrow more, as the repayment on the bonds now correlates negatively with the productivity shock, and issuing more debt promotes consumption smoothing across states L and H. As a result, as θ rises, consumers' welfare increases. This effect remains in place as long as θ is below 0.26, but once it increases beyond this thresholds, positive borrowing becomes unsustainable.

One conclusion we can draw from this analysis is that, even though an increase in the bargaining power of the borrower may promote consump-

Figure 5: Decentralized equilibrium and bargaining power



Notes: The figure plots equilibrium variables in the model with decentralized borrowing, depending on the borrowers' bargaining power. The plots depict expected welfare, default rate and the interest rate.

tion smoothing and raise welfare, bargaining power that is 'too high' will undermine the ability to borrow *ex ante*.

4 Centralized Economy

In the previous section we made two observations. First, bargaining process causes inefficiency as the rate of default under bargaining turns out to be lower than that ensuring full consumption smoothing. Consequently, households are left exposed to productivity shocks. Second, raising the bargaining power of the borrowers does not fully resolve this problem, as the bargaining power that is 'too high' destroys equilibrium with positive borrowing, making it infeasible to borrow ex ante.

In this section we consider a centralized version of our model with bargaining, where all decisions are made by a benevolent social planner who, however, has to comply with the bargaining process. In this economy the planner chooses the amount of borrowing and capital, and internalizes their effect on bond prices and default rates.

4.1 Solution

The timing of decisions is as follows: in period 0 the planner chooses amounts of borrowing and capital; in the beginning of period 1 the renegotiation occurs; given the default rates, the planner then chooses borrowing and capital in period 1. We solve this problem recursively. We start in period 1, after the default rates are announced. Given $\delta_1(L), \delta_1(H)$, and b_0, k_0 chosen in period 0, we determine the planner's decisions over consumption, borrowing and capital from period 1 onward. Given those decisions, we express the values of autarky and repayment as functions of period 0 choices, interest rate, default rate and state: $VR(b_0, k_0, I_0, \delta(s), s)$, $VA(k_0, s)$. Then, given planner's period 1 response functions and the equilibrium definition of I_0, we determine the outcome of the Nash Bargaining Problem and express default rates in each state as functions of date zero variables, $\delta_1(s, b_0, k_0)$. Finally, given the default rate functions we determine the planner's period 0 choices over b_0 and k_0. Below we give a brief technical formulation of this problems.

In a state s of period 1, after a renegotiation, the planner chooses allocations for periods 1 and 2:

$$\{\bar{VR}\}_1^h(s) = u(c_1(s)) + \beta u(c_2(s)) \rightarrow \max \tag{44}$$

$$s.t. \quad c_1(s) + k_1(s) + (1 - \delta_1(s))b_0 I_0 = A_1(s)(1 + Z)k_0^\alpha + b_1(s), \tag{45}$$

$$c_2(s) + b_1(s)I_1^w = A_2(s)(1 + Z)k_1^\alpha. \tag{46}$$

This problem yields first-order conditions identical to those arising in a decentralized version of our economy. In subsection 2.1 we established that for CRRA utility we can directly derive consumption in periods 1 and 2—it is given by (28). Substituting consumption functions into $\{\bar{VR}\}_1(s) = u(c_1(s)) + \beta u(c_2(s))$ we obtain Home value function in period 1 under renegotiation: $VR(b_0, k_0, I_0\ \delta(s))$.

Next, we determine the value function associated with autarky. Under autarky in period 1 the planner solves:

$$\{\bar{VA}\}_1^h(s) \quad = \quad u(c_1(s)) + \beta u(c_2(s)) \to \max \tag{47}$$
$$s.t. \qquad c_1(s) + k_1(s) = A_1(s)k_0^\alpha \tag{48}$$
$$c_2(s) = A_2(s)k_1^\alpha \tag{49}$$

Once again, the first-order conditions mirror those of the decentralized model. As we point out in subsection 2.1, solving this problem and plugging resulting consumption choices into $\{\bar{VA}\}_1(s) = u(c_1(s)) + \beta u(c_2(s))$ gives the value of autarky as function of k_0.

