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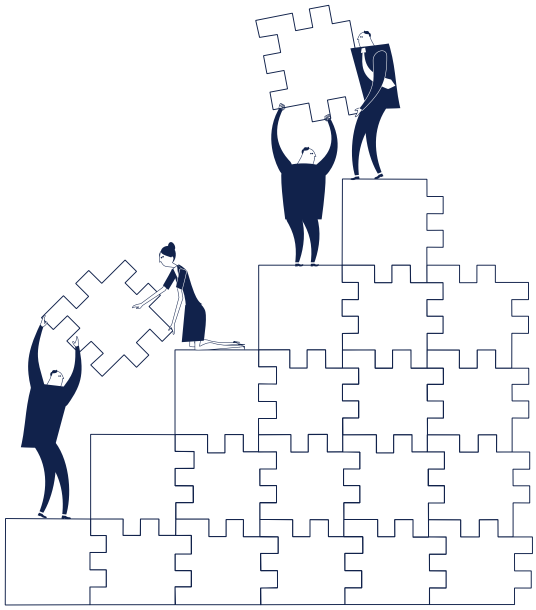

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Credit Risk and Discontinuous Effects of Monetary Reverse Transactions

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Abstract

A central bank possesses various instruments to provide liquidity. These are either outright monetary transactions (OMT) of securities or other refinancing facilities, primarily *repos*, which are executed with standard tenders. The eligible securities (i.e. bonds or equities) need to conform with certain credit risk criteria (i.e., satisfactory credit rating or low default probability). This paper introduces a monetary model to address the role of collateralized securities on the effectiveness of monetary policy. Our results suggest that credit rating downgrading may precipitate into a disproportionate credit contraction.

Keywords: collateralized securities; central banks; endogenous tender rate

JEL Classification: D53, E41, E51, E52, E58

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1 Introduction

Monetary authorities set interest rates and control market liquidity conditions by engaging in appropriate open market operations. These operations are implemented either by outright monetary transactions and/or by reverse transactions, i.e., repos that are settled via tenders. Although outright transactions seem to be the most preferred way to deal with systemic liquidity problems (i.e., ECB's Outright Monetary Transactions to address the EU sovereign debt crisis), it is through reverse transactions that interest rates are set. Reverse transactions involve the purchase and resale of securities at prespecified price and for a short period of time. Nevertheless, these transactions are not different from standard collateralized loans. Collateralized securities ought to have adequate market value, at least equal to the money value of loans. These securities may be bonds or equities that fulfill certain credit risk criteria. Monetary authorities specify credit rating requirements according to which securities are qualified as appropriate collateral. Arguably, the credit risk of collateralized securities plays an important role for the quantity of money that is extended to the real economy.

This study challenges the efficiency of reverse transactions as a monetary instrument when the securities market is distressed. The argument is simple. It is customary for the monetary authorities to enhance market liquidity by offering "cheap" money. However, this might turn to be insufficient, if market participants possess securities of poor credit rating. We show that credit rating downgrading may reverse the intended consequences of OMT of securities for liquidity extension. Put differently, monetary authorities should balance the trade off between securities eligibility and their associated credit risk. We address these anomalies by developing a monetary model with financial assets along the lines suggested by Dubey & Geanakoplos (1992, 2003, 2005, 2006). Monetary economies with incomplete markets and nominal assets, nevertheless, characterized by indeterminacy (Geanakoplos

& Mas-Colell , 1989). The introduction of *fiat* money via cash-in-advance constraints is one way to remove this indeterminacy. Magill & Quinzii (1992) introduce a "sell-all" trading mechanism, where all individuals sell all of their endowments to some Central Exchange that processes and clears all trades. A different way to restore determinacy was proposed by Dubey & Geanakoplos (2006) that made use of outside money (each individual starts with money endowments).Finally, Lin et al. (2015) argue that positive default in equilibrium is sufficient to establish determinacy. This paper follows the second method.

The role of collateralized securities in market based models has been extensively addressed in the literature. However, the precursors to our framework are Geanakoplos & Zame (2014) and Bottazzi et al. (2012) . Geanakoplos & Zame (2014) assume financial assets backed by a durable good and, fundamentally provide the theoretical underpinnings for introducing asset backed securities (ABS) in a general equilibrium context. Equally important is the introduction of financial assets backed by other nominal securities in Bottazzi et al. (2012). The latter introduces naturally repo markets in a general equilibrium framework and explains how rehypothecation of securities by agents that go long in repos increases their leverage. In fact, rehypothecation explains very well the importance of reverse transactions for the transmission of monetary policy.

