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INVESTIGATION OF THE DAMPING OF LIQUIDS IN RIGHT-CIRCULAR CYLINDRICAL TANKS, INCLUDING THE EFFECTS OF A TIME-VARIANT LIQUID DEPTH

By David G. Stephens, H. Wayne Leonard, and Tom W. Perry, Jr.

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Page 30: Figure 7 is in error to the extent that its validity and usefulness is negated. This figure should be replaced with the attached corrected figure 7.

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SUMMARY

An experimental investigation was conducted to determine the effects of several basic variables upon the damping of the fundamental antisymmetric mode of liquids in right-circular cylindrcal tanks without baffles. The variables examined include liquid depth, efflux rate, liquid amplitude, kinematic viscosity, and tank size. The data are presented in dimensionless form and compared with available theory. For the range of variables examined, variations of efflux rate and liquid amplitude were found to have no significant effects on the liquid damping. The following theoretical relationship was found to be adequate for the prediction of the variation of damping with liquid depth, kinematic viscosity, and tank size:

$$\delta = K\nu^{1/2}R^{-3/4}g^{-1/4}\left[1 + 2\left(1 - \frac{h}{R}\right)\operatorname{csch}\left(3.68\frac{h}{R}\right)\right]\left[\tanh^{-1/4}\left(1.84\frac{h}{R}\right)\right]$$

where ν is the kinematic viscosity, R is the cylinder radius, g is the acceleration due to gravity, and h is the liquid depth. However, the constant K was experimentally found in this investigation to have the value 5.23, which is 50 percent higher than the theoretically predicted value.

INTRODUCTION

Oscillations of the liquid propellant in missiles and space flight booster systems may have a marked effect on the stability and structural integrity of the overall configuration. Oscillations of the fuel and oxidizer are particularly critical during phases of flight in which the propellant represents a very large percentage of the total system mass.

The frequency of the fundamental antisymmetric liquid mode and the liquid damping associated with that mode are the basic factors of the fuel-sloshing problem. If the liquid masses are excited at a frequency near that of the fundamental mode, the amplitudes of the oscillations and hence the resulting forces and moments imposed on the vehicle may become excessively large. The magnitudes of these oscillations are, of course, highly dependent upon the amount of liquid damping present. Since the geometry and size of the propellant tankage dictate the frequency spectrum of the liquid (refs. 1 to 3), the solution of the fuel-sloshing problem appears to lie primarily in providing liquid damping.

Many detailed analytical and experimental studies have been conducted to determine the damping present in tanks of various shapes fitted with various types of baffles (refs. 4 to 9). These studies present the total damping of the fundamental antisymmetric mode which is obtained primarily from two sources: one being the relative motion between the liquid and the tank wall and the other being the relative motion between the liquid and such baffles as may be employed as liquid-damping devices. The amount of damping contributed by each of these sources has not been defined because of the limited nature of studies devoted to the determination of the damping present in tanks without baffles. Although the contribution of the smooth-wall damping is assumed to be small, it has been found to be sufficient for certain configurations and therefore requires consideration.

Any attempt to predict the magnitude of the liquid damping must be preceded by a study of the primary variables which may control its magnitude. Of particular importance are the effects of efflux rate, liquid depth, amplitude of oscillation, kinematic viscosity, and tank size.

During the early phases of this experimental investigation, the effects of a time-variant liquid depth (efflux rate) were totally unknown because no analytical or experimental work had been advanced on the subject. Since both the oscillation of the free surface and the movement of the entire liquid mass due to efflux contribute to the relative motion between the liquid and the tank wall, there did exist however a widespread belief that efflux was a variable of considerable importance. Subsequently, the analytical works of Miles (ref. 10) and Nelson (ref. 11) were advanced in an effort to examine the behavior of a draining liquid. The analysis of reference 10 considers the contribution of efflux to the damping of the liquid amplitude whereas reference 11 is concerned with the damping of a velocity potential associated with the liquid motion. These works were compared in reference 12 and were found to be compatible. The analyses predict a very small but always positive increase in the damping of the liquid oscillations due to draining; however, experimental verification of this prediction is not documented in the literature. The effects of liquid amplitude, depth, kinematic viscosity, and tank size have been included in the analyses of references 13, 14, and 15. These theories have been found to underestimate substantially the magnitudes of the available damping, and published experimental data have been too limited to verify the predicted trends.

The purpose of this paper is to present the results of a comprehensive experimental investigation of the damping of liquid oscillations in right-circular cylindrical tanks without baffles. The effects of liquid depth, efflux rate, liquid amplitude, kinematic viscosity, and tank size are examined.

SYMBOLS

A	area of orifice
A_1	area of quiescent liquid surface
B	similarity parameter, $\sqrt{\frac{gR^3}{\nu^2}}$
f	liquid frequency
g	acceleration due to gravity
h	instantaneous liquid depth
h_o	initial liquid depth
K	a constant
n	number of cycles
R	cylinder radius
t	time
α	orifice contraction coefficient
Δ	"deep-tank" damping factor (see eq. (3))
δ	damping factor, $\frac{1}{n} \log_e \frac{\zeta_o}{\zeta_n}$

δ' damping factor due solely to draining

ζ liquid amplitude

ζ_o initial liquid amplitude

ζ_n liquid amplitude of nth cycle

ν liquid kinematic viscosity

Dots over symbols denote differentiation with respect to time.

APPARATUS

The experimental investigation of the damping of liquid oscillations was conducted in two smooth-walled right-circular cylindrical tanks of different sizes securely fastened upright on rigid platforms. The smaller tank, having an inside diameter of 12.0 inches and a height of 52.0 inches was constructed of 1/8-inch-thick plexiglass with a 2.0-inch-thick plexiglass base. The bottom of the tank was designed in such a manner that drain openings ranging from 0.5 to 4.0 inches in diameter could be obtained simply by interchanging orifice inserts in the center of the base. The larger tank, having an inside diameter of 36.0 inches and a height of 72.0 inches, was constructed of 3/8-inch-thick plexiglass and had a 2.5-inch-thick laminated plywood and fiberglass base. A 3.5-inch-diameter square-edged orifice was centered in the base of this tank.

The interchangeable orifice inserts used in the small tank were flat disks of 1/2-inch-thick plexiglass, each having a different-diameter square-edged orifice at its center. The outer diameters of these disks were equal and sized so that they fitted snugly into a 1/2-inch-deep circular recess in the center of the tank bottom. A circular opening, through the center of the tank bottom and larger than any of the orifices, was fitted with a self-sealing, quick-release plug.

The amplitude of the liquid oscillation was sensed by a capacitance-wire transducer system developed by the Langley Instrument Research Division. This device, described fully in the appendix, consists of two capacitance probes mounted parallel to, but insulated from, the tank wall at the antinode points of the fundamental antisymmetric fuel-sloshing mode. These probes yield electrical outputs proportional to the difference in height of the liquid surface at these points and are self-compensating so as to maintain a constant zero level as the tank is drained. From an examination of this transducer signal, the damping of the amplitude of the liquid oscillation may be determined for each of the test conditions established.

Two methods were used to examine these signals; the choice depended upon the test conditions. In cases wherein the liquid depth was maintained constant, the transducer outputs were amplified and fed into a Dampometer, which yields a direct measure of the damping. For test conditions involving a time-variant liquid depth, the transducer signals were recorded as oscillograms and the decay of the oscillation was measured from these records. This technique provided a time history of the entire drain sequence.

PROCEDURE

General

The test procedure was the same for both tank sizes. The tank was filled to a selected depth with tap water at room temperature (approximately 70° F). The liquid was then excited in its fundamental antisymmetric mode by means of a paddle until a desired liquid amplitude ζ was attained. This amplitude, defined as the distance measured along the tank wall between the quiescent surface and the point of maximum surface displacement, was large enough to permit the higher mode transients to decay before the desired initial amplitude ζ_o was reached.

Depth Effect

A study of the relationship between liquid depth and damping was conducted in both the 12- and 36-inch-diameter tanks for discrete depths throughout the complete depth range. In this particular phase of the study, tests were conducted at a fixed value of ζ_o/R equal to 0.056 for both tank sizes. This value of ζ_o/R corresponds to amplitudes of 0.33 inch in the 12-inch-diameter tank and 1.0 inch in the 36-inch-diameter tank. The test procedure involved exciting the liquid, at a given depth, to the desired liquid amplitude; after removal of the excitation, the rate of decay of this amplitude was measured over several cycles of the ensuing oscillation. This rate of decay was then expressed as a damping factor δ defined by

$$\delta = \frac{1}{n} \log_e \frac{\zeta_o}{\zeta_n} \tag{1}$$

where n is the number of cycles over which the decay was measured, ζ_o is the initial amplitude, and ζ_n is the amplitude of the nth cycle.

Efflux-Rate and Liquid-Amplitude Effects

For this study of the effects of efflux rate and liquid amplitude on damping, initial liquid depths of 48 inches and 60 inches were selected for the 12-inch-diameter tank and 36-inch-diameter tank, respectively. Five efflux rates were studied in the small tank with initial amplitudes corresponding to values of ζ_o/R equal to 0.056 and 0.10. In the 36-inch-diameter tank, only one efflux rate was examined with initial amplitudes corresponding to values of ζ_o/R equal to 0.028, 0.056, and 0.083. For this phase of the test program, the orifice insert, corresponding to the desired efflux rate, was installed in the tank. The tank was then filled to the selected initial depth and the liquid was excited. The excitation was then removed and when the oscillation had decayed to the desired initial amplitude, the quick-release plug was actuated and the amplitude-transducer signal was recorded as an oscillogram.