Having obtained $\{\bar{VR}\}_1^h(s)$ and $\{\bar{VA}\}_1^h(s)$, we now go to period 0 and characterize the planner's choice for b_0, k_0, given constraints originating from bargaining. In period 0 the planner solves:

$$E_0W \ = \ u(c_0) \ + \ \beta\Big[\pi VR_1^L(k_0, b_0, I_0, \delta_1(L), L) + +(1-\pi)VR_1^H(k_0, b_0, I_0, \delta_1(H))\Big] \ \to \ \max$$

$$s.t. \qquad A_0(1+Z)k_{-1}^\alpha + b_0 \geq c_0 + k_0, \tag{50}$$
$$u'(c_1(b_0, k_0, I_0, \delta(L), L))I_0b_0 \leq \frac{1-\theta}{\theta}\frac{VR(b_0, k_0, I_0, \delta(L), L) - VA(k_0, L)}{1-\delta_1(L)}, \tag{51}$$
$$u'(c_1(b_0, k_0, I_0, \delta(H), H))I_0b_0 \leq \frac{1-\theta}{\theta}\frac{VR(b_0, k_0, I_0, \delta(H), H) - VA(k_0, H)}{1-\delta_1(H)}, \tag{52}$$
$$I_0 = \frac{I_0^w}{1-\pi\delta_1(L) - (1-\pi)\delta_1(H)}, \tag{53}$$
$$\delta_1(s) \geq 0, \tag{54}$$

for $s = L, H$.

The planner internalizes the effect period 0 borrowing and capital have on bargaining outcomes, and (51) and (52) guarantee that equilibrium allocation complies with the bargaining mechanism. In section 3 we show that without loss of generality we can restrict attention to an equilibrium with full repayment in high state, and default in low. Consider an equilibrium in which condition (52) is not binding, and there is partial default in the low state. Default rate in the low state is determined through,

$$\begin{aligned} & u' \left[c_1(b_0, k_0, I_0(\delta(L), \delta(H) = 0), \delta(L), L)\right] \frac{I_0^w}{1 - \pi\delta_1(L)} b_0 = \\ & = \frac{1-\theta}{\theta} \frac{VR\left[b_0, k_0, I_0(\delta(L), \delta(H) = 0), \delta(L), L\right] - VA\left[k_0, L\right]}{1 - \delta_1(L)}, \end{aligned} \tag{55}$$

and is a function of period 0 borrowing and capital choices. Denote the $\delta(L)$ that solves (55) as $\delta^*(b_0, k_0)$. We can now write the first order condition characterizing the planner's choice of b_0 in period 0:

$$\begin{aligned} u'(c_0) + \beta\pi \frac{\partial VR(L)}{\partial \delta^*} \frac{\partial \delta^*(b_0, k_0)}{\partial b_0} &= \beta I_0 \sum_{s=1}^{2} \pi(s)(1 - \delta_1(s)) \frac{\partial u(c_1(s))}{\partial c_1(s)} - \\ & -\beta \frac{\partial I_0}{\partial \delta^*} \frac{\partial \delta^*(b_0, k_0)}{\partial b_0} \cdot \\ & \cdot \left[\pi \frac{\partial VR(L)}{\partial I_0} + (1-\pi) \frac{\partial VR(H)}{\partial I_0}\right] \end{aligned} \tag{56}$$

The new term on the left-hand side of (56) appears because the planner understands that higher borrowing today will result in higher default rate in the low state, amounting to a gain in tomorrow's expected utility. The new term on the right-hand side emerges because the planner also understands that through default rate borrowing will affect the interest rate on bonds, making repayment costlier and lowering tomorrow's utility.

Substituting derivatives and simplifying, we obtain:

$$\begin{aligned} u'(c_0) &= \beta I_0 \sum_{s=1}^{2} \pi(s)(1 - \delta_1(s)) \frac{\partial u(c_1(s))}{\partial c_1(s)} - \qquad (57) \\ & -\beta \frac{\pi(1-\pi)}{1 - \pi\delta^*(b_0, k_0)} I_0 b_0 \left[u'(c_1(L)) - u'(c_1(H))\right] \frac{\partial \delta^*(b_0, k_0)}{\partial b_0}. \end{aligned}$$

Given that consumption in the low state is lower than in the high state, and that higher debt stock means higher haircut after renegotiation, the right-hand side of (57) is lower compared to that of the first order condition in the decentralized model: the planner has additional incentive to accumulate debt, as higher debt stock will allow the government to bargain for higher default rates in the low state.

