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A COMPUTER PROGRAM FOR THE PREDICTION OF DUCTED FAN PERFORMANCE

by Michael R. Mendenhall and Selden B. Spangler

Prepared by
NIELSEN ENGINEERING & RESEARCH, INC.
Mountain View, Calif.
for Ames Research Center

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SYMBOLS

A	area of duct exit plane, $\pi D^2/4$
A_n	Fourier series coefficients for the radial velocity induced on the reference cylinder by all the internal vortex cylinders, eq. (10)
A_n^*	Fourier series coefficients for the axial velocity induced on the reference cylinder by all the internal vortex cylinders, eq. (11)
A_p	propeller disk area, $\pi(R_p^2 - R_{CB}{}^2)$
B_n	Fourier series coefficients for the radial velocity induced on the reference cylinder by the outer trailing vortex cylinder, eq. (8)
B_n^*	Fourier series coefficients for the axial velocity induced on the reference cylinder by the outer trailing vortex cylinder, eq. (9)
b	propeller chord length, fig. 2
C_M	pitching moment coefficient, M/RAq or $M/\rho n^2 D_p^5$
C_N	normal force coefficient, N/Aq or $N/\rho n^2 D_p^4$
C_n	Glauert series coefficients for γ_D, eq. (6)
C_p	pressure coefficient, eq. (44)
C_T	thrust coefficient, T/Aq or $T/\rho n^2 D_p^4$
c	chord length of duct
c_ℓ	lift coefficient for fan blade section
c_{ℓ_α}	lift curve slope for fan blade section
c_n	Glauert series coefficients for γ_α, eq. (25)
D	diameter of duct in the exit plane
D_n	Fourier series coefficients for the radial velocity induced on the reference cylinder by the centerbody, eq. (12)
D_n^*	Fourier series coefficients for the axial velocity induced on the reference cylinder by the centerbody, eq. (13)

D_p	propeller diameter
E_n	Fourier series coefficients for the radial velocity induced on the reference cylinder by γ, γ_w, and the centerbody, eq. (29)
E_n^*	Fourier series coefficients for the axial velocity induced on the reference cylinder by γ_D, γ, γ_w, and the centerbody, eq. (34)
E_n'	Fourier series coefficients for the radial velocity induced on the reference cylinder by γ_D, γ, γ_w, and the centerbody, eq. (40)
F_n	Fourier series coefficients for the radial velocity induced on the reference cylinder by the duct-bound vorticity, eq. (43)
F_n^*	Fourier series coefficients for the axial velocity induced on the reference cylinder by the duct-bound vorticity, eq. (38)
$F(x)$	scaling factor
G_n^*	Fourier series coefficients for the axial velocity induced on the reference cylinder by the γ_α vorticity, eq. (37)
H_n	Fourier series coefficients for the radial velocity induced on the reference cylinder by the trailing vortex filaments associated with γ_α
h	fan blade thickness, fig. 2
i	incidence angle of fan blade section measured from the line of zero lift, fig. 2
J	advance ratio, V/nD_p
$\bar{J}$	effective advance ratio, $J \cos \alpha$
J'	advance ratio parameter, eq. (3)
$\bar{J}'$	effective advance ratio parameter, $J' \cos \alpha$
ℓ_{CB}	length of centerbody
M	pitching moment
N	number of fan blades or normal force
n	fan rotational speed, rev/sec
$P_{k\ell}$	coefficients for velocity induced by duct-bound vorticity, eq. (16)

q	free-stream dynamic pressure, $\rho V^2/2$
R	radius of duct exit plane, D/2
R_{CB}	radius of fan root (nominally same as centerbody radius at fan station)
R_n	Fourier series coefficients of duct geometric camberline
R_n^*	Fourier series coefficients of the duct effective camberline, eq. (14)
R_p	radius of fan tip
$\bar{r}$	mean radius of equal area element of fan disk
r_c	radius from duct centerline to duct camberline, fig. 1
r_e	radius from duct centerline to effective duct camberline
r_{max}	maximum radius of centerbody
T	thrust force
t	duct thickness
u	induced axial velocity
u_{CB}	axial velocity induced by centerbody singularity distribution
u_{q_d}	axial velocity induced by duct thickness distribution
u_s	duct surface velocity
u_γ	axial velocity induced by the vortex cylinder trailing from the duct trailing edge
u_{γ_w}	total axial velocity induced by all internal vortex cylinders trailing from the fan
u_{γ_D}	axial velocity induced by duct-bound vorticity, γ_D
V	free-stream velocity
$\bar{V}$	effective free-stream velocity, $V \cos \alpha$
v_{CB}	radial velocity induced by centerbody singularity distribution
v_γ	radial velocity induced by the vortex cylinder trailing from the duct trailing edge

v_{γ_w} total radial velocity induced by all the internal vortex cylinders trailing from the fan

v_{γ_D} radial velocity induced by duct-bound vorticity, γ_D

x axial distance from leading edge of duct

x_{CB} location of centerbody nose in duct coordinate system, fig. 1

x_p fan location within duct

$x_{r_{max}}$ location of maximum centerbody radius in duct coordinate system

x_s axial distance measured from c/2, fig. 1

z number of equal area elements making up fan disk area

α free-stream angle of attack

β fan blade section pitch angle, fig. 2

Γ circulation bound to fan blade, eq. (1)

ϵ convergence criterion

γ strength of outer trailing vortex cylinder

γ_D axially symmetric component of duct-bound vorticity, eq. (6)

γ_w strength of w^{th} inner trailing vortex cylinder

γ_α duct-bound vorticity component due to angle of attack, eq. (25)

Δp rise in static and total pressure across actuator disk

θ transformed axial distance, $x = 1/2\ (1 - c \cos \theta)$

ρ free-stream density

ϕ azimuthal angle

ω fan rotational speed, rad/sec

Subscripts

DP	for the complete ducted propeller
D(P)	for the duct in the presence of the propeller
$D(\alpha)$	for the duct at angle of attack
P(D)	for the propeller shrouded by the duct
$_{3/4}$	at 3/4 radius location of the fan blade
tip	at the tip of the fan blade
z	indicating the outer trailing vortex cylinder or outer fan annulus

A COMPUTER PROGRAM FOR THE PREDICTION OF DUCTED FAN PERFORMANCE

By Michael R. Mendenhall and Selden B. Spangler
Nielsen Engineering & Research, Inc.

SUMMARY

This document is a user's manual for a computer program developed to determine the performance of a ducted fan in axial flow and at angle of attack. The program is used to predict the performance at a specified advance ratio and angle of attack of a given fan-duct combination, which is specified by the radial distributions of blade pitch, chord, and thickness; the duct chord, diameter, camber, and thickness distribution; the fan location; and the centerbody geometry. The information obtained from the program is duct and fan thrust and ducted fan normal force and pitching moment coefficient. Radial distributions of fan inflow velocity and blade angle of attack are also obtained. The program calculates the duct surface pressure distribution at any specified azimuthal angle. The program is written in Fortran IV for the IBM 7094 computer and requires approximately two minutes running time per case. Included in this manual are a brief description of the theory and the calculation procedure, descriptions of input and output, program listing, sample cases, and some comparisons with data.

1. INTRODUCTION

This report is one of two documents prepared under Contract NAS2-4953 for the Ames Research Center, NASA. The work on this contract is concerned with development of methods for predicting the aerodynamic performance of ducted fans in uniform flow. This document is a user's manual for the computer program developed under the contract. The second document (ref. 1) describes the analysis on which the computer program is based.

The authors and their associates have done a considerable amount of prior work on ducted fan analysis (refs. 2-6). The final task of that work involved the preparation of a computer program for calculating the aerodynamic characteristics of a ducted fan in a uniform, axial flow (ref. 6). The purpose of the present investigation is to make certain improvements and additions to the computer program of reference 6. The additional analysis required to make these improvements is reported in reference 1. The improvements consist of adding a capability for angle of attack flow, computing duct surface pressure distributions, adding a centerbody model, and removing certain restrictions on advance ratio and nonlinear blade lift characteristics. This report includes a brief discussion of the theoretical approach and the assumed flow model with a summary of the equations used in the program. No derivations are included, but references are given to all the derivations of interest. The actual operation of the program is discussed along with descriptions of input and output. Program listings and sample cases are also included. Some brief comparisons with data are presented to give some information on the range of usefulness of the program. This document completely supersedes reference 6.

2. THEORETICAL APPROACH AND FLOW MODEL

The discussion of this section is intended to describe the theoretical approach in sufficient detail to permit the user of the program to understand the sequence of calculations in the program. The equations in the program are given with references to the source in which each is derived.

The flow model is inviscid and is based on potential flow theory. The approach used is to decouple the axial flow and angle of attack problems, treat each individually, and superimpose the solutions.

2.1 Axial Flow

The analysis for axial flow is basically that described in reference 5 with the modifications and additions described in reference 1. Singularity distributions are used to represent the fan wake, the duct loading, the duct thickness, and the centerbody. The basic axisymmetric flow model is shown in figure 1(a). A duct reference cylinder is defined as that cylinder having the same radius as the duct trailing edge. The reference cylinder is used in conjunction with the boundary condition imposed on the duct.

The duct may have both thickness and camber, and the chord-to-diameter ratio may have any value.[1] The fan configuration is specified by the number of blades (N) and by the radial distribution of chord (b), pitch (β), and thickness (h) as shown in figure 2. The effect of blade camber is not directly considered; however, the pitch angle, β, is assumed to be the angle between the plane of rotation and the zero lift line of the local blade section. Each blade section is assumed to have a lift coefficient slope, c_{ℓ_α}, of 2π up to the point of local blade stall. This point is assumed to be a function only of the blade thickness-to-chord ratio as described in reference 1. After the blade section stalls, the section lift coefficient is assumed to be constant and equal to $c_{\ell_{max}}$. Since blade element theory is used to determine the fan performance, use of the program is limited to ducted fans with relatively low blade solidity. The upper limit on solidity has not been estimated because of the lack of suitable data for comparing predicted and measured performance.

The fan annulus is divided into a number ($z \leq 24$) of equal area annuli in each of which blade element theory is used to describe local blade performance. The bound circulation (Γ) is constant within each annulus and a vortex cylinder with strength γ_w is assumed to be shed from each annulus and to extend downstream along the duct axis. The outer vortex cylinder is assumed to lie along the duct reference cylinder and to be shed from the duct trailing edge.

From equation (5) of reference 5, the bound circulation on the portion of the blade in the w^{th} annulus is

$$\frac{\Gamma_w}{RV} = \frac{1}{2} c_\ell \frac{b_w}{R} \frac{u_w}{V} \left[\left(\frac{\bar{r}_w \omega}{u_w} \right)^2 + 1 \right]^{1/2} \tag{1}$$

[1] There is an upper limit on c/D of approximately 2.5 imposed by the absence of $P_{k\ell}$ values and induced camber coefficients for c/D values greater than 2. See p. 7 for further details.

and the strength of the w^{th} internal trailing vortex cylinder is

$$\frac{\gamma_w}{V} = \left[\left(1 + \sum_{m=w+1}^{z} \frac{\gamma_m}{V}\right)^2 + \frac{N}{\pi J'}\left(\frac{\Gamma_w}{RV} - \frac{\Gamma_{w+1}}{RV}\right)\right]^{1/2} - \left(1 + \sum_{m=w+1}^{z} \frac{\gamma_m}{V}\right) \tag{2}$$

where

$$J' = \frac{V}{R\omega} \tag{3}$$

The strength of the outer vortex cylinder is

$$\frac{\gamma_z}{V} = \left[1 + \frac{N}{\pi J'}\left(\frac{\Gamma_z}{RV}\right)\right]^{1/2} - 1 \tag{4}$$

In many of the following equations, γ is used as a normalizing parameter. When γ is used without a subscript it designates the strength of the outer cylinder, γ_z.

Assuming that the inflow to the fan (u/V) is known, the radial distribution of bound circulation may be calculated using equation (1), and the strength of the vortex cylinders can be obtained from equations (2) and (4). However, the inflow to a fan in a specified duct at a given flight condition is not known a priori. The inflow is made up of the free stream plus the axial velocity components induced by all the singularity distributions. Thus, the inflow to the propeller at a given radial station is

$$\frac{u_m}{V} = 1 + \frac{u_{\gamma_D}}{V} + \frac{u_\gamma}{V} + \frac{u_{q_D}}{V} + \frac{u_{CB}}{V} + \frac{1}{2}\sum_{w=m}^{z-1} \frac{\gamma_w}{V} \tag{5}$$

With the exception of the centerbody-induced flow, the components of the inflow velocity are computed in the same manner as is described in Appendix A of reference 5. The velocity (u_{CB}) induced by the centerbody is not considered in reference 5 but is computed using equation (15) of reference 1. The velocities induced by the centerbody must be corrected to account for the fact that the centerbody is submerged in a free stream greater than V. The details of this correction are presented in reference 1. Note that the inflow profile is dependent on the duct and fan loadings which, in turn, are affected by the inflow profile; therefore, an iterative scheme is used to converge on a solution.

The trailing vortex cylinder, the fan wake vortex cylinders, the centerbody, and the free stream all induce components of flow through the duct camberline. In order to cancel this flow and cause the net flow to be tangent to the camberline, a distribution of bound vorticity (γ_D) is placed on the duct. This bound vorticity is expressed in terms of a Glauert series (eq. (17), ref. 4) as

$$\frac{\gamma_D}{\gamma} = C_o \cot \frac{\theta}{2} + \sum_{n=1}^{5} C_n \sin n\theta \tag{6}$$

and the unknown C_n coefficients are determined from the flow tangency condition on the duct reference cylinder.

The basic relation that is solved to cause the flow to be tangent to the camberline is

$$v_{\gamma_D} + v_{\gamma} + v_{\gamma_w} + v_{CB} = \frac{dr_e}{dx}\left(V + u_{\gamma_D} + u_{\gamma} + u_{\gamma_w} + u_{CB}\right) \tag{7}$$

where dr_e/dx is the effective slope of the camberline. The following procedure is used to solve equation (7) for the unknown C_n coefficients.

The radial velocity (v_γ) induced along the duct reference cylinder by the vortex cylinder trailing from the duct trailing edge is given by equation (C-3) of reference 5. This velocity is expressed in non-dimensional form by a six-term Fourier cosine series as

$$\frac{v_\gamma}{\gamma} = \sum_{n=o}^{5} B_n \cos n\theta \tag{8}$$

The axial velocity (u_γ) induced by the same vortex cylinder is given by equation (23) of reference 3. This velocity is expressed in non-dimensional form by a similar six-term Fourier cosine series as

$$\frac{u_\gamma}{\gamma} = \sum_{n=o}^{5} B_n^* \cos n\theta \tag{9}$$

The radial and axial velocities induced along the duct reference cylinder by all the vortex cylinders trailing from the fan are computed using equation (13) and (20), respectively, of reference 3. The sum of the radial and axial velocities induced by all the internal vortex cylinders is expressed as

$$\frac{v_{\gamma_w}}{V} = \sum_{n=o}^{5} A_n \cos n\theta \tag{10}$$

$$\frac{u_{\gamma_w}}{V} = \sum_{n=o}^{5} A_n^* \cos n\theta \tag{11}$$

The radial and axial velocities induced along the duct reference cylinder by the centerbody singularity distribution are computed using equations (15) and (16) of reference 1. These velocities are expressed as

$$\frac{v_{CB}}{V} = \sum_{n=o}^{5} D_n \cos n\theta \tag{12}$$

$$\frac{u_{CB}}{V} = \sum_{n=o}^{5} D_n^* \cos n\theta \tag{13}$$

The slope of the effective camberline is specified as

$$\frac{dr_e}{dx} = \sum_{n=o}^{3} R_n^* \cos n\theta \tag{14}$$

where the effective camber is made up of a geometric camber and an induced camber as discussed in detail in section 2.3 of reference 6. The induced camber is due to the velocity induced by the source ring distribution representing the duct thickness, and the method of computing this effect is based on the work described in Chapter 2 of reference 7. The induced camber is described by a Fourier series similar to equation (14) where the first four coefficients are tabulated as a function of c/D in reference 6. These coefficients are included in the program in Subroutine CAMBER and are automatically combined with the geometric camber coefficients. The calculation of the geometric camber coefficients is described in section 4.2 of this report.

The axial velocity induced by the bound vorticity, γ_D, can be written in terms of the C_n coefficients as

$$\frac{4D}{c} \frac{u_{\gamma_D}}{\gamma} = \left(\ell n \frac{16D}{c} - 1\right)\left(C_o + \frac{C_1}{2}\right) + \left(\frac{2C_o + C_2}{2}\right) \cos\theta + \left(\frac{C_3 - C_1}{4}\right) \cos 2\theta + \left(\frac{C_4 - C_2}{6}\right) \cos 3\theta + \left(\frac{C_5 - C_3}{8}\right) \cos 4\theta \tag{15}$$

from equation (18) of reference 4. The radial velocity component can be found from equation (1.13) of reference 8 as

$$\frac{v_{\gamma_D}}{\gamma} = \frac{C_o}{2} - \sum_{\ell=o}^{5} \frac{C_\ell}{2} P_{o\ell} + \sum_{k=1}^{5} \cos k\theta \left(-\frac{C_k}{2} + \sum_{\ell=o}^{5} \frac{C_\ell}{2} P_{k\ell}\right) \tag{16}$$

where the $P_{k\ell}$ coefficients are given in tables 2.1 through 2.4 of reference 9. These tables have been extended to c/D values of 2.0, and provision has been made in the program to extrapolate beyond 2.0. However, the variation of $P_{k\ell}$ with c/D is not linear, and the extrapolation should not be made much past 2.5.

Substituting equations (8) through (16) into equation (7) results in

$$\sum_{n=o}^{5} \left(\frac{\gamma}{V} B_n + A_n + D_n\right) \cos n\theta + \frac{{}^{v}\gamma_D}{\gamma}\left(\frac{\gamma}{V}\right)$$

$$= \sum_{m=o}^{3} R_m^* \cos n\theta \left[\sum_{n=o}^{5} \left(\frac{\gamma}{V} B_n^* + A_n^* + D_n^*\right) \cos n\theta + \frac{{}^{u}\gamma_D}{\gamma}\left(\frac{\gamma}{V}\right) + 1 \right] \tag{17}$$

Expanding both sides of equation (17) into Fourier cosine series and equating each of the six harmonics results in six linear algebraic equations in terms of the six unknowns, C_n. With the parameter e defined below,

$$e \equiv \frac{c}{4D} \tag{18}$$

the equation for the zeroth harmonic is

$$\left[(1 - P_{oo}) - 2R_o^* e \left(\ell n \frac{4}{e} - 1\right) - R_1^* e\right] C_o$$

$$+ \left[\frac{R_2^* e}{4} - R_o^* e \left(\ell n \frac{4}{e} - 1\right)\right] C_1 + \left[\frac{R_3^* e}{6} - \frac{R_1^* e}{2} - P_{o2}\right] C_2$$

$$+ \left[\frac{-R_2^* e}{4}\right] C_3 + \left[- \frac{R_3^* e}{6} - P_{o4}\right] C_4 + \left[0\right] C_5$$

$$= 2R_o^* B_o^* + R_1^* B_1^* + R_2^* B_2^* + R_3^* B_3^* - 2B_o$$

$$+ \frac{V}{\gamma} \Big[2R_o^* (D_o^* + A_o^* + 1) + R_1^* (D_1^* + A_1^*) + R_2^* (D_2^* + A_2^*)$$

$$+ R_3^* (D_3^* + A_3^*) - 2(D_o + A_o) \Big] \tag{19}$$

The equation for the first harmonic is

$$\left[P_{10} - 2R_o^*e - 2R_1^*e\left(\ln\frac{4}{e} - 1\right) - R_2^*e\right] C_o$$

$$+ \left[(P_{11} - 1) - R_1^*e\left(\ln\frac{4}{e} - 1\right) + \frac{R_1^*e}{4} + \frac{R_3^*e}{4}\right] C_1$$

$$+ \left[-R_o^*e - \frac{R_2^*e}{3}\right] C_2 + \left[P_{13} - \frac{R_1^*e}{4} - \frac{R_3^*e}{8}\right] C_3$$

$$+ \left[-\frac{R_2^*e}{6}\right] C_4 + \left[P_{15} - \frac{R_3^*e}{8}\right] C_5$$

$$= 2R_o^*B_1^* + 2R_1^*B_o^* + R_1^*B_2^* + R_2^*(B_1^* + B_3^*) + R_3^*(B_2^* + B_4^*) - 2B_1$$

$$+ \frac{V}{\gamma}\Big[2R_o^*(D_1^* + A_1^*) + 2R_1^*(D_o^* + A_o^* + 1) + R_1^*(D_2^* + A_2^*)$$

$$+ R_2^*(D_1^* + D_3^* + A_1^* + A_3^*) + R_3^*(D_2^* + D_4^* + A_2^* + A_4^*)$$

$$- 2(D_1 + A_1)\Big] \qquad (20)$$

The equation for the second harmonic is

$$\left[P_{20} - e(R_1^* + R_3^*) - 2R_2^*e\left(\ln\frac{4}{e} - 1\right)\right] C_o$$

$$+ \left[\frac{R_o^*e}{2} - R_2^*e\left(\ln\frac{4}{e} - 1\right)\right] C_1 + \left[(P_{22} - 1) - \frac{R_1^*e}{3} - \frac{R_3^*e}{2}\right] C_2$$

$$+ \left[\frac{R_2^*e}{8} - \frac{R_o^*e}{2}\right] C_3 + \left[P_{24} - \frac{R_1^*e}{6} + \frac{R_3^*e}{10}\right] C_4 + \left[-\frac{R_2^*e}{8}\right] C_5$$

$$= 2R_o^*B_2^* + R_1^*(B_1^* + B_3^*) + R_2^*(2B_o^* + B_4^*) + R_3^*(B_1^* + B_5^*) - 2B_2$$

$$+ \frac{V}{\gamma}\Big[2R_o^*(D_2^* + A_2^*) + (R_1^* + R_3^*)(D_1^* + A_1^*) + R_1^*(D_3^* + A_3^*)$$

$$+ 2R_2^*(D_o^* + A_o^* + 1) + R_2^*(D_4^* + A_4^*) + R_3^*(D_5^* + A_5^*) - 2(D_2 + A_2)\Big]$$

$$(21)$$

The equation for the third harmonic is

$$\left[P_{3o} - R_2^*e - 2R_3^*e\left(\ell n\,\frac{4}{e} - 1\right)\right] C_o$$

$$+ \left[P_{31} + \frac{R_1^*e}{4} - R_3^*e\left(\ell n\,\frac{4}{e} - 1\right)\right] C_1 + \left[\frac{R_o^*e}{3} - \frac{R_2^*e}{2}\right] C_2$$

$$+ \left[(P_{33} - 1) - \frac{R_1^*e}{8}\right] C_3 + \left[\frac{R_2^*e}{10} - \frac{R_o^*e}{3}\right] C_4 + \left[P_{35} - \frac{R_1^*e}{8}\right] C_5$$

$$= 2R_o^*B_3^* + R_1^*(B_2^* + B_4^*) + R_2^*(B_1^* + B_5^*) + 2R_3^*B_o^* - 2B_3$$

$$+ \frac{V}{\gamma}\Big[2R_o^*(D_3^* + A_3^*) + R_1^*(D_2^* + A_2^* + D_4^* + A_4^*)$$

$$+ R_2^*(D_1^* + A_1^* + D_5^* + A_5^*) + 2R_3^*(D_o^* + A_o^* + 1) - 2(D_3 + A_3)\Big] \quad (22)$$