ANALYTICAL METHODS

Explicit relationships are presented in references 13, 14, and 15 for the determination of the damping factor for the fundamental antisymmetric mode of liquid oscillations in right-circular cylinders at a constant liquid depth. These relationships will be presented and compared in an effort to establish the theoretical dependence of the damping on the variables considered in this investigation. Also included is a presentation of the analytical effect of a time-variant liquid depth as derived in reference 10.

The damping of the oscillations of a slightly viscous liquid resulting from laminar-boundary-layer friction was examined by Miles (ref. 13). An expression was presented for the damping of the fundamental antisymmetric mode of a liquid in a flat-bottom right-circular cylinder having a constant liquid depth. The analysis predicts that the oscillation will be damped according to the relation

$$\delta = K\nu^{1/2}R^{-3/4}g^{-1/4}\left[1 + 2\left(1 - \frac{h}{R}\right)\operatorname{csch}\left(3.68\frac{h}{R}\right)\right]\left[\tanh^{-1/4}\left(1.84\frac{h}{R}\right)\right] \qquad (2)$$

where ν is the kinematic viscosity of the contained liquid, R is the cylinder radius, g is the acceleration due to gravity, h is the liquid depth, and K is a constant with the value 3.52. The bracketed terms become the dominant terms as h/R is decreased below 1.0. As h/R is increased above 1.0, the bracketed terms have a negligible effect and the damping factor is invariant with further depth increases. This invariant or "deep-tank" damping, denoted herein by Δ, is given by the equation

$$\Delta = \delta_{\frac{h}{R}>1} = 3.52\nu^{1/2}R^{-3/4}g^{-1/4} \tag{3}$$

The effect of liquid depth on damping is determined by the ratio of the damping specified by equation (2) to the deep-tank damping of equation (3). This proportional change in damping as a function of depth is defined as the "depth correction factor," as suggested by reference 13.

The damping factor, as given by equation (2), may be related to the expressions developed by Case and Parkinson (ref. 14) for the damping of the fundamental antisymmetric mode resulting from contact between the moving liquid and the side wall and tank bottom (eqs. (42b) and (42c), respectively). Addition of these equations of reference 14 yields an expression identical to equation (2).

The deep-tank damping factor (eq. (3)) was further verified by dimensional analysis in reference 15. It was shown that if two cylindrical tanks are filled to the same relative depth h/R and are subjected to similar excitations, they will generate similar sloshing provided the similarity parameter

$$B = \sqrt{\frac{gR^3}{\nu^2}} \tag{4}$$

is the same in both cases. Furthermore, it was shown that the damping of the fundamental liquid mode in the deep-tank region of a cylindrical tank without baffles is given by

$$\Delta = 3.52B^{-1/2} \tag{5}$$

or

$$\Delta = 3.52\nu^{1/2}R^{-3/4}g^{-1/4} \tag{6}$$

which is identical to the previously presented deep-tank damping equations.

Except for the data of Case and Parkinson (ref. 14), available experimental values of damping were found to be higher than those predicted by the theory. Also the available experimental data have been too limited to verify the predicted variations in damping over a wide range of liquid depths, tank sizes, and kinematic viscosities. Both the individual and combined effects of these variables have been examined in detail in the present investigation. The experimental data of this investigation will be presented in a form similar to the theoretical results to facilitate a comparison of the magnitudes and trends of the theoretical and experimental results.

The effects of liquid efflux on the damping were examined theoretically by Miles (ref. 10). The draining of the liquid from a vertical tank was found to produce a small but always stabilizing damping of the free-surface oscillation. In a right-circular cylinder the damping attributed solely to draining δ' may be approximated by equation (17) of reference 10 where, for $h/R \gg 1$,

$$\delta' = -2\pi\dot{h}(gh)^{-1/2}\left(1.84\frac{h}{R}\right)^{1/2}e^{-3.68\frac{h}{R}} \tag{7}$$

It has been shown (ref. 12) that this result is equivalent to that predicted theoretically by Nelson (ref. 11).

For gravity flow from the tank such that the conditions of Torricelli's theorem are satisfied, that is,

$$\dot{h} = -\alpha(2gh)^{1/2} \tag{8}$$

where α is the orifice contraction coefficient and is determined by the tank and orifice geometry, equation (7) may be reduced to the form

$$\delta' = 2\pi\alpha\left(3.68\frac{h}{R}\right)^{1/2}e^{-3.68\frac{h}{R}} \tag{9}$$

The value of α for draining through a square-edged circular orifice in a cylindrical tank is

$$\alpha = \frac{0.60A}{A_1} \tag{10}$$

where A is the area of the orifice and A_1 is the quiescent surface area. Equation (9) then becomes

$$\delta' = 3.77\frac{A}{A_1}\left(3.68\frac{h}{R}\right)^{1/2}e^{-3.68\frac{h}{R}} \tag{11}$$

For tanks of practical size, the incremental damping as given by equation (11) is extremely small and no experimental data have been published to either substantiate or refute these theoretical predictions.

DATA PRESENTATION AND DISCUSSION OF RESULTS

Damping-Depth Relationships for Zero Efflux Rate

Variations of the damping factor with discrete changes in liquid depth for oscillations in both the 12-inch-diameter and 36-inch-diameter cylinders are indicated by figure 1. The damping-factor ratio δ/Δ is shown as a function of the depth ratio h/R. The data presented were obtained by dividing the experimentally determined damping factors by the average value of damping measured in the deep-tank region. The dimensionless values thus obtained are compared with the analytical depth correction factor from reference 13.

The analytical depth correction factor predicts that a considerable increase in damping occurs as the liquid depth is decreased in the region $h/R < 1$; however, the damping is essentially independent of liquid depth for values of $h/R > 1$. This theory is substantiated by the agreement obtained between the experimental results and the analytical predictions. It should be noted that figure 1 does not give a numerical measure of the damping, it only indicates the damping trends since the data have been normalized by the deep-tank damping values. If, however, a value of damping is known for a given liquid depth, the damping throughout the entire depth range may be determined regardless of the cylinder size.

Damping for Positive Efflux Rates

Efflux rates.- Effects of a positive efflux rate on the damping were studied in both the 12-inch-diameter and 36-inch-diameter cylinders. The efflux rate for each of the orifices was determined by timing incremental changes in the liquid depth during draining. In the 12-inch-diameter tank, five different efflux rates were employed. A composite plot of these efflux rates is presented in figure 2, which shows the variation of liquid depth with time for each of the five orifices. The slopes of the curves in figures 2 and 3 are a measure of the quiescent surface velocities obtained in the test program. For the convenience of the reader, the slope of each curve is presented at $h/R = 2$. Examination of the curve in figure 3, which is for the single efflux rate employed in the 36-inch-diameter cylinder, shows that the quiescent surface velocity associated with this efflux rate lies between the velocities associated with the 1.5- and 2.0-inch-diameter orifices as employed in the 12-inch-diameter cylinder.

In each of the tanks, the time required for the liquid surface to drop from an initial depth h_o to a given depth h was found to be in excellent agreement with that predicted by equation (4.12) of reference 16, which states

$$t = \sqrt{\frac{2}{g}} \frac{A_1}{0.60A} h_o^{1/2}\left[1 - \left(\frac{h}{h_o}\right)^{1/2}\right] \qquad (12)$$

This expression for time may be obtained by substituting the value of α from equation (10) into equation (8) and integrating the resultant equation

$$\dot{h} = -\frac{0.60A}{A_1}(2gh)^{1/2}$$

between $t = 0$ and $t = t$ with the initial condition that at $t = 0$, $h = h_o$.

Since the expression for the damping attributed to efflux (eq. (11)) was derived for the case of a depth-dependent quiescent surface velocity such as that employed in the present investigation, the theoretical predictions of the contribution of efflux to damping were determined from this expression. These theoretical damping values are discussed in a subsequent section entitled "Summary Comparisons."

<u>12-inch-diameter cylinder</u>.- The decay or damping of the free-surface oscillations ($f = 1.73$ cps) in the 12-inch-diameter cylinder are shown in figure 4 where the liquid amplitude is plotted logarithmically as a function of cycle number for each of the five orifices. The data thus presented indicate the timewise behavior of the liquid amplitude throughout the drain sequence. It should be noted that the data presented in this form will yield a straight line only if the damping is independent of the variations in amplitude, liquid depth, and efflux rate encountered during a test run. Furthermore, numerical measures of the damping in terms of logarithmic decrements may be obtained directly from the faired curves by utilizing equation (1).

Data obtained for the tank with orifice diameters of 0.5 and 1.0 inch are presented in figures 4(a) and 4(b), respectively. Only the higher initial amplitude (0.6 inch) was employed since the efflux rates were sufficiently low to allow the liquid amplitude to decay through the spectrum of interest. These figures indicate a linear relationship between the natural logarithm of the amplitude and the cycle number. The logarithmic decrements were obtained directly from the faired curves and the value for each curve is presented in the appropriate figure. In these particular tests, the complete drain time was not investigated because the liquid amplitude had decayed to a value below the range of interest at the completion of 80 cycles.

