Pegs and Ropes: Geometry at Stonehenge

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Abstract

A recent computer-aided-design investigation of the Neolithic 56 Aubrey Hole circuit at Stonehenge has led to the discovery of an astonishingly simple geometrical construction for drawing an approximately regular 56-sided polygon, feasible with a compass and straightedge. In the present work, we prove analytically that the aforementioned construction yields as a byproduct, an extremely accurate method for approximating a regular heptagon, and we quantify the accuracy that prehistoric surveyors may have ideally attained using simple pegs and ropes. We compare this method with previous approximations, and argue that it is likely to be at the same time the simplest and most accurate. Implications of our findings are discussed.

1 Introduction

Our story begins in 1666, when the antiquarian and essayist John Aubrey, walking around the iconic stone structure that gave Stonehenge its name, noticed five small depressions just inside the bank of the encircling earthwork, describing them as ... cavities in the ground from whence one may conjecture stones ... were taken. [1]. These hollows were not mentioned by subsequent writers, and there was no trace of them on the surface when excavations began at Stonehenge in 1919, which is curious for, having seemingly survived for millennia, it is difficult to account for their disappearance in the 250-year period since Aubrey's days. The possibility must therefore remain that small stones had once occupied these positions and that they had been removed not long before the 1600s and that Aubrey's cavities represented relatively recent traces of their removal.

Aubrey's then unpublished manuscript lodged at the Bodleian Library Oxford was to distract Col. William Hawley and his colleague Robert Newall from investigations and restoration work within the interior of the monument, and in 1920 they set about looking for traces of the missing holes. An initial search was done using a steel bar (as a probe). A total of 34 were finally excavated in a continuous arc covering the east and south-east of the circuit and a further 22 located by probing, forming a complete circuit of 56 holes, spaced at quite regular intervals of 16 ft (4.88 m) around the geometrical henge centre. Hawley and Newall called these features 'Aubrey Holes' as a compliment to our pioneer [2]. They are currently interpreted as belonging to the most ancient phase of Stonehenge, to which the bank-and-ditch earthwork, and possibly the unworked Heelstone belong [3]. While it may not be clear that these holes were responsible for the 5 hollows recorded by Aubrey, his sketch and notes certainly led to their discovery.

Averaging just over 1 m in width and 1 m deep the holes were found to have been set on an accurate circle just over 87 m in diameter (fig. 1) running just inside the now much weathered and almost invisible 5,000 year-old chalk bank. There has been considerable debate as to their potential purpose, and not least the significance of the enigmatic number, 56.

Figure 1: Sketch of Stonehenge's plan, the Aubrey Hole circuit evidenced as filled red dots.

A re-examination of all the archaeological evidence in 1995 supported the view that they originally held timber posts [4]. The timbers were removed and over the course of a few hundred years the hollows left by the sockets were variously treated, but eventually most were used for the burial of cremated human remains.

Whatever the number 56 implies it is reasonable to suppose it was not a random occurrence, and that despite the relatively small variations in the spacing that it was the intention of the prehistoric surveyors to create an accurate circuit of 56 holes. This then leaves open the question as to whether the number was desired i.e. that the surveyors were charged with the task of setting out a predetermined array, for whatever purpose, or whether there was something inherent in the method of Neolithic 'peg and rope' survey that might return 56.

Following their excavation, the interest in the Aubrey Holes dwindled, they being no match for the magnificent and puzzling rings of megaliths that occupy the central part of the monument, capturing all of the viewers' attention.

Unexpectedly, focus was suddenly brought back to the Aubrey Holes in 1964 when an English-born American astronomer, Gerald Hawkins, using one of the first available computers investigated the solar and lunar alignments and possible astronomical implications of Stonehenge [5]. Hawkins was intrigued by the number, 56. He noticed that 56/3 is very close

to the 18.61-year period of the rotation of the moon's orbital plane. Since this period is related to eclipse cycles—even though, admittedly, in a rather convoluted way—and since attention to the motions of the sun and moon seemed to be contained in the Stonehenge design, Hawkins devised an ingenious method for predicting eclipses using the Aubrey Holes. He surmised that Stonehenge, at least in its most ancient parts, was a "Neolithic computer", devoted to astronomical use.

The astronomer Fred Hoyle [6] quickly joined Hawkins in supporting this Neolithic computer idea, although the proposal was not widely accepted by archaeologists, with Richard Atkinson, who had recently undertaken several seasons excavation and restoration work at Stonehenge [7], being the first to refute the astronomers position.

