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Comparison of Fission Electric Cell Geometries

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ABSTRACT

Comparison has been made of the relative power-to-weight ratios calculated for fission-electric cell reactor power systems based upon cells of plane, cylindrical, and spherical geometry. It is demonstrated that for systems of equal power output, the choice of cell geometry does not greatly affect the total weight of the system.

I. INTRODUCTION

The electrical efficiencies of fission cells of plane, spherical, and cylindrical geometries have been calculated and summarized previously.[1] If electrical efficiency were the only criterion for choosing among different cell geometries, it is clear from those results that the most desirable fission cell reactor would consist of spherical cells with practically zero fuel layer thickness and large R_2/R_1 ratio. However, for a reactor where weight, and therefore size, must be taken into account, this is not a practical basis for selection. The necessity for limiting the weight of a reactor system for spacecraft use introduces other factors besides efficiency which influence the choice of cell geometry.

[1] Heindl, C. J., *Efficiency of Fission Electric Cells,* Technical Report No. 32-105, Jet Propulsion Laboratory, Pasadena, Calif., May 25, 1961.

The best overall criterion for usefulness of a fission cell reactor in space applications is probably the ratio of electrical power output to weight of the entire system, including reactor, radiator, shielding, coolant, piping and pump, etc. Unfortunately, short of a detailed system design, it is not possible to accurately determine and maximize this ratio with respect to cell geometry. However, if certain assumptions are made about the nature of the reactor and system, a reasonable comparison of power-to-weight ratio for different cell geometries can be achieved; it should be valid for all reactors of the type now being considered for fission electric cell spacecraft power units. This has been done in this Report, for systems exclusive of radiation shielding.

II. ASSUMPTIONS

The weights of the total systems being compared are primarily due to the reactor and radiator, the other components being much lighter. Therefore, if the two major pieces can be properly characterized, then crude approximations are adequate for estimating the weights of the remaining components. For convenience in making the calculations which follow, the reactor weight has been held constant through changes in cell geometry, and the power and radiator weight permitted to change. The rate of waste heat radiation by the radiator is proportional to its area and emissivity, and the fourth power of its temperature. Assuming that in all cases the same radiator construction would be used at the same operating temperatures, the radiator area, and therefore weight, can be taken as directly proportional to the waste heat disposal rate.

The assumptions used can be summarized:

1. The reactors being compared are identical in weight, size, void volume, moderator volume, and fuel loading.

2. Changes in neutron leakage and velocity spectrum, and therefore criticality, due to different void configurations are ignored.

3. The weight of components other than reactor and radiator is almost all due to coolant, piping, and pump. Since these will be much smaller in weight than the radiator, and vary with power in a somewhat similar way, they are included in the $(Wt)_{rad}$ below.

4. Radiator weight is directly proportional to the power being radiated away, which is practically the total reactor power for these low efficiency systems: $(Wt)_{rad} = \alpha P$.

5. Reactor weight is directly proportional to moderator volume, the weight of structure and coolant in the core being taken as some constant fraction of the moderator weight, and their volume fractions ignored:

$$(Wt)_{reac} = \beta \times \text{moderator volume}$$

III. CRITERIA FOR COMPARING POWER-TO-WEIGHT RATIOS

The cell geometries to be compared are parallel-plane, concentric cylinder, and concentric sphere. The calculations below will be limited to cells consisting of electrode pairs, with fissionable material distributed on the outer surface of the inner electrode in the cylindrical and spherical cases. It would of course be possible to construct cells consisting of a series of concentric shells of any number up to a configuration embracing the entire core. However, it is clear that a series of concentric cylindrical or spherical cells of increasing radii simply provide a spectrum of performance ranging from that of the innermost cell to that of parallel plates, as the radii increase towards infinity. The overall result is to achieve power-to-weight ratios somewhere between that of small two-electrode cells and those of parallel-plate arrays; therefore, only two-electrode cells need be examined here.

Under the simplifying assumptions listed above, reactor weights have been forced to equality, but not the electric power output from the reactors. This output is simply (reactor power) × (efficiency), with the efficiency a function of voltage, fuel-layer thickness, and geometry. The reactor power, however, will depend upon conditions of operation. If temperatures and temperature gradients in the moderator are the limiting factor, then it seems reasonable that constant power density,

$$\left(\frac{\text{reactor power}}{\text{moderator volume}}\right)$$

and therefore constant total power, be assumed for all geometries. On the other hand, if power is limited by fuel-layer temperatures, then total power may well be proportional to fuel area, which is a function of geometry. Both of these possibilities are considered below.

The comparisons to be carried out apply specifically to one-region cores consisting of cells upon whose surfaces is uniformly deposited all of the fuel material. It is clear, however, that the results are equally applicable in comparing systems which have the same fraction of their fuel distributed uniformly over the cathode surfaces and the remainder distributed elsewhere. The results are similarly applicable to systems which have identical non-uniformities in fuel-layer thickness over corresponding regions of their cores. Thus, one may draw conclusions about multi-region as well as single-region reactors.

IV. POWER-TO-WEIGHT RATIO COMPARISON OF CELL GEOMETRIES

A. Constant Power Density

In accord with the assumptions listed in Sec. II, two systems of different cell geometry but identical power density can be compared:

$$\frac{\left[\dfrac{\text{electric power}}{\text{system weight}}\right]_1}{\left[\dfrac{\text{electric power}}{\text{system weight}}\right]_2} = \frac{\left[\dfrac{P \times \text{efficiency}}{\text{system weight}}\right]_1}{\left[\dfrac{P \times \text{efficiency}}{\text{system weight}}\right]_2} = \frac{[\text{efficiency}]_1}{[\text{efficiency}]_2}$$

where the assumptions of equal moderator volume and equal power density result in equal reactor power for the two systems. This in turn implies equal radiator weights and, coupled with assumption 3, equal system weights. The efficiencies, of course, depend upon fuel-layer thickness, voltage, and cell geometry.