The equation for the fourth harmonic is

$$\left[P_{4o} - R_3^*e\right] C_o + \left[\frac{R_2^*e}{4}\right] C_1 + \left[P_{42} + \frac{R_1^*e}{6} - \frac{R_3^*e}{2}\right] C_2$$

$$+ \left[\frac{R_o^*e}{4} - \frac{R_2^*e}{4}\right] C_3 + \left[(P_{44} - 1) - \frac{R_1^*e}{15}\right] C_4 + \left[-\frac{R_o^*e}{4}\right] C_5$$

$$= 2R_o^*B_4^* + R_1^*(B_3^* + B_5^*) + R_2^*B_2^* + R_3^*B_1^* - 2B_4$$

$$+ \frac{V}{\gamma}\Big[2R_o^*(D_4^* + A_4^*) + R_1^*(D_3^* + A_3^* + D_5^* + A_5^*)$$

$$+ R_2^*(D_2^* + A_2^*) + R_3^*(D_1^* + A_1^*) - 2(D_4 + A_4)\Big] \quad (23)$$

The equation for the fifth harmonic is

$$\left[P_{50}\right] C_o + \left[P_{51} + \frac{R^*e_3}{4}\right] C_1 + \left[\frac{R^*e_2}{6}\right] C_2 + \left[P_{53} + \frac{R^*e_1}{8} - \frac{R_3^*e}{4}\right] C_3$$

$$+ \left[\frac{R_o^*e}{5} - \frac{R_2^*e}{6}\right] C_4 + \left[(P_{55} - 1) - \frac{R^*e_1}{8}\right] C_5$$

$$= 2R_o^*B_5^* + R_1^*B_4^* + R_2^*B_3^* + R_3^*B_2^* - 2B_5 + \frac{V}{\gamma}\left[2R_o^*(D_5^* + A_5^*)\right.$$

$$\left. + R_1^*(D_4^* + A_4^*) + R_2^*(D_3^* + A_3^*) + R_3^*(D_2^* + A_2^*) - 2(D_5 + A_5)\right] \tag{24}$$

Solution of equations (19) through (24) yields the C_n coefficients defining the duct-bound vorticity, which in turn permit a new inflow profile to be computed from equation (5).

2.2 Angle of Attack

The angle of attack solution is that for a thin, cylindrical duct in a crossflow $V \sin \alpha$. This solution is superimposed on the axial flow solution as follows. The ducted fan is considered to be in a modified axial flow in which the velocity $\overline{V}$ is $V \cos \alpha$. The axial flow analysis discussed in section 2.1 is applied to obtain the axisymmetric portion of the bound and free vorticity and source distributions. The duct is then considered to be in a flow at angle of attack. A nonaxisymmetric vorticity distribution is placed on the duct to cancel the $V \sin \alpha$ crossflow through the duct reference cylinder. The duct-bound vorticity has the form

$$\frac{\gamma_\alpha}{V} = \sin \alpha \cos \phi \left[c_o \cot \frac{\theta}{2} + \sum_{n=1}^{5} c_n \sin n\theta\right] \tag{25}$$

where the c_n coefficients are functions only of c/D and are tabulated in Table I of reference 10. There is in addition a distribution of free trailing filaments caused by the variation of strength around the γ_α rings. The forces and moments on the ducted fan are then computed by considering both the axisymmetric and nonaxisymmetric singularity distributions.

The $V \sin \alpha$ crossflow and the γ_α vorticity induce a nonaxisymmetric inflow into the fan. The resulting circumferentially varying blade loading was examined in reference 5. A solution was obtained, with a number of simplifying assumptions, that indicated two compensating effects occurring to cause the net effect of angle of attack on blade load distribution to be relatively small. On the basis of these results and the complexity of the analysis, the inclusion of angle of attack effects on fan loading in the computer program was not considered justified.

The force and moment equations are derived in references 1 and 5 by considering the force resulting from a velocity acting on a bound vorticity. The viscous drag on the duct, centerbody, and fan blades is not considered in the analysis. The forces and moments on the centerbody and the moment on the fan are small compared to the duct loading and are neglected.

The effective free stream is defined by the variables

$$\bar{J}' = J' \cos \alpha$$
$$\bar{V} = V \cos \alpha \qquad (26)$$

Thus, the fan thrust coefficient[2] is given by equation (10) of reference 5 as

$$C_{T_{P(D)}} = \frac{A_p}{A} \frac{N}{\pi \bar{J}' z} \sum_{n=1}^{z} \frac{\Gamma_n}{R\bar{V}} \cos^2 \alpha \qquad (27)$$

The duct thrust coefficient due to the duct singularity distribution is computed using equation (36) of reference 1.

[2]The equations for the force and moment coefficients given in this section are based on nondimensionalizing the force or moment by q, A, and R. The alternate approach noted in the Symbols List is also provided within the program.

$$C_{T_{D(P)}} = -\pi \frac{c}{D} \frac{\gamma}{\overline{V}} \cos^2 \alpha \left[C_o (4E_o + 2E_1) + 2C_1 E_o \right.$$

$$\left. + \sum_{n=1}^{4} (C_{n+1} E_n - C_n E_{n+1}) \right] \qquad (28)$$

where

$$E_n = \frac{\gamma}{\overline{V}} B_n + A_n + D_n \qquad (29)$$

In the assumed flow model, there is a discontinuity in the duct surface pressure distribution due to the pressure rise across the fan. The increased pressure acting on the inner duct surface aft of the fan causes the following duct thrust coefficient.

$$C_{T_{D(P)}} = \left[1 - \left(\frac{R_p}{R} \right)^2 \right] \frac{\Delta p_z}{\overline{q}} \cos^2 \alpha \qquad (30)$$

One component of the thrust coefficient due to the duct at angle of attack is

$$C_{T_{D(\alpha)}} = \pi \frac{c}{D} \sin^2 \alpha \ (2c_o + c_1) \qquad (31)$$

from equation (39) of reference 1. The second part of the thrust coefficient due to the duct at angle of attack is

$$C_{T_{D(\alpha)}} = -\frac{\pi}{2} \frac{c}{D} \sin^2 \alpha \left[c_o (4H_o + 2H_1) + 2c_1 H_o + \sum_{n=1}^{4} (c_{n+1} H_n - c_n H_{n+1}) \right] \qquad (32)$$

from equation (41) of reference 1. The H_n Fourier series coefficients describe the radial velocity distribution induced at the duct by the trailing vortex filaments associated with γ_α. These Fourier coefficients are defined in equation (40) of the above reference. The total duct thrust coefficient is then the sum of equations (27), (28), (31), and (32).

The duct normal force coefficient is given by equations (48), (51), and (52) of reference 1.

$$C_{N_{DP}} = \frac{\pi}{2}\frac{c}{D} \sin\alpha \cos\alpha \left[(4c_o + 2c_1) + f(c_n, E_n^*) + g(C_n, G_n^*)\right] \qquad (33)$$

where

$$E_n^* = \frac{\gamma}{\overline{V}} (B_n^* + F_n^*) + A_n^* + D_n^* \qquad (34)$$

and

$$f(c_n, E_n^*) = c_o(4E_o^* + 2E_1^*) + 2c_1 E_o^* + \sum_{n=1}^{4} (c_{n+1}E_n^* - c_n E_{n+1}^*) \qquad (35)$$

$$g(C_n, G_n^*) = C_o(4G_o^* + 2G_1^*) + 2C_1 G_o^* + \sum_{n=1}^{4} (C_{n+1}G_n^* - C_n G_{n+1}^*) \qquad (36)$$

The G_n^* are the Fourier coefficients describing the velocity u_{γ_D} which is computed using equation (49) of reference 1.

$$\frac{u_{\gamma_\alpha}}{V} = \sin\alpha \cos\phi \sum_{o}^{5} G_n^* \cos n\theta \qquad (37)$$

The F_n^* are the Fourier coefficients describing the velocity u_{γ_D} which is computed using equation (A-9) of reference 5.

$$\frac{u_{\gamma_D}}{\gamma} = \sum_{o}^{5} F_n^* \cos n\theta \qquad (38)$$

The duct pitching moment coefficient is given by equations (57), (59), (60), and (62) of reference 1 as

$$C_{M_{DP}} = \frac{\pi}{2} \sin\alpha \cos\alpha \left\{ \frac{1}{2} \frac{\gamma}{\overline{V}} \left(\frac{c}{D}\right)^2 \overline{f}(C_n, G_n^*) + \left(\frac{c}{D}\right)^2 (2c_o + c_2) \right.$$

$$+ \frac{1}{2}\left(\frac{c}{D}\right)^2 \overline{g}(c_n, E_n^*) - \frac{c}{D}\left[c_o(4E_o' + 2E_1') + 2c_1E_o'\right.$$

$$\left.\left. + \sum_{n=1}^{4} (c_{n+1}E_n' - c_nE_{n+1}')\right]\right\} \qquad (39)$$

where

$$E_n' = \frac{\gamma}{\overline{V}} (B_n + F_n) + A_n + D_n \qquad (40)$$

$$\overline{f}(C_n, G_n^*) = C_o(4G_o^* + 4G_1^* + 2G_2^*) + C_1(G_1^* - G_3^*) + C_2(2G_o^* - G_4^*)$$

$$+ C_3(G_1^* - G_5^*) + C_4G_2^* + C_5G_3 \qquad (41)$$

$$\overline{g}(c_n, E_n^*) = c_o(4E_o^* + 4E_1^* + 2E_2^*) + c_1(E_1^* - E_3^*) + c_2(2E_o^* - E_4^*)$$

$$+ c_3(E_1^* - E_5^*) + c_4E_2^* + c_5E_3^* \qquad (42)$$

The F_n are the Fourier coefficients describing the velocity v_{γ_D} which is computed using equation (16).

$$\frac{v_{\gamma_D}}{\gamma} = \sum_{o}^{5} F_n \cos n\theta \qquad (43)$$

The duct surface pressure distribution is obtained from the Bernoulli equation

$$C_p = 1 - \left(\frac{u_s}{V}\right)^2 \qquad (44)$$

where u_s is the induced surface velocity distribution. The surface velocity, u_s, is made up of a continuous portion and a discontinuous portion as given by equation (67) of reference 1. The continuous velocity is corrected for duct thickness and the discontinuous velocity is approximated near the leading edge to remove the singularity. These corrections and approximations are explained in detail in reference 1.

3. PROGRAM DESCRIPTION

3.1 Calculation Procedure

The computation proceeds as follows. An initial knowledge of the fan inflow velocity profile is required to compute blade performance, from which all other computations are made. Since the inflow is affected by the as yet undetermined duct-bound vorticity, an iterative procedure is used. An initial, uniform inflow of 2V is assumed, and the blade element calculations are made to determine the fan performance and the fan wake characteristics. The flow tangency condition on the duct reference cylinder is then applied to determine the duct-bound vorticity distribution. With all the singularity distributions known, the fan inflow is calculated. This is compared with the initially assumed inflow. If the two do not agree, a new inflow equal to the average between the initial and computed inflows is determined. The fan and duct-bound vorticity calculations are repeated to obtain a new inflow. The process continues until the inflow velocity in each annulus has converged to within the desired value. When convergence is obtained for all annuli, the program continues to calculate force and moment coefficients and duct surface pressure coefficients if desired.

3.2 Flow of Calculations

This section contains a discussion of the sequence of calculations within the program. The discussion describes the functions that are performed in the main program and each of the subroutines and the basic order in which they are performed. References are given for the various equations or relations used in the parts of the program.

The program consists of a main program and 20 subroutines. The cards in the source deck are identified in Columns 73 to 80 with a four-character identification indicating the number of the subroutine and a three-digit number representing the order of the card within the subroutine.

The main program is relatively short. Its basic functions are to control the flow of the program, and to perform the calculation for the local blade element performance, the calculation for the total fan inflow, the comparison of initial and computed inflows to determine convergence, and the calculation of the new inflow profile if the solution is not converged. The relationship between the main program and the subroutines is shown in figure 3.

The first operation in MAIN is a general initialization of four subroutines. ELLIPS is called to initialize tables of elliptic integrals, LAMBDA is called to initialize tables of the Heuman Lambda Function, PKL is called to initialize tables of $P_{k\ell}$ coefficients at various ratios of duct chord to diameter, and CLALF is called to initialize a table of section $c_{\ell_{max}}$ versus thickness-to-chord ratio. The table of $c_{\ell_{max}}$ versus t/c is determined from figure 2(b) of reference 1.

The program has several subroutines which are necessary for the operation of the program because of the service they perform, but they are not important to the understanding of the sequence of events in the program. These will be discussed here briefly and then omitted from the following program description, although they are shown on figure 3. Subroutine ELLIPS does a table look-up for the complete elliptic integrals of the first and second kind, given the argument of the elliptic integrals. Subroutine LAMBDA does a table look-up for the Heuman Lambda Function, given the two arguments of the function. Subroutine ARCSIN computes the principal value of the angle given the value of the sine of the angle. Subroutine FOURCS does a Fourier analysis of a function, given a table of values of the function. The subroutine computes an n-term cosine series ($n \leq 50$) to fit the given function. The first six terms of the series are used for the calculations in the program.

Subroutine INPUT is called by the main program. The major function of subroutine INPUT is to determine all of the quantities used in the calculation that are functions only of the duct configuration, the centerbody geometry, the fan blade characteristics, or the number of fan annuli. Included in these quantities are the Fourier series coefficients for velocities and vorticities which are functions only of duct chord-to-diameter ratio or duct thickness. This subroutine also reads in the input data, checks it for certain specific errors, and then prints out all of the input information. Now several subroutines must be called to do more specific initialization of parameters which are only dependent on the geometry of the system. Subroutine HUB is called to compute the strength and location of the point source and point sink which describe the centerbody model. ALFRNG is called to compute the G_n^* Fourier coefficients in equation (37) and the H_n Fourier coefficients in equation (32). PRESS is called to set up tables of the duct thickness correction factor and the leading-edge singularity parameter. These tables are used in the calculation of the duct surface pressure distribution. Subroutine INPUT next calls subroutine PKL to look up a table of the $P_{k\ell}$ influence coefficients. These coefficients, used in computing the radial velocity induced by a distribution of vortex rings, are functions only of c/D and are obtained from reference 9. Subroutine CAMBER is called to compute the induced camber coefficients which are combined with the geometric camber coefficients to obtain the total effective camber of the duct.

Subroutine HUB is called again by INPUT to compute the Fourier coefficients D_n and D_n^*. These coefficients are constants for a given duct and centerbody configuration; therefore, they may be computed at this time assuming a unit free stream and then correcting for the increased free stream as they are used.

Subroutine PROP divides the fan annulus into the required number of annuli (which is input), computes the inner, outer, and mean radii of the annuli and interpolates in the blade β, b/R_p, and h/b input tables to obtain values of β, b/R_p, and h/b at the mean radii of the annuli.

Certain components of the inflow to the fan are constants and can be computed at this time. Subroutine GAMCYL computes the nondimensional axial inflow velocities (in the form u/γ) induced at the mean radii of the fan annuli by a vortex cylinder trailing from the duct trailing edge. These are a function only of c/D. The equation solved is equation (A-10) of reference 5. In this equation, it is necessary to use elliptic integrals, an arcsin relation, and the Heuman Lambda Function. These are obtained, respectively, from subroutines ELLIPS, ARCSIN, and LAMBDA. Subroutine SRCRNG computes the nondimensional axial inflow velocity (in the form u/V) induced at the mean radii of the fan annuli by the source rings representing the duct thickness distribution. The equation used is equation (A-6) of reference 5. Subroutine HUB computes the nondimensional axial inflow velocity (in the form u/V) induced at the mean radii of the fan annuli by the source-sink distribution representing the centerbody geometry. The equation used is equation (15) of reference 1. The two latter inflow velocities must be corrected for the increased local axial velocity as is discussed in reference 1.

Subroutine BNCOEF computes the Fourier cosine series coefficients (B_n) for the nondimensional radial velocity (in the form v/γ) induced along the duct reference cylinder by a vortex cylinder trailing from the duct trailing edge. The equation used is equation (C-3) of reference 5. The subroutine ANCOEF computes Fourier series coefficients for three sets of velocities on the duct reference cylinder. The coefficients B_n^* are those of the nondimensional axial velocity (u/γ) induced by a vortex cylinder from the duct trailing edge (eqs. (23) and (25) of ref. 3). The coefficients $(A_n)_w$ and $(A_n^*)_w$ are those of the nondimensional radial (v/γ_w) and axial (u/γ_w) velocities, respectively, induced by the w^{th} inner vortex cylinder trailing from the fan. The equations used are equations (13) and (16), and equations (20) and (21), respectively, of reference 3. These Fourier coefficients are obtained by fitting a 50-term Fourier cosine series to the chordwise velocity distribution and truncating after the sixth term. At this point, the coefficients for all of the velocities induced at the duct reference cylinder by the fan wake, duct thickness, and the centerbody are known, and control returns to the main program.

The blade element performance in each of the fan disk annuli is computed using the local blade lift coefficient obtained from subroutine CLALF. The section c_ℓ includes the effect of local blade stall in the manner described in reference 1. The strengths of the internal vortex cylinders are now computed. The main program calls subroutine CNCOEF. The basic purpose of this subroutine is to apply the boundary condition of no flow through the duct camberline to compute the Fourier coefficients, C_n, representing the duct-bound vorticity γ_D. The six C_n coefficients are obtained from the solution of a matrix equation involving the A_n, A_n^*, B_n, B_n^*, D_n, D_n^* coefficients, the camber coefficients, and the vortex cylinder strengths just computed. The three dependent matrices in this equation are computed and defined in subroutine CNCOEF. The equation is then solved for the C_n through use of the matrix inversion subroutine, MATRIX. If the matrix to be inverted is singular, an error message is printed and the subroutine transfers to a STOP statement.

Now that all the singularity distributions are known, the correction factors which are applied to the duct thickness-induced velocities and the centerbody-induced velocities are computed. Both correction factors need the velocities induced by the vortex cylinder trailing from the trailing edge of the duct and the duct-bound vorticity, γ_D. These induced axial velocities are obtained from subroutines GAMCYL and VTXRNG, respectively.

Subroutine VTXRNG is called again to compute the axial inflow to the fan due to the duct-bound vorticity, γ_D. Equation (A-9) of reference 5 is used with the C_n coefficients just computed.

The main program then sums the inflow velocities in each of the fan annuli obtained from all the singularity distributions and adds to these the free-stream velocity and the discontinuous portion of the velocity difference across each vortex cylinder ($\gamma_w/2$). The resulting total inflow in each annulus is compared with the initially assumed inflow. If all velocities do not agree within the prescribed convergence limit, the new inflow is assumed to be the average between the initial and computed velocities. The program then returns to the computation of new values of the local blade performance. Subroutine CNCOEF is called, new C_n coefficients determined, new corrections for the centerbody-induced velocities and duct-thickness-induced velocities are computed, VTXRNG is called to obtain the fan inflow due to the duct-bound vorticity, and a new total fan inflow obtained. This iterative process is repeated until convergence is obtained for all the fan annuli or until 50 iterations have been completed.

