Leaping and Accelerometry: A Theoretical Approach

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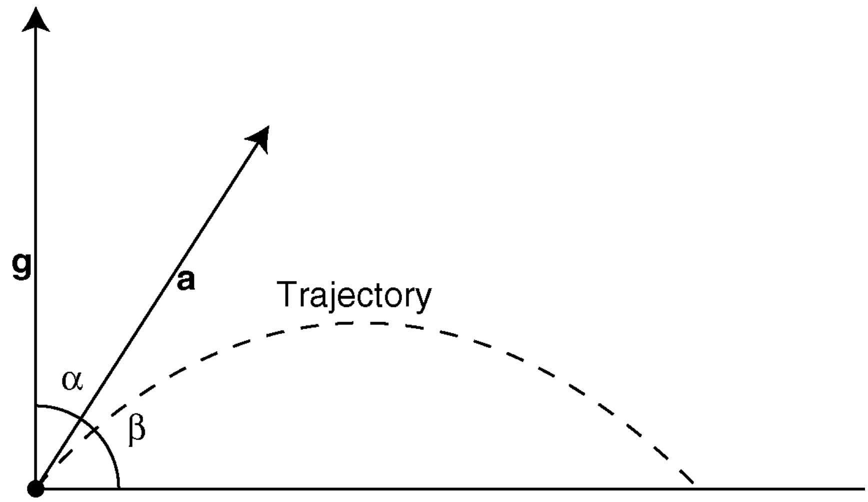
Introduction

Accelerometry has always been a popular method of monitoring locomotor activity but its use is becoming more widespread due to the easy availability of low cost, low power sensors (e.g. 1,2,3). However one of the major problems with interpreting accelerometer data is that rotation of the sensor alters the output and obtaining independent orientation information is currently difficult. For the specific case of leaping locomotion full orientation information is unecessary as long as an independent measure of height change is available such as can be obtained from a sensitive pressure sensor. Therefore a 4 channel logging system recording 3 accelerometry axes combined with a channel measuring pressure could be used to accurately assess leaping locomotion. This approach should also work for any locomotion where acceleration is limited to the vertical plane but in practice it may be restricted by the sensitivity and acquisition characteristics of the pressure measurement system. Ultimately it should be a useful addition to the range of measurements available for remote locomotor monitoring particularly for leaping species such as lemurs and other non-human primates.

Theory

3 axis accelerometry tells us the total acceleration including the 1g vertical acceleration that is always attributable to gravity. Leaping, ignoring any aerodynamic effects, is always a 2D activity and the addition of a pressure sensor to measure actual height should be enough to completely disambiguate the signal even when the orientation of the accelerometer is not known.



From the diagram **g** is the acceleration due to gravity (which is measured as acting upwards by accelerometers); **a** is the acceleration of the animal; α is the angle from the vertical and β is the angle from the horizontal.

The magnitude reading of the accelerometer (r = $\sqrt{x^2 + y^2 + z^2}$) can be found adding the vectors **a** and **g** as vertical and horizontal components:

```
horiz = a Sin[α]

vert = g + a Cos[α]
```

$$\sqrt{\text{horiz}^2 + \text{vert}^2}$$

$$\text{outputOfAccelerometer} = r == \sqrt{(g + a\,\text{Cos}[\alpha])^2 + a^2\,\text{Sin}[\alpha]^2}$$

The reading of the pressure sensor can be differentiated twice to calculate the actual vertical acceleration **b** and this relates to **a**:

```
outputOfPressureSensor = b == a Cos[α]

outputOfPressureSensor1 = Solve[outputOfPressureSensor, α]
```

$$\left\{\left\{\alpha \to -\text{ArcCos}\left[\frac{b}{a}\right]\right\}, \left\{\alpha \to \text{ArcCos}\left[\frac{b}{a}\right]\right\}\right\}$$

```
actualAcceleration1 = outputOfAccelerometer /. outputOfPressureSensor1
```

$$\left\{r == \sqrt{a^2\left(1 - \frac{b^2}{a^2}\right) + (b + g)^2},\ r == \sqrt{a^2\left(1 - \frac{b^2}{a^2}\right) + (b + g)^2}\right\}$$

```
actualAcceleration = Solve[actualAcceleration1, a]
```

$$\left\{\left\{a \to -\sqrt{-2\,b\,g - g^2 + r^2}\right\}, \left\{a \to \sqrt{-2\,b\,g - g^2 + r^2}\right\}\right\}$$