The first order condition for capital investment in period 0 is:

$$u'(c_0) = \beta \sum_{s=1}^{2} \pi(s) R_1(s) \frac{\partial u(c_1(s))}{\partial c_1(s)} + \tag{58}$$
$$+\beta \frac{\pi(1-\pi)}{1-\pi\delta^*(b_0,k_0)} I_0 b_0 \left[u'(c_1(L)) - u'(c_1(H))\right] \frac{\partial \delta^*(b_0,k_0)}{\partial k_0}.$$

The interpretation is similar: the planner now internalizes the effect of capital on the bargaining outcome—the default rate—and how changes of the default rate affect consumption in period 1 directly and through changes in the interest rate.

4.2 Centralized Economy: Numerical Results

We use the calibration adopted in section 3 (describen in subsubsection 3.1.2) to compare features of the centralized economy with those of our baseline model.[15] Figure 6 compares partial equilibria (less the first-order condition for b_0) for centralized and decentralized economies.[16]

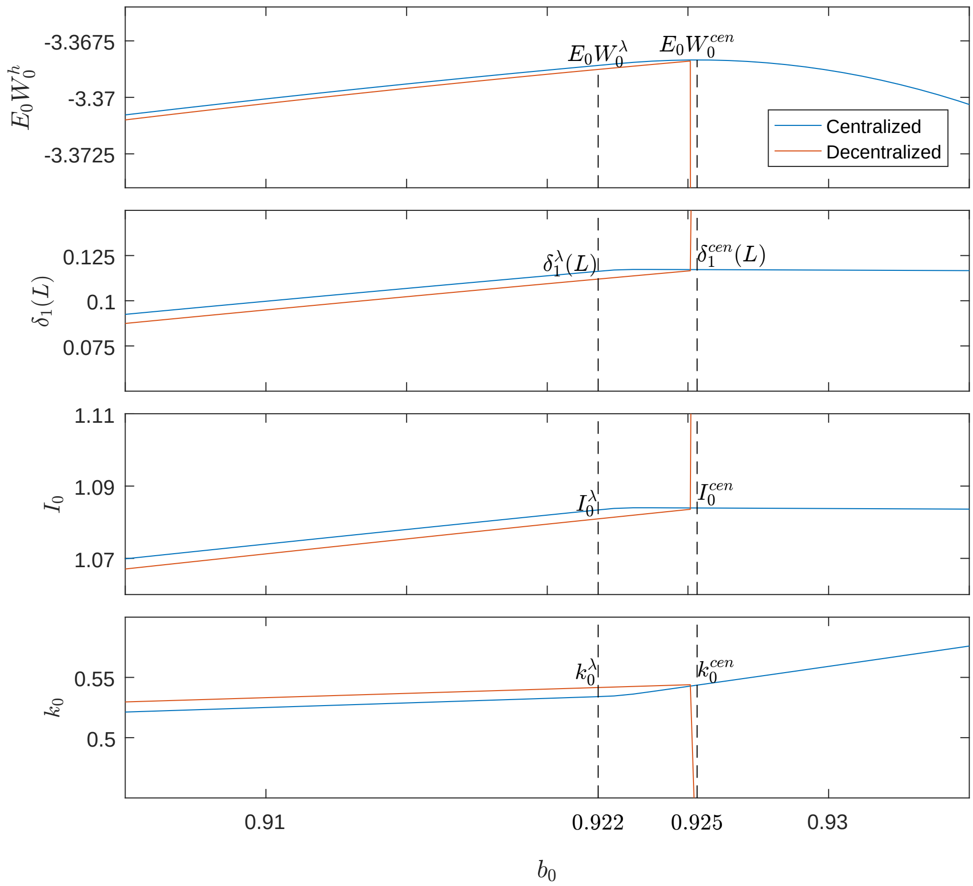
When debt is low, both centralized and decentralized models yield full repayment, as the difference between value functions of autarky and repayment is high. At full repayment there is no need for the planner to manipulate the bargaining process. Thus, solutions for k_0 in the two models coincide—so do welfare functions. For higher levels of debt there emerges a region of risky borrowing (depicted on Figure 6).[17] In that region, for each b_0 the centralized model yields higher expected welfare, higher default rate and higher interest on bonds, as the planner can choose k_0 that improves the outcome of bargaining. In our calibrated model the decentralized equilibrium yields $b_0 = 0.922$, which does not correspond to the peak of expected welfare in neither model. This inefficiency arises because the households do not factor in the effect their borrowing and capital investment have on the bond price schedule. In the centralized version of the model the planner manipulates

[15]To solve for the centralized equilibrium in MATLAB we used the fmincon solver, imposing (51) and (52) as nonlinear constraints.