Stonehenge is built within latitudes where astronomical alignments may be matched, to a varying degree of approximation, with specific features of a number of geometrical figures. As a consequence, one may never know whether an intended geometrical construction casually leads to a coincidence with an astronomically relevant direction, or if, on the contrary, geometry is servicing astronomy. Of course, even if the latter is true, it does not imply that Stonehenge was ever used as an observatory, not to speak of a computer.

As a matter of fact, archaeologists are ill at ease with the idea of assigning to ancient structures a "function", at least before a full assessment of the design of the structures themselves has been reached. They are also acutely aware of the difficulties in collating records from the disparate Stonehenge archive, and re-creating from it a truly reliable groundplan of such a complex monument (remembering that much of the information comes from buried remains which were recorded long before the advent of sophisticated modern excavation, survey control and scientific dating methods). It is therefore understandable that they frown at the uncritical use of this early data for proving specific geometrical, or astronomical, relationships.

Recently, the whole problem of data collecting has been revisited by one of us [3]. When reviewing and critically analyzing all available surveys, one realizes

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how little we know of the design of the most basic features of Stonehenge. In particular, the most difficult information to recover remains the precise measurements of linear dimensions. However, careful gathering of all available survey data has permitted the investigation of geometric relationships to a reasonable accuracy [3]. Amongst the most important of all the Stonehenge datasets is the carefully measured and annotated survey made by John Wood, the architect of Georgian Bath, in the year 1740. It may seem somewhat paradoxical that a survey made over 350 years ago is so valuable in an age in which electronic data measurement and laser survey is commonplace. This is because Wood's plan was made before the collapse of the southwest trilithon in 1797 and the extensive remedial engineering work of the 20th century. Wood's records have been computed to recreate the earliest accurate record of the stones. Using this data we may appreciate more precisely how the original ground plans may have been laid out by the prehistoric surveyors using pegs and ropes. We are then entitled to ask and answer questions about the methods and techniques that would have allowed them to organize space geometrically.

In this paper, we will focus on the Aubrey Hole circuit. We will argue that it was laid down with the specific purpose of drawing a 56-sided polygon, and that a geometrical construction based on the circle and square, readily do-able with pegs and ropes, allows one to trace the polygon to an extremely high accuracy. As a matter of fact, we will show that the method discussed here provides the best known approximation to such a polygon, as well as an exceedingly accurate regular heptagon.

2   The Aubrey Hole circuit

Although not unique, for over 100 timber circles are known from Britain and Ireland [8], few are known to be as old or anywhere near so large or as accurately surveyed as the Aubrey Holes, although a number are clearly complex concentric structures which appear also to employ geometric constructions in both the number of posts and spacing of concentric elements. The number of posts utilised is also variable, none of

the others having 56 [1].

Most of the questioning about the 56 holes that go round close to the inner edge of the boundary bank, concerns their possible use. Ranging from Hawkins' and Hoyle's Neolithic computer, in which the number 56 plays a very important role, through offering pits for communicating with the dead [10], to a more prosaic posthole ring, many hypotheses have been formulated. Rather than speculating upon why there were 56 pits at Stonehenge, a more productive avenue might be to ask how?. Is there a way to arrange 56 holes regularly spaced in a ring using only ropes and pegs? The question deserves attention, because 56 divides by seven, so that drawing a 56 sided polygon—to which the ring of Aubrey Holes of course corresponds—implies drawing (or at least by default including) a heptagon. Since Gauss' proof [11] of the impossibility of constructing the latter using only a straightedge and a compass, we know that tracing a *regular* heptagon—and *a fortiori* a 56 sided polygon—is not possible with pegs and ropes, unless one makes use of a graduated, or marked, ruler, realizing what is called in the jargon a *neusis* construction[12].

How was the 56-sided polygon at Stonehenge worked out, then? In the absence of written evidences, we cannot answer this question, either. We are in fact left with the following choice: either we make a number of assumptions about what we believe the Stonehenge surveyors did, or we simply lay down a set of rules, and try to discover whether a construction can be made according to those rules.

3   Compass and straightedge geometry

These are our requirements.

First, we want a method which is readily reproducible at all scales, based only on geometric proportions, not on measurements.

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[1] The numbers are 8, 10, 13, 14 and 44 [10]. Note that Neolithic surveyors seemed to have a preference for non-constructible polygons, since polygons with 13, 14 or 44 sides cannot be drawn using just straightedge and compass.

Note that this first requirement is just a rephrasing of the basic rule of the Euclidean construction, which only permitted the use of compass and straightedge. In fact, it is easy to conjecture that the classical rules of Greek geometry may well have been a direct consequence of the peg-and-rope surveying techniques. We may also assume that small-scale experiments and 'scale drawings' were made. In the Stonehenge region chalk plaques have been found that display geometric, and in one case mirrored geometric designs [3]. We surmise that strict geometric design was also behind the Aubrey Holes circuit [2].