Equal void volumes have been assumed for all reactors being compared, the void volume being just the vacuum region between electrodes in the fission cells. For different cell geometries, this constant void volume leads to different fuel surface areas; and it follows that the fuel-layer thicknesses must also vary with the geometry in order to achieve equal fuel loadings on these different surface areas. The amount of surface area is obtained from the relation:

$$A \equiv \text{surface area} \equiv (\text{void volume}) \times \left[\frac{\text{surface area}}{\text{void volume}}\right]_{\text{cell}}$$

since the void volume of the core is entirely contained in the fission cells. The second factor above is, for the three basic geometries:

$$\left[\frac{\text{surface area}}{\text{void volume}}\right]_{\text{plates}} = \frac{1}{d}$$

$$\left[\frac{\text{surface area}}{\text{void volume}}\right]_{\text{cylinders}} = \frac{2R_1}{R_2^2 - R_1^2}$$

$$\left[\frac{\text{surface area}}{\text{void volume}}\right]_{\text{spheres}} = \frac{3R_1^2}{R_2^3 - R_1^3}$$

Where d is the separation distance between parallel plane electrodes; R_1 and R_2 are the inner and outer electrode radii, respectively, for concentric cylinders and spheres. The fuel layer has been assumed to be on the outer surface of the inner electrode in the two latter cases. To provide equal fuel loadings in the different reactors, the fuel layer thicknesses, τ, must be related by:

$$\tau_{pl}\, A_{pl} = \tau_{cyl}\, A_{cyl} = \tau_{sph}\, A_{sph}$$

or:

$$\tau_{pl} = \frac{2R_1 d}{(R_2^2 - R_1^2)}\, \tau_{cyl}$$

$$\tau_{pl} = \frac{3R_1^2 d}{(R_2^3 - R_1^3)}\, \tau_{sph}$$

In order to utilize these fuel thickness expressions in calculating efficiency ratios, it is necessary to determine the relationship between d, R_1, and R_2 for equivalent reactors of different cell geometries. In addition to the assumptions of equal fuel loadings, void and moderator volumes, and weights as indicated above, this equivalence is further defined to mean that the fission cells in the reactors being compared must operate at equal voltages. Since the core of least weight in any geometry is that one which has the smallest void volume, it can be assumed that the electrode separation will always be chosen close to the minimum value which will withstand breakdown at operating voltage. Thus the additional equivalence criterion can be restated as the requirement for achieving the same voltage before breakdown (or significant current leakage) occurs in the different cell geometries. Unfortunately, a good criterion for selecting minimum electrode separations to hold specified voltages is not presently known; and until experiments are actually carried out, it can only be guessed at. However, it is possible to select two extreme cases for present purposes:

1. Equal separation distances between electrodes in all geometries:

$$R_2 - R_1 = d$$

2. Equal maximum electric field between electrodes in all cases:

$$E_{max} = E_{constant} = -\frac{V}{d} \quad \text{plane electrodes}$$

$$E_{max} = E(R_1) = -\frac{V}{R_1} \frac{1}{\ln\left(\frac{R_2}{R_1}\right)} \quad \text{cylindrical electrodes}$$

$$E_{max} = E(R_1) = -\frac{V\left(\frac{R_2}{R_1}\right)}{(R_2 - R_1)} \quad \text{spherical electrodes}$$

$$\therefore \left[\frac{1}{d}\right]_{pl} = \left[\frac{1}{R_1} \frac{1}{\ln\left(\frac{R_2}{R_1}\right)}\right]_{cyl} = \left[\frac{\frac{R_2}{R_1}}{(R_2 - R_1)}\right]_{sph}$$

For any specified void fraction (which determines R_1/R_2), either of the two criteria above will give unique values of R_1 and R_2 for the cylindrical or spherical cell reactor which is equivalent to a reactor containing parallel plate cells of separation distance d.

In obtaining R_1/R_2 from a given void fraction, it is assumed that the cylindrical or spherical cells are in close-packed array in the core. While this arrangement cannot actually be achieved, it would presumably be approached as closely as practical engineering considerations permit in any final core design, since it gives the maximum fuel surface area per unit of core volume. In an infinite array of close-packed elements, the fraction of total volume contained within the cells is:

$$\left[\frac{\text{cell volume}}{\text{total volume}}\right]_{\text{cylinders}} = 0.908$$

$$\left[\frac{\text{cell volume}}{\text{total volume}}\right]_{\text{spheres}} = 0.739$$

From these, the corresponding expressions for void fraction follow immediately:

$$[V.\ F.]_{cyl} = \left[\frac{\text{cell volume}}{\text{total volume}}\right]_{cyl} \times \left[\frac{\text{void volume/cell}}{\text{cell volume}}\right]_{cyl} = 0.908 \left[\frac{R_2^2 - R_1^2}{R_2^2}\right]$$

or

$$\frac{R_1}{R_2} = 1 - \left(\frac{[V.\ F.]_{cyl}}{0.908}\right)^{1/2}$$

$$[V.\ F.]_{sph} = 0.739 \left[\frac{R_2^3 - R_1^3}{R_2^3}\right]$$

or

$$\frac{R_1}{R_2} = 1 - \left(\frac{[V.\ F.]_{sph}}{0.739}\right)^{1/3}$$

The values of R_1/R_2 for cylinders and spheres are listed in Tables I and II for a series of void fractions between 0.2 and 0.7. Combining these results with the electrode separation relations above gives the values of R_1 and R_2 (in terms of d) for cylindrical cell and spherical cell reactors equivalent to a parallel plate cell reactor with electrode separation distance d. This has been done for both equal electrode separation and equal maximum electric field configurations; the R_1 and R_2 so determined are listed in Tables I and II.