Data obtained for the tank with orifice diameters of 1.5, 2.0, and 3.0 inches are presented in figures 4(c), 4(d), and 4(e), respectively. For these tests, two initial amplitudes were employed for each orifice since the associated efflux rates were too high to permit the liquid amplitude to traverse the desired spectrum during a test run. Again, the figures indicate a linear relationship between the natural logarithm of the liquid amplitude and the cycle number. The data show no significant changes in the damping factor with changes in initial amplitude. Since the effects of initial amplitude and efflux were the primary objectives in this particular series of tests, no data are presented for liquid depths less than 8 inches, corresponding to $h/R = 1.33$. Thus, the increase in damping for values of $h/R < 1$ is not reflected in these data.

A comparison of all the data of figure 4 shows no apparent dependence of the damping factor on efflux rate or initial amplitude. Some variations are evident but they are of such a random nature as to suggest that they emanate from experimental errors, day-to-day changes in the condition of the tank wall, and other such secondary quantities.

<u>36-inch-diameter cylinder</u>.- Variations of liquid amplitude with cycle number, while undergoing draining corresponding to the depth-time curve of figure 3, are presented in figure 5 for the 36-inch-diameter cylinder ($f = 0.999$ cps). The form of the presentation is identical to that employed in figure 4. In this case, three initial amplitudes were examined as an additional check on the effects of liquid amplitude on the damping. Furthermore, the data represent oscillations of the liquid throughout the range of liquid depths corresponding to $0.44 < h/R < 3.3$. Consequently, a significant increase in the slope of the faired curves (an increase in damping), as predicted by the depth correction factor and shown in figure 1, is evident at the higher cycle numbers. The value of damping, as presented in the figure, was obtained from the linear portion of each of the faired curves. Again, the data show no significant changes in the damping with variations in the initial amplitude.

Summary Comparison

A summary of the experimentally determined damping factors for liquid in the 12-inch-diameter and the 36-inch-diameter cylinders for both zero and positive efflux rates are presented in table I. Also included is the theoretical prediction for the increase in damping attributed to draining as calculated from equation (11). Since equation (11) is applicable for values of $h/R \gg 1$, a conservative lower limit was arbitrarily set at $h/R = 2$ for the calculation of the values presented. Higher values of h/R would have resulted in progressively smaller values of the incremental damping due to efflux.

The data as presented in table I summarize the effects of the initial liquid amplitude and the liquid surface velocity, associated with efflux, on the damping. Variations of damping with amplitude appear to be small, and the random nature of the variations indicates experimental errors rather than an amplitude dependency. The lack of dependence of damping upon amplitude was of course illustrated previously by the fact that the faired data of figures 4 and 5 are straight lines.

In considering the effects of efflux on damping in the 12-inch-diameter tank, small variations of the damping factors are noted but again the variations are of such a random nature that no direct dependence can be concluded. For example, the damping factors obtained for the highest efflux rate in the 12-inch-diameter cylinder are identical to the damping factor associated with zero efflux. The damping factors for the intermediate efflux rates are shown to lie both above and below the value obtained with zero efflux.

The results presented for the 36-inch-diameter tank show approximately a 20-percent difference in the damping between the draining and zero-efflux cases. In view of the consistency exhibited in the more comprehensive data obtained in the 12-inch-diameter tank, it is believed that the variations in the data obtained in the 36-inch-diameter tank represent inherent experimental variations rather than significant physical changes. Due to the extremely low total damping present in the 36-inch-diameter tank, secondary quantities, such as changes in wall condition due to sedimentary film, may yield experimental variations in damping of the order of magnitude observed. The damping-depth data for the 36-inch-diameter tank shown in figure 1 of the present paper exhibit scatter approaching this magnitude. As indicated by the values obtained from equation (11), the increase in damping, calculated at $h/R = 2$, is so small as to be unmeasurable with the procedures and instrumentation employed in the test program.

Damping—Viscosity—Tank-Size Relationships

Experimentally determined deep-tank damping factors for liquids having kinematic viscosities ranging from 1×10^{-5} ft^2/sec to 90×10^{-5} ft^2/sec in right-circular cylindrical tanks having diameters ranging from 3 inches to 10 feet are presented in figure 6 in terms of the variables of equation (3).

The data points presented represent various test conditions. Points 1, 3, and 5 were obtained with SAE 10-W oil, kerosene, and water, respectively, in the 12-inch-diameter tank. These three test conditions illustrate the effects of a wide range of kinematic viscosities upon the damping in a given cylinder. Damping factors obtained by Case and Parkinson (ref. 14) for water in cylinders having diameters of 3 inches and 6 inches are presented as points 2 and 4, respectively. Both of these points appear low when compared with the other data; however, the cylinders employed by Case and Parkinson were highly polished in an effort to reduce the damping to a minimum. A 30-inch-diameter cylinder with kerosene and water as the test liquids was used to determine the damping factors presented as points 6 and 7, respectively. The apparatus and procedures used in obtaining these points are the same as those described in reference 4 for the 30-inch tank. Point number 8 is the deep-tank damping factor obtained for water in the 36-inch-diameter cylinder of the present investigation. The range of cylinder diameters considered is extended by point 9, which was drawn from unpublished data furnished by Convair/Astronautics, and is an estimate of the damping in a 10-foot-diameter tank. The solid straight line is the faired curve for this collection of data points. The equation of this faired curve was found to be

$$\Delta = 5.23\nu^{1/2}R^{-3/4}g^{-1/4} \tag{13}$$

Equation (13) is identical to the equation for deep-tank damping previously presented as equation (3) with the exception that the empirical constant 5.23, developed from the experimental results, is 50 percent higher than that predicted by reference 13. In view of the wide range of tank radii and liquid kinematic viscosities examined in figure 6, it is believed that equation (13) will adequately predict the deep-tank damping of liquids in cylinders of general size.

General Application of Results

It has been shown (fig. 4) that neither a time-variant liquid depth nor the amplitude of the liquid oscillations within the test range has any important effect on the damping of liquid oscillations in rigid right-circular cylinders. These variables may then for practical purposes be neglected in the consideration of a general expression for the damping. The effects of discrete changes in liquid depth were shown by figure 1 as a depth correction factor and are consistent with the theory of Miles. Furthermore, a relationship was presented in figure 6 which was shown to be sufficient to predict the deep-tank damping of liquids in cylinders regardless of the liquid kinematic viscosity or cylinder radius. As previously stated, if the damping is known at a given liquid depth, the damping at any liquid depth may be determined by applying the depth correction factor of figure 1 to the known value. Since figure 6 does predict the deep-tank damping of liquids in cylinders of general size, figures 1 and 6 together serve as a means of determining the damping, throughout the complete depth range, of liquids in cylindrical tanks.

A more direct approach to the solution of the problem seems to be a combination of the equations associated with figures 1 and 6 to arrive at a general free-surface damping equation. When the effects of the three pertinent variables - liquid depth, kinematic viscosity, and tank size - are combined, the damping of the fundamental antisymmetric mode of liquids contained in upright circular cylinders is specified by

$$\delta = 5.23\nu^{1/2}R^{-3/4}g^{-1/4}\left[1 + 2\left(1 - \frac{h}{R}\right)\operatorname{csch}\left(3.68\frac{h}{R}\right)\right]\left[\tanh^{-1/4}\left(1.84\frac{h}{R}\right)\right] \tag{14}$$

CONCLUSIONS

An investigation has been conducted to determine the effects of several basic variables upon the damping of free-surface liquid oscillations in right-circular cylindrical tanks without baffles. The investigation was conducted by considering the theoretical relationship for damping developed by Miles which states

$$\delta = K\nu^{1/2}R^{-3/4}g^{-1/4}\left[1 + 2\left(1 - \frac{h}{R}\right)\operatorname{csch}\left(3.68\frac{h}{R}\right)\right]\left[\tanh^{-1/4}\left(1.84\frac{h}{R}\right)\right]$$

where ν is the kinematic viscosity, R is the cylinder radius, g is the acceleration due to gravity, h is the liquid depth, and K is a constant. The individual effects of each of the variables were experimentally investigated and may be summarized as follows:

1. The damping of the fundamental antisymmetric liquid mode is independent of liquid depth for depths greater than 1 radius but increases as the depth is decreased below 1 radius as predicted by the bracketed terms in the theoretical damping equation.

2. The damping was experimentally found to vary with liquid kinematic viscosity and tank size according to $\nu^{1/2}$ and $R^{-3/4}$, respectively, as predicted by theory.

3. The constant K was experimentally found to have the numerical value of 5.23, which is 50 percent higher than that given by Miles' theory.

4. A time-variant liquid depth does not have an important effect on the liquid damping.

5. The damping is independent of liquid amplitude within the range of amplitudes covered in this investigation.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., April 26, 1962.

APPENDIX

AMPLITUDE TRANSDUCER

An electrical transducer was developed to sense the amplitude of the liquid oscillations under study in this investigation. The transducer was designed to operate both for constant liquid depths and for cases wherein liquid was being drained from the container.