So we have two solutions for the acceleration - only one of them is likely to be at all sensible.

```
actualAngle1 = outputOfPressureSensor /. actualAcceleration[[1]]
```

$$b == -\sqrt{-2\,b\,g - g^2 + r^2}\ \text{Cos}[\alpha]$$

```
actualAngle2 = FullSimplify[Solve[actualAngle1, α]]
```

$$\left\{\left\{\alpha \to -\text{ArcCos}\left[-\frac{b}{\sqrt{-g\,(2\,b + g) + r^2}}\right]\right\}, \left\{\alpha \to \text{ArcCos}\left[-\frac{b}{\sqrt{-g\,(2\,b + g) + r^2}}\right]\right\}\right\}$$

```
actualAngle3 = outputOfPressureSensor /. actualAcceleration[[2]]
```

$b == \sqrt{-2\,b\,g - g^2 + r^2}\ \mathrm{Cos}[\alpha]$

```
actualAngle = Flatten[Append[actualAngle2, actualAngle3]]
```

$$\left\{\alpha \to -\mathrm{ArcCos}\left[-\frac{b}{\sqrt{-g\,(2\,b+g)+r^2}}\right],\ \alpha \to \mathrm{ArcCos}\left[-\frac{b}{\sqrt{-g\,(2\,b+g)+r^2}}\right],\ \alpha \to -\mathrm{ArcCos}\left[\frac{b}{\sqrt{-g\,(2\,b+g)+r^2}}\right],\ \alpha \to \mathrm{ArcCos}\left[\frac{b}{\sqrt{-g\,(2\,b+g)+r^2}}\right]\right\}$$

We can also produce a combined solution which should help us choose which of the 4 solutions is correct

```
angleAccelerationSolution =
 FullSimplify[Solve[{outputOfPressureSensor, outputOfAccelerometer}, {a, α}]]
```

$$\left\{\left\{a \to -\frac{b}{\sqrt{\frac{b^2}{-g\,(2\,b+g)+r^2}}},\ \alpha \to -\mathrm{ArcCos}\left[-\sqrt{\frac{b^2}{-g\,(2\,b+g)+r^2}}\right]\right\},\right.$$

$$\left\{a \to -\frac{b}{\sqrt{\frac{b^2}{-g\,(2\,b+g)+r^2}}},\ \alpha \to \mathrm{ArcCos}\left[-\sqrt{\frac{b^2}{-g\,(2\,b+g)+r^2}}\right]\right\},$$

$$\left\{a \to \frac{b}{\sqrt{\frac{b^2}{-g\,(2\,b+g)+r^2}}},\ \alpha \to -\mathrm{ArcCos}\left[\sqrt{\frac{b^2}{-g\,(2\,b+g)+r^2}}\right]\right\},$$

$$\left.\left\{a \to \frac{b}{\sqrt{\frac{b^2}{-g\,(2\,b+g)+r^2}}},\ \alpha \to \mathrm{ArcCos}\left[\sqrt{\frac{b^2}{-g\,(2\,b+g)+r^2}}\right]\right\}\right\}$$

Obviously we usually describe trajectory angles from the horizontal and that can be achieved using this relationship:

```
angleFromHorizontalDefinition = α -> 90 Degree - β
```

$\alpha \to 90\,° - \beta$

Test with some real values

This allows us to see whether one of the solutions is obviously valid given sensible input criteria.

■ Example 1

Acceleration of 1 g at 45° to horizontal e.g. during landing phase

$45\,\frac{\pi}{180}$ // N

0.785398

```
outputOfAccelerometer /. {g → 9.81, a → 9.81, α → 0.7853981633974483`}
```

$r == 18.1265$

```
outputOfPressureSensor /. {g → 9.81, a → 9.81, α → 0.7853981633974483`}
```

```
b == 6.93672
```

And reversing the calculation

```
actualAcceleration /. {g → 9.81, r -> 18.126516427871447`, b -> 6.936717523440031`}
```

```
{{a → -9.81}, {a → 9.81}}
```

```
actualAngle /. {g → 9.81, r -> 18.126516427871447`, b -> 6.936717523440031`}
```

```
{α → -2.35619, α → 2.35619, α → -0.785398, α → 0.785398}
```

```
angleAccelerationSolution /. {g → 9.81, r -> 18.126516427871447`, b -> 6.936717523440031`}
```

```
{{a → -9.81, α → -2.35619}, {a → -9.81, α → 2.35619},
 {a → 9.81, α → -0.785398}, {a → 9.81, α → 0.785398}}
```