Second, we want a method which is easily remembered and executed. Here, we run into problems, because we have to define what *easy* may mean.

The best definition we may think of is in terms of *operational complexity*. One construction may be deemed easier than another one, if the former needs the execution of a smaller number of different actions than the latter.

Third, we require that the construction be sufficiently accurate.

The Aubrey Hole circuit has a diameter very close to 87 m. The 56 holes are thus 4.88 m apart, on average, and the perimeter of the polygon is approximately 273.18 m. In general, such geometrical constructions start from a circle, and produce a chord which is the length of the side of the sought-after polygon [13]. The chord must then be transferred an appropriate number of times around the circle to generate the whole polygon. Assume that the construction allows for the first side with a 1 cm accuracy—i.e. a 0.2 % accuracy in our case. After transferring it 55 more times around the circuit, one has spanned a length of 272.72 m, 46 cm less than the "ideal" perimeter. Given the unavoidable errors introduced by the peg-and-rope technique—although the latter are random, non-systematic errors, with a tendency to averaging out—about 50 cm is likely to be an ac-

[2]As a matter of fact, one may draw a 56 sided polygon by first drawing an octagon, and then subdividing each side into seven parts by stretching a piece of rope. If the rope turns out to be too long, one just has to alter it until the correct one-seventh length is found by trial and error. Even though one would indeed be able to trace a 56 sided polygon this way, the method is clearly cumbersome.

ceptable inaccuracy. However, 10 times more, a mere 10 cm over a 5 m length, would give rise to an inaccuracy of about 5 m—a whole side!—at the end of the process, which would definitely render the construction useless. Note, by the way, that if one wants to reach an accuracy of the order of 0.1 % for a 56 sided polygon, one has consistently to reach a (relative) accuracy about ten times larger for the heptagon.

In the Appendix, we will provide two examples: a construction which is simple—according to our definition—but not accurate enough, and another one which is very accurate, but not simple.

It may seem very unlikely that a geometrical construction yielding both a simple and very accurate 56-sided polygon capable of being laid out on the ground using pegs and ropes, exists. Astonishingly, it does.
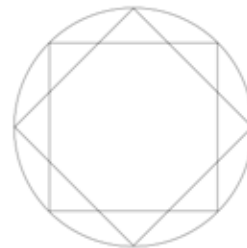

Figure 2: Step 1: Inscribe in a circle a square, then a second one rotated by 45° and standing on a corner.

4 A story of pegs and ropes

The construction in question has been discovered by one of us empirically, with the help of a CAD programme, and published in his very recent book on Stonehenge [3]. Here, we will recall how the construction works, and give an explicit, analytical evaluation of the accuracy of the approximation it provides to the heptagon and the 56-sided polygon. To begin with, we draw a circle, and inscribe two squares rotated by 45° one with the respect to the other (Fig. 2). We might note that a line between any two opposing corners of a square may be considered a potential
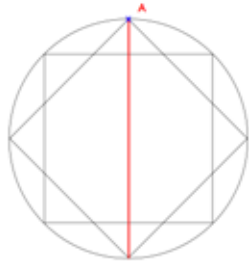
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Figure 3: Step 2: Drop a diameter from the topmost vertex A.

surveyors base-line, i.e. a practical origin for the survey passing through the exact centre of our intended array. It is not difficult find several groups of opposing Aubrey Holes within the known (excavated) series that mark corners of quite accurate squares, however it is also apparent that some within the array deviate from ideal positions [3].

The eight points where the squares and the circle intersect define an octagon, as well as eight vertices of the 56-sided polygon. Draw a diameter from point A (Fig. 3) then set a peg in point B, as in figure 4, and pull a rope between B and C, the latter being the point where the diameter through A cuts the side of the square opposite to A (Fig. 4).
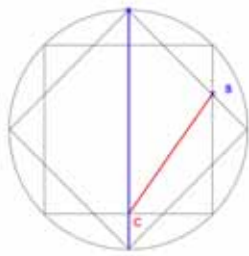

Figure 4: Step 3: Placing the pin of the compass at point B, open it to join point C, where the diameter through A cuts the side of the first square.