Finally, the fuel layer thicknesses of the equivalent reactor cells must be found before efficiencies can be calculated. These are determined from R_1, R_2, and the equal fuel volume relation obtained previously:

$$\tau_{pl} = \tau_{cyl} \left(\frac{2R_1 d}{R_2^2 - R_1^2} \right) = \tau_{sph} \left(\frac{3R_1^2 d}{R_2^3 - R_1^3} \right)$$

The ratios τ_{pl}/τ_{cyl} and τ_{pl}/τ_{sph} have been found for the series of void fractions indicated above, for both equal electrode separation and equal maximum electric field assumptions. They are also included in Tables I and II.

The calculation of efficiencies as a function of voltage and fuel-layer thickness now follows immediately, utilizing the digital computer program described previously.[1] Ratios of the efficiencies for equivalent systems are then taken; and, as indicated earlier, these efficiency ratios are approximately equal to the desired ratios of power per unit weight for equivalent systems on a constant power density assumption. The efficiency ratios are shown in Fig. 1 through 8 for reactors with cell radii and layer thickness ratios listed in Tables I and II, over the range of voltages and fuel layer thicknesses which appear possible in an actual reactor design.

B. Constant Power Per Unit Area of Fuel Surface

If the assumption is made that reactor power is limited by heat removal from the fuel layer, the ratio of the electric power per unit weight for two equivalent reactor systems of different geometry is no longer simply equal to their efficiency ratio, as it was under the constant power density assumption.

In accord with the assumptions listed in Sec. II, the ratio of the electric power per unit weight in this situation can be written:

$$\frac{\left[\dfrac{\text{electric power}}{\text{system weight}}\right]_1}{\left[\dfrac{\text{electric power}}{\text{system weight}}\right]_2} = \frac{\left[\dfrac{\epsilon P}{(Wt)_{reac} + \alpha P}\right]_1}{\left[\dfrac{\epsilon P}{(Wt)_{reac} + \alpha P}\right]_2} = \frac{\epsilon_1 P_1 \left[(Wt)_{reac} + \alpha P_2\right]}{\epsilon_2 P_2 \left[(Wt)_{reac} + \alpha P_1\right]} = \frac{\epsilon_1 A_1 \left[(Wt)_{reac} + \alpha \left(\dfrac{P}{A}\right) A_2\right]}{\epsilon_2 A_2 \left[(Wt)_{reac} + \alpha \left(\dfrac{P}{A}\right) A_1\right]}$$

Where (P/A) is the power per unit area, here assumed constant.

It can be seen from the above expression that if the radiator weight is very much larger than the reactor weight, this ratio again reduces to ϵ_1/ϵ_2; the particular assumption made about reactor power limitations is not significant. However, where reactor weight is dominating, the ratio becomes

$$\frac{\epsilon_1 A_1}{\epsilon_2 A_2} = \frac{\epsilon_1 \tau_2}{\epsilon_2 \tau_1}$$

Actual cases will lie between these two extremes.

Without detailed knowledge of actual values for α and (P/A), it is not possible to obtain power-to-weight ratios for the constant power per unit fuel area case as was done above for the constant power density case and displayed in Fig. 1 through 8.

However, the two limiting cases of power-to-weight ratios,

$$\frac{\epsilon_1}{\epsilon_2} \text{ for } (Wt)_{rad} \gg (Wt)_{reac}$$

and

$$\frac{\epsilon_1\, \epsilon_2}{\epsilon_2\, \tau_1} \text{ for } (Wt)_{reac} \gg (Wt)_{rad}$$

are immediately available from the results of Sec. IV-A. The first limiting case is as already plotted in Fig. 1 through 8; the second is shown in Fig. 9 through 16.

It was found impractical, and in some instances impossible, to manage the entire problem analytically; so a program to obtain numerical solution was prepared for the IBM 704 computer. This program determines R_1/R_2 and τ for equivalent reactors as shown above, then calculates their electrical efficiencies just as was done previously.[1] The results shown in Fig. 1 through 16 were obtained by means of this program.

V. CONCLUSIONS

Considerable increases in the electrical efficiencies of fission electric cells occur in proceeding from plane electrodes, to cylindrical electrodes, to spherical electrodes; as observed earlier.[1] The purpose of the present work was to ascertain how much effect this apparent improvement would have in an actual reactor system for space applications.

It is seen from the results displayed here that the increasing electrical efficiency due to geometric factors is unfortunately accompanied by a decreasing [fuel surface/void volume] ratio which is due to these same geometric factors. Maintaining a constant fuel volume on the decreasing element area requires increasing the fuel layer thickness which, of course, results in a reduction of electrical efficiency. Thus, for reactors of most practical interest (void fraction < 0.5, $\tau \geq 1.0$), the intended gains are largely lost.

It seems clear, therefore, that the choice of fission-cell geometry in a reactor for space application cannot be expected to greatly influence the actual power-to-weight ratio of the system; the choice of geometry can be made purely on a basis of engineering practicability.

ACKNOWLEDGMENT

The author wishes to express his indebtedness to Mr. Sidney Silver of the Applied Mathematics Section, Jet Propulsion Laboratory, for the digital computer programming and computations referred to in this Report.