The device, shown schematically in figure 7, consists basically of two capacitance probes extending vertically from above the liquid surface to the tank bottom. These probes are positioned 180° apart on the antinodes of the wave. The operation of the system is based upon the measurement of small changes in electrical capacitance resulting from the differences in liquid height at the probes caused by the liquid oscillations.

The probes, made of Supersil silicone magnet wire (size 32, heavy HS) manufactured by American Super-Temperature Wires, Inc., are in effect variable capacitors. The magnet wire is one plate, the silicone coating is the dielectric, and the liquid serves as the other plate and is common to both probes. A metal plate in the base of the tank, and connected between adjacent arms of a resistive bridge, provides the common return current path to the bridge. The conductor plates (magnet wires) are connected to the other ends of these adjacent arms and form adjacent capacitive arms of the bridge. The bridge output is then amplified and fed into a phase-sensitive demodulator of a conventional carrier amplifier from which the signals may be recorded as oscillograms. Connected in this manner, the transducer output reflects only the phase unbalance resulting from differences in liquid surface height at the probes. Thus the system is equally effective in cases wherein liquid is being drained from the tank since the capacitance in each of the probes decreases an equal amount and does not generate an unbalance in the bridge.

The capacitance sensitivity of the probes is approximately 60μf per inch amplitude in tapwater and has been found to be linear throughout depth changes of up to 6 feet. This device has been used to measure liquid amplitudes from the order of 0.01 inch up to greater than 6 inches. The complete unit (probes, bridge, carrier amplifier, and oscillograph recorder) when calibrated as a unit has been found to be accurate within ±1.5 percent.

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TABLE I

SUMMARY OF DAMPING FOR ZERO AND POSITIVE EFFLUX RATES

Cylinder diameter, in.	Orifice diameter, in.	Initial liquid amplitude, ζ_o, in.	Damping factor, δ	Incremental damping, δ' (eq. (11))
12	0	0.6	0.0128	0
	0.5	0.6	0.0141	0.000011
	1.0	0.6	0.0128	0.000045
	1.5	0.3	0.0130	0.00010
		0.6	0.0130	
	2.0	0.3	0.0127	0.00018
		0.6	0.0127	
	3.0	0.3	0.0128	0.00041
		0.6	0.0119	
36	0	1.0	0.00500	0
	3.5	0.5	0.00390	0.000062
		1.0	0.00375	
		1.5	0.00417	

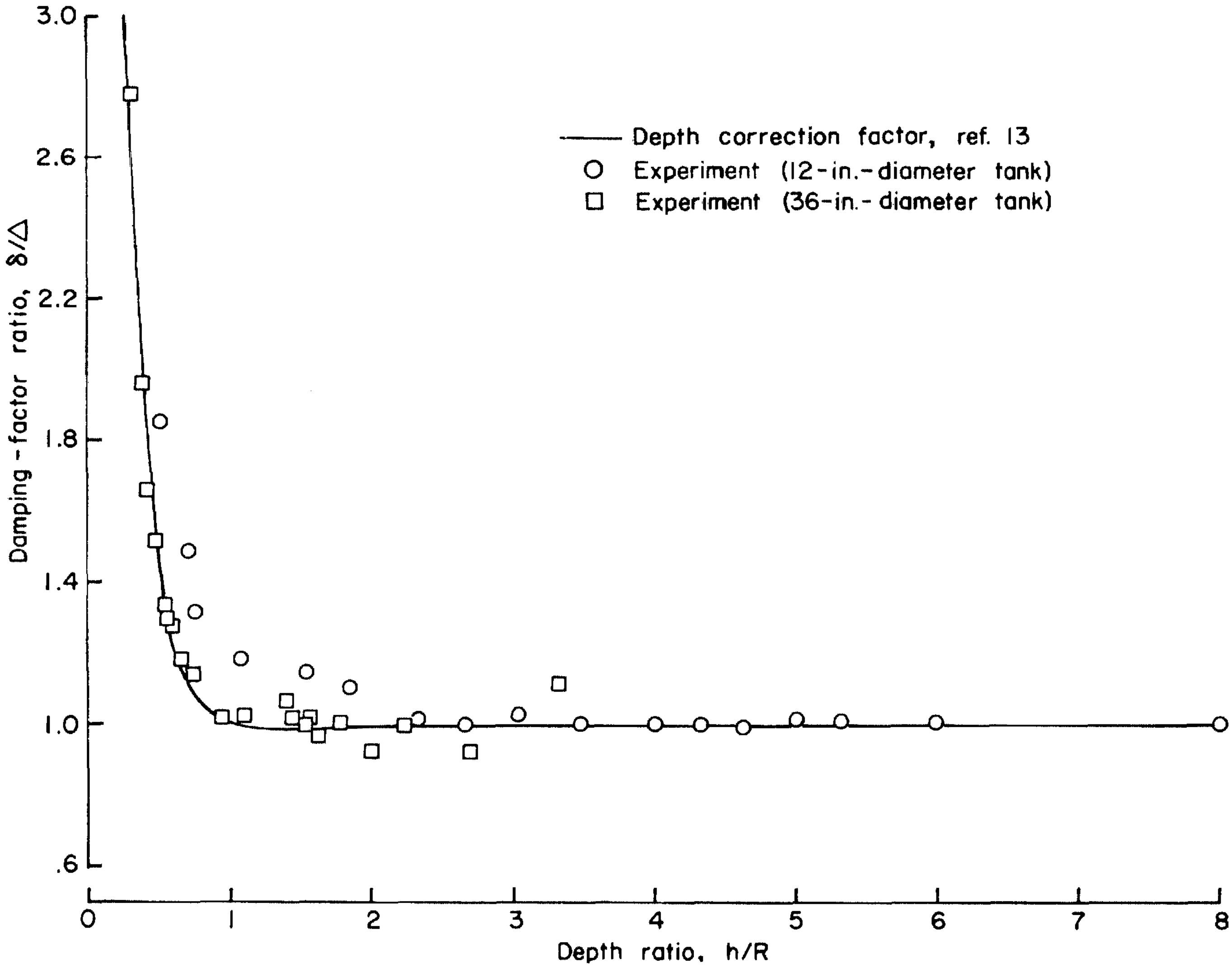
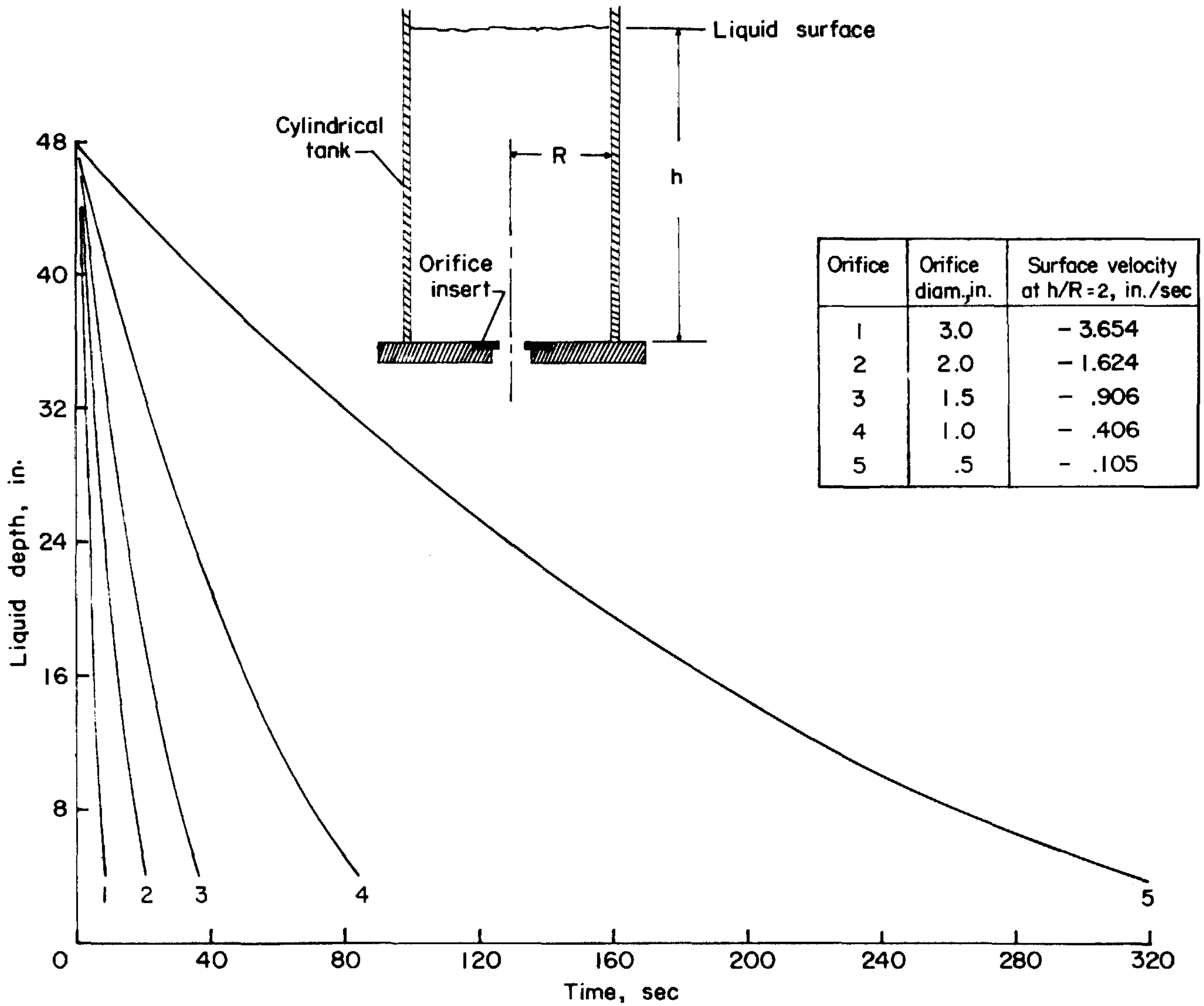


Figure 1.- Experimental and analytical depth correction factor for liquid damping in right-circular cylindrical tanks.