Using BC as the radius, draw a circle centred on B. Call D_1 and D_2 the points where this circle cuts the original circle (Fig. 5.) Perform the same action,

using the point symmetrical to B with respect to the axis AC as pivot (Fig. 6.) The heptagon's side is the segment D_3D_1. But we are not yet done. Repeat the same construction from the other three points equivalent to B. We obtain figure 7, in which 24 vertices of the 56-sided polygon are found. In fact, each pair of points between two successive vertices of the squares delimits one side of the 56-sided polygon.
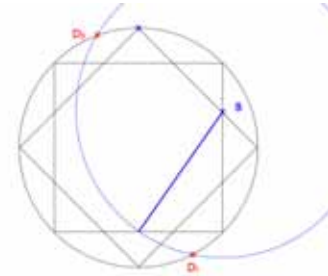

Figure 5: Step 4: Using BC as the radius, draw an arc and mark the points D_1 and D_2 where the arc cuts the initial circle.

5 An analytical proof

In this section, we will prove that the preceding geometrical construction yields an approximation for the heptagon as well as for 56-sided polygon we are after, and we will explicitly compute both polygons' sides.

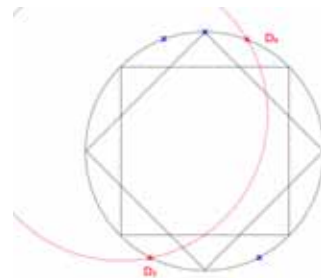We will do it using an analytical geometry approach.

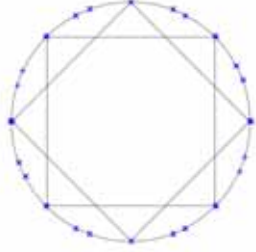Figure 6: Step 5: the same as in step 4, from the point mirror symmetric of B.

Figure 7: Step 6: Same as steps 4 and 5 from the remaining points equivalent to B. The 24 marked points are vertices of the 56-sided polygon.

Let us choose the Cartesian axis in such a way that the circle's centre is at the origin, and its radius is unity. Then, points A and B have coordinates $(0,1)$ and $(\sqrt{2}/2, 1 - \sqrt{2}/2)$, respectively. The side of the hexagon is then given by twice the abscissa of point D_1, which is the intersection point of the unit circle at the origin, and of the circle centred in B and having radius $R = BC = \sqrt{3/2}$. Lengthy but straightforward algebra yields

$$x_{D_1} = \frac{x_B}{2} + x_B\frac{1 - R^2}{2d^2} \pm \frac{y_B}{2d^2}\sqrt{[(R + 1)^2 - d^2][d^2 - (1 - R)^2]} \tag{1}$$

where $d^2 = x_B^2 + y_B^2$, and (x_B, y_B) are the coordinates of point B.

Using the values above, and choosing the positive root, one finds the heptagon side S_h

$$S_h = 2x_{D_1} = \frac{1}{4}\left(\sqrt{2} - 1 + \sqrt{15 - 4\sqrt{2}}\right)$$

$$\approx 0.8677173844, \tag{2}$$

whence the heptagon angle α_h,

$$\alpha_h = 2\arcsin(S_h/2) \approx 51.4253858° \tag{3}$$

to be compared with the exact values

$$\alpha_{h-exact} = 360°/7 \approx 51.4285714°, \tag{4}$$

$$S_{h-exact} = 2\sin(360°/14) \approx 0.867767478, \tag{5}$$

which means a relative angular error of 6.2×10^{-5}, or 0.0062 %.

Since $1/56 = 1/7 - 1/8$, once one has the angle of the heptagon, subtracting the angle of the octagon yields the angle of the 56 sided polygon. Indeed, each arc is drawn starting from peg positions at 45° from each other. Thus,

$$\alpha_{56} = \alpha_h - 45° \tag{6}$$

a relation that we will explicitly check in the Appendix. Then, one finds

$$\alpha_{56} \approx 6.425385806° \tag{7}$$

so that the side of the 56-sided polygon is

$$S_{56} = 2\sin\alpha_{56} \approx 0.11208538, \tag{8}$$

the exact result being

$$S_{56-exact} = 2\sin(360°/112) \approx 0.112140894, \tag{9}$$

a 0.05 % relative difference.

An alternative calculation of α_{56} will be presented in the Appendix.

As we recalled above, the 56 Aubrey Holes span a polygon inscribed in a circle of diameter 87 m. Thus, the side of the polygon measures 4.876 m. This must be compared with the exact average value 4.878 m. The difference, as we said, is about 0.05 %, or 2.4 mm. Over the whole circuit, the difference between the "ideal" and approximate perimeter would be 13.5 cm within the present approximation, obviously immaterial for practical purposes.