Section 2 sets out a monetary model where open market operations occur. Section 3 defines the equilibrium and the existence result is proved. Section 4 examines the continuity properties of monetary operations. Finally, Section 5 offers our concluding results.

2 The model

Consider an exchange economy that extends over two periods $t \in \{0, 1\}$. There are n possible states at date 1, $S = \{s_0, s_1, \ldots, s_n\}$, with s_0 denoting the state at date 0. In each state there are L consumption commodities available, and hence the commodity space is $\mathcal{R}^{L \times S}$. On this stochastic structure, we assume a continuum of individuals defined as the positive and compact subset of real line I. Following Aumann (1964), we equip the measurable space $(I, \mathcal{I})$ with the standard Lebesgue measure ν. Individuals are *atomless*, however we additionally assume that I is partitioned into k *types* having strictly positive measure, $I = \cup_{j \leq k} I_j$ with $I_i \cap I_j = \emptyset$ for all i, j. Individuals have preferences representable by utility functions $U^i : \mathcal{R}_+^{L \times S} \mapsto \mathcal{R}$, dictated by the axioms of von Neumann and Morgenstern theorem, i.e. state separable utility functions. Each individual is endowed with an initial bundle of commodities and initial holdings of *fiat* money, $w = (w^i(s), m(s)^i) \in \mathcal{R}^{L+1 \times S}$. We assume that endowments are bounded away to infinity. Individuals sharing the same *type* are identical with respect to their characteristics, i.e. preferences and endowments, while all *types* are symmetric with respect to their information given by a common prior ψ over the states. In this setup, given the price system of commodities $p = (\ldots, p(s), \ldots) \in \mathcal{R}_+^{L \times S}$, individuals opt for a consumption plan $x^i = (\ldots, x^i(s), \ldots) \in \mathcal{R}_+^{L \times S}$.

The securities market. There are $J \leq S$ nominal securities available, described by their state dependent payoff at date 1. A nominal security is defined by the function $y_j : S \setminus \{s_0\} \mapsto R_+$ and the *asset payoffs matrix* by $y = (\ldots, y(s), \ldots) \in \mathcal{R}^{J \times S \setminus \{s_0\}}$. Payoffs are denominated in *fiat* money and assume no-arbitrage security prices given by the vector $\pi = (\ldots, \pi(s), \ldots) \in \mathcal{R}_+^J$. Typically, an individual will hold a portfolio of securities $\theta^i \in \mathcal{R}^J$.

The central bank. In our exchange economy *fiat* money is the stipulated means of exchange. There is a monetary authority, which we call the *central bank* (or simply the bank), having the exclusive permission to print paper money. Moreover, central bank is the sole financial intermediary that lends money, if needed. Specifically, all consumption decisions should defer a Clower type cash-in-advance constraint. Essentially we postulate that individuals cannot spend more for purchases than their money possessions (outside money). Whenever they are willing to spend more than their money holdings, the central bank creates inside money by credit, i.e. it gives loans to individuals. Finally, the central bank is also be responsible to implement monetary policy by regulating money balances by appropriately defined monetary operations.

What is different in this model is that credit is provided by reverse transactions. In short, an individual i borrows c^i monetary units by issuing bonds of face value μ^i. The central bank buys the bonds, once they are backed by securities of equal value and of certain credit quality (probability of default). All in all, the monetary policy parameters will be given by the pair (M, d), where $M \ll +\infty$ is the supply of credit for period t and $d \in [0, 1]$ is the threshold probability of default of the collateralized securities. The *ex ante* nominal interest rate is endogenously determined by the

$$1 + r = \int_I \frac{\mu^i}{M}. \tag{1}$$

The credit risk. Individuals may default, i.e. they may overdraft, becoming unable to deliver their obligations. In an no-arbitrage economy with no credit risk, *state prices* (or risk neutral probabilities) λ_j for each security y_j can be calculated as the inner product $\lambda_j \cdot \mathbf{y}'_j = \pi_j$, i.e. the expected payoff (no discounting). Allowing credit risk the expected payoff becomes

$$\psi_j \cdot \mathbf{0} + (1 - \psi_j)\lambda_j y'_j = \pi_j \Rightarrow \psi_j = 1 - \frac{\pi_j}{\lambda_j \cdot y_j}, \tag{2}$$

with scalar $\psi_j \geq 0$ denoting the probability of default of security y_j. When ψ_j is strictly positive, π_j should encompass the risk premium borne by individual holders.