[16]The reader may wonder why expected welfare functions, default rates and interest schedules of the two solutions do not coincide. But recall that the planner optimizes not only over b_0, but also over k_0. If we allowed households to choose k_0, then the planner's solution over b_0 would give default, interest and welfare functions similar to those observed in a decentralized case (and solution would be at the perk of expected welfare function).

[17]We only depict equilibria associated with default in low state and full repayment in high: as discussed in subsection 3.1, this is the combination that improves consumption smoothing.

Figure 6: Centralized vs. Decentralized



Notes: The figure plots partial equilibrium values (less first-order condition for borrowing), depending on the amount of debt. The figure contrast the results for decentralized and centralized models. The plots depict expected welfare, default rate, interest rate and the amount of capital investment.

the renegotiation surplus of period 1 to improve domestic bargaining stance and tilt the outcome of renegotiation toward more default.

When debt exceeds a threshold of ≈ 0.924 the decentralized economy starts to display properties depicted on Figure 4 of subsection 3.2, and positive borrowing ceases to be feasible. This happens because with high debt burden fully defaulting on debt becomes more beneficial, and Home's surplus from renegotiation $(\{\bar{V}R\}_1^h(s) - \{\bar{V}A\}_1^h(s))$ drops. This improves Home's bargaining stance and allows for more default in both states. But as discussed in subsection 3.2, such improvement undermines Home's ability to borrow in the first place.

If in period 0 the households could commit to repaying high portion of debt in period 1, despite favorable stance in the bargaining process, they would choose to do so because this strategy would increase welfare ex ante. However, the households cannot make such a commitment, so the foreign creditors are not willing to lend.

Now consider the centralized model. Unlike households, the planner has access to a form of inter-period commitment: she can choose k_0 and through it influence her bargaining stance in period 1. Specifically, the planner chooses higher capital that raises consumption in period 1 and reduces $u'(c_1(L))$, making repayment less costly in terms of utility losses. Mathematically, the planner chooses k_0 to equilibrate

$$u'(c_1(H))I_0b_0 = \frac{1-\theta}{\theta}\left[\{VR(H)\}_1^h - \{VA(H)\}_1^h\right].$$

This allows to sustain high levels of borrowing regardless of the bargaining power.

Table 2: Comparison with the centralized economy

Variable	Decentralized	Optimal δ-eqm	Centralized
Welfare home	-3.369	-3.361	-3.368
b_0	0.9218	0.9220	0.9253
δ_1^L	0.112	0.292	0.117
I_0	1.081	1.195	1.084
k_0	0.542	0.541	0.544
c_0	0.8739	0.8750	0.8754
$c_1(L)$	0.8226	0.8750	0.8232
$c_1(H)$	0.928	0.875	0.926

Notes: The table compares equilibrium allocations for the baseline decentralized model, the optimal δ-equilibrium and the centralized model.

Table 2 compares the decentralized equilibrium with bargaining, the maximum achieved under δ-equilibrium and the centralized equilibrium. The planner improves upon the expected welfare of the decentralized model. However, the δ-equilibrium still yields the highest welfare. Notice that in the centralized economy period 0 borrowing, consumption and capital investment are the highest. The planner borrows more to raise current consumption and invest into more capital, so that in period 1 bargaining results in partial repayment and thereby allows for borrowing in the first place. At the same time, higher level of debt still improves the outcome of bargaining for the planner allowing for higher default rate compared to the decentralized case.

Compared to the centralized model, the optimal δ-equilibrium delivers a higher default rate pared with lower level of borrowing and capital. Since in

δ-equilibrium there is no bargaining, raising capital and debt does not yield an improvement. Thus, in the centralized model the fact that the planner is constrained by bargaining process prevents full consumption smoothing.

5 Macroprudential Policy and Taxing Capital Flows

In the previous section we have established that the presence of a social planner can alleviate inefficiencies that occur in a decentralized model with bargaining. By manipulating capital investment the planner can ensure that positive borrowing is feasible and raise equilibrium default rates. We now ask whether a government of a decentralized economy could play a similar role through macroprudential regulation.