6 Discussion and Conclusions

A shrine erected:
a holy shrine it is, its interior is like a maze;
a shrine whose interior is a twisted thread,
a thing unknown to man,
a shrine whose lower station is the roving iku-constellation,
a holy shrine whose upper station moves toward the chariot-constellation,
a turbulent flood-wave...
its melam is awesome. [16]

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Let ... the child of the sun-god Utu,
light up for him the netherworld, the place of dark-
ness!
Let him set up the threshold there (as bright) as the
moon
(for) all mankind, whatever their names be,
(for) those whose statues were fashioned in days of
yore,
(for) the heroes, the young men and the ... !
From there the strong and the mighty will march out.
Without him no light would be there during the
month ne-IZI-gar, during the festi[val of the gh]osts.
[17]

———

[These are they who] hold the measuring cord in
Ament, and they go over therewith the fields of the
KHU (i.e., the beatified spirits). [Ra saith to them]:–
'Take ye the cord, draw it tight, and mark out the
limit (or, passage) of the fields of Amentet, the KHU
whereof are in your abodes, and the gods whereof
are on your thrones.' The KHU of NETERTI are
in the Field of Peace, [and] each KHU hath been
judged by him that is in the cord. Righteousness is to
those who are (i.e., who exist), and unrighteousness
to those who are not. Ra saith unto them:–'What is
right is the cord in Ament, and Ra is content with
the stretching (or, drawing) of the same. Your pos-
sessions are yours, O ye gods, your homesteads are
yours, O ye KHU. Behold ye, Ra maketh (or, wor-
keth) your fields, and he commandeth on your behalf
that there may be sand (?) with you.' [18]

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the fourfold siding, fourfold cornering,
measuring, fourfold stacking,
halving the cord, stretching the cord
in the sky, on the earth,
the four sides, the four corners, as it is said. [19]

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Atkinson wrote in 1956 that 'one thing upon which it is agreed is that it [Stonehenge] is primarily a 'temple', a structure in which it was possible for man to establish contact and communication with extramundane forces or beings. [9]' The preceding excerpts, written down by different cultures in different epochs all show that ancient societies considered the foundation of a 'temple' as a sacred action, inasmuch as it was a repetition of the divine creation of the Cosmos. As such, the laying down of the foundations was a ritualized act, to be performed according to the primeval gestures of the gods.

Creation is the act of turning Chaos into Order. The sacred space is the earthly image of the Cosmos, of the orderly Universe moulded from the disordered matter. But above all, a sacred enclosure is an opening by which communication between the world of the gods and the world of humans is made possible. Historian of religions Mircea Eliade wrote: "Every sacred space implies a hierophany, an irruption of the sacred that results in detaching a territory from the surrounding milieu and making it qualitatively different." [20]

Geometry is what ensures that the sacred space is detached from the surrounding. The first act, drawing a circle, makes the detachment a reality. Inscribing the square fixes the four cardinal directions– all sacred space must be *oriented* with respect to heaven, because all the architectonic models come from heaven.

Known examples from palaeolithic and megalithic engravings show that symmetry was the main feature associated with geometry. The spirals and lozenges, clearly exhibit a deep interest for figures produced with geometric constructions in which the same shape could be reproduced at will at different scales, by mean of repeated expansions or contractions[3]. As Paolo Zellini noticed [21], one of the strongest drives for scientific research, the search for unity in multiplicity, found a powerful tool in the geometrical and mathematical procedures of similarity transformations. For instance, a square constructed on the diagonal of a smaller square will have double area, and the construction can be iterated *ad infinitum*, producing a spiraling structure made of area-doubling squares (Fig. 8).

The power of geometry reveals itself in its ability to reproduce ever changing objects of identical shape through the repetition of simple operations. That is also the main characteristic of the construction of an approximate regular 56-sided polygon, and of an

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[3]Cf. Ref. [3] for a geometrical construction of the Bush Barrow lozenge
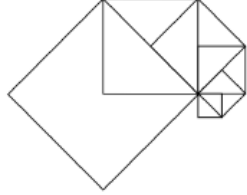
Figure 8: Infinite spiral generated by area-doubling squares.

approximate heptagon as a byproduct, discussed in this work.

We will never be able to prove that the construction proposed in [3], and discussed here, was actually used for crafting the template of the Aubrey Hole circuit. We will never know whether, as Hawkins suggested, the 56-sided polygon inside the bank of Stonehenge has anything to do with the polygon that, according to Plutarch, the Pythagoreans said belonged to Typhon [22], although the simple fact that this complex polygon was mentioned in Classical antiquity suggests that the knowledge of its construction belongs to even earlier times which may even hark back to the dawn of the pantheon of Indo-European tradition.