We put emphasis on the credit risk of *inside money*, i.e. the risk the central bank is exposed when buys debt. It is anticipated that rational individuals will prefer to default whenever the value of collateralized securities hold by the central bank falls short in value compared to their debt.

The collateralized securities. Not all securities can play effectively the role of the collateral. Securities are normally priced to reflect their credit risk, hence high risk securities are sold at discount. The central bank would preferably not accept securities that are associated to a high probability of default. Formally, monetary authorities stipulate some threshold probability of default d that would be used as an eligibility criterion for securities accepted for collateral. It is imperative to examine the effectiveness of monetary parameter d viz. the securities market conditions.

Define by $\bar{\theta}_j = \int_I \max\{\theta^i_j, 0\}$ the long market portfolio for security j and by $\bar{\theta} = (\bar{\theta}_j)_{j\in J}$ the overall long market portfolio. The securities market value in nominal terms can be calculated for finite security prices π, and equals to $F = \bar{\theta} \cdot \pi$, with $F \in \mathcal{R}_{++}$. It is also useful to calculate the relative size securities possess in the total market size.

Definition 1 *The securities distribution $\rho = \left(\frac{\bar{\theta}_j \cdot \pi_j}{F}\right)_{j\in J}$ gives the relative shares the securities hold across the market.*

In securities distribution we primarily interested in their distribution with respect to their credit risk. Therefore, it is useful to partition the market size in security equivalence classes with respect to their credit risk. Define the set of mutually disjoint sets $J_\psi = \{j' \in J | \psi_j = \psi\}$. The relative size of class J_ψ will be $\rho_\psi = \sum_{j\in J_\psi} \rho_j$. Some securities classes may possess an important share of the market, i.e. sovereign debt which share a small probability of default compared to the equities that may share a higher one. Equivalently, we could use a partition according to credit ratings classification, nevertheless default probabilities are far more versatile for our purposes.

Another fact that we would like to model explicitly is the bimodal and sometimes multi-modal distribution of securities with respect to their credit risk. There are different groups of securities with diverse credit characteristics. The following definition guarantees that securities are not distributed uniformly across ψ. Let $\mathbf{1}_d$ be the indicator function, taking value 1 when $\psi \leq d$, and $\mathbf{1}_d \cdot \rho = \sum_{\substack{j \in J_\psi \\ \psi \leq d}} \rho_j$.

Definition 2 *The securities distribution ρ will be called unbalanced if for some d and $d' \in \mathcal{V}$ neighborhood of d, there exists $\epsilon > 0$ such that $|\mathbf{1}_d \cdot \rho - \mathbf{1}_{d'} \cdot \rho| > \epsilon$,*

For a small change in the probability of default criterion, the measure of the securities that conform the threshold value changes abruptly. The central bank defines what securities will be eligible for collateralization by appropriately choosing the probability of default threshold. For a low probability of default threshold, only the high quality securities can play the role, while as the d increases, more and more securities become eligible.

Definition 3 *The value of eligible market portfolio is*

$$F^e = \mathbf{1}_d \cdot \rho \cdot F \quad \text{for} \quad \mathbf{1}_d = \begin{cases} 1 & \text{if} \quad \psi_j \leq d \\ 0 & \text{if} \quad \psi_j > d \end{cases} . \tag{3}$$

Eventually, the tender rate [eq. (1)] is sensitive to changes in d and thus is restated as

$$r^t\Big((M,d);\rho\Big) = \int_I \frac{\mu^i}{\min\{M, F^e\}} - 1. \tag{4}$$

The formation of the nominal interest rate depends on both policy parameters M and d. When the money supply M is less than the eligible market value F^e, the probability of default threshold plays no role and the formula is reduced to the standard case of eq.(1).

There are sufficient eligible securities to use as collateral. To the contrary, when $M > F^e$ then individuals have no sufficient collateral to borrow the total M.

2.1 The market mechanism

For clarity, we provide the actions taken by individuals in the different stages of the mechanism.