After convergence is obtained, the main program calls OUTPUT which immediately prints out the fan loading and inflow parameters. The force and moment coefficients are computed and output, and if the duct surface pressure coefficients are to be computed, subroutine PRESS is called. In PRESS, the velocity induced on the duct reference cylinder by all the singularity distributions is computed. This includes the axial velocity induced by the duct-bound vorticity, γ_α, associated with the duct at angle of attack. These induced velocities are obtained from subroutine ALFRNG which uses equation (49) of reference 1. After summing all the continuous and discontinuous velocity components, Bernoulli's equation is used to compute the duct surface pressure coefficients. Control returns to OUTPUT where the pressure coefficients are printed. This completes the series of calculations and control is returned to the main program.

The above sequence of operations is arranged such that the calculations performed by INPUT provide all the parameters dependent only on the duct and propeller configuration. Thus, if performance of a given configuration at a number of advance ratios or angles of attack is desired, an index is read from a card following completion of the calculations in OUTPUT, the next values of J and α read, and control is returned to the part of the main program just after INPUT has been called, which avoids some recomputation. If a new configuration is to be analyzed, control returns to the statement in the main program calling INPUT, where a complete new case is read in.

3.3 Use of Program

The program is written in Fortran IV for the IBM 7094 computer. No tapes other than the standard input and output tapes are required. Typical execution times are approximately two minutes for the first case and one minute for each additional run utilizing the same geometry. A complete description of the input and output is presented in sections 4 and 5 respectively, and a program listing is given in section 7.

4. DESCRIPTION OF INPUT

This section contains a description of the input for the computer program. The information required on the cards is described in section 4.1. The remainder of the subsections below describe the manner in which some of the input quantities are determined for a given ducted fan configuration.

4.1 Input Data

The format of the input cards making up a normal run is shown in figure 4. In this figure the variable is given as well as the card column in which the punching of the value of the variable is begun and the format in which it is punched. In all input the format is set up for four figures to the right of the decimal point. If more significant figures are required, the decimal point may be moved to the left some appropriate amount. As long as the number is punched in the specified field of 10 columns, the decimal point may be punched in any column.

Card No. 1 contains any alphabetic and numeric information that is desired for identification purposes. This information is printed at the top of each page of output.

Card No. 2 contains information about the geometry of the duct. The first item (c/D) is the chord-to-diameter ratio. This value should be greater than zero and less than about 2.5. This upper limit is approximate and is necessary only because some coefficients built into the program must be extrapolated for $c/D > 2$. The next items on this card are the axial location of the fan within the duct (x_p/c), the maximum thickness-to-chord ratio of the airfoil section of the duct (t/c),[3] the ratio of the duct radius at the trailing edge to the fan radius (R/R_p), and the ratio of the centerbody radius at the fan station to the fan tip radius (R_{CB}/R_p) (or the fan hub-tip ratio). The last item is the convergence criterion (ϵ). The iteration will stop when the inflow profiles agree within $\epsilon \times 10^2$ percent. A suggested range for ϵ is $0.001 \leq \epsilon \leq 0.01$.

Card No. 3 contains the four Fourier coefficients (R_n) of a cosine series fit to the duct camberline. If the duct is uncambered, these four values are all zero.[4]

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[3]The value of t/c must be within the range $0.06 \leq t/c \leq 0.24$ because of correction factors used in the calculation of duct pressure distribution. If t/c is outside this range, the pressure distribution will be slightly in error, but all other calculations will be correct.

[4]A detailed discussion of the manner of computing these four coefficients is given in section 4.2.

Card No. 4 contains the information necessary to define the centerbody. The first item is the centerbody length expressed as a fraction of the duct chord (ℓ_{CB}/c). The second item is the location of the centerbody nose in the duct coordinate system (x_{CB}/c). If the centerbody extends forward of the duct leading edge, $x_{CB}/c < 0$. The next item is the maximum radius of the centerbody expressed as a fraction of the centerbody length (r_{max}/ℓ_{CB}) and the last item on this card is the x location of r_{max} ($x_{r_{max}}$) in the duct coordinate system. Preparation of the input on this card must meet certain specifications which are discussed in detail in section 4.3.

Card No. 5 contains six quantities. The first, NBLD, is the number of fan blades. This number must be greater than zero. The second item, NZ, is the number of equal area annuli into which the fan annulus is divided. The limitation on NZ is $2 \leq NZ \leq 24$. The third item, NZP, is the number of stations in the input table of fan blade characteristics. The limitations on NZP are $2 \leq NZP \leq 24$. The next quantity is IR, the number of x-stations at which the duct surface pressure distribution is to be calculated. At a given x/c, the pressure coefficient is computed on the inside and outside surface of the duct. The value of IR must be in the range $1 \leq IR \leq 25$.[5] The next item, NPRES, is an index which controls the form of the surface pressure coefficient. If NPRES = 1, the surface pressure coefficient is based on free-stream dynamic pressure. If NPRES = 2, the surface pressure coefficient is based on the propeller tip speed. The last quantity, NPRINT, is an output index which controls the quantity of output. For normal runs, NPRINT = 0. If NPRINT = 1, an extra page of optional output is developed which contains a table of the components of the inflow to the propeller. This optional output is described in the output section of this report. Another output option is available, but the authors recommend that it not be used, as it is useful only for diagnostic purposes. If NPRINT = 10, a page listing all of the Fourier cosine series coefficients for the u and v

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[5]The input controlling the pressure distribution calculation has been set up for ease of repetitive running with the same configuration. Consequently, IR must be 1 or greater, whether or not any pressures are to be computed. An index on the last card indicates the requirement for computing pressures.

velocities induced at the duct reference cylinder is printed. If NPRINT = 11, both of the above optional pages are output. In most cases of interest, NPRINT will be identically zero.

Card No. 6 contains the fan radii (r/R_p) at which the blade characteristics are to be input. There are NZP entries in this table, so if NZP > 8, the other entries in the table must be included on following cards. For example, if NZP = 17, three cards are required. The first two cards will have eight fields each and the third card one field. The values must be put in order of increasing radius ratio. The first entry in the table should be the value of r/R_p at the root or hub, which nominally is equal to R_{CB}/R_p, the value on Card No. 2. The last value in the table should be $r/R_p \approx 1.0$. There cannot be more than 24 entries in the table.

Card No. 7 contains values of the fan blade chord (b/R_p) at the radial stations corresponding to the values on Card No. 6. With eight fields per card, there will be as many cards here as in the r/R_p case (Card No. 6).

Card No. 8 contains the fan blade pitch angle (β) in degrees at the radial stations corresponding to the values on Card No. 6. The pitch angle is the blade twist angle measured from the plane of rotation to the zero lift line of the local blade section. The number of cards here will be the same as the number of r/R_p cards (Card No. 6).

Card No. 9 contains the blade section thickness-to-chord ratio at the radial stations corresponding to those on Card No. 6. The number of cards here will be the same as the number of r/R_p cards (Card No. 6).

Card No. 10 contains the values of x/c at which the duct surface pressure coefficients are to be calculated. This card should contain IR values, and if IR > 8, the ninth and succeeding values should be put on following cards in the same format as shown for Card No. 10. The values should be put in ascending order with the limitation $0 \leq x/c \leq 1.0$. Since the inner duct surface pressure distribution has a discontinuity at the fan due to the pressure jump aft of the fan, it is desirable to include x/c values just upstream and downstream of the fan. No numerical problem occurs, however, if the x/c value exactly at the fan station is input, since the program uses the total pressure upstream of the fan to compute the inner duct pressure coefficient at the fan station.

Card No. 11 is the last card required for a particular configuration. The first item on this card is a run identification number, NRUN. The only restrictions on NRUN are that NRUN $\leq$ 9999 and NRUN $\neq$ 0. The second quantity is the index NPHI, which is the number of azimuth stations around the duct at which the surface pressure coefficients are to be calculated. The limitation is that $0 \leq$ NPHI ≤ 5. If NPHI = 0, no pressure calculation will be performed for this run. The next quantity is the advance ratio, J. The advance ratio must be greater than zero.[6] The duct angle of attack, α, in degrees is the next item. The only requirement is that $\cos \alpha \neq 0$. The remaining five fields on the card should contain the azimuth angles, ϕ, in degrees, at which pressure distributions are to be calculated. There should be NPHI values of ϕ, and due to the symmetry of the flow, $0 \leq \phi \leq 180^{\circ}$.

If only one run is to be made, the input deck is complete with Card No. 11. If two or more runs are to be made, the card arrangement following Card No. 11 for stacking runs is as follows:

(1) If the same ducted fan configuration is to be investigaged at a different advance ratio or angle of attack or if different values of ϕ are of interest, only one additional card is required to initiate the new run. This card has the same format as Card No. 11. The run number can be changed to identify the new run. Each succeeding run with the same configuration requires the one additional card, and this card can be repeated as many times as desired.

(2) If a different duct and/or fan configuration is to be investigated, the card following Card No. 11 must be blank. The new case is then loaded as a deck consisting of Card Nos. 1 through 11.

A sample set of input data illustrating the program options is shown in figure 6.

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[6]A value of J = 0 causes numerical problems in the machine. To compute a zero advance ratio (hover) case, set J = 0.01.

4.2 Duct Camber and Thickness

The duct thickness and camber are required on the second and third cards. A brief discussion is given below with regard to the selection of these parameters. In order to give a reasonably wide latitude to the use of the program while at the same time not storing a great quantity of airfoil data in the program, the specifications of duct thickness and camber were separated.

The thickness distribution is taken to be the local profile thickness as measured from the camberline. For purposes of including thickness effects in the performance calculations, the thickness distribution is specified within the program as that of a symmetrical four-digit NACA airfoil which has an analytical expression for the variation of thickness with distance along the chord. This distribution is completely specified by one parameter, the ratio of maximum thickness to chord. In selecting a value for a given duct configuration, it is suggested that the $(t/c)_{max}$ value be chosen which will yield the best fit to the actual duct shape along the inner forward portion of the duct, while still giving a reasonable fit over the other portions of the section. Tables of thickness distribution for various $(t/c)_{max}$ can be obtained from Appendix I of reference 11.

The camber coefficients required in the third card are computed according to the methods given below. Figure 1 indicates the notation and coordinate systems. The axial coordinates are x, measured from the leading edge with the range 0 to c; θ, measured from the leading edge with the range 0 to π; and x_s, measured from $x = c/2$ with the range $-c/2$ to $+c/2$. The relation between θ and x_s is given by

$$\cos\theta = -2x_s/c \tag{45}$$

The local slope of the geometric camberline is

$$\frac{dr_c}{dx_s} = \sum_{n=o}^{3} R_n \cos(n\theta) \tag{46}$$

This can be integrated to give the camberline shape as

$$\frac{r_c - R}{c} = -R_o\left(\frac{1 + \cos\theta}{2}\right) + R_1\left(\frac{1 - \cos^2\theta}{4}\right)$$

$$+ R_2\left(\frac{1}{6} + \frac{1}{2}\cos\theta - \frac{1}{3}\cos^3\theta\right)$$

$$+ R_3\left(-\frac{1}{4} + \frac{3}{4}\cos^2\theta - \frac{1}{2}\cos^4\theta\right) \tag{47}$$

The camberline radius at the trailing edge is identically that of the duct reference cylinder, which passes through the trailing-edge radius; that is, at $\theta = \pi$, $r_c - R = 0$. Four other points along the camberline must be selected and their r_c values put into the above equation to obtain four simultaneous equations for the four R_n coefficients. It is suggested on the basis of past experience that these points be nearly equally spaced for best results; that is, at $x/c = 0$, $1/4$, $1/2$, and $3/4$. If these four points are chosen, the R_n coefficients can be calculated from the following equations.

$$\begin{bmatrix} R_o \\ R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} (-1\frac{2}{3}) & (1\frac{1}{3}) & (0) & (-1\frac{1}{3}) \\ (-3\frac{1}{3}) & (5\frac{1}{3}) & (-4) & (5\frac{1}{3}) \\ (-2) & (4) & (0) & (-4) \\ (-1\frac{1}{3}) & (5\frac{1}{3}) & (-8) & (5\frac{1}{3}) \end{bmatrix} \times \begin{bmatrix} \left(\frac{r_c - R}{c}\right)_o \\ \left(\frac{r_c - R}{c}\right)_{1/4} \\ \left(\frac{r_c - R}{c}\right)_{1/2} \\ \left(\frac{r_c - R}{c}\right)_{3/4} \end{bmatrix} \tag{48}$$

If the camberline is not well behaved (for example, if it should have a hook near the leading edge), four other values of x/c may be necessary to give the best overall fit to the actual camberline. Several tries may be necessary to select the right combination of four points.

If the duct has no camber, all the R_n coefficients are input equal to zero.

4.3 Centerbody

The centerbody shape is approximated by a Rankine body, figure 5. Because the centerbody model is so simple, there are certain restrictions on the input. It is necessary that x_{CB}/c, r_{max}/ℓ_{CB}, ℓ_{CB}/c, and $x_{r_{max}}/c$ be input such that the centerbody half-length (distance between the nose and the point of maximum radius) is greater than the maximum radius; that is

$$x_{r_{max}} - x_{CB} > r_{max}$$

or in terms of the nondimensional ratios,

$$\left(\frac{x_{r_{max}}}{c}\right)\left(\frac{c}{\ell_{CB}}\right) - \left(\frac{x_{CB}}{c}\right)\left(\frac{c}{\ell_{CB}}\right) > \left(\frac{r_{max}}{\ell_{CB}}\right)$$

The effect of the centerbody on the ducted fan performance cannot be eliminated entirely from the program, but it can be made negligibly small by inputing the appropriate geometry. Thus, if the user wishes to remove the effect of the centerbody, the centerbody parameters should be input according to the following rules:

$$\frac{x_{CB}}{c} = \frac{x_p}{c} - 0.02$$

$$\frac{\ell_{CB}}{c} = 0.04$$

$$\frac{r_{max}}{\ell_{CB}} = 0.005$$

$$\frac{x_{r_{max}}}{c} = \frac{x_p}{c}$$

If these values are used, the area inside the fan hub radius (πR_{CB}^2) will be unloaded unless R_{CB} is set equal to zero. It should be noted that because of the formulation of the program, the leading edge of the centerbody should be forward of the fan station in order to avoid numerical problems; that is

$$\frac{x_{CB}}{c} < \frac{x_p}{c}$$

Generally, in approximating the centerbody shape, the user should try to fit the portion between the nose and the fan station best. This will give the best results for the inflow to the fan. If possible, the total length should be approximated, but if compromise is necessary, the portion ahead of the fan is more important than that aft of the fan. If the maximum centerbody radius occurs aft of the fan, some judgment must be used to decide on the position and value of the maximum radius. Remembering that the purpose of the centerbody model is to approximate the blockage effect on the inflow to the fan, the best centerbody model is a compromise to give the closest fit to both the maximum radius and the radius at the fan station.

4.4 Fan Blade Characteristics

A simple method to account for local blade stall is described in reference 1. This method involves a table of $c_{\ell_{max}}$ versus t/c derived from figure 2(b) of reference 1. This table is built into Subroutine CLALF. If the user wishes to change this table, he should use the following procedure.

The local section thickness-to-chord ratio is called array TCB with 10 entries ranging from TCB(1) = 0 to TCB(10) = 0.34. The blade section $c_{\ell_{max}}$ is called array CLMX, with 10 entries corresponding to the TCB array. The table is easily changed by changing corresponding entries of TCB and CLMX. It is advised that the total number of entries (10) in the table be unchanged. If the user would like to remove the effect of blade stall and assume that the blade section c_{ℓ_α} is constant and equal to 2π at any angle of attack, every entry in the array CLMX should be set equal to some large number, for example, 10.0.

5. DESCRIPTION OF OUTPUT

The nominal output consists of three pages of data, to which can be added one additional page of optional output. The pages are numbered and each page has the run number at the top plus the identification information which is punched in the first card of the input deck.

A typical set of output including one page of optional output is shown in the sample case (figure 7).

The first page is numbered PAGE 0. It contains a listing of the input data plus a table of definitions of symbols. The input listed on the line entitled DUCT GEOMETRY is the duct chord-to-diameter ratio (C/D), the axial position of the fan station as a fraction of duct chord (XP/C), the duct maximum thickness-to-chord ratio (T/C), the ratio of the duct trailing-edge radius to fan radius (RTE/RP), and the ratio of centerbody radius in the plane of the fan to the fan radius (RCB/RP). The next line lists the geometric camber coefficients, R_n. The next group of data indicates the fan blade geometry input into the program. The number of blades is given, and a table of (nondimensional) blade chord (B/RP), blade pitch angle (BETA), and thickness-to-chord ratio (TH/CHD) versus nondimensional radius (R/RP) is given. This information is followed by the centerbody geometry which consists of the ratio of centerbody length to the duct chord (LCB/C), the axial location of the centerbody nose (XCB/C), the maximum radius of the centerbody in fraction of centerbody length (RMAX/LCB), and the axial location of the maximum radius (X(RMAX)/C). The next line contains the convergence criterion (EPSILON). This is followed by a table of symbols and their definitions.

Page 1 is optional output (NPRINT = 1) and is so noted at the top of the page. This page lists the inflow velocity results obtained after convergence of the iterative process. For convenience, the duct characteristics printed on the previous page are again printed here at the top of the page with the addition of the ratio of the fan disk area to the duct exit area (AP/A). The next line lists the four Fourier coefficients of the effective camber, R_n^*. Following these are the duct angle of attack in degrees (ALPHA), the advance ratios $J = V/nD_p$ and $J' = V/\omega R$, and the values of $J \cos \alpha$ and $J' \cos \alpha$. The following table lists the induced inflow velocity in each of the annuli (N). At the center of each of the annuli, denoted R/RP, the inflow is made up of the velocities induced by the duct thickness (UQD/V), the duct-bound vorticity (UGD/V), the vortex cylinder trailing from the fan tip (UG/V), and the centerbody (UCB/V). The final column indicates the local blade bound vorticity, Γ/RV, (GAMMA/RV). The numbers in parentheses above UQD/V and UCB/V are the correction factors for the thickness distribution and the centerbody, respectively. The last output on the page is the number of iterations required for convergence, and the convergence criterion. When the duct is at angle of attack, the reference V in the table is actually $V \cos \alpha$ or $\overline{V}$.

The next page, page 2, lists the fan and duct performance. Again, the duct characteristics are listed at the top of the page, together with the angle of attack and advance ratios. The table that follows lists at each of the fan annuli centers (R/RP) the total inflow velocity (U/V), the strength of the vortex cylinder shed from the outer radius of the annulus (GAM/V), the angle of attack of the blade, in degrees, as measured from the line of zero lift of the blade, and the total pressure rise divided by the free-stream dynamic head (DELTA P/Q). When the duct is at angle of attack, the nondimensional inflow results, blade bound vorticity, and fan pressure rise are based on $\overline{V} = V \cos \alpha$, rather than V. Following the table are various force and moment coefficients preceded by note (A) or (B). The thrust and normal force and moment coefficients opposite (A) are defined as

$$C_T = \frac{T}{qA}$$

$$C_N = \frac{N}{qA}$$

$$C_M = \frac{M}{qAR}$$

where the moment center is at the center of the duct ($x = c/2$, $r = 0$). The coefficients opposite (B) are defined using the fan rotational speed and tip diameter as

$$C_T = \frac{T}{\rho n^2 D_p^4}$$

$$C_N = \frac{N}{\rho n^2 D_p^4}$$

$$C_M = \frac{M}{\rho n^2 D_p^5}$$

The propeller thrust coefficient is CTP(D). The duct thrust coefficient based only on the thrust on the duct-bound singularity distributions is given as CTD(P). The sum of these two is CTDP. The duct-thrust coefficient including the pressure thrust due to the fan tip pressure rise acting on the inner duct surface aft of the propeller as well as the thrust on the singularity distributions is noted CTD(P)'. The sum of this duct thrust and the propeller thrust is noted CTDP'. The normal-force coefficient is CNDP and the moment coefficient is CMDP.

The thrust coefficients with and without the pressure thrust added to the inner duct surface are presented in the output. The results in reference 5 show that inclusion of the pressure thrust on the duct gives better agreement with measured duct-thrust coefficients and pressure distributions on the inner duct surface. Therefore, all thrust results presented in section 9, Data Comparisons, include the pressure thrust on the duct. Both thrust results are included in the output so that the user can see the full effect of the pressure rise aft of the fan.

The last items on page 2 are several notes. Notes (A) and (B) are self-explanatory. The note denoted with an asterisk (*) corresponds to any asterisk printed in the table. This note is printed to bring attention to the fact that the blade sections are stalled in the annuli so noted.

The last page of output, page 3, contains the duct surface pressure distribution if it has been requested (NPHI $\neq$ 0). Again, the duct characteristics and flow conditions are printed at the top of the page. The next line lists the azimuth angles, ϕ, in degrees. Directly under the azimuth angles are the corresponding pressure coefficients on the inside (CP(IN)) and outside (CP(OUT)) surfaces of the duct. The pressure coefficients are based on the free-stream dynamic pressure (NPRES = 1)

$$c_p = 1 - \left(\frac{u_s}{V}\right)^2$$

or the propeller tip speed (NPRES = 2)

$$C_p = \frac{V^2 - u_s^2}{2n^2D_p^2}$$

according to the note printed as the last line on this page.