The scenario discussed here, with its deep geometric elegance and fascinating mixture of extreme simplicity and accuracy, is in full agreement with all we know of the ways our ancestors felt about building a temple, as well as of their attitude towards using geometry for rationalizing reality. Of course it may not be true but, given the accepted uncertainties of our knowledge of prehistoric cosmology, we do have here a tangible and credible possibility; we should certainly like to imagine the foundation of Stonehenge in this way.

Appendix

Let us first discuss an alternative computation of the angle α_{56} subtending the side of the 56-sided polygon.

The angle α_{56} is half the difference between the angles of the larger and smaller triangles in fig. 9.
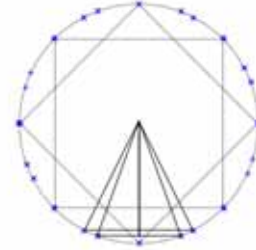

Figure 9: Computation of the angle α_{56} subtending the side of the 56-sided polygon.

These angles are respectively equal to the approximate heptagonal angle $\alpha_h = 2\arcsin(S_h/2)$, and to $\alpha_s = 2\arcsin(\delta/2)$, where $\delta = 2x_{D_4} = \frac{1}{4}\left(1 - \sqrt{2} + \sqrt{15 - 4\sqrt{2}}\right)$, D_4 being as in fig. 6 above. Then

$$\alpha_{56} = \arcsin\left[\frac{1}{8}\left(\sqrt{2} - 1 + \sqrt{15 - 4\sqrt{2}}\right)\right]$$

$$- \arcsin\left[\frac{1}{8}\left(1 - \sqrt{2} + \sqrt{15 - 4\sqrt{2}}\right)\right]$$

or

$$\alpha_{56} \approx 25.712692903° - 19.287307097° \approx 6.425385806°,$$

as found previously. The agreement is due to the relation $\alpha_s = 90° - \alpha_h$, which is easily verified.

As we mentioned in the text, drawing a perfect regular heptagon is impossible with pegs and cords. However, various methods can be devised for drawing an *approximate* heptagon, at varying complexity and approximation levels.

We will quickly describe two methods for approximating a heptagon. The first one is rather simple, in the sense that few actions have to be performed to obtain the result. However, the approximation obtained would, in our opinion, make it useless for Stonehenge. The second one, on the contrary, attains a very good approximation, but requires a rather involved geometric construction, with a large number of different actions.
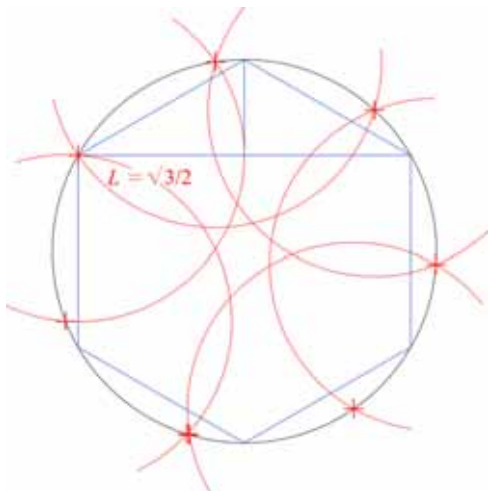
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Figure 10: Sketch of the geometrical construction yielding a regular heptagon to a low approximation.

Figure 11: Sketch of the geometrical construction yielding a regular heptagon to a high approximation.

The first technique requires that a hexagon be inscribed in the unit circle.

Then (Fig. 10) a chord of length $2L$ is traced between any pair of alternate vertices of the hexagon. The length $L = \sqrt{3}/2$, and transferring it along the circle as shown in fig. 10, one can construct an approximate heptagon, since $\sqrt{3}/2 \approx 0.866$, and the side of the regular heptagon is 0.8678, the difference being about 0.2 %. Hence this construction, extended to yield a 56 sided polygon, would imply an error of about 1.6 % for the side of the latter, or 8 cm, which, after reporting this length 56 times, would be shorter than the "ideal" perimeter by 4.6 m, a whole inter-hole distance! The second geometrical construction yields a much better accuracy, at the price of a considerable complexity.

The construction requires drawing a pentagon, as well as its incircle and circumcircle. Let us call R_1 and R_2, respectively, their radii. Then, the ratio $R_2/R_1 = 2/\phi \approx 1.23607$, where $\phi = (\sqrt{5}+1)/2$ is the Golden Mean. Drawing a pentagon is readily feasible with pegs and cords.