Date 0

In security markets, individual i bids β_j^i for acquiring security j while offers for sale γ_j^i contracts of the same security. *Wash* sales are permitted, i.e. individuals can be at both sides of the market simultaneously. Security prices obtained are

$$\pi_j = \begin{cases} \int_I \beta_j^i / \int_I \gamma_j^i & \text{if} \quad \int_I \gamma_j^i > 0 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

For prices π, the portfolio possessed by individual i will be

$$\theta_j^i = \begin{cases} \frac{\beta_j^i}{\pi_j} - \gamma_j^i & \text{if} \quad \pi_j > 0 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Then, spot commodity markets open. Each individual i places a bid of money $b^i(0)$ for the purchase of commodities and offers a quantity $q^i(0)$ of each commodity to be disposed for sale. For $t = 0$,

$$p_l(t) = \begin{cases} \int_I b_l^i(t) / \int_I q_l^i(t) & \text{if} \quad \int_I q_l^i(t) > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

The consumption plan for commodity l is

$$x_l^i(t) = \begin{cases} w_l^i(t) - q_l^i(t) + \frac{b_l^i(t)}{p_l(t)} & l \in L \\ -\sum_{l\in L} b_l^i(t) + \sum_{l\in L} p_l(t) q_l^i(t) & m \end{cases} \tag{8}$$

Actions taken at date 0 are determined by the budget set of individual i:

$$\begin{aligned} \Sigma^i(0) = \{ & ((\beta^i, \gamma^i), c^i, (b^i(0), q^i(0))) \in \mathcal{R}^{2J} \times \mathcal{R}_+ \times \mathcal{R}^{2L} \\ & ; c^i(0) \geq 0, q_l^i(0) \leq w_l^i(0), \sum_{l\in L} b_l^i(0) + \pi\theta^i \leq m^i(0) + \frac{\mu^i}{1+r^t}, \\ & \beta^i, \gamma^i \geq 0\} \end{aligned} \tag{9}$$

Notice that the *tender rate* r^t determines the money market conditions in the budget constraint (9).

Date 1

The spot commodity markets open. No transactions take place in security markets. The budget set of individual i is

$$\begin{aligned} \Sigma^i(1) = \Big\{ & (b^i(1), q^i(1))) \in \mathcal{R}^{2L} \; ; q_l^i(1) \leq w_l^i(1), \sum_{l\in L} b_l^i(1) \leq m^i(1) + p(0) q^i(0) \\ & + \theta^i y - \min\{\mu^i, \theta^i \tilde{y}^i\} \Big\} \end{aligned}$$

for $\theta^i \tilde{y}^i$ the value of collateralized securities, owned by trader i. Individuals pay back their debt only if the value of collateralized securities exceeds the debt. Otherwise, they always prefer to default.

Here, the revenues from sales have no value for the individuals since there is no more trade. Money balances that are collected in date 1, are used by individuals to collect their fraction of gold from the central bank that has intrinsic value and can be consumed immediately.

In summary, the strategy set of individual i is $\Sigma^i = \Sigma^i(0) \times \Sigma^i(1)$, with generic element σ^i.

Figure 1: The timeline of market mechanism

The payoff function. The payoff function of individual i in period t is no different from her utility function, $V^i\big(\sigma\big) = U^i\Big(x\big(\sigma\big)\Big)$.

3 Monetary equilibrium with credit risk

The equilibrium concept for the model is defined.

Definition 4 *We say that a strategy profile $\sigma^* \in \Sigma$ is a strategic monetary equilibrium with credit risk if for a set of monetary parameters (M, d) for all $i \in I$:*

1. $V^i(\sigma^{i*}, \sigma^{-i*}) \geq V^i(\sigma^i, \sigma^{-i*})$ *for all* $\sigma^i \in \Sigma^i$

2. $\int_I w^i(s) = \int_I x^i(s)$ *for all* $s \in S$

3. $\int_I \theta^i = 0$

4. $1 + r^t = \int_I \frac{\mu^i}{\min\{M, F^e\}}$

The strategic equilibrium requires the market clearance of (2) spot commodity markets, (3) security markets and (4) credit (inside money) market.

Next we prove that the *strategic monetary equilibrium with credit risk* always exists. We state the following assumptions:

A1. V^i is continuous, strictly monotone and quasi-concave

A2. $m^i(t) > 0$ and $m^i(t) \ll \infty$

A3. We consider no-arbitrage security prices, i.e. $\pi \in intH_+ \subset R^J$, with H the subspace of R^J spanned by vectors $y(s)$.