Orifice	Orifice diam.,in.	Surface velocity at h/R = 2, in./sec
1	3.0	-3.654
2	2.0	-1.624
3	1.5	-.906
4	1.0	-.406
5	.5	-.105

Figure 2.- Variation of liquid depth with time in 12-inch-diameter tank.

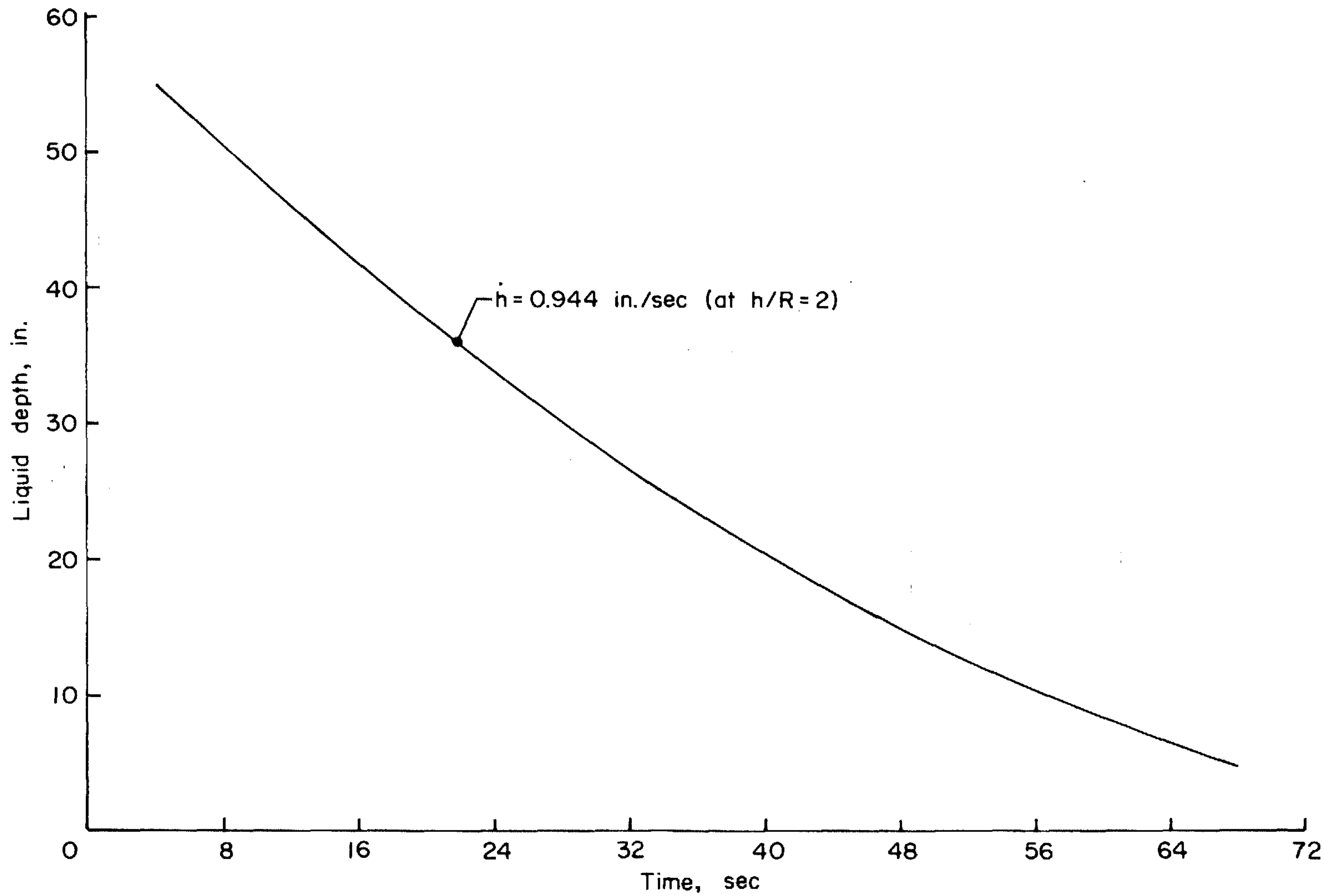
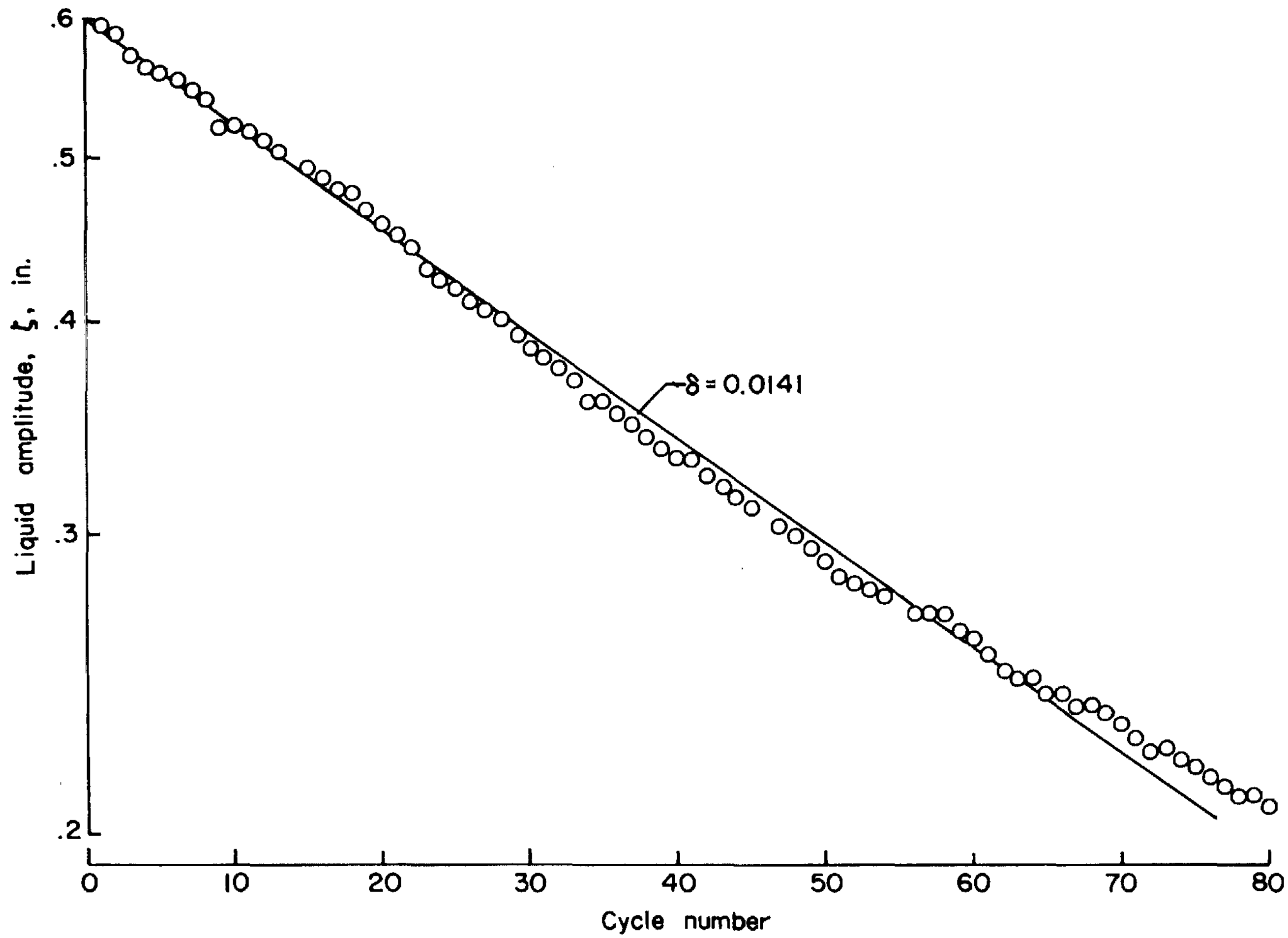
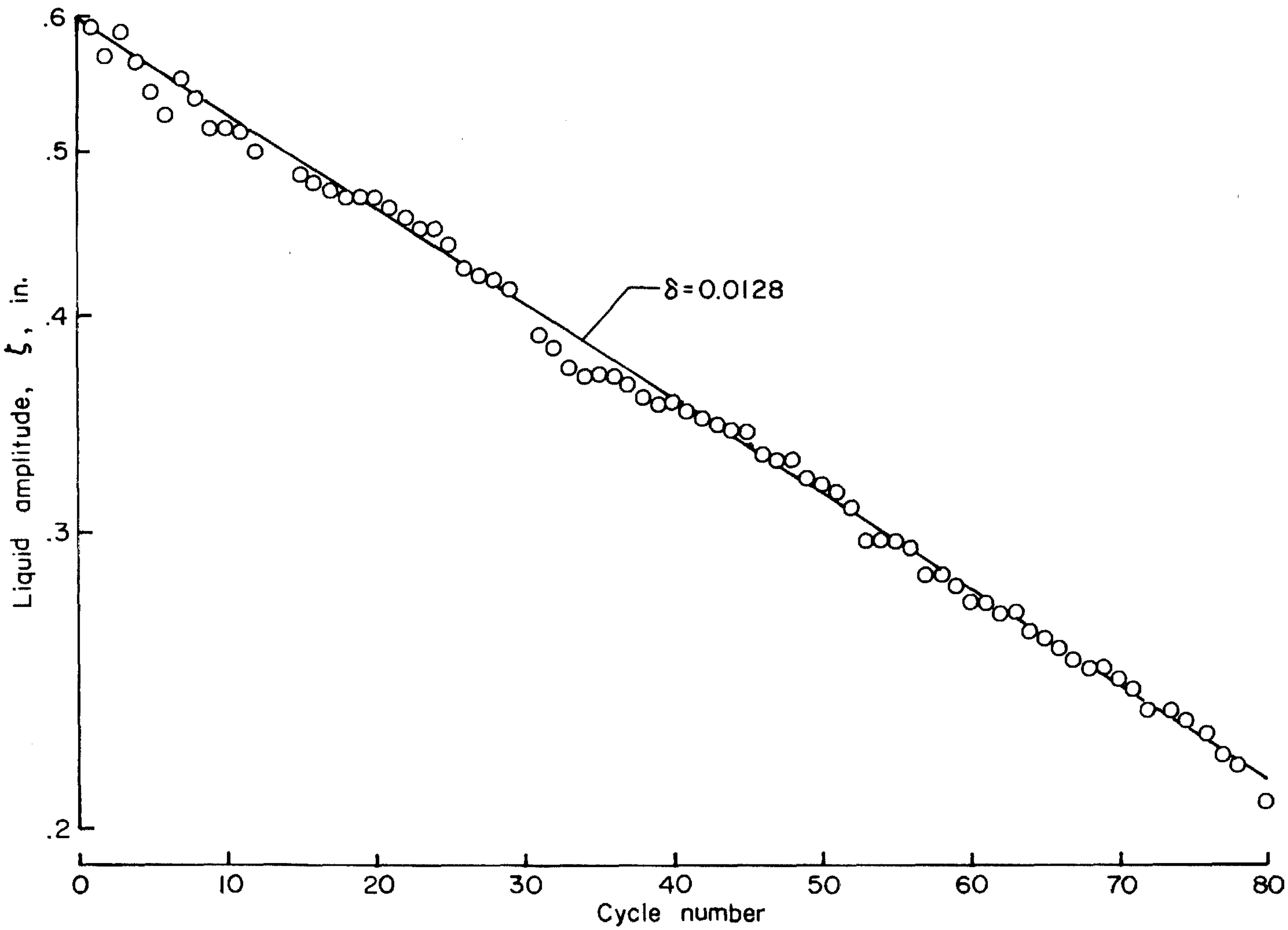