Table 1. Cells with equal electrode separation

	Cylindrical Cells				Spherical Cells			
Void Fraction	R_1/R_2	R_1	R_2	τ_{pl}/τ_{cyl}	R_1/R_2	R_1	R_2	τ_{pl}/τ_{sph}
0.2	0.883027	7.548993d	8.548993d	0.937880	0.90016	9.01608d	10.01608d	0.896841
0.3	0.834288	5.034562	6.034562	0.909658	0.84065	5.27551	6.27551	0.832271
0.4	0.747979	2.967923	3.967923	0.855821	0.77125	3.37159	4.37159	0.754193
0.5	0.670328	2.033318	3.033318	0.802630	0.68644	2.18918	3.18918	0.655162
0.6	0.582416	1.394727	2.394727	0.736110	0.57300	1.34192	2.34192	0.518053
0.7	0.478618	0.917979	1.917979	0.647386	0.37515	0.60038	1.60038	0.278527

Table 2. Cells with equal maximum electric field

	Cylindrical Cells				Spherical Cells			
Void Fraction	R_1/R_2	R_1	R_2	τ_{pl}/τ_{cyl}	R_1/R_2	R_1	R_2	τ_{pl}/τ_{sph}
0.2	0.883027	8.03859d	9.10345d	0.88076	0.90016	10.016076d	11.126989d	0.807307
0.3	0.834288	5.51998	6.61640	0.82966	0.84065	6.275514	7.465069	0.699648
0.4	0.747979	3.44400	4.60441	0.73752	0.77125	4.371590	5.668186	0.581672
0.5	0.670328	2.50006	3.72961	0.65278	0.68644	3.189185	4.645976	0.449728
0.6	0.582416	1.84993	3.17630	0.55498	0.57300	2.341922	4.087122	0.296844
0.7	0.478618	1.35715	2.83556	0.43789	0.37515	1.600384	4.265984	0.104489

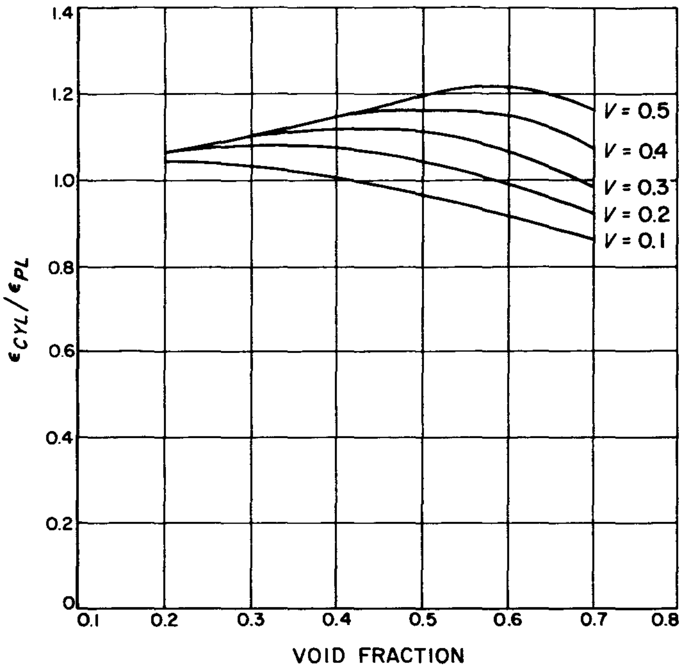

Fig. 1. $\epsilon_{cyl}/\epsilon_{pl}$ vs void fraction, equal electrode separation, τ = 1.0

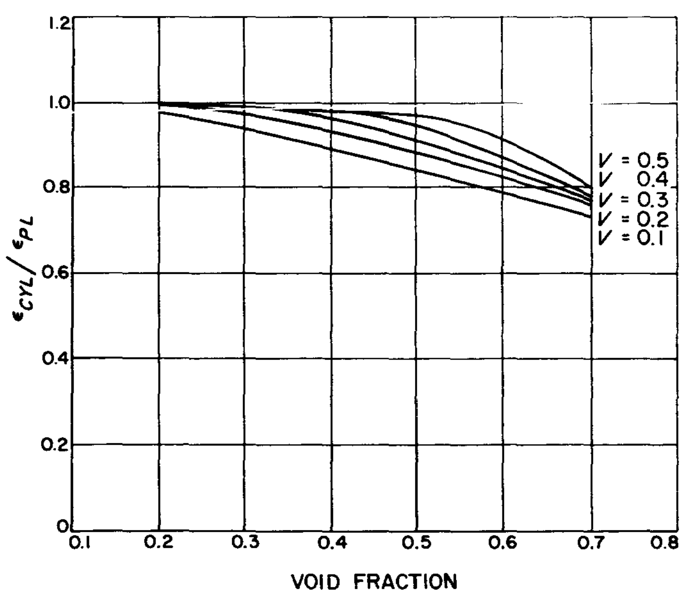

Fig. 2. $\epsilon_{cyl}/\epsilon_{pl}$ vs void fraction, equal electric field, τ = 1.0