Assumption A3 is a rather mild assumption. In fact, having assumed an atomless economy, no individual can unilaterally affect security prices.

We define the proper subset of Σ,

$$\Sigma^f = \Big\{\sigma \in \Sigma | \text{ s.t. } \int_I w^i(s) = \int_I x^i(s) \text{ and } \int_I \theta^i = 0\Big\}.$$

The subset Σ^f is bounded since Σ is bounded. It is also closed and hence compact. Each individual aims to maximize its payoff for known policy parameters (M, d). The best response correspondence $B^i : \Sigma^f \times R_+ \times [0,1] \mapsto \Sigma^i$ maps the best response strategy for some strategy of the rest of the economy σ^{-i}. Define $B(\sigma) := \prod_{k \leq K} B^k(\sigma)$ for the k types.

Lemma 1 *The set of feasible strategy profiles Σ^f is nonempty, convex and compact.*

Proof. By Assumption (A2) it is easy to verify that the strategy sets are non empty. Clearly, by the linearity of constraints are also convex. For compactness, it suffices to prove that the strategy sets are compact.

Types of individuals have positive measure, hence we proceed by considering an arbitrary individual of type k. The action set of k at date 0 is bounded. Indeed, money supply is bounded from above, i.e. $M \ll \infty$, hence for the type k, $c^k \leq M$. By assumption (A2), outside money is finite as well. Hence, both $\sum_{l\in L} b^k(0)$ and $\sum_{j\in J} \beta^k(0)$ are bounded away to infinity. Also, $q_l^k(0) < w_l^k(0)$ for all $l \in L$.

Following Werner (1985), nonarbitrage security prices (Assumption A3) are bounded and individuals are constrained to choose finite portfolios. As result, $\pi\theta^k \leq m^i(0) + \frac{\mu^i}{1+r}$, therefore $\Sigma^k(0)$ is compact. Using the same reasoning $\Sigma^k(1)$ is compact as well. The result follows for the Cartesian product of all types $\Sigma = \times_{k\leq K}\Sigma^k$. The Σ^f is obviously closed and proper subset of Σ, hence it is compact. □

Lemma 2 *The best response correspondence B is nonempty and convex valued.*

Proof. First we prove that B is convex. Consider two strategies for an individual of arbitrary type k, σ^k and σ'^k with both $(\sigma^k, \sigma^{-k}), (\sigma'^k, \sigma^{-k}) \in \Sigma^f$. By (A1), V is quasi-concave, hence there is $\phi \in (0, 1)$ such that

$$V^k(\phi\sigma^k + (1-\phi)\sigma^{-k}, \sigma^{-k}) \geq \min\{V^k(\sigma^k, \sigma^{-k}), V^k(\sigma'^k, \sigma^{-k})\}.$$

We conclude that the strategy $\phi\sigma^k + (1-\phi)\sigma^{-k}$ is an element of B^k, hence the latter is convex. The Cartesian product of convex sets preserves convexity.

The correspondence B is also nonempty. We proved that Σ^f is compact. By (A1) V^k is continuous. By Weierstrass theorem, it is known that a continuous function has always a maximal element in a compact set, and therefore it is nonempty. □

Lemma 3 *The best response correspondence B is upper hemicontinuous (u.h.c).*

Proof. By lemma (1), the set of feasible strategies is nonempty, convex and compact. By assumption (A1), V^i is continuous. By applying the Maximum theorem (Berge , 1963) we easily obtain that $B^k(\sigma^{-i}; M, d) = \{\sigma'^i \in \Sigma^i | V^i(\sigma'^i, \sigma^{-i}) \geq V^i i(\sigma^i, \sigma^{-i})\}$ is upper hemicontinuous. □

Theorem 1 *The strategic monetary equilibrium with credit risk always exists.*

Proof. By lemmas (2) and (3) we know that the best response correspondence is convex valued and u.h.c. It is also compact as a proper subset of a compact set. We apply the Kakutani's theorem to the correspondence B, thus we obtain a fixed point $\sigma^* \in B(\sigma^*)$. □

4 Continuity properties of monetary operations

The presence of outside money (money endowments) guarantees that *fiat* money has positive value and monetary policy may have real effects to the economy. An increase in money supply creates more opportunities for trade and the opposite occurs when the central bank mops up excess liquidity. The key difference from other monetary models is that here the monetary policy is effective but in a discontinuous fashion. The argument goes as follows. A small decrease in d (threshold of probability of default) may discard a large share of eligible securities for collateralization. Individuals possessing these non-eligible securities are essentially blocked from participating to money tenders and thus less money is channeled into the economy. As a result, a small change in monetary parameter d may affect abruptly the disposed money income and the budget set of individuals may get enormously tightened (or relaxed for an increase in d).