Figure 3.- Variation of liquid depth with time in 36-inch-diameter tank. Orifice diameter, 3.5 inches.



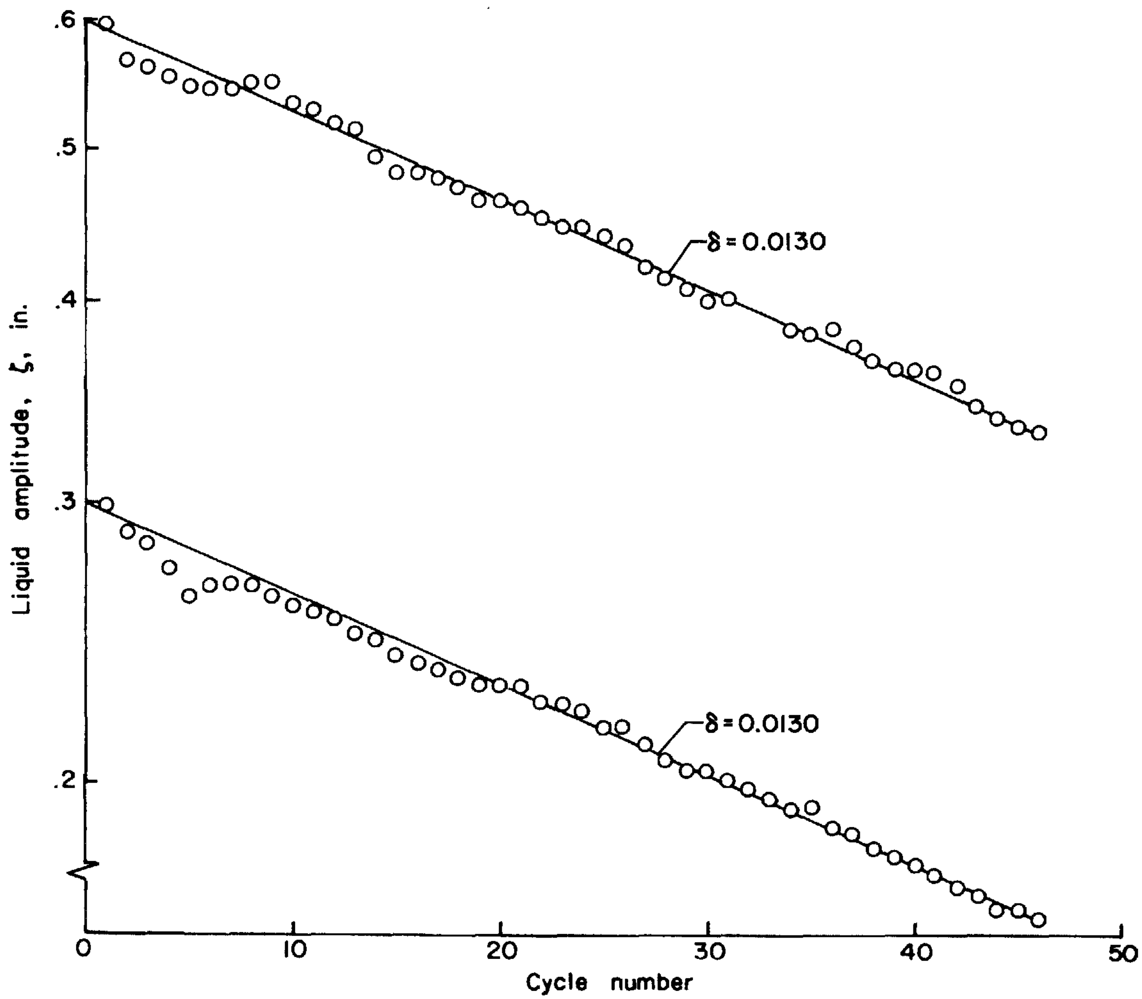
(a) Orifice diameter, 0.5 inch.

Figure 4.- Damping of free-surface oscillations in 12-inch-diameter tank with positive efflux rates. (f = 1.73 cps.)