We go back to the decentralized model of section 2 and introduce a tax on capital investment, τ, imposed in period 0. The revenue from tax collection is given back to households in the form of a lump-sum transfer:

$$c_0 + k_0(1+\tau) = w_0 l_0 + tr + R_0 k_{-1} + b_0, \tag{59}$$

$$k_0 \tau = tr. \tag{60}$$

When $\tau < 0$, the government imposes a capital subsidy financed through a lump-sum tax.

The presence of τ alters the relative price of capital faced by households, modifying their first-order condition with regard to capital investment:

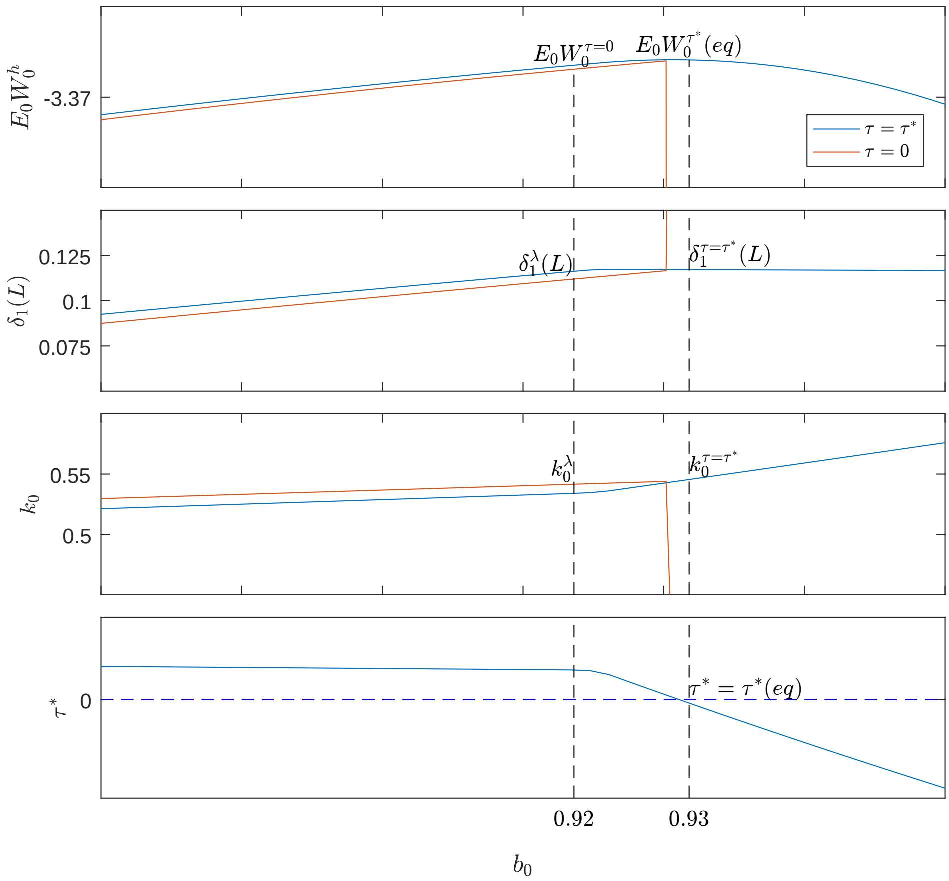
$$\frac{\partial u(c_0)}{\partial c_0} \cdot (1+\tau) = \beta \sum_{s=1}^{2} \pi(s) R_1(s) \frac{\partial u(c_1(s))}{\partial c_1(s)}, \tag{61}$$

while all other budget constraints and first-order conditions remain unchanged.

We solve numerically for the optimal τ^*. Figure 7 depicts partial equilibrium (less the first-order condition for b_0), in which for each level of b_0 we calculate a unique value of τ that maximizes households' welfare. The dashed lines indicate equilibrium in the decentralized model with $\tau = 0$ and the equilibrium in an economy where τ is chosen optimally to maximize welfare. We observe that welfare-improving τ is negative, meaning that the government chooses to subsidize capital investment. This result is in line with observations made in section 4, in which we documented that a social planner would choose capital that exceeds equilibrium value derived for the decentralized economy with no macroprudential policy.