We define a family of correspondences that attribute the effect of monetary policy upon the budget set of individuals. For arbitrary individual i define the correspondence, $\mathcal{M}^i : \mathcal{R}_+ \mapsto S$ with $S \subset \mathcal{R}^{2J} \times \mathcal{R}_+ \times \mathcal{R}^{2L}$, which we call *money supply* correspondence. For different values of M the correspondence maps the budget of i at date 0. Respectively, define the *quality of collateral* correspondence, $\mathcal{QC}^i : [0,1] \mapsto S$, which maps the budget set of i for different values of the probability of default threshold d. Overall, the monetary policy instruments will be attributed by the *policy mix* rule given by the pair $(\mathcal{M}^i, \mathcal{QC}^i)$. We examine the continuity properties of the policy mix rule. Once continuous, small changes of the policy parameters will not cause abrupt variations to the purchasing power of individuals.

We start by characterizing the money supply correspondence and prove that it is a continuous correspondence.

Lemma 4 *For arbitrary individual i, the money supply correspondence $\mathcal{M}^i$ is compact-valued.*

Proof. Fix M to be bounded away from infinity. Notice that F^e is always finite for finite security prices. By equation (4) it is $1 + r^t = \int_I \mu^i / \min\{M, F^e\}$. Substituting into the budget constraint (9) it becomes

$$\sum_{l \in L} b_l^i(0) + \pi\theta^i \leq m^i(0) + \frac{\mu^i}{\int_I \mu^i} \min\{M, F^e\}$$

Trivially, the budget set at date 0 is closed. It suffices to prove that it is also bounded. From the right hand side of the inequality, outside money m^i is always bounded away from the infinity. Fix $\bar{\mu} = max\{\mu^1, \mu^2, \dots, \mu^k\}$ for the k types. Evidently, $\frac{\bar{\mu}}{\int_I \mu^i}$ is bounded by 1 and therefore inside money for all individual is bounded by $\frac{\bar{\mu}}{\int_I \mu^i} \min\{M, F^e\}$. As a result the budget set is bounded, hence by the Heine-Borel theorem it is compact. □

Lemma 5 *For arbitrary individual i, the money supply correspondence $\mathcal{M}^i$ has a closed graph.*

Proof. If the money supply is inactive, i.e. $M > F^e$, the lemma is trivially satisfied. We examine the case where $M < F^e$. Let (M_n) be a sequence such that $M_n \to M$ and sequence of actions (σ_n^i) where $\sigma_n^i \to \sigma^i$ and $\sigma_n^i \in \mathcal{M}^i(M_n)$. If the function $\mathcal{M}^i$ has a closed graph it ought to be that in the limit $\sigma^i \in \mathcal{M}^i(M)$.

Since Σ^i is bounded and closed then there exists an open neighborhood of $\mathcal{M}^i(M)$, V_ϵ[1] and some $\bar{n}$ such that for all $n \geq \bar{n}$ it is $\mathcal{M}^i(M_n) \in V_\epsilon$. It follows that $\sigma_n^i \in V_\epsilon$ for all $n \geq \bar{n}$. Hence there is a converging subsequence $\sigma_{nq}^i \to v$ such that $v \in V_\epsilon$.

Suppose now that the function has not a closed graph, i.e. $\sigma^i \notin \mathcal{M}^i(M)$. Then, there is a closed neighborhood of σ^i, $V_{\epsilon'}$ which evidently it is that there is no σ_n^i belonging in $V_{\epsilon'}$, hence $v \notin V_{\epsilon'}$. For $\epsilon < \epsilon'$ this contradicts that there is a converging subsequence which arises from the closedness of Σ^i. □

The next lemma is necessary to prove the lower hemi-continuity of the money supply, i.e. a small decrease in money supply M will not implode the money budget of individuals. Lower hemi-continuity is equivalent to the openess of the lower inverse image of the mapping.