This completes the output for one complete run. The pages that follow depend on how runs are stacked. If the same duct and fan configuration is to be run at new values of advance ratio, angle of attack, or azimuth angles, the additional output will be the same as above starting with page 1. If some change has been made in the configuration, then the complete set of output from page 0 listing the new input data will follow.

6. DESCRIPTION OF ERROR MESSAGES AND STOPS

A number of error messages and stops are built into the program so that if certain conditions occur, the computation will stop and some indication of why it was stopped will be printed on the output.

Immediately after reading in the input data, several checks are made on specific quantities. If NPHI > 5, it is set equal to 5 and execution continues with no error message.

If the number of propeller blades (NBLD) is less than or equal to zero, the following message is printed:

"NUMBER OF BLADES IN ERROR, NBLD = -xx"

If the convergence criterion, ϵ, is less than or equal to zero, the following message is printed:

"CONVERGENCE CRITERION MUST BE GREATER THAN 0.0"

If the angle of attack, α, is greater than or equal to 90°, the following message is printed:

"ANGLE OF ATTACK MUST BE LESS THAN 90.0 DEGREES"

If the advance ratio, J, is less than or equal to zero, the following message is printed:

"ADVANCE RATIO MUST BE GREATER THAN 0.0"

In Subroutine CLALF the propeller blade thickness-to-chord ratios are tested. If any do not fall in the range $0.0 \leq t/c \leq 0.34$, the following message is printed:

"THE BLADE THICKNESS-TO-CHORD RATIO IS OUTSIDE THE RANGE 0.0 TO 0.34"

Two error messages associated with computation of a centerbody shape are built into Subroutine HUB. If the centerbody geometry is input such that the requirements listed in section 4.3 are not met, the following message is printed:

"INPUT CENTERBODY DIMENSIONS IN ERROR"

If Subroutine HUB is unable to converge on the source and sink locations, the following message is printed:

"SUBROUTINE HUB UNABLE TO COMPUTE CENTERBODY GEOMETRY"

If any or all of the above messages are printed, the following message is also printed:

"ERROR - EXECUTION TERMINATED"

and the program terminates execution at a STOP statement in the Main program.

The above errors are due to errors in the input data and should be simple to correct. If the program fails to converge on the inflow profile in 50 iterations, two more iterations are made. The results of iterations number 51 and 52 are output, including the optional page showing the components of the induced inflow velocities. The following messages are then printed:

"PROGRAM DID NOT CONVERGE ON INFLOW PROFILE"

"ERROR - EXECUTION TERMINATED"

and the computation terminates at the STOP statement in the Main program.

The above error is rare and has occurred in the authors' experience only under conditions of small advance ratio, $J \cos \alpha$, and small blade pitch angles. If all the input appears correct, increase $J \cos \alpha$ by increasing J or decreasing α and rerun the same case again. Also check the convergence criterion (ϵ), and consider increasing it to relax the tolerance on convergence. Experience has shown that ϵ

should not be larger than 0.01. There is approximately 10 percent difference between force and moment coefficients computed with $\epsilon = 0.01$ and $\epsilon = 0.10$. An example of a case that consistently did not converge was the Doak ducted fan (ref. 12) with 11° pitch at advance ratios less than 0.5 in axial flow.

If the duct thickness-to-chord ratio is outside the range $0.06 \leq t/c \leq 0.24$, the following warning is printed:

"DUCT PRESSURE DISTRIBUTION CALCULATION ASSUMES
T/C = x.xxx"