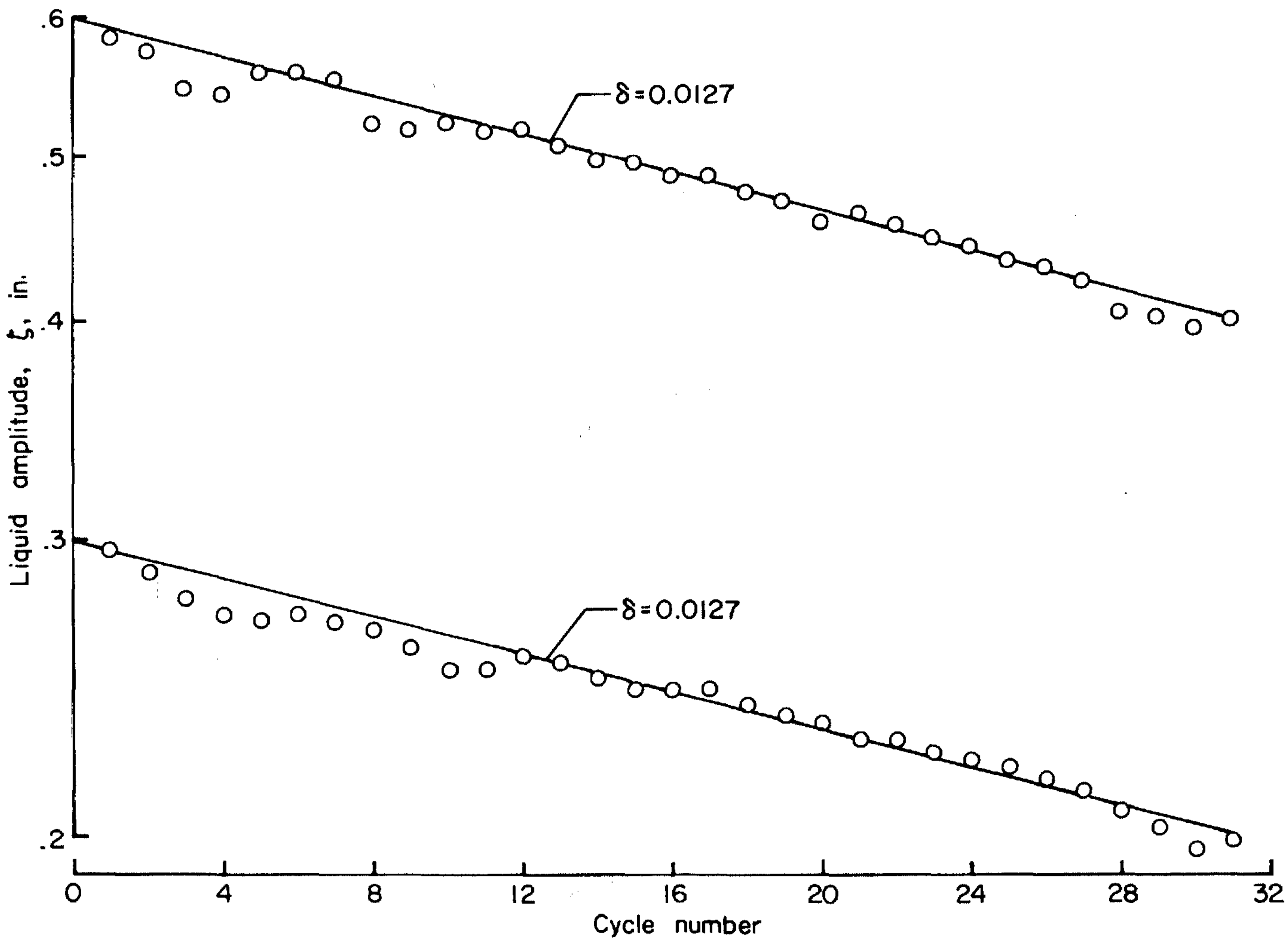
(b) Orifice diameter, 1.0 inch.

Figure 4.- Continued.



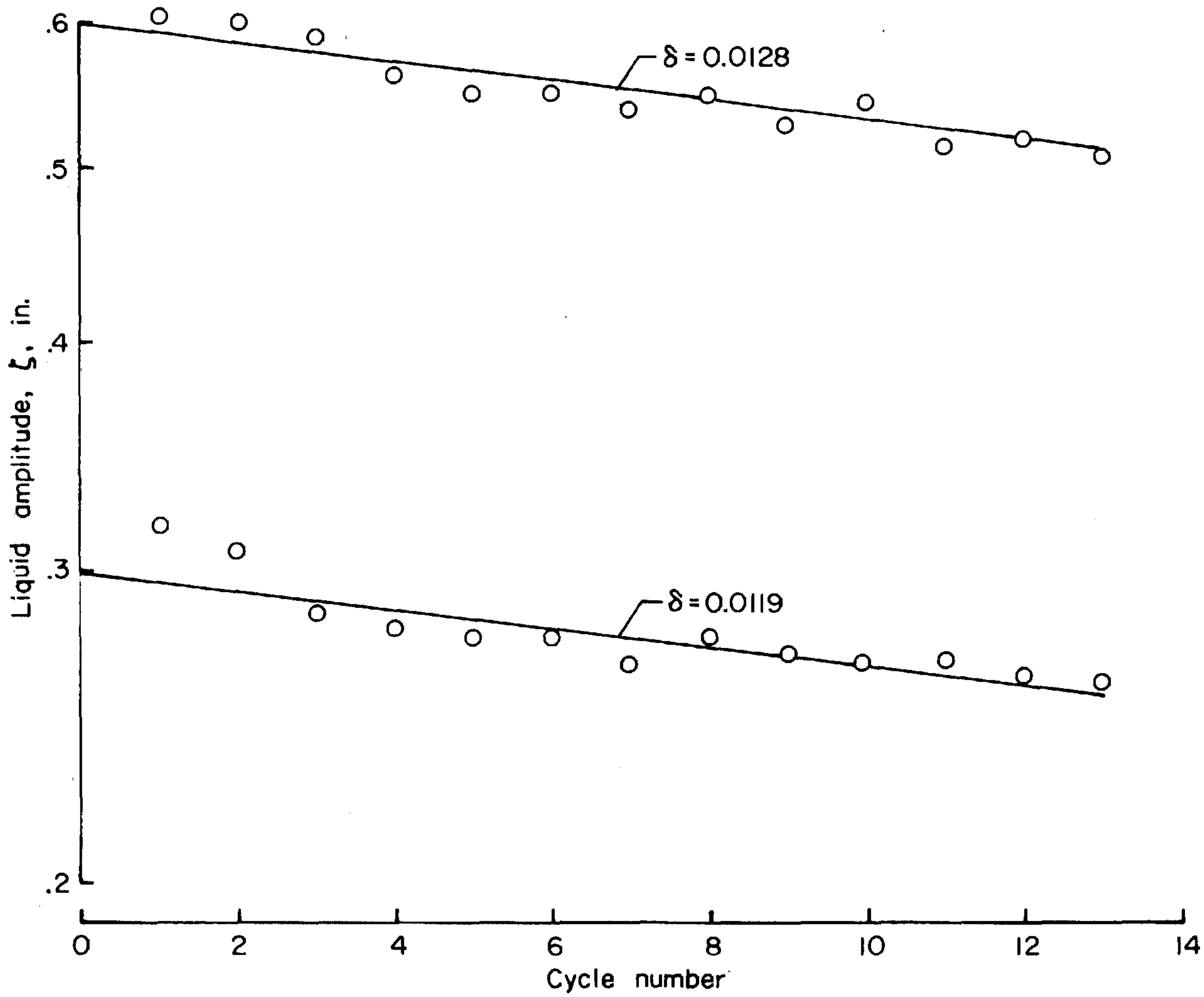
(c) Orifice diameter, 1.5 inches.

Figure 4.- Continued.



(d) Orifice diameter, 2.0 inches.

Figure 4.- Continued.



(e) Orifice diameter, 3.0 inches.

Figure 4.- Concluded.