■ Example 2

Acceleration of 2.5 g at 25° to horizontal e.g. during takeoff phase.

$$(90 - 25)\ \frac{\pi}{180}\ //\ N$$

```
1.13446
```

```
outputOfAccelerometer /. {g → 9.81, a → 9.81 × 2.5, α → 1.1344640137963142`}
```

```
r == 30.0178
```

```
outputOfPressureSensor /. {g → 9.81, a → 9.81 × 2.5, α → 1.1344640137963142`}
```

```
b == 10.3647
```

And reversing the calculation

```
actualAcceleration /. {g → 9.81, r -> 30.017784586699946`, b -> 10.364712869190654`}
```

```
{{a → -24.525}, {a → 24.525}}
```

```
actualAngle /. {g → 9.81, r -> 30.017784586699946`, b -> 10.364712869190654`}
```

```
{α → -2.00713, α → 2.00713, α → -1.13446, α → 1.13446}
```

```
angleAccelerationSolution /. {g → 9.81, r -> 30.017784586699946`, b -> 10.364712869190654`}
```

```
{{a → -24.525, α → -2.00713}, {a → -24.525, α → 2.00713},
 {a → 24.525, α → -1.13446}, {a → 24.525, α → 1.13446}}
```

■ Example 3

Acceleration of 2.5 g at -10° to horizontal e.g. when leaping downwards

In[115]:= $$(90 - (-10))\ \frac{\pi}{180}\ //\ N$$

Out[115]= `1.74533`

In[116]:= `outputOfAccelerometer /. {g → 9.81, a → 9.81 × 2.5, α → 1.7453292519943295`}`

Out[116]= `r == 24.7822`

```
In[117]:= outputOfPressureSensor /. {g → 9.81, a → 9.81 × 2.5, α → 1.7453292519943295`}

Out[117]= b == -4.25872
```

And reversing the calculation

```
In[118]:= actualAcceleration /. {g → 9.81, r -> 24.78216310264578`, b -> -4.258721557281466`}

Out[118]= {{a → -24.525}, {a → 24.525}}

In[119]:= actualAngle /. {g → 9.81, r -> 24.78216310264578`, b -> -4.258721557281466`}

Out[119]= {α → -1.39626, α → 1.39626, α → -1.74533, α → 1.74533}

In[120]:= angleAccelerationSolution /. {g → 9.81, r -> 24.78216310264578`, b -> -4.258721557281466`}

Out[120]= {{a → 24.525, α → -1.74533}, {a → 24.525, α → 1.74533},
           {a → -24.525, α → -1.39626}, {a → -24.525, α → 1.39626}}
```

Example 4

Acceleration of 1.5 g at -85° to horizontal e.g. when leaping steeply downwards

$$(90 - (-85)) \frac{\pi}{180} \ // \ N$$

```
3.05433

outputOfAccelerometer /. {g → 9.81, a → 9.81 × 1.5, α → 3.0543261909900767`}

r == 5.01574

outputOfPressureSensor /. {g → 9.81, a → 9.81 × 1.5, α → 3.0543261909900767`}

b == -14.659
```

And reversing the calculation

```
actualAcceleration /. {g → 9.81, r -> 5.015739949889638`, b -> -14.659004982420036`}

{{a → -14.715}, {a → 14.715}}

actualAngle /. {g → 9.81, r -> 5.015739949889638`, b -> -14.659004982420036`}

{α → -0.0872665, α → 0.0872665, α → -3.05433, α → 3.05433}

angleAccelerationSolution /. {g → 9.81, r -> 5.015739949889638`, b -> -14.659004982420036`}

{{a → 14.715, α → -3.05433}, {a → 14.715, α → 3.05433},
 {a → -14.715, α → -0.0872665}, {a → -14.715, α → 0.0872665}}
```

So, it looks like the answer where a and α are positive is the correct one.

References

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2. Sellers, W.I. & Crompton, R.H. 2004 Automatic monitoring of primate locomotor behaviour using accelerometers. Folia Primatologia 75, 279 -293.
3. Byrnes, G., Lim, N. T.-L. & Spence, A. J. 2008 Take-off and landing kinetics of a free-ranging gliding mammal, the Malayan colugo (Galeopterus variegatus). Proceedings of the Royal Society B: Biological Sciences 275, 1007-1013.