Table 3 compares allocations obtained in the baseline model with bargaining, in the optimal δ-equilibrium, in the centralized model with bargaining

Figure 7: Optimal capital subsidy



Notes: The figure plots partial equilibrium values (less first-order condition for borrowing), depending on the amount of debt. The figure contrast the results for decentralized model with no capital subsidy, and the decentralized model where capital subsidy is chosen to maximize welfare. The plots depict expected welfare, default rate, the amount of capital investment and the optimal subsidy.

and in the decentralized model with optimal capital subsidy. The centralized economy and the economy with capital subsidy display higher levels of debt an capital than those observed in the decentralized economy, which, again, stipulates the role played by capital investment financed through extra foreign borrowing. Furthermore, borrowing and capital in these two models exceed the corresponding values documented for the optimal δ-equilibrium. This suggests that the pareto-improving effect of capital is tied to bargaining process: in economy with optimal default there is no need to commit to repayment through capital over-accumulation.

Table 3: Capital subsidy: comparison with previous results

Variable	Decentralized	Optimal δ-eqm	Centralized	Optimal τ
Welfare home	-3.3688	-3.3606	-3.3683	-3.3684
b_0	0.9218	0.9220	0.9253	0.9259
d_1^L	0.1120	0.2917	0.1172	0.1172
I_0	1.0809	1.1946	1.0839	1.0839
k_0	0.542	0.541	0.544	0.545
c_0	0.8739	0.8750	0.8754	0.8741
$c_1(L)$	0.8226	0.8750	0.8232	0.8239
$c_1(H)$	0.928	0.875	0.926	0.927
$c_1(H)$	-	-	-	-0.004

Notes: The table compares equilibrium allocations for the baseline decentralized model, the optimal δ-equilibrium, the centralized model and the decentralized model with optimal subsidy.

6 Concluding Remarks

In this paper we studied the nexus between private borrowing and centralized renegotiation and concluded that in such an environment bargaining generates suboptimal outcomes. The problem is twofold. First, expected default rates are too low compared to the Pareto optimum that would allow private agents to hedge against output fluctuations. Second, if the anticipated haircuts in the event of default are different from the optimum (either too low or too high), the volume of external debt extended is inefficiently low or even zero.

Macroprudential policy that favorably distorts the investment rate in capital can resolve both problems stated above: higher capital investment acts as an ex ante assurance that repayment of a large portion of debt will occur ex post. At the same time, higher investment necessitates more borrowing, which then leads to higher expected default rates that, as we argue, can be Pareto-improving. This contributes to the discussion concerning reforms that should be undertaken by troubled economies, a discussion that is especially current for countries that sought assistance with the 'troika' and where then obliged to approve a package of reforms.

Our argument contributes to the literature on macroprudential regulation, particularly to the discussion about over- and underborrowing. Bianchi and Mendoza (2015) and Jeanne and Korinek (2010) show that the fact that private agents do not internalize the effect their borrowing has on collateral prices amplifies booms and busts along the business cycles. They conclude that policies that combat overborrowing are welfare-improving. By contrast, Schmitt-Grohe and Uribe (2016) show that in a framework where multiple equilibria are possible a similar environment with collateral constraints will generate underborrowing, as agents will attempt to self-insure against bad times.

Unlike this literature, we do not incorporate collateral constraints, but include partial defaults and bargaining. The externality present in our model leads to underborrowing because debt contracts imply that repayment will occur even if the economy is in a severe economic downturn. This renders private sector borrowing less effective in smoothing consumption across states and leads private agents to borrow less. This inefficiency is exacerbated in an environment where private agents do not internalize the effect their borrowing has on equilibrium bargaining outcomes, a result that echos Kim and Zhang (2012) who also examine a model of decentralized borrowing and conclude that it leads to suboptimal debt levels and impaired welfare. Our argument regarding default rates that are 'too low' complements that of Krugman (1988), who stress that in some circumstances creditors are better off forgiving a large portion of the debt rather than financing the debtor country in hopes that it would repay in full in the future.

A limitation of our approach is that our model does not incorporate in-equilibrium costs resulting from partial default—the output loss in our model triggers default, but default (given orderly renegotiation) does not cause further output losses. We would argue that a successful renegotiation over a haircut and provision of financial aid does not in itself depress economic activity.[18] At the same time, we recognize that even orderly restructuring of bank debt may initiate bank panic and financial turmoil. A future research in this area could incorporate the feedback between losses associated with private debt restructuring and the bargaining process.