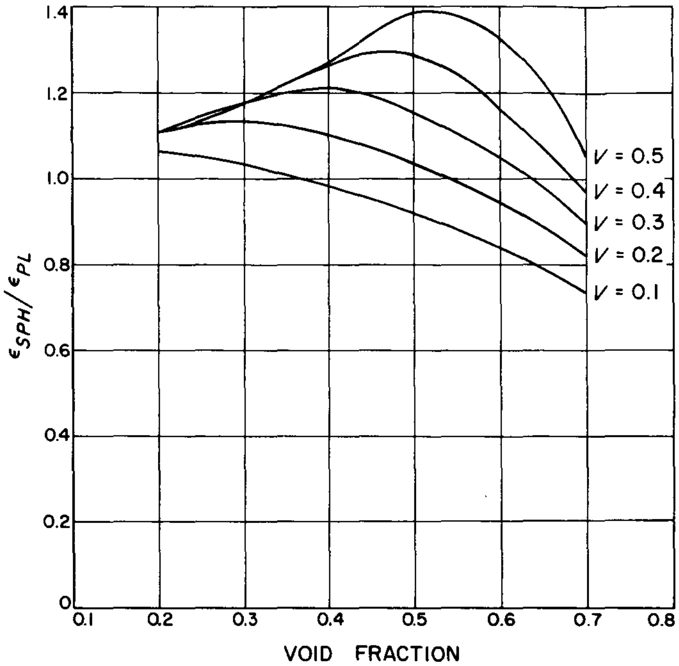

Fig. 3. $\epsilon_{sph}/\epsilon_{pl}$ vs void fraction, equal electrode separation, τ = 1.0

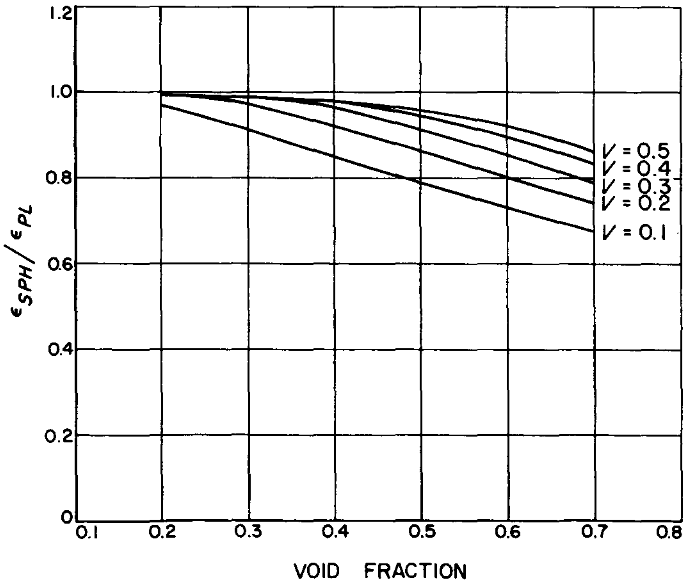

Fig. 4. $\epsilon_{sph}/\epsilon_{pl}$ vs void fraction, equal electric field, τ = 1.0

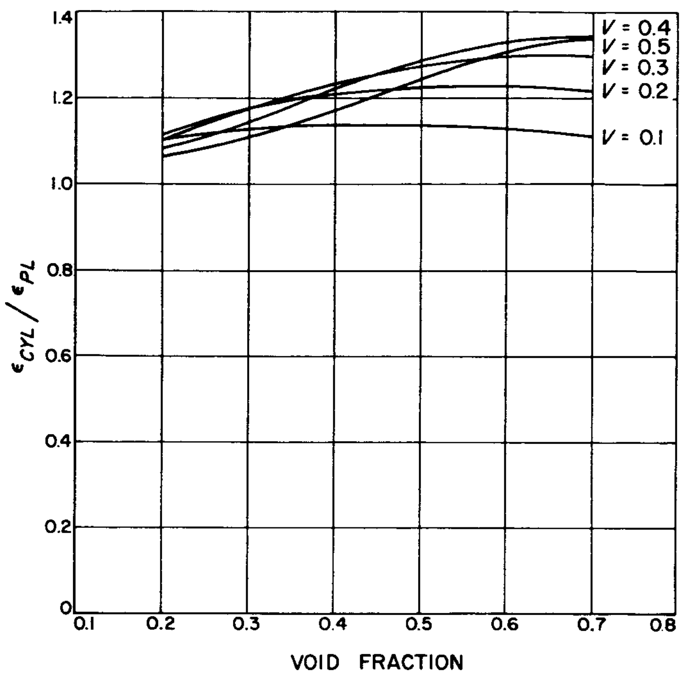

Fig. 5. $\epsilon_{cyl}/\epsilon_{pl}$ vs void fraction, equal electrode separation, $\tau = 0.5$

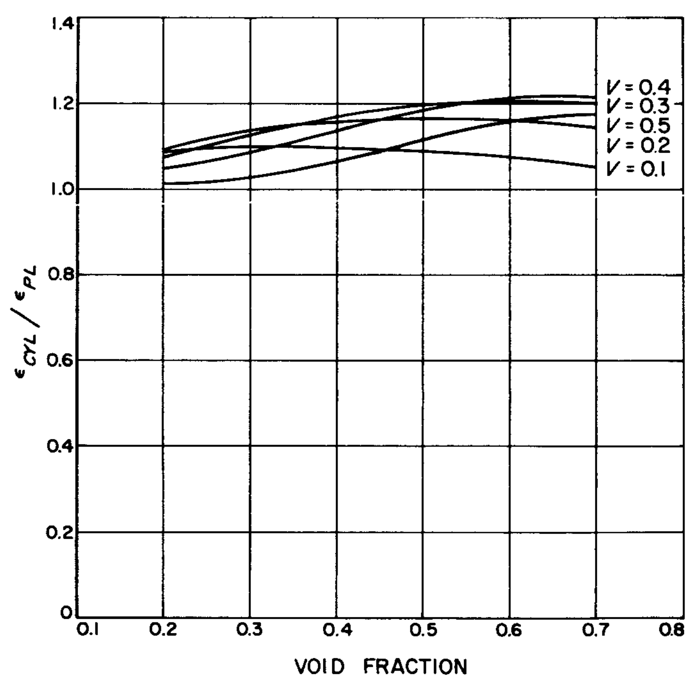

Fig. 6. $\epsilon_{cyl}/\epsilon_{pl}$ vs void fraction, equal electric field, $\tau = 0.5$