Next, selecting the point A (Fig. 11) as the centre, draw a circle of radius $AB = a = R_2 - R_1$. From the centre O of the structure, a third circle is drawn, of radius $R_3 = R_1 - a$. An equilateral triangle is then inscribed into this circle.
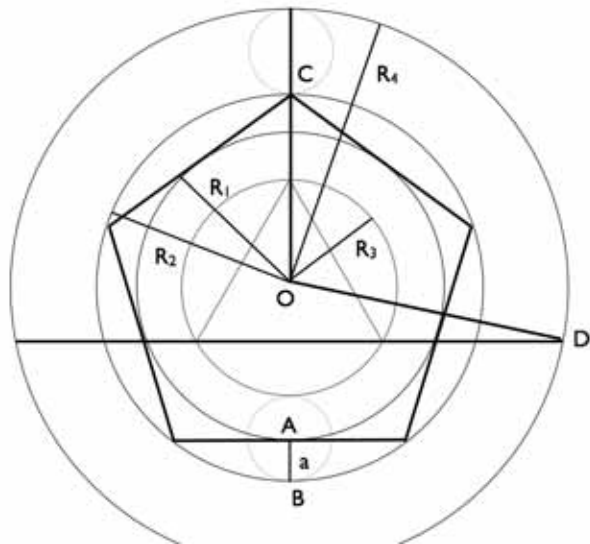
A fourth circle, of radius $R_4 = R_2 + 2a$ has to be drawn, and the base of the equilateral triangle is extended until it cuts the latter circle. The intersection of the base with the circles yields two vertices of the heptagon.

Finally, the angle α_h of the approximate heptagon can be computed from $2\alpha_h = 90° + \arcsin[1/(\phi r_4)]$, where $r_4 = 2R_4/R_2$. Letting $\phi = 2R_1/R_2$, yields $\alpha_h = 51.46048°$, to be compared with the exact value $\alpha_{h-exact} = 360°/7 \approx 51.42857°$. The difference is about 0.062%, a very good approximation (even though nowhere as good as the one discussed in this work), yielding a heptagon side equal to $S_h \approx 0.86827$, as well as a 56 sided polygon of side approximately 0.11270. At Stonehenge, this means that the Aubrey holes spacing would measure 4.902 m, and the total perimeter 274.54 m, about 1.5 m longer than the "ideal" value. Of course, the larger this discrepancy, the more numerous the corrections needed, and the less effective the tracing method.
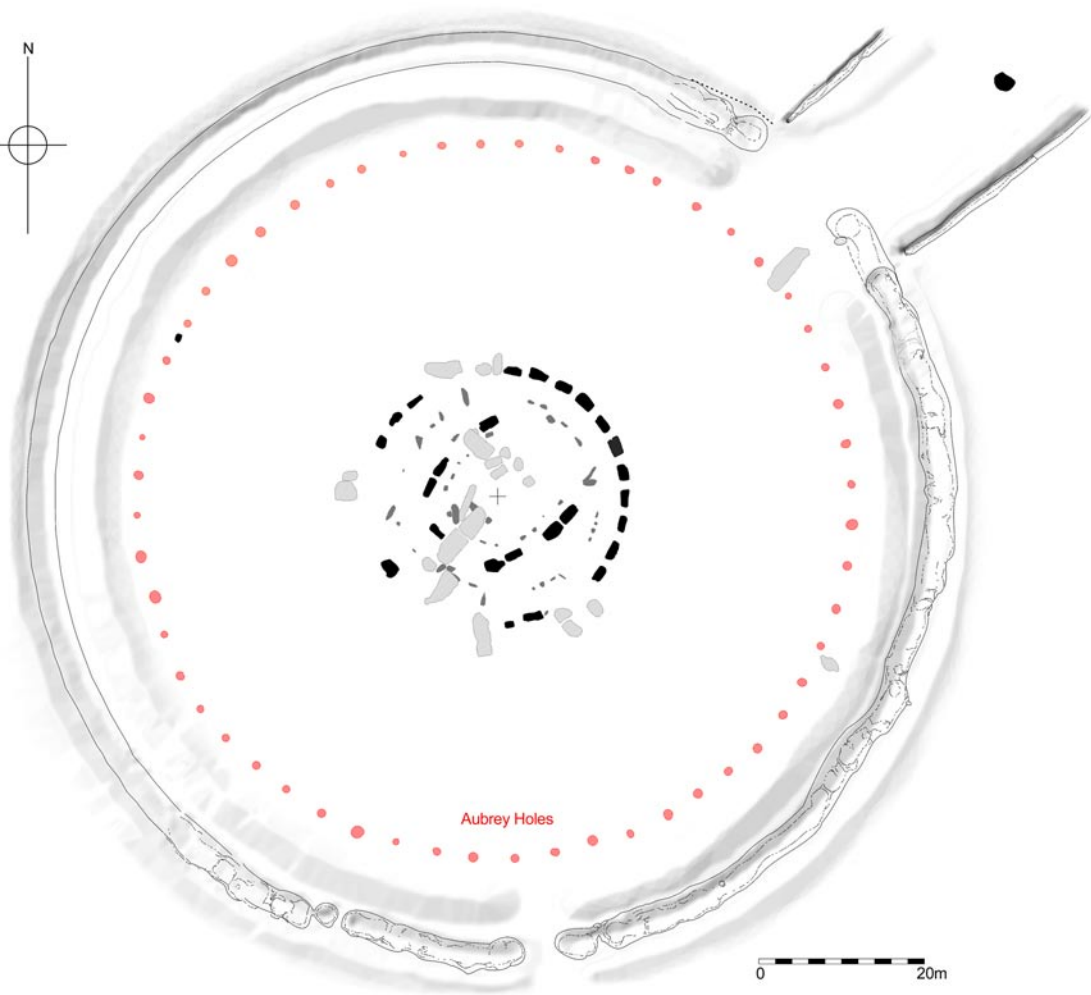
The approximate construction of the heptagon just described allows for a high enough accuracy, though