7. PROGRAM LISTING

The ducted fan performance analysis program is written in Fortran IV for the IBM 7094 computer. The program consists of the main program and 20 subroutines. Each source deck is identified in columns 73-80 by a four-character identification (NExx) indicating the number of the subroutine and a three-digit number sequencing the cards within the subroutine. The program listing is given in the following pages. The table below will act as a table of contents for the program listing.

```
C      DUCTED PROPELLER ANALYSIS PROGRAM
C
      DIMENSION B(6),BS(6),SA(6),SAS(6),P(6,6)
      DIMENSION C(6),A(25,6),AS(25,6),D(6),DS(6),SC(6),GS(6),H(6)
      DIMENSION RB(25),BR(25),BTA(25),TCBLD(25),RA(25),XPRES(25),PHI(5)
      DIMENSION UG(25),UQD(25),UGD(25),UV(25),UCB(25),VCB(25),
     1          UGP(25,25),UGA(25),GV(25),GRV(25),CPP(5,25),CPM(5,25)
      DIMENSION ALPHA(25),STALL(25),JSTL(25),TALK(20)
C
      COMMON MZZZ,CD,R0,R1,R2,R3,PI,B,BS,SA,SAS,P
      COMMON/NEAR1/ NRUN,NBLD,NZ,MZ,NPRES,IR,NTIME,NERR,NPAG,NPHI,NPRINT
      COMMON/NEAR2/ C,A,AS,D,DS,SC,GS,H
      COMMON/NEAR3/ RRP,XP,Z,BLD,RB,BR,BTA,TCBLD,TC,RCBRP,APA,ALF,XPRES,
     1              RA,XCB,XR,ELCBC,RMAX,PHI,CORJ,CORCB
      COMMON/NEAR4/ UG,UQD,UGD,UV,UCB,VCB,UGP,UGA,GV,GAM,GRV,CPP,CPM
      COMMON/NEAR5/ ARJ,ARJP,EPS,RAD,CL,ALPHA,STALL,JSTL,TALK
C
  120 FORMAT(2I5,7F10.6)
  997 FORMAT (///10X,42HPROGRAM DID NOT CONVERGE ON INFLOW PROFILE///)
  998 FORMAT (///10X,28HERROR - EXECUTION TERMINATED///)
C
C       INITIALIZATION OF SUBROUTINES
C
      DUM=0.0
      MZZZ=0
      CALL ELLIPS (DUM,DUM,DUM)
      CALL LAMBDA (DUM,DUM,DUM)
      CALL PKL (DUM,P)
      CALL CLALF (0)
      MZZZ=1
C
      PI=3.1415926
      RAD=180./PI
C
   29 NERR=0
      CALL INPUT
      IF (NERR) 28,28,999
C
   28 CSALF=COS(ALF/RAD)
      ARJV=ARJ
      ARJVP=ARJP
      ARJ=ARJ*CSALF
      ARJP=ARJP*CSALF
      DO 30 K=1,NZ
   30 UV(K)= 2.0
      NTIME=1
      CORCB=2.0
   31 DO 32 K=1,NZ
      BPI=RB(K)/ARJP/UV(K)
      ABPI=ATAN (BPI)
      ALPHA(K)=(ABPI-BTA(K))*RAD
      BPI=BPI*BPI
      BPI=SQRT (BPI+1.0)
      J=K
      CALL CLALF  (J)
      IF (NERR-1) 25,999,999
   25 CONTINUE
      GRV(K)=0.5*CL*BR(K)*UV(K)*BPI
   32 CONTINUE
```

```
   33 GV(NZ)=GRV(NZ)*BLD/PI/ARJP+1.0
      AGV= 1.
      IF(GV(NZ))133,233,233
  133 GV(NZ)=-GV(NZ)
      AGV=-1.
  233 GV(NZ)=AGV*SQRT (GV(NZ))-1.
      AGV= 1.
      SGV=1.0
   34 DO 35 J=1,MZ
      K=NZ-J
      L=K+1
      SGV=SGV+GV(L)
      SGV2=SGV*SGV
      GRVD=GRV(K)-GRV(L)
      GRVD=GRVD*BLD/PI/ARJP
      DUM=SGV2+GRVD
      AGV=1.0
      IF (DUM.LT.0.0) AGV=-1.0
      GV(K)=AGV*SQRT(AGV*DUM)-SGV
   35 CONTINUE
      DO 38 N=1,6
      SA(N)=0.
   38 SAS(N)=0.
      DO 40 N=1,6
      DO 39 M=1,MZ
      SA(N)=SA(N)+A(M,N)*GV(M)
   39 SAS(N)=SAS(N)+AS(M,N)*GV(M)
   40 CONTINUE
C
      DO 41 N=1,6
      SA(N)=SA(N) + D(N)*CORCB
   41 SAS(N)=SAS(N) + DS(N)*CORCB
      GAM=GV(NZ)
   97 CALL CNCOEF (GAM,C)
      DO 141 N=1,6
      SA(N)=SA(N) - D(N)*CORCB
  141 SAS(N)=SAS(N) - DS(N)*CORCB
C
C       COMPUTE CORRECTION FOR LOW ADVANCE RATIOS  (CORJ)
C
      DUM=XPRES(1)
      DUMM=RA(1)
      XPRES(1)=XP
      RA(1)=1.0
      CALL GAMCYL (CD,XP,1,RB,UG,1,XPRES,UGP,RRP,RA)
      IRC=-1
      CALL VTXRNG (CD,XP,IRC,RB,C,UGD,XPRES,P)
      CORJ=UGD(1)*GAM + UGP(1,1)*GAM + 1.0
       IF (CORJ.LT.1.0) CORJ=1.0
C
C       COMPUTE CORRECTION FOR CENTERBODY INDUCED VELOCITIES (CORCB)
C
      RA(1)=0.0
      CALL GAMCYL (CD,XP,1,RA,UG,0,XPRES,UGP,RRP,RA)
      IRC=1
      CALL VTXRNG (CD,XP,IRC,RA,C,UGD,XPRES,P)
      CORCB=UGD(1)*GAM+UGP(1,1)*GAM+1.0
      DO 142 J=1,MZ
  142 CORCB=CORCB+GV(J)/2.0
```

```
       IF (CORCB.LT.1.0) CORCB=1.0
      XPRES(1)=DUM
      RA(1)=DUMM
C
      CDP=CD*RRP
      CALL VTXRNG (CDP,XP,NZ,RB,C,UGD,XPRES,P)
   98 IRT=0
      SUMG=0.
      DO 42 J=1,MZ
   42 SUMG=SUMG+GV(J)
      DO 45 J=1,MZ
      UNV=1.+(UG(J)*GAM)+(UGD(J)*GAM)+UQD(J)*CORJ+UCB(J)*CORCB+SUMG/2.0
      SUMG=SUMG-GV(J)
      DELV=UV(J)-UNV
      DELV=DELV/UNV
      DELV=ABS (DELV)
      IF(DELV-EPS) 45,45,44
   44 IRT=IRT+1
   45  UV(J)=(UV(J)+UNV)/2.
      J=NZ
      UNV=1.+(UG(J)*GAM)+(UGD(J)*GAM)+UQD(J)*CORJ+UCB(J)*CORCB
      DELV=UV(NZ)-UNV
      DELV=DELV/UNV
      DELV=ABS (DELV)
       UV(NZ)=(UV(NZ)+UNV)/2.
      IF(DELV-EPS) 46,46,47
   47 IRT=IRT+1
   46 IF(IRT) 50,60,50
   50 NTIME=NTIME+1
      IF (NTIME-51) 31,60,60
   60 CALL OUTPUT
      IF (NTIME-51) 68,69,68
   69 NERR=0
      GO TO 31
   68 IF (NERR) 70,70,996
   70 READ (5,120) NRUN,NPHI,ARJ,ALF,(PHI(J),J=1,5)
      ARJP=ARJ/PI/RRP
      NPAG=0
      IF (NPHI.GT.5) NPHI=5
      IF (NRUN) 28,29,28
C
C      ERROR STOP
  996 WRITE (6,997)
  999 WRITE (6,998)
      STOP
C
      END
```

```
      SUBROUTINE INPUT
C
      DIMENSION B(6),BS(6),SA(6),SAS(6),P(6,6)
      DIMENSION C(6),A(25,6),AS(25,6),D(6),DS(6),SC(6),GS(6),H(6)
      DIMENSION RB(25),BR(25),BTA(25),TCBLD(25),RA(25),XPRES(25),PHI(5)
      DIMENSION UG(25),UQD(25),UGD(25),UV(25),UCB(25),VCB(25),
     1          UGP(25,25),UGA(25),GV(25),GRV(25),CPP(5,25),CPM(5,25)
      DIMENSION ALPHA(25),STALL(25),JSTL(25),TALK(20)
      DIMENSION RE(4)
C
      COMMON MZZZ,CD,R0,R1,R2,R3,PI,B,BS,SA,SAS,P
      COMMON/NEAR1/ NRUN,NBLD,NZ,MZ,NPRES,IR,NTIME,NERR,NPAG,NPHI,NPRINT
      COMMON/NEAR2/ C,A,AS,D,DS,SC,GS,H
      COMMON/NEAR3/ RRP,XP,Z,BLD,RB,BR,BTA,TCBLD,TC,RCBRP,APA,ALF,XPRES,
     1              RA,XCB,XR,ELCBC,RMAX,PHI,CORJ,CORCB
      COMMON/NEAR4/ UG,UQD,UGD,UV,UCB,VCB,UGP,UGA,GV,GAM,GRV,CPP,CPM
      COMMON/NEAR5/ ARJ,ARJP,EPS,RAD,CL,ALPHA,STALL,JSTL,TALK
C
  101 FORMAT(15H1     RUN NUMBER,I5,49X,4HPAGE,I3//)
  102 FORMAT(12H          INPUT/5X,65HDUCT GEOMETRY...     C/D       XP/C
     1   T/C       RTE/RP     RCB/RP)
  103 FORMAT(10X,20HCAMBER COEFFICIENTS.,4F10.6//)
  104 FORMAT(5X,22HPROPELLER GEOMETRY... ,I3,1X,6HBLADES//)
  105 FORMAT(25X,34HR/RP         B/RP        BETA     TH/CHD)
  106 FORMAT (/5X,47HDEFINITION OF SYMBOLS USED IN TABULAR OUTPUT...//)
  107 FORMAT(10X,66HR/RP       RADIAL PROPELLER STATION IN FRACTION OF PR
     1OPELLER RADIUS)
  108 FORMAT(10X,57HB/RP       PROPELLER CHORD IN FRACTION OF PROPELLER R
     1ADIUS)
  109 FORMAT(10X,36HBETA       PROPELLER PITCH IN DEGREES)
  110 FORMAT (10X,50HTH/CHD     PROPELLER BLADE THICKNESS-TO-CHORD RATIO)
  111 FORMAT (/5X,22HCENTERBODY GEOMETRY...2X5HLCB/C,5X5HXCB/C,3X8HRMAX/
     1LCB,2X9HX(RMAX)/C/25X,4F10.5)
  112 FORMAT (/5X,35HCONVERGENCE CRITERION...  EPSILON =,F7.5)
  114 FORMAT (10X1HV9X20HFREE STREAM VELOCITY)
  122 FORMAT(10X,31HU          TOTAL INFLOW VELOCITY)
  124 FORMAT(10X,48HGAM/V      STRENGTH OF INTERNAL VORTEX CYLINDER N)
  125 FORMAT(10X,34HALPHA      ANGLE OF ATTACK, DEGREES)
  126 FORMAT(10X,60HDELTA P/Q RISE IN TOTAL PRESSURE ACROSS PROPELLER NO
     1RMALIZED/25X,31HON FREE STREAM DYNAMIC PRESSURE)
  127 FORMAT(10X,53HCTP(D)     THRUST COEFFICIENT ON PROPELLER IN THE DUC
     1T)
  128 FORMAT(10X,40HCTD(P)     THRUST COEFFICIENT ON THE DUCT)
  129 FORMAT(10X,34HCTDP       TOTAL THRUST COEFFICIENT)
  130 FORMAT(10X,65HCTD(P)'    THRUST COEFFICIENT ON DUCT INCLUDING PRESS
     1URE THRUST ON/25X,29HTHE DUCT AFT OF THE PROPELLER)
  131 FORMAT (10X,60HCTDP'       TOTAL THRUST COEFFICIENT INCLUDING PRESSU
     1RE THRUST)
  132 FORMAT (10X,40HCNDP        TOTAL NORMAL FORCE COEFFICIENT)
  133 FORMAT (10X,43HCMDP        TOTAL PITCHING MOMENT COEFFICIENT)
  134 FORMAT (10X,23HJ           ADVANCE RATIO)
  135 FORMAT (10X,43HJ'          RATIO OF V TO PROPELLER TIP SPEED)
  148 FORMAT(20A4)
  149 FORMAT(10X,20A4//)
C
  240 FORMAT (F10.3,F8.3,F10.4,F8.3,F8.4,2(2X,1PE12.5))
  241 FORMAT (I10,6F10.6)
  244 FORMAT (5X,6(1PE13.6))
  250 FORMAT(20X,5F10.6//)
```

```
  251 FORMAT(20X,2F10.6,F10.3,F10.5)
  520 FORMAT (2I5,7F10.4)
  521 FORMAT (8F10.4)
  522 FORMAT (10I5)
C
  750 FORMAT (///10X,33HNUMBER OF BLADES IN ERROR, NBLD =,I5)
  751 FORMAT (///10X,46HCONVERGENCE CRITERION MUST BE GREATER THAN 0.0)
  752 FORMAT (///10X,46HANGLE OF ATTACK MUST BE LESS THAN 90.0 DEGREES)
  753 FORMAT (///10X,38HADVANCE RATIO MUST BE GREATER THAN 0.0)
C
   20 READ (5,148) TALK
      READ (5,521) CD,XP,TC,RRP,RCBRP,EPS
      READ (5,521) R0,R1,R2,R3
      READ (5,521) ELCBC,XCB,RMAX,XR
      READ (5,522) NBLD,NZ,NZP,IR,NPRES,NPRINT
      READ (5,521)  (RB(J),J=1,NZP)
      READ (5,521)  (BR(J),J=1,NZP)
      READ (5,521)  (BTA(J),J=1,NZP)
      READ (5,521)  (TCBLD(J),J=1,NZP)
      READ (5,521) (XPRES(N),N=1,IR)
      READ (5,520) NRUN,NPHI,ARJ,ALF,(PHI(J),J=1,5)
C
C       CHECK INPUT DATA
C
      IF (NPHI.GT.5) NPHI=5
      IF (NBLD) 700,700,701
  700 WRITE (6,750) NBLD
      NERR=1
  701 IF (EPS) 702,702,703
  702 WRITE (6,751)
      NERR=1
  703 IF (ALF-90.0) 704,705,705
  705 WRITE (6,752)
      NERR=1
  704 IF (ARJ) 706,706,707
  706 WRITE (6,753)
      NERR=1
  707 IF (NERR) 708,708,999
  708 CONTINUE
      NPAG=0
      WRITE (6,101) NRUN,NPAG
      WRITE (6,149) TALK
C
C       INITIALIZATION OF SUBROUTINES
C
      MZZZ=0
      CALL HUB (CD,XR,XCB,ELCBC,RMAX,IR,XPRES,RB,RRP,UCB,VCB)
      IF (NERR.GE.1) GO TO 999
      CALL ALFRNG (CD,50,SC,UGA,XPRES)
      DO 22 J=1,6
      H(J)=UGA(J+6)
   22 GS(J)=UGA(J)
      CALL PRESS
      MZZZ=1
      Z=NZ
      MZ=NZ-1
      CALL PKL (CD,P)
      ARJP=ARJ/PI/RRP
      BLD=NBLD
```

```
      APA=1./(RRP*RRP)-(RCBRP*RCBRP/RRP/RRP)
      WRITE (6,102)
      WRITE (6,250) CD,XP,TC,RRP,RCBRP
      WRITE (6,103) R0,R1,R2,R3
      WRITE (6,104) NBLD
      WRITE (6,105)
      DO 21 J=1,NZP
   21 WRITE (6,251) RB(J),BR(J),BTA(J),TCBLD(J)
      WRITE (6,111) ELCBC,XCB,RMAX,XR
      WRITE (6,112) EPS
C
C      COMPUTE INDUCED CAMBER COEFFICIENTS
C
      CALL CAMBER (CD,TC,RE)
      R0=R0-RE(1)
      R1=R1-RE(2)
      R2=R2-RE(3)
      R3=R3-RE(4)
C
      WRITE (6,106)
      WRITE (6,107)
      WRITE (6,108)
      WRITE (6,109)
      WRITE (6,110)
      WRITE (6,114)
      WRITE (6,122)
      WRITE (6,134)
      WRITE (6,135)
      WRITE (6,124)
      WRITE (6,125)
      WRITE (6,126)
      WRITE (6,127)
      WRITE (6,128)
      WRITE (6,129)
      WRITE (6,130)
      WRITE (6,131)
      WRITE (6,132)
      WRITE (6,133)
C
C      COMPUTE THE FOURIER COEFFICIENTS DUE TO THE CENTERBODY
C
      MZZZ=-1
      N=50
      CALL HUB (CD,XP,XCB,ELCBC,RMAX,N,XPRES,RB,RRP,UCB,VCB)
      MZZZ=1
      DO 40 K=1,6
      D(K)=VCB(K)
   40 DS(K)=UCB(K)
C
      CALL PROP (NZP,RB,BR,BTA,TCBLD,RRP,RCBRP,NZ,RA)
C
C      COMPUTE THE CONSTANT PORTION OF THE INFLOW PROFILE
C
      CDP=CD*RRP
      CALL GAMCYL (CDP,XP,NZ,RB,UG,0,XPRES,UGP,RRP,RA)
      CALL SRCRNG  (CD,XP,TC,NZ,RB,UQD)
      CALL HUB (CD,XP,XCB,ELCBC,RMAX,NZ,XPRES,RB,RRP,UCB,VCB)
C
C      COMPUTE THE FOURIER COEFFICIENTS DUE TO THE TRAILING VORTICITY
```

```
C
      CRBN=CD*2.
      CALL BNCOEF (CRBN,B)
      NOUT=6
      N=50
      NCYL=NZ-1
      CALL ANCOEF (NCYL,N,RA,XP,CD,RRP,NRUN,0,BS,A,AS)
      DO 30 K=1,NZ
      BR(K)=BR(K)/RRP
      RB(K)=RB(K)/RRP
      BTA(K)=90.0-BTA(K)
   30 BTA(K)=BTA(K)/RAD
  999 RETURN
      END
```

```
      SUBROUTINE PKL (CD,P)
C
      DIMENSION P(6,6),A(6,6,7),C(7)
C
      COMMON MZZZ
C
      IF (MZZZ) 1,2,1
    2 DO 10 K=1,6
      DO 10 L=1,6
      DO 10 M=1,7
   10 A(K,L,M)=0.0
C
      A(1,1,2) = .02683
      A(1,3,2) = .01343
      A(1,5,2) =-.00001
      A(2,1,2) = .05366
      A(2,2,2) = .02987
      A(2,4,2) =-.00304
      A(3,1,2) = .00608
      A(3,3,2) = .00403
      A(3,5,2) =-.00099
      A(4,1,2) =-.00005
      A(4,2,2) =-.00101
      A(4,4,2) = .00148
      A(4,6,2) =-.00049
      A(5,1,2) =-.00001
      A(5,3,2) =-.00049
      A(5,5,2) = .00079
      A(6,4,2) =-.00029
      A(6,6,2) = .00049
C
      A(1,1,3) = .07281
      A(1,3,3) = .03644
      A(1,5,3) =-.00004
      A(2,1,3) = .14561
      A(2,2,3) = .08526
      A(2,4,3) =-.01252
      A(2,6,3) = .00006
      A(3,1,3) = .02491
      A(3,3,3) = .01655
      A(3,5,3) =-.00411
      A(4,1,3) =-.00016
      A(4,2,3) =-.00417
      A(4,4,3) = .00609
      A(4,6,3) =-.00200
      A(5,1,3) =-.00013
      A(5,3,3) =-.00206
      A(5,5,3) = .00318
      A(6,2,3) = .00002
      A(6,4,3) =-.00120
      A(6,6,3) = .00197
C
      A(1,1,4) = .11925
      A(1,3,4) = .05945
      A(1,5,4) = .00019
      A(2,1,4) = .23849
      A(2,2,4) = .14668
      A(2,4,4) =-.02765
      A(2,6,4) = .00021
```

```
      A(3,1,4) = .05488
      A(3,3,4) = .03702
      A(3,5,4) =-.00965
      A(4,1,4) = .00071
      A(4,2,4) =-.00922
      A(4,4,4) = .01418
      A(4,6,4) =-.00463
      A(5,1,4) =-.00042
      A(5,3,4) =-.00482
      A(5,5,4) = .00732
      A(6,1,4) =-.00005
      A(6,2,4) = .00005
      A(6,4,4) =-.00278
      A(6,6,4) = .00450
C
      A(1,1,5) = .16016
      A(1,3,5) = .07907
      A(1,5,5) = .00107
      A(2,1,5) = .32032
      A(2,2,5) = .20609
      A(2,4,5) =-.04629
      A(2,6,5) = .00035
      A(3,1,5) = .09186
      A(3,3,5) = .06338
      A(3,5,5) =-.01766
      A(4,1,5) = .00402
      A(4,2,5) =-.01544
      A(4,4,5) = .02592
      A(4,6,5) =-.00855
      A(5,1,5) =-.00072
      A(5,3,5) =-.00883
      A(5,5,5) = .01338
      A(6,1,5) =-.00025
      A(6,2,5) = .00008
      A(6,4,5) =-.00512
      A(6,6,5) = .00814
C
      A(1,1,6) = .22278
      A(1,3,6) = .10634
      A(1,5,6) = .00536
      A(2,1,6) = .44556
      A(2,2,6) = .30830
      A(2,4,6) =-.08513
      A(2,6,6) =-.00059
      A(3,1,6) = .17105
      A(3,3,6) = .12346
      A(3,5,6) =-.03845
      A(4,1,6) = .02020
      A(4,2,6) =-.02784
      A(4,4,6) = .05692
      A(4,6,6) =-.01932
      A(5,1,6) = .00079
      A(5,3,6) =-.01858
      A(5,5,6) = .02935
      A(6,1,6) =-.00123
      A(6,2,6) =-.00011
      A(6,4,6) =-.01087
      A(6,6,6) = .01620
C
```

```
      A(1,1,7) = .26624
      A(1,3,7) = .12146
      A(1,5,7) = .01214
      A(2,1,7) = .53248
      A(2,2,7) = .38891
      A(2,4,7) =-.11898
      A(2,6,7) =-.00429
      A(3,1,7) = .24533
      A(3,3,7) = .18468
      A(3,5,7) =-.06210
      A(4,1,7) = .04662
      A(4,2,7) =-.03871
      A(4,4,7) = .09563
      A(4,6,7) =-.03416
      A(5,1,7) = .00738
      A(5,3,7) =-.02992
      A(5,5,7) = .05270
      A(6,1,7) =-.00194
      A(6,2,7) =-.00088
      A(6,4,7) =-.01918
      A(6,6,7) = .02998
C
      C(1)=0.0
      C(2)=0.25
      C(3)=0.50
      C(4)=0.75
      C(5)=1.00
      C(6)=1.50
      C(7)=2.00
      GO TO 71
C
    1 DO 20 J=1,7
      IF(C(J)-CD) 20,19,21
   20 CONTINUE
      GO TO 21
   19 M=J
      GO TO 100
   21 M=J-1
      N=J
      DELT=C(N)-C(M)
      DIFF=CD-C(M)
      DELTA=DIFF/DELT
      DO 30 K=1,6
      DO 30 L=1,6
      P(K,L)=A(K,L,M)+(DELTA*(A(K,L,N)-A(K,L,M)))
   30 CONTINUE
      GO TO 71
  100 DO 40 K=1,6
      DO 40 L=1,6
      P(K,L)=A(K,L,M)
   40 CONTINUE
   71 RETURN
      END
```

```
      SUBROUTINE ELLIPS (AKSQ,TK,TE)
C
C     SUB. ELLIPS -- TABLE LOOK-UP OF ELLIPTIC INTEGRALS
C
      DIMENSION CKK(100),CK(100),CE(100)
      COMMON MZZZ
      IF (MZZZ) 73,10,3
   10 CONTINUE
C
C        CKK = ARGUMENT OF ELLIPTIC INTEGRALS
C
      CKK(  1) = 0.00
      CKK(  2) =   .01
      CKK(  3) =   .02
      CKK(  4) =   .03
      CKK(  5) =   .04
      CKK(  6) =   .05
      CKK(  7) =   .06
      CKK(  8) =   .07
      CKK(  9) =   .08
      CKK( 10) =   .09
      CKK( 11) =   .10
      CKK( 12) =   .11
      CKK( 13) =   .12
      CKK( 14) =   .13
      CKK( 15) =   .14
      CKK( 16) =   .15
      CKK( 17) =   .16
      CKK( 18) =   .17
      CKK( 19) =   .18
      CKK( 20) =   .19
      CKK( 21) =   .20
      CKK( 22) =   .21
      CKK( 23) =   .22
      CKK( 24) =   .23
      CKK( 25) =   .24
      CKK( 26) =   .25
      CKK( 27) =   .26
      CKK( 28) =   .27
      CKK( 29) =   .28
      CKK( 30) =   .29
      CKK( 31) =   .30
      CKK( 32) =   .31
      CKK( 33) =   .32
      CKK( 34) =   .33
      CKK( 35) =   .34
      CKK( 36) =   .35
      CKK( 37) =   .36
      CKK( 38) =   .37
      CKK( 39) =   .38
      CKK( 40) =   .39
      CKK( 41) =   .40
      CKK( 42) =   .41
      CKK( 43) =   .42
      CKK( 44) =   .43
      CKK( 45) =   .44
      CKK( 46) =   .45
      CKK( 47) =   .46
      CKK( 48) =   .47
```

```
      CKK( 49) =  .48
      CKK( 50) =  .49
      CKK( 51) =  .50
      CKK( 52) =  .51
      CKK( 53) =  .52
      CKK( 54) =  .53
      CKK( 55) =  .54
      CKK( 56) =  .55
      CKK( 57) =  .56
      CKK( 58) =  .57
      CKK( 59) =  .58
      CKK( 60) =  .59
      CKK( 61) =  .60
      CKK( 62) =  .61
      CKK( 63) =  .62
      CKK( 64) =  .63
      CKK( 65) =  .64
      CKK( 66) =  .65
      CKK( 67) =  .66
      CKK( 68) =  .67
      CKK( 69) =  .68
      CKK( 70) =  .69
      CKK( 71) =  .70
      CKK( 72) =  .71
      CKK( 73) =  .72
      CKK( 74) =  .73
      CKK( 75) =  .74
      CKK( 76) =  .75
      CKK( 77) =  .76
      CKK( 78) =  .77
      CKK( 79) =  .78
      CKK( 80) =  .79
      CKK( 81) =  .80
      CKK( 82) =  .81
      CKK( 83) =  .82
      CKK( 84) =  .83
      CKK( 85) =  .84
      CKK( 86) =  .85
      CKK( 87) =  .86
      CKK( 88) =  .87
      CKK( 89) =  .88
      CKK( 90) =  .89
      CKK( 91) =  .90
      CKK( 92) =  .91
      CKK( 93) =  .92
      CKK( 94) =  .93
      CKK( 95) =  .94
      CKK( 96) =  .95
      CKK( 97) =  .96
      CKK( 98) =  .97
      CKK( 99) =  .98
      CKK(100) =  .99
C
C        CK = COMPLETE ELLIPTIC INTEGRALS OF FIRST KIND
C
      CK(  1) = 1.570796
      CK(  2) = 1.574746
      CK(  3) = 1.578740
      CK(  4) = 1.582780
```

```
CK(  5) = 1.586868                NE03 119
CK(  6) = 1.591003                NE03 120
CK(  7) = 1.595188                NE03 121
CK(  8) = 1.599423                NE03 122
CK(  9) = 1.603710                NE03 123
CK( 10) = 1.608049                NE03 124
CK( 11) = 1.612441                NE03 125
CK( 12) = 1.616889                NE03 126
CK( 13) = 1.621393                NE03 127
CK( 14) = 1.625955                NE03 128
CK( 15) = 1.630576                NE03 129
CK( 16) = 1.635257                NE03 130
CK( 17) = 1.640000                NE03 131
CK( 18) = 1.644806                NE03 132
CK( 19) = 1.649678                NE03 133
CK( 20) = 1.654617                NE03 134
CK( 21) = 1.659624                NE03 135
CK( 22) = 1.664701                NE03 136
CK( 23) = 1.669850                NE03 137
CK( 24) = 1.675073                NE03 138
CK( 25) = 1.680373                NE03 139
CK( 26) = 1.685750                NE03 140
CK( 27) = 1.691208                NE03 141
CK( 28) = 1.696749                NE03 142
CK( 29) = 1.702374                NE03 143
CK( 30) = 1.708087                NE03 144
CK( 31) = 1.713889                NE03 145
CK( 32) = 1.719785                NE03 146
CK( 33) = 1.725776                NE03 147
CK( 34) = 1.731865                NE03 148
CK( 35) = 1.738055                NE03 149
CK( 36) = 1.744351                NE03 150
CK( 37) = 1.750754                NE03 151
CK( 38) = 1.757269                NE03 152
CK( 39) = 1.763898                NE03 153
CK( 40) = 1.770647                NE03 154
CK( 41) = 1.777519                NE03 155
CK( 42) = 1.784519                NE03 156
CK( 43) = 1.791650                NE03 157
CK( 44) = 1.798918                NE03 158
CK( 45) = 1.806328                NE03 159
CK( 46) = 1.813884                NE03 160
CK( 47) = 1.821593                NE03 161
CK( 48) = 1.829460                NE03 162
CK( 49) = 1.837491                NE03 163
CK( 50) = 1.845694                NE03 164
CK( 51) = 1.854075                NE03 165
CK( 52) = 1.862641                NE03 166
CK( 53) = 1.871400                NE03 167
CK( 54) = 1.880361                NE03 168
CK( 55) = 1.889533                NE03 169
CK( 56) = 1.898925                NE03 170
CK( 57) = 1.908547                NE03 171
CK( 58) = 1.918410                NE03 172
CK( 59) = 1.928526                NE03 173
CK( 60) = 1.938908                NE03 174
CK( 61) = 1.949568                NE03 175
CK( 62) = 1.960521                NE03 176
CK( 63) = 1.971783                NE03 177
```

```
      CK( 64) = 1.983371
      CK( 65) = 1.995303
      CK( 66) = 2.007598
      CK( 67) = 2.020279
      CK( 68) = 2.033369
      CK( 69) = 2.046894
      CK( 70) = 2.060882
      CK( 71) = 2.075363
      CK( 72) = 2.090373
      CK( 73) = 2.105948
      CK( 74) = 2.122132
      CK( 75) = 2.138970
      CK( 76) = 2.156516
      CK( 77) = 2.174827
      CK( 78) = 2.193971
      CK( 79) = 2.214022
      CK( 80) = 2.235068
      CK( 81) = 2.257205
      CK( 82) = 2.280549
      CK( 83) = 2.305232
      CK( 84) = 2.331409
      CK( 85) = 2.359264
      CK( 86) = 2.389016
      CK( 87) = 2.420933
      CK( 88) = 2.455338
      CK( 89) = 2.492635
      CK( 90) = 2.533335
      CK( 91) = 2.578092
      CK( 92) = 2.627773
      CK( 93) = 2.683551
      CK( 94) = 2.747073
      CK( 95) = 2.820752
      CK( 96) = 2.908337
      CK( 97) = 3.016112
      CK( 98) = 3.155875
      CK( 99) = 3.354141
      CK(100) = 3.695637
C
C        CE = COMPLETE ELLIPTIC INTEGRALS OF SECOND KIND
C
      CE(  1) = 1.570796
      CE(  2) = 1.566862
      CE(  3) = 1.562913
      CE(  4) = 1.558948
      CE(  5) = 1.554969
      CE(  6) = 1.550973
      CE(  7) = 1.546963
      CE(  8) = 1.542936
      CE(  9) = 1.538893
      CE( 10) = 1.534833
      CE( 11) = 1.530758
      CE( 12) = 1.526665
      CE( 13) = 1.522555
      CE( 14) = 1.518428
      CE( 15) = 1.514284
      CE( 16) = 1.510122
      CE( 17) = 1.505942
      CE( 18) = 1.501743
      CE( 19) = 1.497526
```

```
CE( 20) = 1.493290                                    NE03 237
CE( 21) = 1.489035                                    NE03 238
CE( 22) = 1.484761                                    NE03 239
CE( 23) = 1.480466                                    NE03 240
CE( 24) = 1.476152                                    NE03 241
CE( 25) = 1.471818                                    NE03 242
CE( 26) = 1.467462                                    NE03 243
CE( 27) = 1.463086                                    NE03 244
CE( 28) = 1.458688                                    NE03 245
CE( 29) = 1.454269                                    NE03 246
CE( 30) = 1.449827                                    NE03 247
CE( 31) = 1.445363                                    NE03 248
CE( 32) = 1.440876                                    NE03 249
CE( 33) = 1.436366                                    NE03 250
CE( 34) = 1.431832                                    NE03 251
CE( 35) = 1.427274                                    NE03 252
CE( 36) = 1.422691                                    NE03 253
CE( 37) = 1.418083                                    NE03 254
CE( 38) = 1.413450                                    NE03 255
CE( 39) = 1.408791                                    NE03 256
CE( 40) = 1.404105                                    NE03 257
CE( 41) = 1.399392                                    NE03 258
CE( 42) = 1.394652                                    NE03 259
CE( 43) = 1.389883                                    NE03 260
CE( 44) = 1.385086                                    NE03 261
CE( 45) = 1.380259                                    NE03 262
CE( 46) = 1.375402                                    NE03 263
CE( 47) = 1.370515                                    NE03 264
CE( 48) = 1.365596                                    NE03 265
CE( 49) = 1.360645                                    NE03 266
CE( 50) = 1.355661                                    NE03 267
CE( 51) = 1.350644                                    NE03 268
CE( 52) = 1.345592                                    NE03 269
CE( 53) = 1.340505                                    NE03 270
CE( 54) = 1.335382                                    NE03 271
CE( 55) = 1.330223                                    NE03 272
CE( 56) = 1.325024                                    NE03 273
CE( 57) = 1.319788                                    NE03 274
CE( 58) = 1.314511                                    NE03 275
CE( 59) = 1.309192                                    NE03 276
CE( 60) = 1.303832                                    NE03 277
CE( 61) = 1.298428                                    NE03 278
CE( 62) = 1.292979                                    NE03 279
CE( 63) = 1.287484                                    NE03 280
CE( 64) = 1.281942                                    NE03 281
CE( 65) = 1.276350                                    NE03 282
CE( 66) = 1.270707                                    NE03 283
CE( 67) = 1.265013                                    NE03 284
CE( 68) = 1.259263                                    NE03 285
CE( 69) = 1.253458                                    NE03 286
CE( 70) = 1.247595                                    NE03 287
CE( 71) = 1.241671                                    NE03 288
CE( 72) = 1.235684                                    NE03 289
CE( 73) = 1.229632                                    NE03 290
CE( 74) = 1.223512                                    NE03 291
CE( 75) = 1.217321                                    NE03 292
CE( 76) = 1.211056                                    NE03 293
CE( 77) = 1.204714                                    NE03 294
CE( 78) = 1.198290                                    NE03 295
```

```
      CE( 79) = 1.191781
      CE( 80) = 1.185183
      CE( 81) = 1.178490
      CE( 82) = 1.171697
      CE( 83) = 1.164798
      CE( 84) = 1.157787
      CE( 85) = 1.150656
      CE( 86) = 1.143396
      CE( 87) = 1.135998
      CE( 88) = 1.128451
      CE( 89) = 1.120742
      CE( 90) = 1.112856
      CE( 91) = 1.104775
      CE( 92) = 1.096478
      CE( 93) = 1.087938
      CE( 94) = 1.079121
      CE( 95) = 1.069986
      CE( 96) = 1.060474
      CE( 97) = 1.050502
      CE( 98) = 1.039947
      CE( 99) = 1.028595
      CE(100) = 1.015994
C
      GO TO 30
    3 IF(AKSQ-.99)20,20,21
   21 PARA=0.25*(1.0-AKSQ)
  700 TEST = 1.00E-07
      IF(PARA-TEST)701,702,702
  701 PARA=TEST
  702 ZLP=ALOG(4./PARA)
      TK=ZLP*0.5*(1.+PARA)-PARA
      TE=1.0+(ZLP*PARA)-PARA
      GO TO 30
   20 JA=100.0*AKSQ
      JA=1+JA
      IF(CKK(JA)-AKSQ)22,23,22
   23 TK=CK(JA)
      TE=CE(JA)
      GO TO 30
   22 CON=(AKSQ-CKK(JA))/(CKK(JA+1)-CKK(JA))
      TK=CK(JA)+CON*(CK(JA+1)-CK(JA))
      TE=CE(JA)+CON*(CE(JA+1)-CE(JA))
      GO TO 30
   73 IF(AKSQ-.01)721,721,720
  721 PARA=.25*AKSQ
      GO TO 700
  720 AKSQ=1.-AKSQ
      GO TO 20
   30 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE LAMBDA (XP,ZP,YP)
C
C        TABLE LOOK-UP OF HEUMAN LAMBDA FUNCTION
C
      DIMENSION Y(19,19),X(19),Z(19)
      COMMON MZZZ
      IF (MZZZ) 20,10,20
   10 CONTINUE
C
C        X = ARCSIN K  (DEGREES)
C
      X( 1)=  0.000000
      X( 2)=  5.000000
      X( 3)= 10.000000
      X( 4)= 15.000000
      X( 5)= 20.000000
      X( 6)= 25.000000
      X( 7)= 30.000000
      X( 8)= 35.000000
      X( 9)= 40.000000
      X(10)= 45.000000
      X(11)= 50.000000
      X(12)= 55.000000
      X(13)= 60.000000
      X(14)= 65.000000
      X(15)= 70.000000
      X(16)= 75.000000
      X(17)= 80.000000
      X(18)= 85.000000
      X(19)= 90.000000
C
C        Z = BETA  (DEGREES)
C
      Z( 1)=  0.000000
      Z( 2)=  5.000000
      Z( 3)= 10.000000
      Z( 4)= 15.000000
      Z( 5)= 20.000000
      Z( 6)= 25.000000
      Z( 7)= 30.000000
      Z( 8)= 35.000000
      Z( 9)= 40.000000
      Z(10)= 45.000000
      Z(11)= 50.000000
      Z(12)= 55.000000
      Z(13)= 60.000000
      Z(14)= 65.000000
      Z(15)= 70.000000
      Z(16)= 75.000000
      Z(17)= 80.000000
      Z(18)= 85.000000
      Z(19)= 90.000000
C
C        Y = HEUMAN LAMBDA FUNCTION
C
      Y( 1, 1)=  0.000000
      Y( 1, 2)=   .087156
      Y( 1, 3)=   .173648
      Y( 1, 4)=   .258819
```

```
Y( 1, 5)=   .342020        NE04 060
Y( 1, 6)=   .422618        NE04 061
Y( 1, 7)=   .500000        NE04 062
Y( 1, 8)=   .573576        NE04 063
Y( 1, 9)=   .642788        NE04 064
Y( 1,10)=   .707107        NE04 065
Y( 1,11)=   .766044        NE04 066
Y( 1,12)=   .819152        NE04 067
Y( 1,13)=   .866025        NE04 068
Y( 1,14)=   .906308        NE04 069
Y( 1,15)=   .939693        NE04 070
Y( 1,16)=   .965926        NE04 071
Y( 1,17)=   .984808        NE04 072
Y( 1,18)=   .996195        NE04 073
Y( 1,19)=  1.000000        NE04 074
Y( 2, 1)=  0.000000        NE04 075
Y( 2, 2)=   .086990        NE04 076
Y( 2, 3)=   .173318        NE04 077
Y( 2, 4)=   .258327        NE04 078
Y( 2, 5)=   .341370        NE04 079
Y( 2, 6)=   .421815        NE04 080
Y( 2, 7)=   .499050        NE04 081
Y( 2, 8)=   .572487        NE04 082
Y( 2, 9)=   .641567        NE04 083
Y( 2,10)=   .705765        NE04 084
Y( 2,11)=   .764592        NE04 085
Y( 2,12)=   .817600        NE04 086
Y( 2,13)=   .864388        NE04 087
Y( 2,14)=   .904599        NE04 088
Y( 2,15)=   .937930        NE04 089
Y( 2,16)=   .964135        NE04 090
Y( 2,17)=   .983037        NE04 091
Y( 2,18)=   .994624        NE04 092
Y( 2,19)=  1.000000        NE04 093
Y( 3, 1)=  0.000000        NE04 094
Y( 3, 2)=   .086495        NE04 095
Y( 3, 3)=   .172332        NE04 096
Y( 3, 4)=   .256858        NE04 097
Y( 3, 5)=   .339430        NE04 098
Y( 3, 6)=   .419419        NE04 099
Y( 3, 7)=   .496219        NE04 100
Y( 3, 8)=   .569244        NE04 101
Y( 3, 9)=   .637940        NE04 102
Y( 3,10)=   .701786        NE04 103
Y( 3,11)=   .760298        NE04 104
Y( 3,12)=   .813034        NE04 105
Y( 3,13)=   .859602        NE04 106
Y( 3,14)=   .899660        NE04 107
Y( 3,15)=   .932934        NE04 108
Y( 3,16)=   .959244        NE04 109
Y( 3,17)=   .978597        NE04 110
Y( 3,18)=   .991511        NE04 111
Y( 3,19)=  1.000000        NE04 112
Y( 4, 1)=  0.000000        NE04 113
Y( 4, 2)=   .085677        NE04 114
Y( 4, 3)=   .170704        NE04 115
Y( 4, 4)=   .254434        NE04 116
Y( 4, 5)=   .336231        NE04 117
Y( 4, 6)=   .415475        NE04 118
```

```
Y( 4, 7)=    .491565                                          NE04 119
Y( 4, 8)=    .563926                                          NE04 120
Y( 4, 9)=    .632010                                          NE04 121
Y( 4,10)=    .695307                                          NE04 122
Y( 4,11)=    .753346                                          NE04 123
Y( 4,12)=    .805703                                          NE04 124
Y( 4,13)=    .852010                                          NE04 125
Y( 4,14)=    .891969                                          NE04 126
Y( 4,15)=    .925384                                          NE04 127
Y( 4,16)=    .952226                                          NE04 128
Y( 4,17)=    .972787                                          NE04 129
Y( 4,18)=    .988015                                          NE04 130
Y( 4,19)=   1.000000                                          NE04 131
Y( 5, 1)=   0.000000                                          NE04 132
Y( 5, 2)=    .084549                                          NE04 133
Y( 5, 3)=    .168458                                          NE04 134
Y( 5, 4)=    .251092                                          NE04 135
Y( 5, 5)=    .331827                                          NE04 136
Y( 5, 6)=    .410054                                          NE04 137
Y( 5, 7)=    .485184                                          NE04 138
Y( 5, 8)=    .556657                                          NE04 139
Y( 5, 9)=    .623939                                          NE04 140
Y( 5,10)=    .686540                                          NE04 141
Y( 5,11)=    .744012                                          NE04 142
Y( 5,12)=    .795963                                          NE04 143
Y( 5,13)=    .842073                                          NE04 144
Y( 5,14)=    .882119                                          NE04 145
Y( 5,15)=    .916018                                          NE04 146
Y( 5,16)=    .943918                                          NE04 147
Y( 5,17)=    .966343                                          NE04 148
Y( 5,18)=    .984410                                          NE04 149
Y( 5,19)=   1.000000                                          NE04 150
Y( 6, 1)=   0.000000                                          NE04 151
Y( 6, 2)=    .083124                                          NE04 152
Y( 6, 3)=    .165625                                          NE04 153
Y( 6, 4)=    .246882                                          NE04 154
Y( 6, 5)=    .326288                                          NE04 155
Y( 6, 6)=    .403252                                          NE04 156
Y( 6, 7)=    .477203                                          NE04 157
Y( 6, 8)=    .547600                                          NE04 158
Y( 6, 9)=    .613936                                          NE04 159
Y( 6,10)=    .675748                                          NE04 160
Y( 6,11)=    .732623                                          NE04 161
Y( 6,12)=    .784220                                          NE04 162
Y( 6,13)=    .830282                                          NE04 163
Y( 6,14)=    .870676                                          NE04 164
Y( 6,15)=    .905441                                          NE04 165
Y( 6,16)=    .934867                                          NE04 166
Y( 6,17)=    .959607                                          NE04 167
Y( 6,18)=    .980779                                          NE04 168
Y( 6,19)=   1.000000                                          NE04 169
Y( 7, 1)=   0.000000                                          NE04 170
Y( 7, 2)=    .081425                                          NE04 171
Y( 7, 3)=    .162247                                          NE04 172
Y( 7, 4)=    .241870                                          NE04 173
Y( 7, 5)=    .319707                                          NE04 174
Y( 7, 6)=    .395191                                          NE04 175
Y( 7, 7)=    .467777                                          NE04 176
Y( 7, 8)=    .536953                                          NE04 177
```