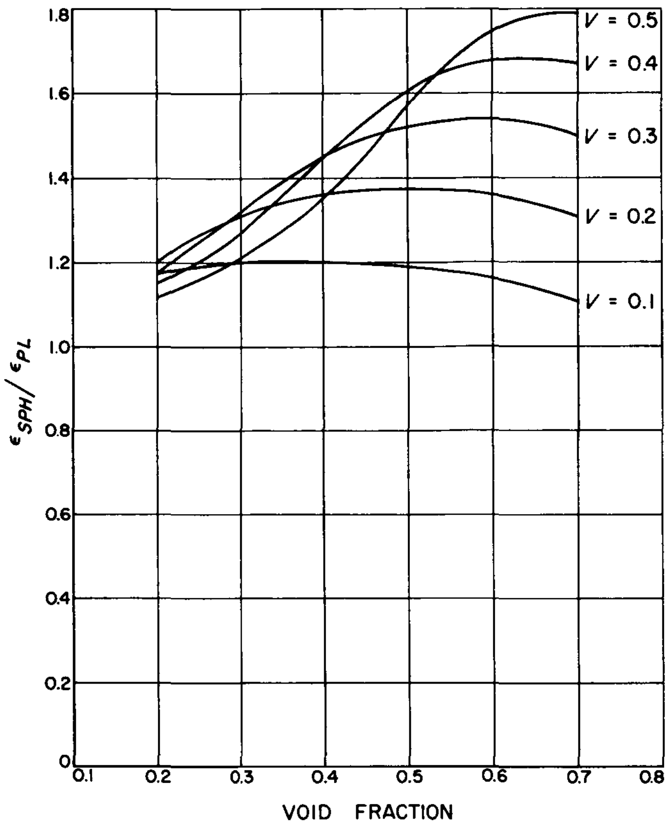

Fig. 7. $\epsilon_{sph}/\epsilon_{pl}$ vs void fraction, equal electrode separation, τ = 0.5

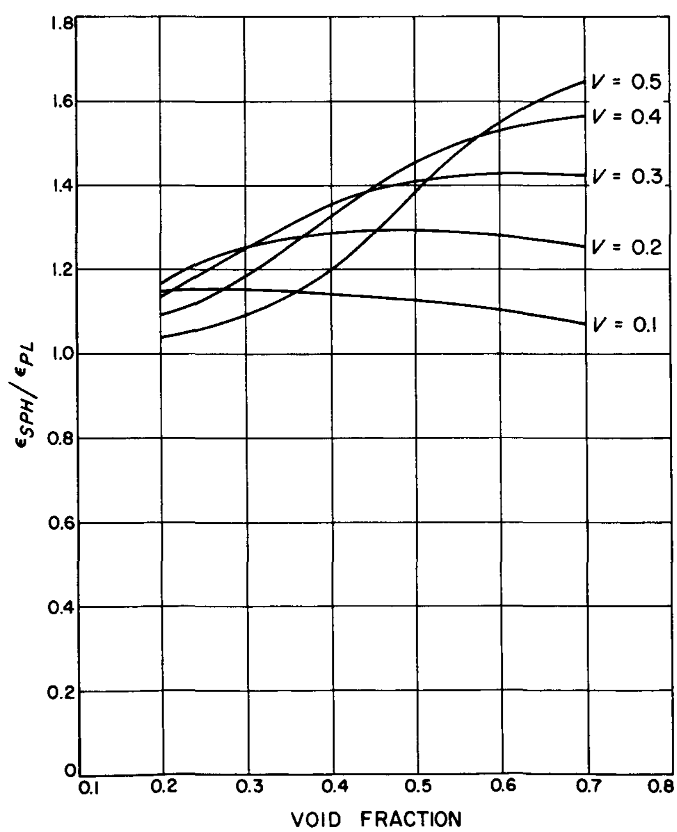

Fig. 8. $\epsilon_{sph}/\epsilon_{pl}$ vs void fraction, equal electric field, τ = 0.5

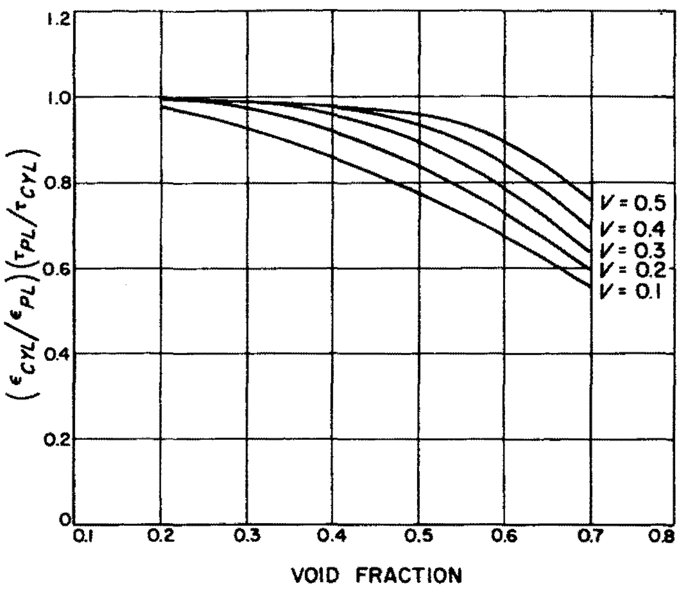

Fig. 9. $(\epsilon_{cyl}/\epsilon_{pl})(\tau_{pl}/\tau_{cyl})$ vs void fraction, equal electrode separation, τ = 1.0