[18]Furthermore, as argued by Yeyati and Panizza (2011), empirically sovereign default events often signal the beginning of economic recovery.

References

Acharya, V., Drechsler, I., and Schnabl, P. (2014). A pyrrhic victory? bank bailouts and sovereign credit risk. *The Journal of Finance*, 69(Issue 6).

Aguiar, M. and Gopinath, G. (2006). Defaultable debt, interest rates and the current account. *Journal of International Economics*, 69(1):64–83.

Arellano, C. (2008). Default risk and income fluctuations in emerging economies. *American Economic Review*, 98(3):690–712.

Arellano, C. and Bai, Y. (2014). Renegotiation Policies in Sovereign Defaults. *American Economic Review*, 104(5):94–100.

Arellano, C. and Kocherlakota, N. (2014). Internal debt crises and sovereign defaults. *Journal of Monetary Economics*, 68(S):S68–S80.

Arteta, C. and Hale, G. (2008). Sovereign debt crises and credit to the private sector. *Journal of International Economics*, 74(1):53–69.

Asonuma, T. and Trebesch, C. (2016). Sovereign Debt Restructurings: Preemptive Or Post-Default. *Journal of the European Economic Association*, 14(1):175–214.

Balteanu, I. and Erce, A. (2014). Banking crises and sovereign defaults in emerging markets: exploring the links. Working Papers 1414, Banco de Espana.

Bianchi, J. and Mendoza, E. G. (2015). Optimal time-consistent macroprudential policy. BIS Working Papers 516, Bank for International Settlements.

De Walque, G., Pierrard, O., and Rouabah, A. (2010). Financial (in)stability, supervision and liquidity injections: A dynamic general equilibrium approach*. *The Economic Journal*, 120(549):1234–1261.

D'Erasmo, P. and Mendoza, E. G. (2016). Optimal Domestic (and External) Sovereign Default. NBER Working Papers 22509, National Bureau of Economic Research, Inc.

Dubey, P., Geanakoplos, J., and Shubik, M. (2005a). Default and punishment in general equilibrium. *Econometrica*, 73(1):1–37.

Dubey, P., Geanakoplos, J. D., and Shubik, M. (2005b). Default and punishment in general equilibrium. *Econometrica*, 73.

Eaton, J. and Gersovitz, M. (1981). Debt with potential repudiation: theoretical and empirical analysis. *The Review of Economic Studies*, 48:289–309.

Fuentes, M. and Saravia, D. (2010). Sovereign defaulters: Do international capital markets punish them? *Journal of Development Economics*, 91(2):336–347.

Geanakoplos, J. D. and Polemarchakis, H. (1986). Existence, regularity and constrained suboptimality of competitive allocations when the asset market is incomplete. In Heller, W., Starr, R., and Starrett, D., editors, *Uncertainty, Information and Communication: Essays in Honour of K. J. Arrow, Vol. III*, pages 65–95. Cambridge University Press.

Goodhart, C., Sunirand, P., and Tsomocos, D. (2005). A risk assessment model for banks. *Annals of Finance*, (1):197–224.

Goodhart, C., Tsomocos, D. P., and Peiris, M. U. (2016). Debt, recovery rates and the greek dilemma. *Saïd Business School WP*, 15.

Goodhart, C. A. E., Sunirand, P., and Tsomocos, D. (2006). A model to analyse financial fragility. *Economic Theory*, 27:107–142.

Honohan, P. (2009). Resolving irelands banking crisis. *The Economic and Social Review*, 40(2).

Jeanne, O. and Korinek, A. (2010). Managing Credit Booms and Busts: A Pigouvian Taxation Approach. CEPR Discussion Papers 8015, C.E.P.R. Discussion Papers.

Jeske, K. (2006). Private international debt with risk of repudiation. *Journal of Political Economy*, 114(3):576–593.

Kim, Y. J. and Zhang, J. (2012). Decentralized borrowing and centralized default. *Journal of International Economics*, 88(1):121–133.

Krugman, P. (1988). Financing vs. forgiving a debt overhang. *Journal of Development Economics*, 29(3):253–268.

Na, S., Schmitt-Grohe, S., Uribe, M., and Yue, V. Z. (2014). A Model of the Twin Ds: Optimal Default and Devaluation. NBER Working Papers 20314, National Bureau of Economic Research, Inc.