being less accurate than the one discussed in the body of the text, and clearly operationally more complex.

The most accurate construction of the regular heptagon's side is due to Röber, and has been reported by Hamilton [14]. Hamilton states that Röber's diagram is not very complex, but since Hamilton's paper does not contain any diagram, understanding the construction is essentially impossible. In note 1082 of the Mathematical Gazette, Youngman [15] provides another accurate approximation. He compares his with Röber's, claiming that the latter's approximation "needs rather elaborate drawing".

References

[1] Monumenta Britannica. The manuscript is housed in the Bodleian Library, Oxford. Although the manuscript was widely circulated amongst antiquarians and scholars, it was not published in full until 1981: J. Fowles (ed.) *Monumenta Britannica or A Miscellany of British Antiquities by John Aubrey* (Little, Brown and Company, 1981). See p.80.

[2] W. Hawley, The Antiquaries Journal **1**, 19-41 (1921)

[3] A. Johnson, *Solving Stonehenge* (Thames & Hudson, 2008)

[4] R.M.J. Cleal, K.E. Wlaker and R. Montague, *Stonehenge in its landscape. Twentieth-century excavations*, English Heritage Archaeological Report **10** (English Heritage, 1995)

[5] G.S. Hawkins, *Stonehenge Decoded* (Doubleday, 1965)

[6] F. Hoyle, *On Stonehenge* (W.H Freeman & Company, 1977)

[7] R.J.C Atkinson, Antiquity **40**, 212-216 (1966)

[8] A. Gibson, *Stonehenge & Timber Circles* (Tempus, 1998).

[9] R.J.C. Atkinson, *Stonehenge*, Revised Edition (Penguin Books, 1979).

[10] B.C. Ray, History of Religions **26**, 225-278 (1987)

[11] C.F. Gauss, § 365 and 366 in *Disquisitiones Arithmeticae* (Leipzig, Germany, 1801. Reprinted Yale University Press, 1965)

[12] C. Johnson, The Mathematical Gazette **59**, 17-21 (1975)

[13] L.E. Dickson, Ch. 8 in *Monographs on Topics of Modern Mathematics Relevant to the Elementary Field* (Ed. J. W. A. Young) pp. 352-386 (Dover, 1955)

[14] W.R. Hamilton, Philosophical Magazine **27**, 124-132 (1864)

[15] C.E. Youngman, The Mathematical Gazette **17**, 266-267 (1933)

[16] *Myths of Enki, The Crafty God*, S.N. Kramer and J. Meier trans. (Oxford University Press, 1989)

[17] M.E. Cohen, *The Cultic Calendars of the Ancient Near East*, p. 103 (CDL Press, 1993)

[18] E.A. Wallis Budge *The Egyptian Heaven and Hell*, Parts One and Two, p. 149 (Kessinger Publishing, 2004)

[19] *Popol Vuh*, p.63, D. Tedlock trans. (Touchstone, 1996)

[20] M. Eliade, *The Sacred and the Profane* (Harcourt Brace Jovanovich, 1987)

[21] P. Zellini, *Gnomon. Un indagine sul numero* (Adelphi, 1999)

[22] Plutarch, *Moralia – Isis and Osiris*: see http://penelope.uchicago.edu/Thayer/E/Roman/ Texts/Plutarch/Moralia/Isis_and_Osiris*/B.html

N

Aubrey Holes

0   20m

D_2

B

D_1

D_4

D_3

$L = \sqrt{3}/2$

R_4

R_1

R_2

R_3

C

O

D

A

a

B