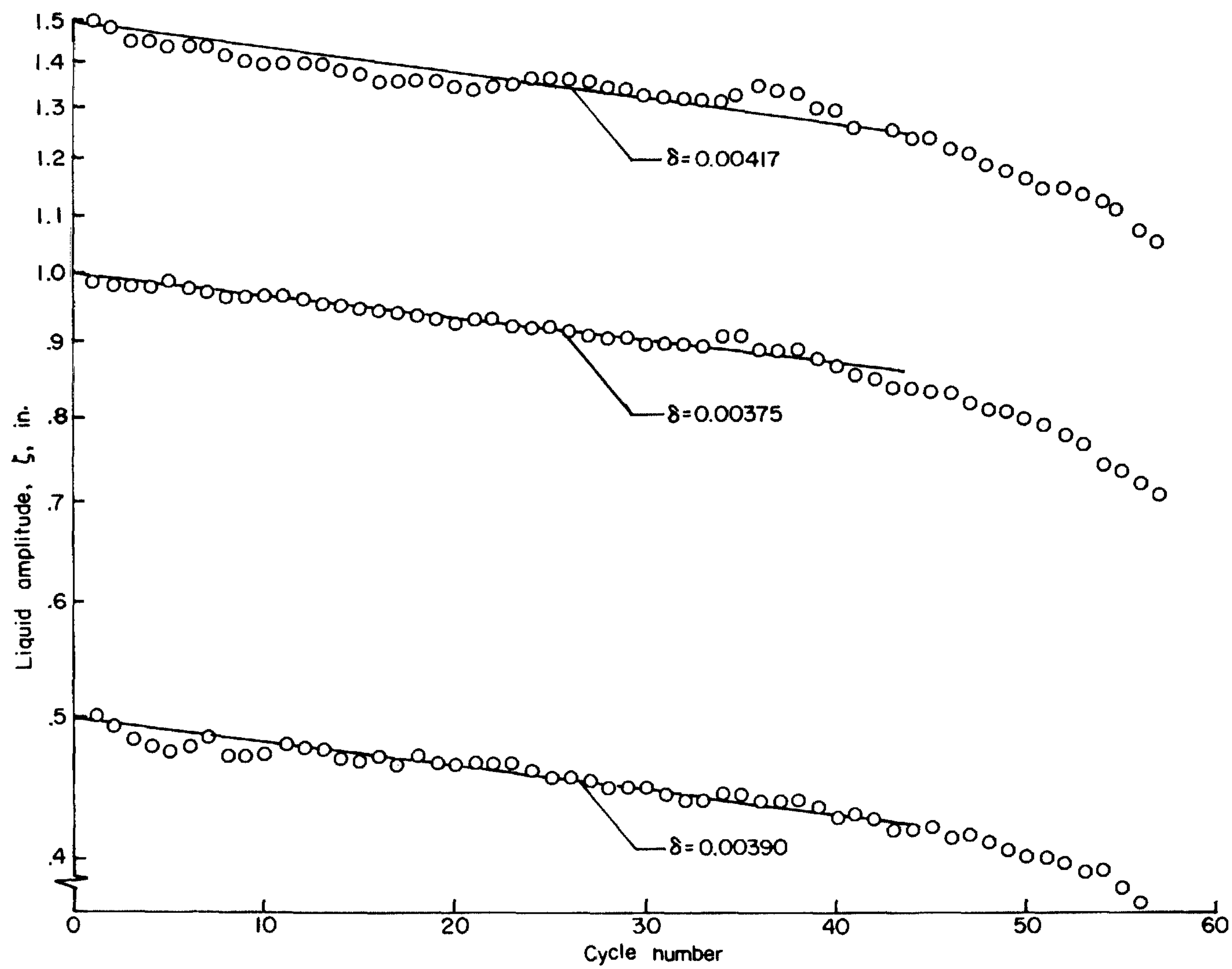
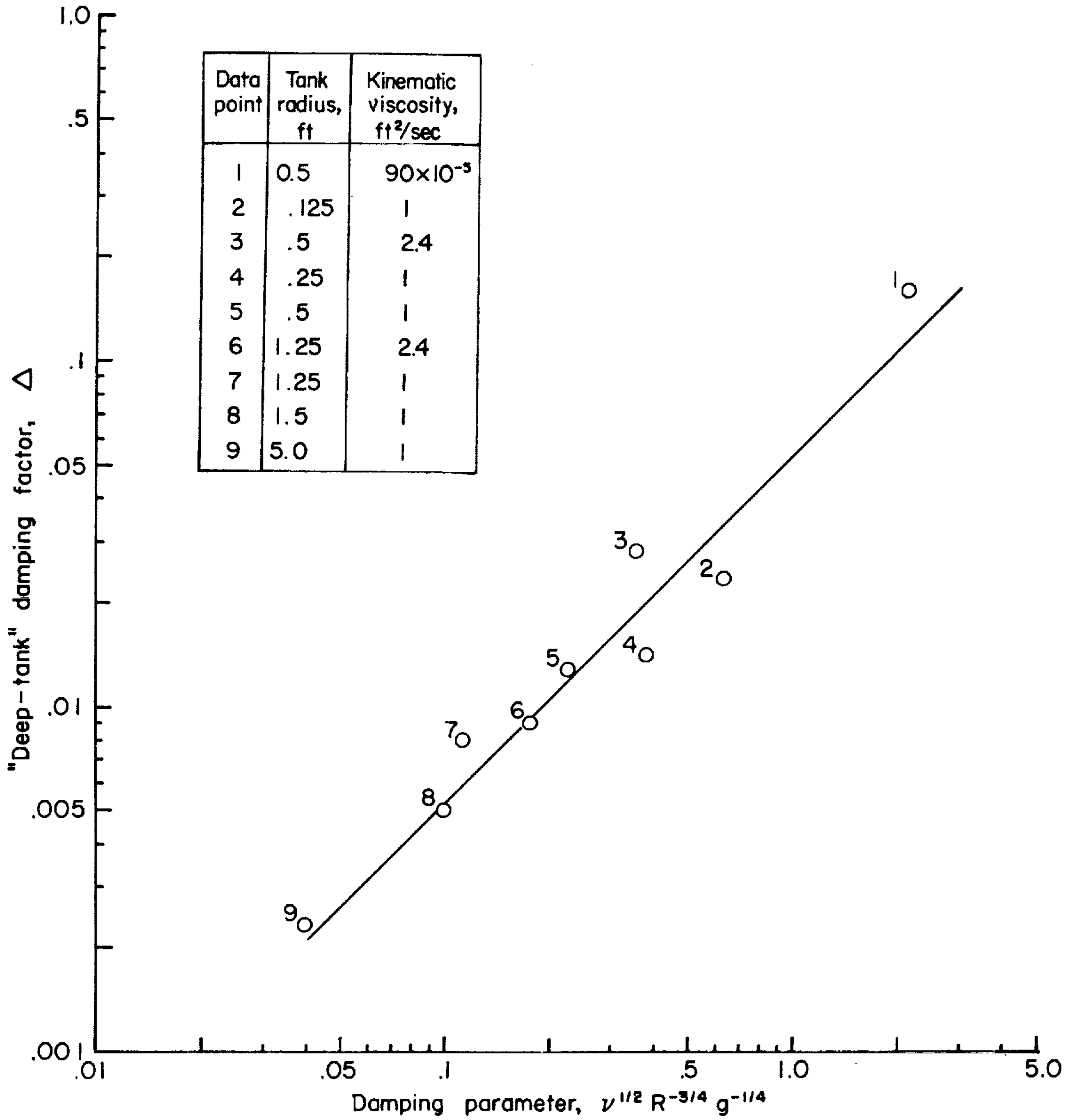
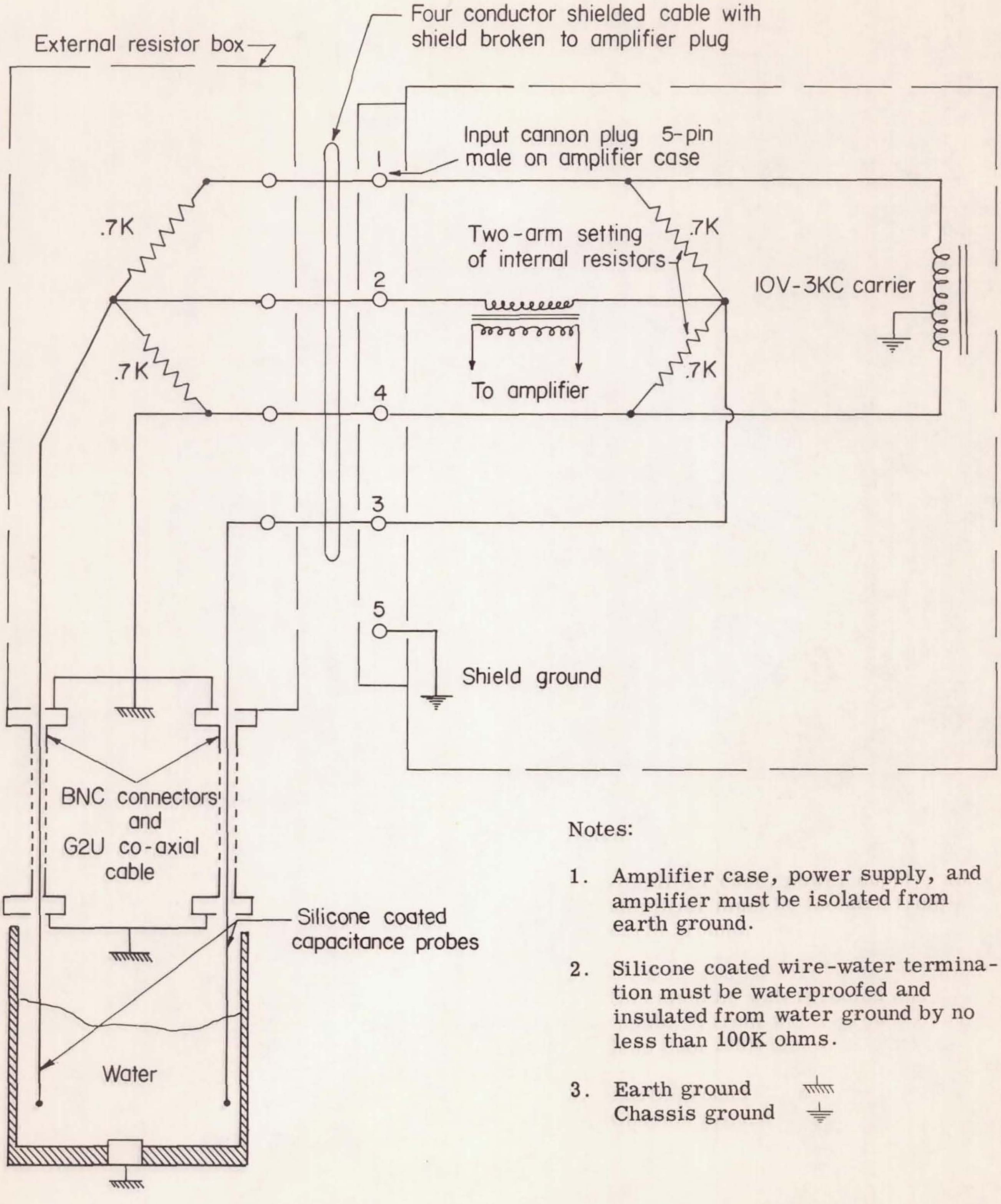


Figure 5.- Damping of free-surface oscillations in 36-inch-diameter tank with positive efflux rate. (Orifice diameter, 3.5 inches; f = 0.999 cps.)



Data point	Tank radius, ft	Kinematic viscosity, ft²/sec
1	0.5	90×10^{-5}
2	.125	1
3	.5	2.4
4	.25	1
5	.5	1
6	1.25	2.4
7	1.25	1
8	1.5	1
9	5.0	1

Figure 6.- Damping—viscosity—tank-size relationships for liquids in cylinders.



Notes:

1. Amplifier case, power supply, and amplifier must be isolated from earth ground.

2. Silicone coated wire-water termination must be waterproofed and insulated from water ground by no less than 100K ohms.

3. Earth ground
   Chassis ground

Figure 7.- Schematic diagram of amplitude transducer.

NASA-Langley, 1963