Lemma 6 *For arbitrary individual i, the lower inverse set $\{M \in \mathcal{R}_+ | \mathcal{M}^i(M) \cap U\}$ is open for every open $U \in S$.*

[1] $V_\epsilon = \{\sigma^i | \inf_{\psi \in \mathcal{M}^i(M)} d(\psi, \sigma^i) < \epsilon\}$, under the standard metric.

Proof. Openess requires that for every open subset U with $\mathcal{M}^i(M) \cap U \neq \emptyset$ if we get an arbitrary element M' in a neighborhood in M it is $\mathcal{M}^i(M') \cap U \neq \emptyset$. Suppose this is not the case. Hence, for open subset U with $\mathcal{M}^i(M) \cap U \neq \emptyset$ and M' in a neighborhood $\mathcal{V}$ of M, it is $\mathcal{M}^i(M') \cap U = \emptyset$.

Therefore, there is an increasing converging sequence $\{M_n\}$ such that $\{M_n\} \to M$ with $\mathcal{M}^i(M_n) \cap U = \emptyset$. Clearly, the budget set under the constraint

$$\sum_{l \in L} b_l^i(0) + \pi\theta^i \leq m^i(0) + \frac{\mu^i}{\int_I \mu^i} M_n$$

will converge to the budget set under the constraint

$$\sum_{l \in L} b_l^i(0) + \pi\theta^i \leq m^i(0) + \frac{\mu^i}{\int_I \mu^i} M.$$

It is also clear that for sufficiently large n there is a M_n very close to M that $M_n \in U$, i.e. $\mathcal{M}(M_n) \cap U \neq \emptyset$. Literally, for some $M_n \leq M$ the budget set under M_n is a proper subset of the budget set under M. Contradiction. Hence all M' are interior and the set is open. □

The next proposition proves continuity.

Proposition 1 *For arbitrary individual i, the money supply correspondence $\mathcal{M}^i$ is continuous.*

Proof. By the Closed Graph Theorem (see Aliprantis & Border (2007) p.561) if the range of $\mathcal{M}^i$ is a Hausdorff topological space and compact then it suffices to prove that its graph is closed to be upper-hemicontinuous. The range S is trivially compact (see lemma 4) and Hausdorff space as a subset of a rectangle of reals. By lemma 5, $\mathcal{M}^i$ is also closed. Hence, $\mathcal{M}^i$ is upper hemicontinuous. In addition, by theorem (see Hildenbrand (1974) p.27 and lemma 6 the $\mathcal{M}^i$ is lower hemicontinuous everywhere. The result follows. □

Proposition 1 verifies that for small adjustments in money supply M, there is not going to have an abrupt deterioration or improvement in the purchasing power of individuals. The next lemma illustrates that small changes in d may cause a significant change in the endogenously determined tender rate.

Lemma 7 *When the securities distribution ρ is unbalanced and $M > F^e$, then the tender rate r^t is not continuous function in d.*

Proof. Suppose that r^t is a continuous function in d. Then for arbitrary d, and $d' \in \mathcal{V}_d$ in an neighborhood of d, it must be the case that for every $\epsilon > 0$ it is $|r^t(d) - r^t(d')| < \epsilon$. That is,

$$\int_I \frac{\mu^i}{\mathbf{1}_{d'} \cdot \rho \cdot F} - \epsilon < \int_I \frac{\mu^i}{\mathbf{1}_d \cdot \rho \cdot F} < \int_I \frac{\mu^i}{\mathbf{1}_{d'} \cdot \rho \cdot F} + \epsilon$$
$$\frac{1}{\mathbf{1}_{d'} \cdot \rho \cdot F} \int_I \mu^i - \epsilon < \frac{1}{\mathbf{1}_d \cdot \rho \cdot F} \int_I \mu^i$$

or

$$\frac{1}{\mathbf{1}_{d'} \cdot \rho} - \frac{F}{\int_I \mu^i} \epsilon < \frac{1}{\mathbf{1}_d \cdot \rho}$$
$$\mathbf{1}_{d'} \cdot \rho - \alpha > \mathbf{1}_d \cdot \rho \tag{10}$$

for $\alpha = \frac{\int_I \mu^i}{F\epsilon} > 0$.