Y(7, 9)=	.602244	NE04 178
Y(7,10)=	.663225	NE04 179
Y(7,11)=	.719533	NE04 180
Y(7,12)=	.770883	NE04 181
Y(7,13)=	.817093	NE04 182
Y(7,14)=	.858117	NE04 183
Y(7,15)=	.894095	NE04 184
Y(7,16)=	.925409	NE04 185
Y(7,17)=	.952751	NE04 186
Y(7,18)=	.977159	NE04 187
Y(7,19)=	1.000000	NE04 188
Y(8, 1)=	0.000000	NE04 189
Y(8, 2)=	.079476	NE04 190
Y(8, 3)=	.158377	NE04 191
Y(8, 4)=	.236134	NE04 192
Y(8, 5)=	.312192	NE04 193
Y(8, 6)=	.386013	NE04 194
Y(8, 7)=	.457086	NE04 195
Y(8, 8)=	.524935	NE04 196
Y(8, 9)=	.589127	NE04 197
Y(8,10)=	.649283	NE04 198
Y(8,11)=	.705094	NE04 199
Y(8,12)=	.756337	NE04 200
Y(8,13)=	.802903	NE04 201
Y(8,14)=	.844820	NE04 202
Y(8,15)=	.882297	NE04 203
Y(8,16)=	.915757	NE04 204
Y(8,17)=	.945873	NE04 205
Y(8,18)=	.973573	NE04 206
Y(8,19)=	1.000000	NE04 207
Y(9, 1)=	0.000000	NE04 208
Y(9, 2)=	.077307	NE04 209
Y(9, 3)=	.154073	NE04 210
Y(9, 4)=	.229767	NE04 211
Y(9, 5)=	.303869	NE04 212
Y(9, 6)=	.375880	NE04 213
Y(9, 7)=	.445330	NE04 214
Y(9, 8)=	.511786	NE04 215
Y(9, 9)=	.574862	NE04 216
Y(9,10)=	.634231	NE04 217
Y(9,11)=	.689642	NE04 218
Y(9,12)=	.740932	NE04 219
Y(9,13)=	.788051	NE04 220
Y(9,14)=	.831085	NE04 221
Y(9,15)=	.870277	NE04 222
Y(9,16)=	.906056	NE04 223
Y(9,17)=	.939042	NE04 224
Y(9,18)=	.970039	NE04 225
Y(9,19)=	1.000000	NE04 226
Y(10, 1)=	0.000000	NE04 227
Y(10, 2)=	.074953	NE04 228
Y(10, 3)=	.149408	NE04 229
Y(10, 4)=	.222878	NE04 230
Y(10, 5)=	.294884	NE04 231
Y(10, 6)=	.364976	NE04 232
Y(10, 7)=	.432729	NE04 233
Y(10, 8)=	.497760	NE04 234
Y(10, 9)=	.559735	NE04 235
Y(10,10)=	.618381	NE04 236

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Y(10,11)=    .673501                                                    NE04 237
Y(10,12)=    .724985                                                    NE04 238
Y(10,13)=    .772830                                                    NE04 239
Y(10,14)=    .817155                                                    NE04 240
Y(10,15)=    .858217                                                    NE04 241
Y(10,16)=    .896419                                                    NE04 242
Y(10,17)=    .932311                                                    NE04 243
Y(10,18)=    .966576                                                    NE04 244
Y(10,19)=   1.000000                                                    NE04 245
Y(11, 1)=   0.000000                                                    NE04 246
Y(11, 2)=    .072455                                                    NE04 247
Y(11, 3)=    .144464                                                    NE04 248
Y(11, 4)=    .215587                                                    NE04 249
Y(11, 5)=    .285399                                                    NE04 250
Y(11, 6)=    .353500                                                    NE04 251
Y(11, 7)=    .419519                                                    NE04 252
Y(11, 8)=    .483125                                                    NE04 253
Y(11, 9)=    .544038                                                    NE04 254
Y(11,10)=    .602038                                                    NE04 255
Y(11,11)=    .656976                                                    NE04 256
Y(11,12)=    .708785                                                    NE04 257
Y(11,13)=    .757496                                                    NE04 258
Y(11,14)=    .803241                                                    NE04 259
Y(11,15)=    .846269                                                    NE04 260
Y(11,16)=    .886942                                                    NE04 261
Y(11,17)=    .925731                                                    NE04 262
Y(11,18)=    .963204                                                    NE04 263
Y(11,19)=   1.000000                                                    NE04 264
Y(12, 1)=   0.000000                                                    NE04 265
Y(12, 2)=    .069861                                                    NE04 266
Y(12, 3)=    .139334                                                    NE04 267
Y(12, 4)=    .208034                                                    NE04 268
Y(12, 5)=    .275597                                                    NE04 269
Y(12, 6)=    .341676                                                    NE04 270
Y(12, 7)=    .405958                                                    NE04 271
Y(12, 8)=    .468167                                                    NE04 272
Y(12, 9)=    .528076                                                    NE04 273
Y(12,10)=    .585512                                                    NE04 274
Y(12,11)=    .640369                                                    NE04 275
Y(12,12)=    .692612                                                    NE04 276
Y(12,13)=    .742291                                                    NE04 277
Y(12,14)=    .789537                                                    NE04 278
Y(12,15)=    .834576                                                    NE04 279
Y(12,16)=    .877717                                                    NE04 280
Y(12,17)=    .919353                                                    NE04 281
Y(12,18)=    .959944                                                    NE04 282
Y(12,19)=   1.000000                                                    NE04 283
Y(13, 1)=   0.000000                                                    NE04 284
Y(13, 2)=    .067226                                                    NE04 285
Y(13, 3)=    .134126                                                    NE04 286
Y(13, 4)=    .200380                                                    NE04 287
Y(13, 5)=    .265684                                                    NE04 288
Y(13, 6)=    .329751                                                    NE04 289
Y(13, 7)=    .392328                                                    NE04 290
Y(13, 8)=    .453192                                                    NE04 291
Y(13, 9)=    .512167                                                    NE04 292
Y(13,10)=    .569122                                                    NE04 293
Y(13,11)=    .623985                                                    NE04 294
Y(13,12)=    .676745                                                    NE04 295
```

Y(13,13)=	.727455	NE04 296
Y(13,14)=	.776237	NE04 297
Y(13,15)=	.823283	NE04 298
Y(13,16)=	.868846	NE04 299
Y(13,17)=	.913240	NE04 300
Y(13,18)=	.956826	NE04 301
Y(13,19)=	1.000000	NE04 302
Y(14, 1)=	0.000000	NE04 303
Y(14, 2)=	.064614	NE04 304
Y(14, 3)=	.128968	NE04 305
Y(14, 4)=	.192809	NE04 306
Y(14, 5)=	.255897	NE04 307
Y(14, 6)=	.318009	NE04 308
Y(14, 7)=	.378946	NE04 309
Y(14, 8)=	.438541	NE04 310
Y(14, 9)=	.496661	NE04 311
Y(14,10)=	.553214	NE04 312
Y(14,11)=	.608153	NE04 313
Y(14,12)=	.661480	NE04 314
Y(14,13)=	.713246	NE04 315
Y(14,14)=	.763552	NE04 316
Y(14,15)=	.812552	NE04 317
Y(14,16)=	.860443	NE04 318
Y(14,17)=	.907464	NE04 319
Y(14,18)=	.953885	NE04 320
Y(14,19)=	1.000000	NE04 321
Y(15, 1)=	0.000000	NE04 322
Y(15, 2)=	.062100	NE04 323
Y(15, 3)=	.124009	NE04 324
Y(15, 4)=	.185540	NE04 325
Y(15, 5)=	.246517	NE04 326
Y(15, 6)=	.306778	NE04 327
Y(15, 7)=	.366180	NE04 328
Y(15, 8)=	.424604	NE04 329
Y(15, 9)=	.481959	NE04 330
Y(15,10)=	.538183	NE04 331
Y(15,11)=	.593247	NE04 332
Y(15,12)=	.647159	NE04 333
Y(15,13)=	.699961	NE04 334
Y(15,14)=	.751731	NE04 335
Y(15,15)=	.802581	NE04 336
Y(15,16)=	.852654	NE04 337
Y(15,17)=	.902119	NE04 338
Y(15,18)=	.951166	NE04 339
Y(15,19)=	1.000000	NE04 340
Y(16, 1)=	0.000000	NE04 341
Y(16, 2)=	.059779	NE04 342
Y(16, 3)=	.119433	NE04 343
Y(16, 4)=	.178839	NE04 344
Y(16, 5)=	.237883	NE04 345
Y(16, 6)=	.296459	NE04 346
Y(16, 7)=	.354475	NE04 347
Y(16, 8)=	.411857	NE04 348
Y(16, 9)=	.468546	NE04 349
Y(16,10)=	.524506	NE04 350
Y(16,11)=	.579721	NE04 351
Y(16,12)=	.634200	NE04 352
Y(16,13)=	.687972	NE04 353
Y(16,14)=	.741089	NE04 354

```
Y(16,15)=    .793624                                NE04 355
Y(16,16)=    .845669                                NE04 356
Y(16,17)=    .897332                                NE04 357
Y(16,18)=    .948733                                NE04 358
Y(16,19)=   1.000000                                NE04 359
Y(17, 1)=   0.000000                                NE04 360
Y(17, 2)=    .057773                                NE04 361
Y(17, 3)=    .115479                                NE04 362
Y(17, 4)=    .173054                                NE04 363
Y(17, 5)=    .230436                                NE04 364
Y(17, 6)=    .287571                                NE04 365
Y(17, 7)=    .344410                                NE04 366
Y(17, 8)=    .400915                                NE04 367
Y(17, 9)=    .457055                                NE04 368
Y(17,10)=    .512813                                NE04 369
Y(17,11)=    .568181                                NE04 370
Y(17,12)=    .623166                                NE04 371
Y(17,13)=    .677782                                NE04 372
Y(17,14)=    .732059                                NE04 373
Y(17,15)=    .786036                                NE04 374
Y(17,16)=    .839759                                NE04 375
Y(17,17)=    .893286                                NE04 376
Y(17,18)=    .946677                                NE04 377
Y(17,19)=   1.000000                                NE04 378
Y(18, 1)=   0.000000                                NE04 379
Y(18, 2)=    .056256                                NE04 380
Y(18, 3)=    .112490                                NE04 381
Y(18, 4)=    .168682                                NE04 382
Y(18, 5)=    .224814                                NE04 383
Y(18, 6)=    .280867                                NE04 384
Y(18, 7)=    .336826                                NE04 385
Y(18, 8)=    .392679                                NE04 386
Y(18, 9)=    .448417                                NE04 387
Y(18,10)=    .504034                                NE04 388
Y(18,11)=    .559529                                NE04 389
Y(18,12)=    .614903                                NE04 390
Y(18,13)=    .670162                                NE04 391
Y(18,14)=    .725315                                NE04 392
Y(18,15)=    .780373                                NE04 393
Y(18,16)=    .835352                                NE04 394
Y(18,17)=    .890270                                NE04 395
Y(18,18)=    .945145                                NE04 396
Y(18,19)=   1.000000                                NE04 397
Y(19, 1)=   0.000000                                NE04 398
Y(19, 2)=    .055556                                NE04 399
Y(19, 3)=    .111111                                NE04 400
Y(19, 4)=    .166667                                NE04 401
Y(19, 5)=    .222222                                NE04 402
Y(19, 6)=    .277778                                NE04 403
Y(19, 7)=    .333333                                NE04 404
Y(19, 8)=    .388889                                NE04 405
Y(19, 9)=    .444444                                NE04 406
Y(19,10)=    .500000                                NE04 407
Y(19,11)=    .555556                                NE04 408
Y(19,12)=    .611111                                NE04 409
Y(19,13)=    .666667                                NE04 410
Y(19,14)=    .722222                                NE04 411
Y(19,15)=    .777778                                NE04 412
Y(19,16)=    .833333                                NE04 413
```

```
      Y(19,17)=    .888889
      Y(19,18)=    .944444
      Y(19,19)=   1.000000
C
      GO TO 90
   20 DX = 5.0
      DZ = 5.0
      FNX=XP/DX
      N=FNX
      N=N+1
      FNZ=ZP/DZ
      M=FNZ
      M=M+1
      C1=(ZP-Z(M))/DZ
      C2=(XP-X(N))/DX
      A=(Y(N,M+1)-Y(N,M))*C1
      B=(Y(N+1,M)-Y(N,M))*C2
      D=C1*C2*(Y(N+1,M+1)-Y(N+1,M)+Y(N,M)-Y(N,M+1))
      YP=Y(N,M)+A+B+D
   90 RETURN
      END
```

```
      SUBROUTINE HUB (CD,XR,XCB,ELCBC,H,IR,X,RB,RRP,U,V)
C
C       CALCULATION OF VELOCITIES INDUCED BY THE CENTERBODY
C          **  CENTERBODY IS REPRESENTED AS A RANKINE BODY  **
C
      DIMENSION X(25),RB(25),U(25),V(25),FU(50),FV(50),D(6),DS(6)
      COMMON MZZZ
C
  700 FORMAT (///10X,36HINPUT CENTERBODY DIMENSIONS IN ERROR///)
  701 FORMAT (///10X,52HSUBROUTINE HUB UNABLE TO COMPUTE CENTERBODY GEOM
     1ETRY///)
C
      IF (MZZZ) 3,1,2
    1 XIZ=(XR-XCB)/ELCBC
C
C       COMPUTE LOCATION OF POINT SOURCE
C
      IF (XIZ-H) 10,10,11
   10 NERR=1
C
C       ERROR MESSAGE
      WRITE (6,700)
      RETURN
   11 H2=H*H
      XIZ2=XIZ*XIZ
      A=XIZ-H
      DO 20 J=1,20
      AA=XIZ*H2*SQRT(A*A + H2)
      AA=XIZ2-SQRT(AA)
      AA=SQRT(AA)
      DEL=ABS(AA-A)/AA
      IF (DEL-.001) 30,30,19
   19 A=(AA+A)/2.
   20 CONTINUE
C
C       ERROR MESSAGE
C
      WRITE (6,701)
      NERR=1
      RETURN
   30 A=AA
      EMV=((XIZ2-A*A)**2)/(4.*XIZ*A)
      RETURN
    2 IF (IR) 100,100,200
C
C       COMPUTE VELOCITIES INDUCED AT THE DUCT REFERENCE CYLINDER
C
  100 W=0.5/CD/ELCBC
      NX=-IR
      DO 40 J=1,NX
      XI=X(J)-XCB-XIZ*ELCBC
      XI=XI/ELCBC
   35 XPA=XI+A
      XMA=XI-A
   34 RXPA=SQRT(XPA*XPA + W*W)
      RXMA=SQRT(XMA*XMA + W*W)
      RXPA3=RXPA**3
      RXMA3=RXMA**3
      U(J)=EMV*(XPA/RXPA3 - XMA/RXMA3)
```