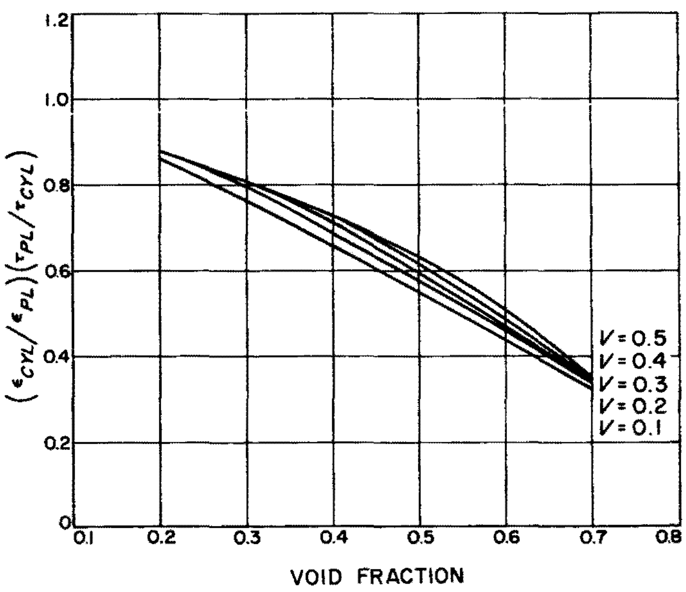

Fig. 10. $(\epsilon_{cyl}/\epsilon_{pl})(\tau_{pl}/\tau_{cyl})$ vs void fraction, equal electric field, τ = 1.0

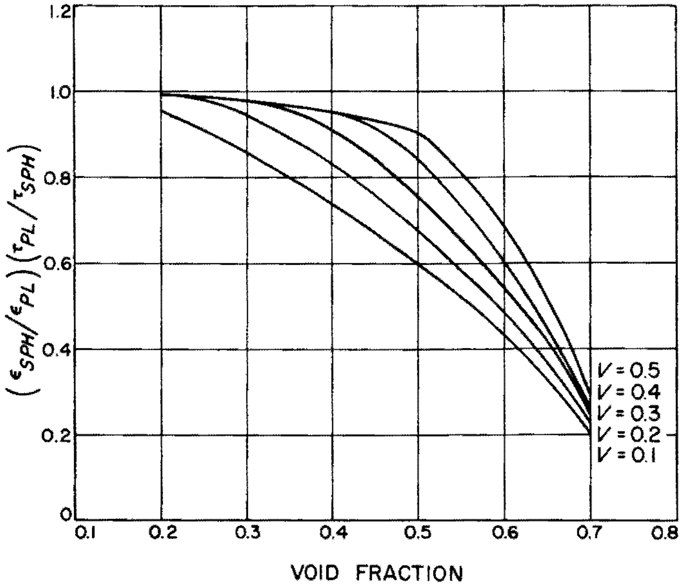

Fig. 11. $(\epsilon_{sph}/\epsilon_{pl})(\tau_{pl}/\tau_{sph})$ vs void fraction, equal electrode separation, $\tau = 1.0$

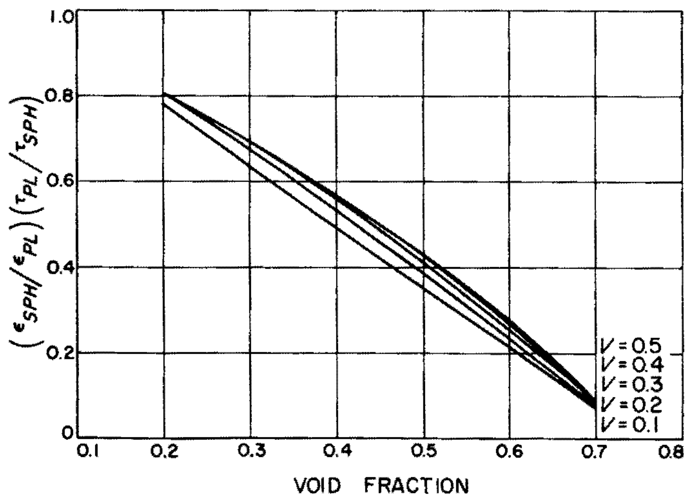

Fig. 12. $(\epsilon_{sph}/\epsilon_{pl})(\tau_{pl}/\tau_{sph})$ vs void fraction, equal electric field, $\tau = 1.0$

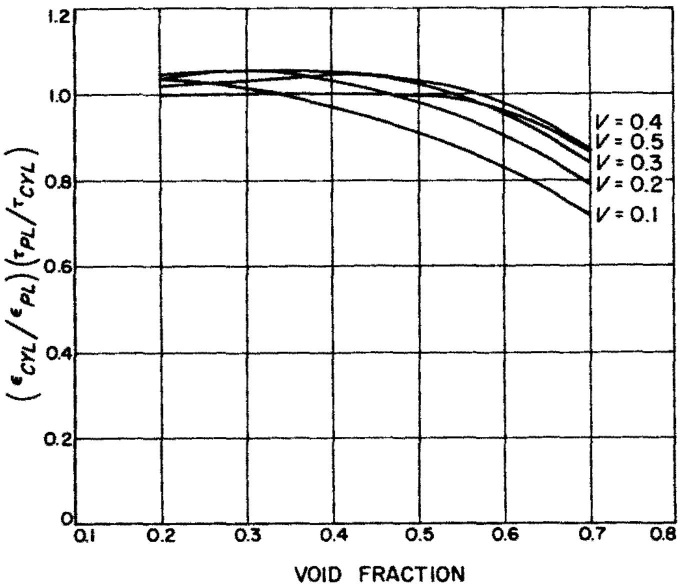

Fig. 13. $(\epsilon_{cyl}/\epsilon_{pl})(\tau_{pl}/\tau_{cyl})$ vs void fraction, equal electrode separation, $\tau = 0.5$