However this is $|\mathbf{1}_d \cdot \rho - \mathbf{1}_{d'} \cdot \rho| > \alpha$, which violates the assumption of unbalancedness, when we set $\epsilon = \alpha$. □

The next proposition is the main result of the paper. It shows that monetary policy mix is not continuous. For, it suffices to show that the *quality of collateral* correspondence is not lower hemi-continuous.

Proposition 2 *For unbalanced securities distribution and arbitrary individual i. the QC^i correspondence is not l.h.c.*

Proof. To prove this result suppose a sequence (d_n) in the domain of QC^i. The correspondence will be not l.h.c. at some d if for some $d' \in \mathcal{V}_d$, with $\mathcal{V}_d$ a neighborhood of d, and open set $\mathcal{O} \subset S$ we have $QC^i(d) \cap \mathcal{O} \neq \emptyset$ but $QC^i(d') \cap \mathcal{O} = \emptyset$.

By lemma 7 we can always find some d such that the tender rate to be discontinuous. This induces that the budget set

$$\sum_{l \in L} b_l^i(0) + \pi\theta^i \leq m^i(0) + \frac{\mu^i}{1 + r^t}$$

varies discontinuously, as well. For a small change in the neighborhood of d, say d', the tender rate will abruptly change imploding (or exploding) the budget set. Hence, we can always find an open set $\mathcal{O}$, defined as an open ball of the suprema of the budget set for sufficiently small radius δ, $\mathcal{O} = \mathcal{B}(\sup \Sigma^i(0), \delta)$, such that $QC^i(d') \cap \mathcal{O} = \emptyset$. $\square$

A small change in the threshold probability of default might discard many securities that share the same probability of default. More technically, the rank of the qualified asset subspace decreases abruptly. In turn, the effective money supply will change drastically.

5 Concluding Remarks

The sovereign debt crisis in eurozone countries that followed the sub-prime crisis of 2008 has been partly caused by the deflation of real assets prices used for secured loans and by a significant downside correction in financial assets. Financial intermediaries were lacking liquidity, interbank lending markets were strained, bond yields were soaring and there was an escalating danger for the liquidity problem to become an insolvency problem. The European central bank as lender of last resort, intervened by a series of unconventional monetary policies to inject liquidity and correct the interbank market failure. Apart from the extensive OMT, the European central bank adopted a longer term refinancing facility, i.e., repos maturing after three to thirty six months and broaden the pool of acceptable collateralized securities.

Our model provides the conceptual framework to address the effect of these different monetary policies. The discontinuous effect of monetary reverse transactions caused by the change of the quality of collateral threshold has important implications. First, an enlargement of the pool of eligible collateral reduces liquidity problems and increase financial stability. Financial market conditions have a decisive role in the effectiveness of monetary policy. When there is an increase in money supply, and it is higher than the market value of eligible collateralized assets, then the monetary policy is ineffective. The central bank can mitigate liquidity risks only if there is a sufficient pool of collateral to make monetary policy effective.

Insofar the analysis has neglected the role collateral haircut, i.e., the need for over-collateralization. This can simply accommodated by introducing an average haircut rate $h \in (0, 1)$. Thus, the tender rate takes the form,

$$r^t\Big((M, d); \rho\Big) = \int_I \frac{\mu^i}{\min\{M, hF^e\}} - 1. \tag{11}$$

Suppose that it is the case that $M > F^e$ and the central bank decides to broaden the accepted collateral by accepting asset backed securities but for a significant haircut. If the accompanied haircut is too high then it might turn out that $M > hF^e$ and, thus, nothing changes. In conclusion, it is not only the quality of collateral that affects the interest rate but also the "velocity of collateral" as attributed by the haircut rate.

The broadening of the central bank's list of accepted collateral, however, involves several risks. One is that the central bank repos absorb securities of poor quality, while all the good quality securities are directed to private repos. Financial intermediaries change their lending behavior while their balance sheets may now include assets of lower quality and thus higher sensitivity to financial distress. The continuing degradation of the quality of collateral and the longer maturities may affect adversely the financial conditions of the economy. To avoid these risks, we suggest that the quality of collateral or the repos maturity should be indexed to the interbank market conditions (e.g., interbank rate spread), accompanied by efficient monitoring mechanisms. Last, the central bank should pursue to smooth out the distribution of securities, eliminating the presence of large shares of low quality items that may cause discontinuous jumps to the tender rate.

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