```
      V(J)=EMV*W*(1./RXPA3 - 1./RXMA3)
   40 CONTINUE
      RETURN
C
C       COMPUTE INFLOW VELOCITIES
C
  200 XI=XR-XCB-XIZ*ELCBC
      XI=XI/ELCBC
      DO 60 J=1,IR
      W=RB(J)*.5/CD/ELCBC/RRP
   65 XPA=XI+A
      XMA=XI-A
   64 RXPA=SQRT(XPA*XPA + W*W)
      RXMA=SQRT(XMA*XMA + W*W)
      RXPA3=RXPA**3
      RXMA3=RXMA**3
      U(J)=EMV*(XPA/RXPA3 - XMA/RXMA3)
      V(J)=EMV*W*(1./RXPA3 - 1./RXMA3)
      PSI=W*W/2. - EMV*(XPA/RXPA - XMA/RXMA)
   60 CONTINUE
      RETURN
C
C       COMPUTE D(N) AND D-STAR(N) FOURIER COEFFICIENTS
C
    3 W=0.5/CD/ELCBC
      N=IR
      CN=N
      PI=3.1415926
      DTH=PI/(CN-1.)
      TH=-DTH
      DO 80 J=1,N
      TH=TH+DTH
      CSTH=COS(TH)
      XG=0.5*(1.-CSTH)
      XI=XG-XCB-XIZ*ELCBC
      XI=XI/ELCBC
      XPA=XI+A
      XMA=XI-A
      RXPA=SQRT(XPA*XPA+W*W)
      RXMA=SQRT(XMA*XMA+W*W)
      RXPA3=RXPA**3
      RXMA3=RXMA**3
      FU(J)=EMV*(XPA/RXPA3-XMA/RXMA3)
      FV(J)=EMV*W*(1./RXPA3-1./RXMA3)
   80 CONTINUE
      NOUT=6
      CALL FOURCS (FU,FU,N,NOUT,0)
      CALL FOURCS (FV,FV,N,NOUT,0)
      DO 81 J=1,NOUT
      D(J)=FV(J)
      DS(J)=FU(J)
      U(J)=DS(J)
   81 V(J)=D(J)
      RETURN
      END
```

```
      SUBROUTINE CAMBER (CD,TC,RP)
C
C      SUBROUTINE TO COMPUTE INDUCED CAMBER COEFFICIENTS
C
      DIMENSION CDRAT(5),DCD(4,5),RP(4)
C
      DATA CDRAT/0.0,0.25,0.50,0.75,1.0/
      DATA (DCD(1,J),J=1,5)/0.0,0.00149,0.00316,0.00505,0.00696/
      DATA (DCD(2,J),J=1,5)/0.0,0.10897,0.22236,0.33414,0.43940/
      DATA (DCD(3,J),J=1,5)/0.0,0.03558,0.07223,0.11022,0.14928/
      DATA (DCD(4,J),J=1,5)/0.0,-0.00203,-0.00539,-0.00948,-0.01375/
C
      DO 2 J=1,5
      N=J
      IF (CDRAT(J)-CD) 2,3,4
    2 CONTINUE
    4 M=N-1
      DEL=CDRAT(N)-CDRAT(M)
      DIF=CD-CDRAT(M)
      DELTA=DIF/DEL
      DO 10 K=1,4
      RP(K)=DCD(K,M) + DELTA*(DCD(K,N)-DCD(K,M))
   10 CONTINUE
      GO TO 20
    3 M=N
      DO 6 K=1,4
    6 RP(K)=DCD(K,M)
   20 DO 21 K=1,4
   21 RP(K)=RP(K)*TC
      RETURN
      END
```

```
      SUBROUTINE PROP (NZP,RB,BR,BTA,TCBLD,RRP,RCBRP,NZ,RA)
C
C       SUBROUTINE TO COMPUTE PROPELLER GEOMETRY PARAMETERS
C
      DIMENSION RB(25),BR(25),TCBLD(25),X(25),Y(25),Z(25),ZZ(25),BTA(25)
     1          ,RA(25)
C
      P=RCBRP*RCBRP
      D=1.-P
      X(1)=RCBRP
      ZN=NZ
      MZ=NZ+1
      DO 20 J=2,MZ
      K=J-1
      AK=K
      E=AK/ZN*D+P
   20 X(J)=SQRT (E)
      DO 21 J=1,NZ
   21 RA(J)=X(J+1)
      X(1)=(RCBRP+RA(1))/2.0
      DO 22 J=2,NZ
   22 X(J)=(RA(J)+RA(J-1))/2.0
      K=1
      RB(NZP+1)=RB(NZP)
      BR(NZP+1)=BR(NZP)
      BTA(NZP+1)=BTA(NZP)
      TCBLD(NZP+1)=TCBLD(NZP)
      DO 39 J=1,NZ
   30 IF (RB(K)-X(J)) 31,32,33
   32 Y(J)=BR(K)
      Z(J)=BTA(K)
      ZZ(J)=TCBLD(K)
      GO TO 39
   31 K=K+1
      GO TO 30
   33 DEL=X(J)-RB(K-1)
      DEL=DEL/(RB(K)-RB(K-1))
      Y(J)=DEL*(BR(K)-BR(K-1))
      Y(J)=Y(J)+BR(K-1)
      Z(J)=DEL*(BTA(K)-BTA(K-1))
      Z(J)=Z(J)+BTA(K-1)
      ZZ(J)=DEL*(TCBLD(K)-TCBLD(K-1))
      ZZ(J)=ZZ(J)+TCBLD(K-1)
   39 CONTINUE
      DO 40 J=1,NZ
      RB(J)=X(J)
      BR(J)=Y(J)
      BTA(J)=Z(J)
      TCBLD(J)=ZZ(J)
   40 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CLALF  (J)
C
C     COMPUTATION OF BLADE SECTION LIFT COEFFICIENT
C
      DIMENSION TCB(10),CLMX(10)
C
      DIMENSION B(6),BS(6),SA(6),SAS(6),P(6,6)
      DIMENSION RB(25),BR(25),BTA(25),TCBLD(25),RA(25),XPRES(25),PHI(5)
      DIMENSION ALPHA(25),STALL(25),JSTL(25),TALK(20)
C
      COMMON MZZZ,CD,R0,R1,R2,R3,PI,B,BS,SA,SAS,P
      COMMON/NEAR1/ NRUN,NBLD,NZ,MZ,NPRES,IR,NTIME,NERR,NPAG,NPHI,NPRINT
      COMMON/NEAR3/ RRP,XP,Z,BLD,RB,BR,BTA,TCBLD,TC,RCBRP,APA,ALF,XPRES,
     1              RA,XCB,XR,ELCBC,RMAX,PHI,CORJ,CORCB
      COMMON/NEAR5/ ARJ,ARJP,EPS,RAD,CL,ALPHA,STALL,JSTL,TALK
C
  101 FORMAT (////////////70H    THE BLADE THICKNESS-TO-CHORD RATIO IS OUT
     1SIDE THE RANGE 0.0 TO .34)
      IF (MZZZ) 2,1,2
    1 CONTINUE
C
C     TABLE OF VALUES OF CLMAX VERSUS BLADE THICKNESS-TO-CHORD RATIO
C
      TCB(1)=0.0
      TCB(2)=.06
      TCB(3)=.08
      TCB(4)=.10
      TCB(5)=.12
      TCB(6)=.15
      TCB(7)=.18
      TCB(8)=.21
      TCB(9)=.24
      TCB(10)=.34
C
      CLMX(1)=0.9
      CLMX(2)=0.9
      CLMX(3)=1.2
      CLMX(4)=1.45
      CLMX(5)=1.6
      CLMX(6)=1.5
      CLMX(7)=1.35
      CLMX(8)=1.3
      CLMX(9)=1.25
      CLMX(10)=1.1
      RETURN
C
    2 CONTINUE
   19 IF (TCB(10)-TCBLD(J)) 21,20,20
   20 IF (TCB(1)-TCBLD(J)) 22,22,21
   21 WRITE (6,101)
      NERR=1
      RETURN
   22 N=1
   23 IF (TCB(N)-TCBLD(J)) 24,25,26
   24 N=N+1
      GO TO 23
   25 CLMAX=CLMX(N)
      GO TO 27
   26 DEL=(TCBLD(J)-TCB(N-1))/(TCB(N)-TCB(N-1))
```

```
      CLMAX=CLMX(N-1)+DEL*(CLMX(N)-CLMX(N-1))
   27 CONTINUE
      CL=2.*PI*ALPHA(J)/RAD
      STALL(J)=0.0
   28 IF (CL-CLMAX) 29,30,30
   30 CL=CLMAX
      STALL(J)=1.0
   29 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ARCSIN(ARG,PHI,INDEX)
C
C     SUBROUTINE FOR COMPUTING ARC SINE
C
C
C     ARG IS THE SINE OF THE ANGLE.
C     PHI IS THE ANGLE (PRINCIPAL VALUE,  -PI/2 TO PI/2)
C         OUTPUT FROM SUBROUTINE.
C     INDEX IS 1 IF ANGLE IS TO BE IN RADIANS.
C     INDEX IS 0 IF ANGLE IS TO BE IN DEGREES.
C
      HPI=1.5707963
      A=1.0-ARG*ARG
      IF (A) 5,2,1
    2 IF (ARG) 4,3,3
    3 PHI=HPI
      GO TO 6
    4 PHI=-HPI
      GO TO 6
    1 PHI=ATAN (ARG/SQRT (A))
      GO TO 6
    5 WRITE (6,100)
  100 FORMAT(/5X30HERROR...SIN X GREATER THAN 1.0//)
      GO TO 2
    6 IF(INDEX) 7,7,8
    7 PHI=90.*PHI/HPI
    8 RETURN
      END
```

```fortran
      SUBROUTINE MATRIX (A,NR)
      DIMENSION A( 6, 7),B(19,19),N(12)
      NC=NR+1
      DO 1 J=1,NR
    1 N(J)=0
      DO 2 I=1,NR
      CON=0.0
      II=I-1
      DO 3 J=1,NR
      JJ=J-1
      IF (II) 4,4,5
    5 DO 6 K=1,II
      IF (J-N(K)) 6,3,6
    6 CONTINUE
    4 CONA=A(J,1)
      IF (CONA) 8,9,9
    8 CONA=-CONA
    9 IF (CONA-CON) 3,3,11
   11 CON=CONA
      N(I)=J
    3 CONTINUE
      IF(CON) 500,600,500
  600 WRITE (6,601) I
  601 FORMAT(13H SINGULAR, I=,I3)
      STOP
  500 NN=N(I)
      DO 12 J=1,NR
   12 A(J,NC)=0.0
      A(NN,NC)=1.0
      DIV=A(NN,1)
      DO 13 L=1,NC
   13 A(NN,L)=A(NN,L)/DIV
      DO 14 L=1,NR
      IF (L-NN) 15,14,15
   15 CMULT=A(L,1)
      DO 16 J=1,NC
   16 A(L,J)=A(L,J)-CMULT*A(NN,J)
   14 CONTINUE
      DO 17 L=1,NR
      DO 18 J=1,NR
   18 A(L,J)=A(L,J+1)
   17 CONTINUE
    2 CONTINUE
      DO 19 J=1,NR
      DO 20 L=1,NR
      NN=N(J)
   20 B(J,L)=A(NN,L)
   19 CONTINUE
      DO 21 J=1,NR
      DO 22 L=1,NR
      NN=N(J)
   22 A(L,NN)=B(L,J)
   21 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FOURCS  (F,B,N,NOUT,NPRINT)
C
C     SUB. FOURCS -- FOURIER COSINE SERIES DETERMINATION
C     FROM RALSTON AND WILF -- MATH. METHODS FOR DIGITAL COMPUTERS
C     CHAPT. 24 BY G. GOERTZEL
C
      DIMENSION F(50),FF(50),B(50)
      COMMON MZZZ
  700 FORMAT (1H /)
  701 FORMAT (4E15.7)
  702 FORMAT(5X5HTHETA6X10HFOURIER FN6X11HORIGINAL FN4X12HFOURIER COEF/)
      PI=3.1415927
      IF(NOUT-N)10,10,11
   11 NOUT=N
   10 ZRO=0.
      ONE=1.0
      TWO=2.0
      IZRO=0
      IONE=1
      ITWO=2
      HPI=1.5707963
      NH=N
      NS=ITWO*(NH-IONE)
      FNS=NS
      FR=TWO/FNS
      PIN=PI*FR
      DO 201 I=1,NH
      CJ=I-IONE
      CSI=COS (PIN*CJ)
      CCSI=CSI+CSI
      CA=ZRO
      CB=ZRO
      DO 202 J=2,NS
      JK=NS-J+ITWO
      IF(JK-NH) 220,220,221
  221 JK=ITWO*NH-JK
  220 CC=F(JK)+CA*CCSI-CB
      CB=CA
  202 CA=CC
      FF(I)=FR*(F(1)+CA*CSI-CB)
      IF(I-IONE) 290,291,290
  291 FF(I)=FF(I)/TWO
  290 JN=NS-I-I+IONE
      IF(JN) 203,204,204
  203 FF(I)=FF(I)/TWO
  204 CONTINUE
  201 CONTINUE
      IF (NPRINT-1)2,6,7
    6 M=N
      GO TO 1
    7 M=NOUT
    1 CN=N
      WRITE (6,700)
      WRITE (6,702)
      DTH=PI/(CN-1.0)
      TH=-DTH
      DO 3 I=1,N
      TH=TH+DTH
      SUM=0.0
```

```
      DO 4 J=1,M                                                        NE11 060
      CJ=J-1                                                            NE11 061
    4 SUM=SUM+ FF(J)*COS (CJ*TH)                                        NE11 062
    3 WRITE (6,701) TH, SUM,F(I), FF(I)                                 NE11 063
    2 DO 8 K=1,NOUT                                                     NE11 064
    8 B(K)=FF(K)                                                        NE11 065
      RETURN                                                            NE11 066
      END                                                               NE11 067
```

```
      SUBROUTINE SRCRNG (CD,XSC,TC,NRP,RP,UQD)
C
C        VELOCITY INDUCED BY DISTRIBUTION OF SOURCE RINGS
C
      DIMENSION TH(205),QD(205),DI(3),A(6),RP(25),UQD(25)
      COMMON MZ
      PI=3.1415926
      A(1)=TC* 2.969
      A(2)=-TC*1.26
      A(3)=-TC*3.516
      A(4)=TC* 2.843
      A(5)=-TC*1.015
      DQ =200.
      DTI=200.
   14 DT=PI/DQ
      XS=XSC-.5
      N=DQ
      NM=N
      N=N+1
      K=N+4
      DO 15 J=1,K
      TH(J)=0.
   15 QD(J)=0.
      DO 19 J=2,N
      TH(J)=TH(J-1)+DT
  120 T=TH(J)
      ST=SIN (T)
      SN2=SIN (T/2.)
      QD(J)=A(1)/2./SN2
      DO 18 I=2,5
      IN=I-1
      EN=IN
      INX=IN+IN-2
      ENX=INX
      QD(J)=QD(J)+EN*A(I)*(SN2**ENX)
   18 CONTINUE
      QD(J)=QD(J)*ST
   19 CONTINUE
      TH(N+1)=PI+1.
      QD(N+1)=QD(N)
   22 QD(1)=A(1)
      TH(1)=0.
   31 DTP=PI/DTI
      TP=0.
      H=DTP/3.
      UI=0.
      CST=-2.*XS
      CST2=CST*CST
   40 DO 69 JR=1,NRP
      R=RP(JR)
   50 K=1
      NT=1
   51 DO 60 J=K,3
      CTP=COS (TP)
      ZWP=CD*(CTP-CST)
      PP=(ZWP*ZWP)+((R+1.)*(R+1.))
      PM=(ZWP*ZWP)+((R-1.)*(R-1.))
      AK2=4.*R/PP
      CALL ELLIPS (AK2,ZK,ZE)
```

```
      PP=SQRT (PP)                                                      NE12 060
      U=ZWP*ZE/PP/PM                                                    NE12 061
   52 IF(TP-TH(NT))55,54,53                                             NE12 062
   53 NT=NT+1                                                           NE12 063
      GO TO 52                                                          NE12 064
   54 Q=QD(NT)                                                          NE12 065
      GO TO 56                                                          NE12 066
   55 Q=QD(NT-1)+(TP-TH(NT-1))*(QD(NT)-QD(NT-1))/(TH(NT)-TH(NT-1))      NE12 067
   56 DI(J)=Q*U                                                         NE12 068
   60 TP=TP+DTP                                                         NE12 069
      K=2                                                               NE12 070
      UI=UI+H*(DI(1)+4.*DI(2)+DI(3))                                    NE12 071
      DI(1)=DI(3)                                                       NE12 072
      IF(TP-PI)51,51,61                                                 NE12 073
   61 UV=UI*CD/PI                                                       NE12 074
      UQD(JR)=UV                                                        NE12 075
      TP=0.                                                             NE12 076
      UI=0.                                                             NE12 077
   69 CONTINUE                                                          NE12 078
      RETURN                                                            NE12 079
      END                                                               NE12 080
```

```
      SUBROUTINE VTXRNG (CD,XSC,IR,RP,CN,UG,XPRES,P)
C        VELOCITY INDUCED BY DISTRIBUTION OF VORTEX RINGS
C
      DIMENSION TH(205),GD(205),DI(3),RP(25),UG(25),CN(6),XPRES(25)
      DIMENSION FU(50),FV(50),CP(6),P(6,6)
C
      COMMON MZZZ
C
      C0=CN(1)
      C1=CN(2)
      C2=CN(3)
      C3=CN(4)
      C4=CN(5)
      C5=CN(6)
C
C       SET UP A TABLE OF GAMMA D/GAMMA * SIN(THETA) VERSUS THETA
C
      DG=100.
      DTI=200.
      PI=3.1415926
   13 DT=PI/DG
      N=DG
      NM=N
      N=N+1
      K=N+4
      DO 12 J=1,K
      TH(J)=0.
      ST=0.0
   12 GD(J)=0.
      DO 19 J=2,N
      TH(J)=TH(J-1)+DT
  120 T=TH(J)
      CT=COS (T/2.)
      CT=CT/SIN (T/2.)
      A=C0*CT
      ST=SIN (T)
      B=C1*ST
      C=C2*SIN (2.*T)
      D=C3*SIN (3.*T)
      E=C4*SIN (4.*T)
      F=C5*SIN (5.*T)
      GD(J)=A+B+C+D+E+F
      GD(J)=GD(J)*ST
   19 CONTINUE
      TH(N+1)=PI+1.
      GD(N+1)=GD(N)
   22 GD(1)=2.*C0
      TH(1)=0.
   31 DTP=PI/DTI
      TP=0.
      H=DTP/3.
      UI=0.
C
      IF (MZZZ) 200,10,10
C
   10 IF (IR) 70,11,11
C
   11 XS=XSC-.5
      CST=-2.*XS
```

```
      CST2=CST*CST
C
C      COMPUTE INFLOW VELOCITIES
C
   40 DO 69 JR=1,IR
      R=RP(JR)
   50 K=1
      NT=1
   51 DO 60 J=K,3
      STP=SIN (TP)
      CTP=COS (TP)
      CTP2=CTP*CTP
      X2=CST2-(2.*CST*CTP)+CTP2
      X2=X2*CD*CD
      PP=X2+((R+1.)*(R+1.))
      PM=X2+((R-1.)*(R-1.))
      AK2=4.*R/PP
      CALL ELLIPS (AK2,ZK,ZE)
      PM=2.*(R-1.)/PM
      PP=1./SQRT (PP)
      U=ZK-ZE*(1.+PM)
      U=U*PP
   52 IF(TP-TH(NT))55,54,53
   53 NT=NT+1
      GO TO 52
   54 G=GD(NT)
      GO TO 56
   55 G=GD(NT-1)+(TP-TH(NT-1))*(GD(NT)-GD(NT-1))/(TH(NT)-TH(NT-1))
   56 DI(J)=G*U
   60 TP=TP+DTP
      K=2
      UI=UI+H*(DI(1)+4.*DI(2)+DI(3))
      DI(1)=DI(3)
      IF(TP-PI)51,51,61
   61 UG(JR)=CD/2./PI*UI
      TP=0.
      UI=0.
   69 CONTINUE
      GO TO 999
C
C      COMPUTE VELOCITIES INDUCED AT THE DUCT REFERENCE CYLINDER
C
   70 IR=-IR
      DO 99 JR=1,IR
      X=XPRES(JR)
      K=1
      NT=1
      XS=X-.5
      CST=-2.*XS
      CST2=CST*CST
   77 DO 71 J=K,3
      CTP=COS (TP)
      CTP2=CTP*CTP
      X2=CST2-(2.*CST*CTP)+CTP2
      X2=X2*CD*CD
      PP=X2+4.
      AK2=4./PP
      CALL ELLIPS (AK2,ZK,ZE)
      PP=1./SQRT(PP)
```