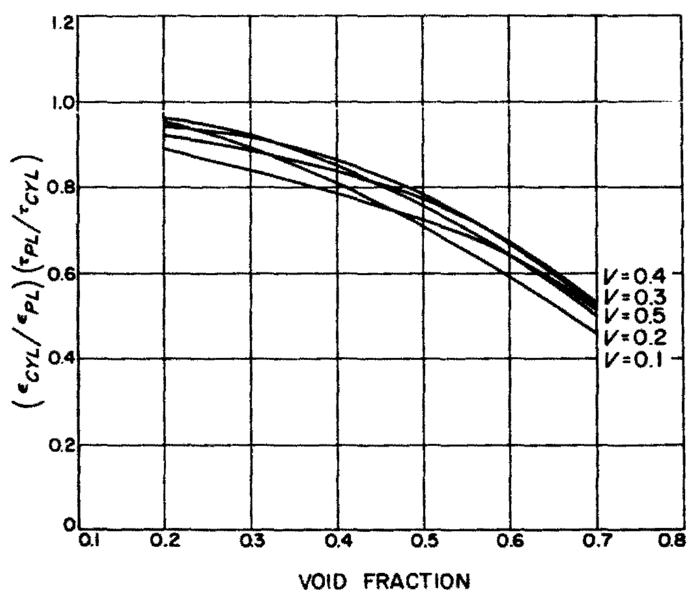

Fig. 14. $(\epsilon_{cyl}/\epsilon_{pl})(\tau_{pl}/\tau_{cyl})$ vs void fraction, equal electric field, $\tau = 0.5$

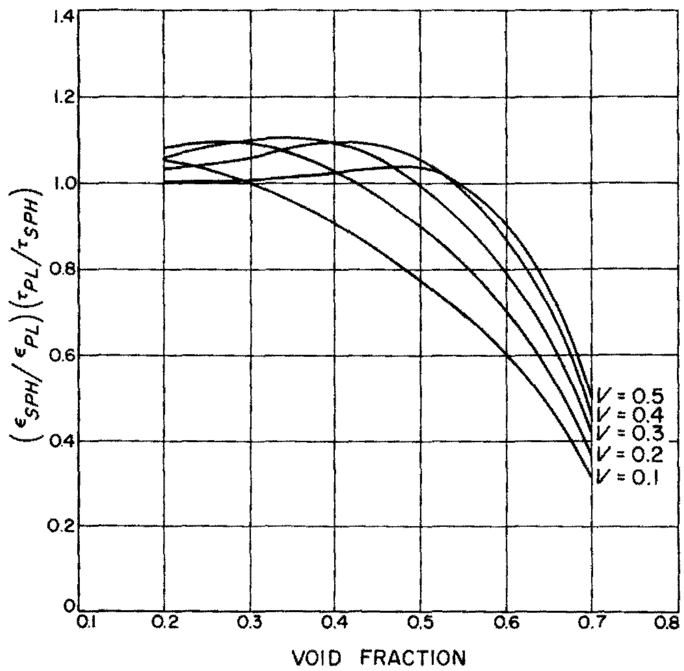

Fig. 15. $(\epsilon_{sph}/\epsilon_{pl})(\tau_{pl}/\tau_{sph})$ vs void fraction, equal electrode separation, $\tau = 0.5$

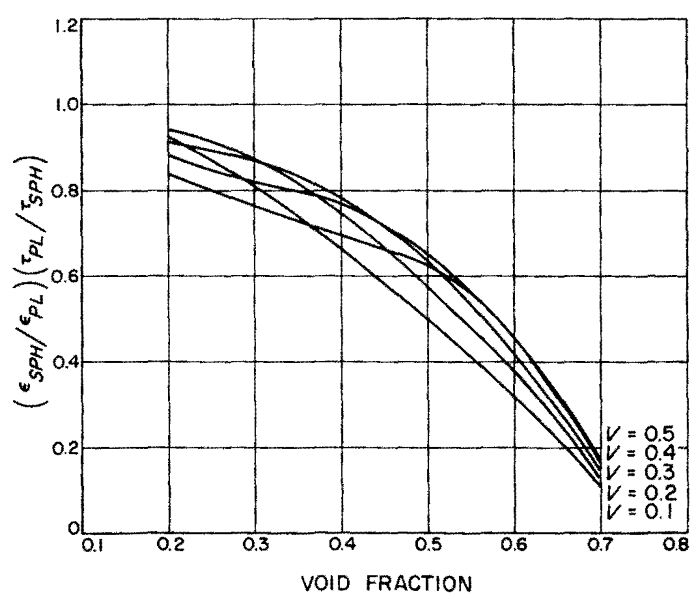

Fig. 16. $(\epsilon_{sph}/\epsilon_{pl})(\tau_{pl}/\tau_{sph})$ vs void fraction, equal electric field, $\tau = 0.5$