Panizza, U., Sturzenegger, F., and Zettelmeyer, J. (2009). The Economics and Law of Sovereign Debt and Default. *Journal of Economic Literature*, 47(3):651–98.

Peiris, M. and Vardoulakis, A. (2013). Savings and default. *Economic Theory*, 54(1):153–180.

Peiris, M. U. and Tsomocos, D. P. (2015). International monetary equilibrium with default. *Journal of Mathematical Economics*, 56:47 – 57.

Reinhart, C. M. and Rogoff, K. S. (2011a). From Financial Crash to Debt Crisis. *American Economic Review*, 101(5):1676–1706.

Reinhart, C. M. and Rogoff, K. S. (2011b). From Financial Crash to Debt Crisis. *American Economic Review*, 101(5):1676–1706.

Reinhart, C. M. and Rogoff, K. S. (2013). Banking crises: An equal opportunity menace. *Journal of Banking & Finance*, 37(11):4557–4573.

Rose, A. K. (2005). One reason countries pay their debts: renegotiation and international trade. *Journal of Development Economics*, 77(1):189–206.

Schmitt-Grohe, S. and Uribe, M. (2016). Multiple Equilibria in Open Economy Models with Collateral Constraints: Overborrowing Revisited. NBER Working Papers 22264, National Bureau of Economic Research, Inc.

Schreger, J. and Du, W. (2014). Sovereign Risk, Currency Risk, and Corporate Balance Sheets. Working Paper 209056, Harvard University OpenScholar.

Shubik, M. and Wilson., C. (1977). The optimal bankruptcy rule in a trading economy using fiat money. *Journal of Economics*, 37:337–354.

Tomz, M. and Wright, M. L. J. (2007). Do Countries Default in 'Bad Times'? *Journal of the European Economic Association*, 5(2-3):352–360.

Trebesch, C. and Zabel, M. (2016). The Output Costs of Hard and Soft Sovereign Default. CESifo Working Paper Series 6143, CESifo Group Munich.

Tsomocos, D. P. (2003). Equilibrium analysis, banking and financial instability. *Journal of Mathematical Economics*, 39:619–655.

UBS (2011). Euro break-up the consequences. *UBS Investment Research*. UBS Limited.

Walsh, K. (2015a). Portfolio choice and partial default in emerging markets: A quantitative analysis. *working paper*.

Walsh, K. (2015b). A theory of portfolio choice and partial default. *working paper*.

Whelan, K. (2014). *Journal of Macroeconomics*, 39(PB):424–440.

Wright, M. L. (2006). Private capital flows, capital controls, and default risk. *Journal of International Economics*, 69(1):120 – 149.

Yeyati, E. L. and Panizza, U. (2011). The elusive costs of sovereign defaults. *Journal of Development Economics*, 94(1):95–105.

Yue, V. Z. (2010). Sovereign default and debt renegotiation. *Journal of International Economics*, 80(2):176 – 187.

Appendix A: Decentralized borrowing, centralized renegotiation: historical cases

Appendix A:.1 Ireland 2008-2012

In the period preceding 2008 Ireland experienced construction boom. Irish banks funded growing demand for loans by borrowing abroad, raising net foreign borrowing to 60 percent of GDP compared to 10 percent in 2003.[19] In 2008, following a sharp decline in house prices, the largest Irish bank, Anglo Irish Bank, started rapidly loosing funds. Fears of looming banking crisis prompted Irish government to take action: on September 30, 2008 it announced that it had extended guarantees to deposits of six largest Irish banks. Following this announcement, the CDS spreads on government bonds soared. In an effort to save the banks the government took further actions that included buying non-performing loans from the banks via the National Asset Management Agency (NAMA) set up in 2009, and eventually nationalizing five of the six biggest banks. In 2010, following further increases in spreads on sovereign bonds, the Irish government lost access to financial market and in November 2010 was forced to request assistance from the 'troika' (the EC, the ECB and the IMF), reaching an agreement on a 67.5 billion euro financial support. I July 2012 Ireland regained partial access to the financial market. In 2013 Ireland exchanged 25 billion high interest (about 8%) promissory notes used to bail out Anglo Irish Bank for lower cost (about 3%) long term government debt.[20]

[19] For detailed accounts of Irish crisis, see Whelan (2014), Honohan (2009)

[20] Source: OECD Sovereign Borrowing Outlook 2014