```
      U=(ZK-ZE)*PP
   72 IF (TP-TH(NT)) 75,74,73
   73 NT=NT+1
      GO TO 72
   74 G=GD(NT)
      GO TO 76
   75 G=GD(NT-1)+(TP-TH(NT-1))*(GD(NT)-GD(NT-1))/(TH(NT)-TH(NT-1))
   76 DI(J)=G*U
   71 TP=TP+DTP
      K=2
      UI=UI+H*(DI(1)+4.*DI(2)+DI(3))
      DI(1)=DI(3)
      IF (TP-PI) 77,77,78
   78 UG(JR)=CD/2./PI*UI
      TP=0.
      UI=0.
   99 CONTINUE
      GO TO 999
C
C       COMPUTE F(N) AND F-STAR(N) FOURIER COEFFICIENTS
C
  200 MZZ=MZZZ
      MZZZ=1
      DO 210 J=1,6
  210 CP(J)=0.0
      DUM=0.0
      DO 211 L=1,6
  211 DUM=DUM + CN(L)*P(1,L)
      CP(1)=(CN(1)-DUM)/2.0
      DO 212 K=2,6
      DO 213 L=1,6
  213 CP(K)=CP(K) + CN(L)*P(K,L)
  212 CP(K)=(CP(K)-CN(K))/2.0
      N=IR
      EN=N
      DTA=PI/(EN-1.)
      TA=-DTA
      DO 220 JR=1,N
      TA=TA+DTA
      CSTA=COS(TA)
      CSTA2=CSTA*CSTA
      DUM=0.0
      DO 215 K=2,6
      J=K-1
      TJ=J
  215 DUM=DUM+CP(K)*COS(TJ*TA)
      FV(JR)=CP(1)+DUM
      K=1
      NT=1
  207 DO 201 J=K,3
      CTP=COS(TP)
      CTP2=CTP*CTP
      X2=CSTA2-(2.*CSTA*CTP)+CTP2
      X2=X2*CD*CD
      PP=X2+4.0
      AK2=4./PP
      CALL ELLIPS (AK2,ZK,ZE)
      PP=1./SQRT(PP)
      U=(ZK-ZE)*PP
```

```
  202 IF (TP-TH(NT)) 205,204,203
  203 NT=NT+1
      GO TO 202
  204 G=GD(NT)
      GO TO 206
  205 G=GD(NT-1)+(TP-TH(NT-1))*(GD(NT)-GD(NT-1))/(TH(NT)-TH(NT-1))
  206 DI(J)=G*U
  201 TP=TP+DTP
      K=2
      UI=UI+H*(DI(1)+4.*DI(2)+DI(3))
      DI(1)=DI(3)
      IF (TP-PI) 207,207,208
  208 FU(JR)=CD/2./PI*UI
      TP=0.0
      UI=0.0
  220 CONTINUE
      NOUT=6
      NPR=0
      CALL FOURCS (FU,FU,N,NOUT,NPR)
      DO 216 J=1,6
      UG(J)=FU(J)
  216 UG(J+6)=CP(J)
      MZZZ=MZZ
C
  999 RETURN
      END
```

```
      SUBROUTINE ALFRNG (CD,IR,SC,UGA,XPRES)
C     AXIAL VELOCITY INDUCED BY A DISTRIBUTION OF ALPHA VORTEX RINGS
C
      DIMENSION TH(205),GA(205),DI(3),XPRES(25),SC(6),UGA(25),ST(6)
      DIMENSION TCD(7),TSC(6,7)
      DIMENSION GS(6),F(50),HF(6),FH(50),DVT(3)
C
      COMMON MZZZ
C
      DATA TCD/ 0.,0.5,0.75,1.0,1.5,2.0,5.0/
      DATA (TSC(I,1),I=1,6)/1.7,0.,0.,0.,0.,0./
      DATA (TSC(I,2),I=1,6)/1.0978,-0.2382,-0.0175,0.,0.,0./
      DATA (TSC(I,3),I=1,6)/0.92,-0.345,-0.042,0.,0.,0./
      DATA (TSC(I,4),I=1,6)/0.7955,-0.4151,-0.0680,0.,0.,0./
      DATA (TSC(I,5),I=1,6)/0.5611,-0.4834,-0.1230,-0.0121,0.,0./
      DATA (TSC(I,6),I=1,6)/0.5640,-0.5045,-0.1666,-0.0317,0.0035,0./
      DATA (TSC(I,7),I=1,6)/0.3566,-0.4590,-0.2607,-0.1246,-0.0494,0./
C
      IF (MZZZ) 2,1,2
    1 CONTINUE
C
C      COMPUTE SC(N) COEFFICIENTS
C
      DO 11 J=1,7
      JSC=J
      IF (CD-TCD(J)) 4,3,11
   11 CONTINUE
    3 DO 5 N=1,6
    5 SC(N)=TSC(N,JSC)
      GO TO 10
    4 IF (JSC-1) 6,6,7
    6 RETURN
    7 DELSC=(TCD(JSC)-TCD(JSC-1))/(TCD(JSC)-CD)
      J=JSC
      DO 8 N=1,6
    8 SC(N)=(TSC(N,J-1)-TSC(N,J))/DELSC + TSC(N,J)
C
C     SET UP A TABLE OF GAMMA ALPHA/V*SIN(THETA) VERSUS THETA
C
   10 PI=3.1415926
      DG=200.
      DT=PI/DG
      K=201
      DO 12 J=1,K
      TH(J)=0.0
   12 GA(J)=0.0
      DO 20 J=2,K
      TH(J)=TH(J-1)+DT
      T=TH(J)
      CTN=COS(T/2.)/SIN(T/2.)
      CT=COS(T)
      ST(1)=SIN(T)
      ST(2)=2.*ST(1)*CT
      DO 13 N=3,5
      NM=N-1
      NMM=N-2
   13 ST(N)=2.*ST(NM)*CT-ST(NMM)
      GA(J)=SC(1)*CTN
      DO 14 N=2,6
```

```
   14 GA(J)=GA(J)+SC(N)*ST(N-1)
      GA(J)=GA(J)*ST(1)
   20 CONTINUE
      GA(1)=2.*SC(1)
      TH(K+1)=PI+1.
      GA(K+1)=GA(K)
C
C       COMPUTE G-STAR(N) AND H(N) FOURIER COEFFICIENTS
C
      MZZZ=1
      DTI=200.
      DTP=PI/DTI
      H=DTP/3.
      TP=0.0
      UI=0.0
      VIT=0.0
      R=1.0
      R32=R**1.5
      RP2=(R+1.)**2
      N=IR
      CN=N
      DTA=PI/(CN-1.)
      TA=-DTA
      DO 50 JR=1,N
      TA=TA+DTA
      CSTA=COS(TA)
      K=1
      NT=1
   47 DO 41 J=K,3
      CTP=COS(TP)
      XI=(CTP-CSTA)*CD
      X2=XI*XI
      DEN=X2+RP2
      AK2=4.*R/DEN
C
      CALL ELLIPS (AK2,ZK,ZE)
C
      AKP=1.0-AK2
      AKQ=2.0-AK2
      AK4=AK2*AK2
      AYE=((8.0*R*AKQ/AK4) + (2.*R) + 2.0 - 4./AK2*(2.*R+1.))*ZE
      AYE=AYE + AKP/AK2*(8.*R + 4. - 16.*R/AK2)*ZK
      U=AYE/SQRT(DEN)/(X2+(R-1.)*(R-1.))
      AK=SQRT(AK2)
      AKZ=4.*R/RP2
      SBETA=SQRT((1.-AKZ)/(1.-AK2))
C
      CALL ARCSIN (SBETA,BETA,0)
      CALL ARCSIN (AK,ASK,0)
      CALL LAMBDA (ASK,BETA,HLMB)
C
      EPSR=1.
      AVT=4.*(ZK-ZE)/AK+2.*PI*AK/SQRT(AKZ)*(1.-HLMB)*SQRT((1.-AKZ)/(AKZ
     1   -AK2))
      AVT=AVT*XI/(8.*PI*R32) + EPSR/4.
   42 IF (TP-TH(NT)) 45,44,43
   43 NT=NT+1
      GO TO 42
   44 G=GA(NT)
```

```
      GO TO 46
   45 G=GA(NT-1)+(TP-TH(NT-1))*(GA(NT)-GA(NT-1))/(TH(NT)-TH(NT-1))
   46 DI(J)=G*U
      DVT(J)=G*AVT
   41 TP=TP+DTP
      K=2
      UI=UI+H*(DI(1)+4.*DI(2)+DI(3))
      DI(1)=DI(3)
      VIT=VIT+H*(DVT(1)+4.*DVT(2)+DVT(3))
      DVT(1)=DVT(3)
      IF (TP-PI) 47,47,48
   48 F(JR)=-CD*UI/2./PI
      FH(JR)=CD*VIT
      TP=0.0
      UI=0.0
      VIT=0.0
   50 CONTINUE
      NOUT=6
      NPR=0
      CALL FOURCS (F,F,N,NOUT,NPR)
      CALL FOURCS (FH,FH,N,NOUT,NPR)
      DO 51 J=1,NOUT
      GS(J)=F(J)
      HF(J)=FH(J)
      UGA(J+6)=HF(J)
   51 UGA(J)=GS(J)
      MZZZ=0
      RETURN
    2 CONTINUE
      TP=0.0
      UI=0.0
C
C     CALCULATE THE AXIAL VELOCITY INDUCED BY THE ABOVE VORTEX DISTRIBUT
C
   70 DO 99 JR=1,IR
      X=XPRES(JR)
      K=1
      NT=1
      XS=X-.5
      CST=-2.*XS
      R=1.0
   77 DO 71 J=K,3
      CTP=COS(TP)
      X2=(CTP-CST)*CD
      X2=X2*X2
      DEN=X2+(R+1.)*(R+1.)
      AK2=4.*R/DEN
      CALL ELLIPS (AK2,ZK,ZE)
      AKP=1.0-AK2
      AKQ=2.0-AK2
      AK4=AK2*AK2
      AYE=((8.0*R*AKQ/AK4) + (2.*R) + 2.0 - 4./AK2*(2.*R+1.))*ZE
      AYE=AYE + AKP/AK2*(8.*R + 4. - 16.*R/AK2)*ZK
      U=AYE/SQRT(DEN)/(X2+(R-1.)*(R-1.))
   72 IF (TP-TH(NT)) 75,74,73
   73 NT=NT+1
      GO TO 72
   74 G=GA(NT)
      GO TO 76
```

```
 75 G=GA(NT-1)+(TP-TH(NT-1))*(GA(NT)-GA(NT-1))/(TH(NT)-TH(NT-1))
 76 DI(J)=G*U
 71 TP=TP+DTP
    K=2
    UI=UI+H*(DI(1)+4.*DI(2)+DI(3))
    DI(1)=DI(3)
    IF (TP-PI) 77,77,78
 78 UGA(JR)=-CD*UI/2./PI
    TP=0.0
    UI=0.0
 99 CONTINUE
999 RETURN
    END
```

```
      SUBROUTINE GAMCYL (CD,XC,N,RP,UG,IR,XPRES,UGP,RRP,RA)
C
C     SUBROUTINE TO COMPUTE THE AXIAL VELOCITIES INDUCED BY CONSTANT
C       STRENGTH VORTEX CYLINDERS
C
      DIMENSION RP(25),UG(25),XPRES(25),UGP(25,25),RA(25)
      COMMON MZ
C
      PI=3.1415926
      NR=N
      CR=CD*2.
      IF (IR) 2,1,2
    1 XG=(XC-1.)*CR
C
C      COMPUTE THE VELOCITY INDUCED AT THE PROPELLER STATION
C
   20 DO 30 J=1,NR
      R=RP(J)
      IF(R) 22,21,22
   21 U=SQRT ((XG*XG)+1.)
      U=XG/U+1.0
      U=0.5*U
      GO TO 30
   22 XG2=XG*XG
      RP1=(R+1.)*(R+1.)
      RM1=(R-1.)*(R-1.)
      SNB=SQRT (XG2+RM1)
      SNB=XG/SNB
      SKG2=4.*R/(XG2+RP1)
      CALL ELLIPS(SKG2,ZK,ZE)
      SKG=SQRT (SKG2)
      CALL ARCSIN(SKG,ASKG,0)
      CALL ARCSIN(SNB,BETA,0)
      NDEX=0
      IF (BETA) 23,24,24
   23 BETA=-BETA
      NDEX=1
   24 CALL LAMBDA(ASKG,BETA,HLMB)
      IF(NDEX) 25,26,25
   25 HLMB=-HLMB
      NDEX=0
   26 CONTINUE
      U=.25*HLMB+.5
      TM=SKG*XG*ZK/(4.*PI)
      TM=TM/SQRT (R)
      U=U+TM
   30 UG(J)=U
      GO TO 99
    2 NCYL=N
C
C      COMPUTE THE VELOCITY INDUCED AT THE REFERENCE CYLINDER BY
C        THE TRAILING VORTEX CYLINDERS
C
      DO 60 J=1,NCYL
      RGR=RA(J)/RRP
      RGR2=(RGR+1.)*(RGR+1.)
      RGRM=(RGR-1.)*(RGR-1.)
      RTRGR=SQRT(RGR)
      IF (J-NCYL) 31,32,32
```

```
 32 XP=1.0
    RGR=1.0
    RGR2=4.0
    RGRM=0.0
    GO TO 33
 31 XP=XC
 33 DO 60 JR=1,IR
    X=XPRES(JR)
    XG=X-XP
    XGR=XG*CR
    XGR2=XGR*XGR
    IF (XGR) 38,42,38
 42 IF (RGR-1.0) 142,242,342
142 UG=0.0
    GO TO 60
242 UG=0.25
    GO TO 60
342 UG=0.50
    GO TO 60
 38 CONTINUE
    GKSQ=4.*RGR/(XGR2+RGR2)
    GK=SQRT(GKSQ)
    CALL ELLIPS (GKSQ,ZK,ZE)
    NDEX=0
    SNB=XGR/SQRT(XGR2+RGRM)
    CALL ARCSIN (SNB,BETA,0)
    CALL ARCSIN (GK,ASK,0)
    IF (BETA) 34,35,35
 34 BETA=-BETA
    NDEX=-1
 35 CALL LAMBDA (ASK,BETA,HLMB)
    IF (NDEX) 36,37,37
 36 HLMB=-HLMB
    BETA=-BETA
    NDEX=0
 37 CONTINUE
    IF (XGR) 41,42,43
 43 NDEX=-1
    HLMB=-HLMB
    XGR=-XGR
 41 IF (RGR-1.0) 141,241,241
141 UG=GK*XGR*ZK/(4.*PI*RTRGR)-.25*HLMB
    GO TO 45
241 UG=GK*XGR*ZK/(4.*PI*RTRGR)+0.50+.25*HLMB
 45 IF (NDEX) 46,60,60
 46 XGR=-XGR
    NDEX=0
    HLMB=-HLMB
 47 IF (RGR-1.0) 147,247,347
147 UG=-UG
    GO TO 60
247 UG=-UG+0.5
    GO TO 60
347 UG=-UG+1.0
 60 UGP(J,JR)=UG
 99 RETURN
    END
```

```
      SUBROUTINE BNCOEF (C,BGI)
C
C        FOURIER ANALYSIS OF RADIAL VELOCITY INDUCED BY AN
C             ACTUATOR DISK IN THE EXIT PLANE  --  B(N)
C
      DIMENSION BG( 6,2),BGI(6),BGO( 6)
      COMMON MZZZ
      PI=3.1415927
      MZZZ=-10
      N=100
      KMAX=6
      DO 10 K=1,KMAX
   10 BGI(K)=0.0
      EPS=PI/180.
      PIM=PI-EPS
      CN=N
      DELTH=.5*PIM/CN
      NP=N+1
      DO 6 I=1,NP
      DO 3 J=1,2
      FI=I-2
      FJ=J
      TH=FI*PIM/CN+FJ*DELTH
      STH=SIN (TH)
      CTH=COS (TH)
   72 XG=-.5*C*(1.+CTH)
   73 XGSQ=XG*XG
      GKSQ=XGSQ/(4.+XGSQ)
      GK=SQRT (4./(4.+XGSQ))
      CALL ELLIPS (GKSQ,ZKG,ZCG)
      BGC=.5/PI*((GK-2./GK)*ZKG+2./GK*ZCG)
      STHK=0.
      CTHK=1.
      DO 2 K=1,KMAX
      BG(K,J)=BGC*CTHK
      CON=STHK*CTH+CTHK*STH
      CTHK=CTHK*CTH-STHK*STH
    2 STHK=CON
    3 CONTINUE
      DO 5 K=1,KMAX
      IF(I-1)4,5,4
    4 BGI(K)=BGI(K)+DELTH*(BG(K,1)+(BGO(K)+BG(K,1)+BG(K,2))/3.)
    5 BGO(K)=BG(K,2)
    6 CONTINUE
      EXTRA=-EPS/PI*ALOG(4./EPS*SQRT (2./C))
      SI=-1
      DO 7 K=1,KMAX
      SI=-SI
    7 BGI(K)=2.*(BGI(K)+SI*EXTRA)/PI
      BGI(1)=.5*BGI(1)
      MZZZ=1
      RETURN
      END
```

```
      SUBROUTINE ANCOEF(NCYL,N,RA,XP,CD,RRP,NRUN,NPRINT,BS,A,AS)
      DIMENSION F(50),BS(6),A(25,6),AS(25,6),RA(25)
      COMMON MZZZ
  142 FORMAT (1H1,4X10HRUN NUMBER,I5,49X4HPAGE,I3,2H-F,//)
  242 FORMAT (10X55HFOURIER COSINE SERIES COEFFICIENTS  ---  OUTPUT INDE
     1X =,I3)
      NPAGE = 0
      NOUT = 6
   20 CR=CD*2.0
      PI=3.1415927
   21 CN=N
      DTH=PI/(CN-1.)
      TH=-DTH
C
C        COMPUTE B-STAR(N) FOURIER COEFFICIENTS
C
   22 DO 30 J=1,N
      TH=TH+DTH
      CSTH=COS (TH)
      XG=-.5*CR*(1.+CSTH)
      GKSQ=XG*XG
      GKSQ=4.0+GKSQ
      GKSQ=4.0/GKSQ
      CALL ELLIPS (GKSQ,ZK,ZE)
      GK=SQRT (GKSQ)
   30 F(J)=0.25+(GK*XG/4.*ZK/PI)
      IF(NPRINT) 171,171,170
  170 NPAGE = NPAGE + 1
      WRITE (6,142) NRUN,NPAGE
      WRITE (6,242) NPRINT
  171 CALL FOURCS (F,F ,N,NOUT,NPRINT)
      DO 31 J=1,NOUT
   31 BS(J)=F(J)
C
C        COMPUTE A(N) FOURIER COEFFICIENTS
C
   32 DO 40 J=1,NCYL
      RRA=RRP/RA(J)
      RRAP=RRA+1.0
      RRAP=RRAP*RRAP
      RRA4=4.0*RRA
      RASQT=SQRT (RRA)
      TH=-DTH
   33 DO 38 K=1,N
      TH=TH+DTH
      CSTH=COS (TH)
      XR=CR*(1.-CSTH)*.5
      XG=XR-(XP*CR)
      XG=XG*RRA
      XGSQ=XG*XG
      GKSQ=RRAP+XGSQ
      GKSQ=RRA4/GKSQ
      GK=SQRT (GKSQ)
      CALL ELLIPS(GKSQ,ZK,ZE)
      F(K)=((ZE-ZK)/GK+(GK*ZK/2.))/PI/RASQT
   38 CONTINUE
      IF(NPRINT) 173,173,172
  172 NPAGE = NPAGE + 1
      WRITE (6,142) NRUN,NPAGE
```

```
      WRITE (6,242) NPRINT
  173 CALL FOURCS (F,F,N,NOUT,NPRINT)
      DO 39 K=1,NOUT
   39 A(J,K)=F(K)
   40 CONTINUE
C
C        COMPUTE A-STAR(N) FOURIER COEFFICIENTS
C
   42 DO 50 J=1,NCYL
      RRA=RRP/RA(J)
      RPRIM=1.0/RRA
      RRAP=RRA+1.0
      RRAP=RRAP*RRAP
      RRA4=4.0*RRA
      RASQT=SQRT (RRA)
      TH=-DTH
   43 DO 48 K=1,N
      TH=TH+DTH
      CSTH=COS (TH)
      XR=CR*(1.0-CSTH)*.5
      XG=XR-(XP*CR)
      XG=XG*RRA
      XGSQ=XG*XG
      GKSQ=RRAP+XGSQ
      GKSQ=RRA4/GKSQ
      GK=SQRT (GKSQ)
      CALL ELLIPS (GKSQ,ZK,ZE)
      NDEX=0
      SNB=RRA-1.0
      SNB=SNB*SNB
      SNB=XGSQ+SNB
      SNB=1.0/SQRT (SNB)
      SNB=XG*SNB
      CALL ARCSIN (SNB,BETA,0)
      CALL ARCSIN (GK,ASK,0)
      XG=XG*RPRIM
      IF(BETA) 62,63,63
   62 BETA=-BETA
      NDEX=-1
   63 CALL LAMBDA (ASK,BETA,HLMB)
      IF(NDEX) 64,65,65
   64 HLMB=-HLMB
      NDEX=0
      BETA=-BETA
   65 CONTINUE
      IF(XG) 71,72,73
   72 IF(RPRIM-1.0) 74,75,76
   74 F(K)=0.0
      GO TO 48
   75 F(K)=-.25
      GO TO 48
   76 F(K)=-.50
      GO TO 48
   73 NDEX=-1
      HLMB=-HLMB
      XG=-XG
   71 IF(RPRIM-1.0) 77,78,78
   77 F(K)=-GK*XG*ZK/4.0/PI*RASQT
      F(K)=F(K)+(.25*HLMB)
```

```
      GO TO 79
   78 F(K)=-GK*XG*ZK/4.0/PI*RASQT
      F(K)=F(K)-.5-(.25*HLMB)
   79 IF(NDEX) 80,48,48
   80 XG=-XG
      NDEX=0
      IF(RPRIM-1.) 81,82,83
   81 F(K)=-F(K)
      GO TO 48
   82 F(K)=-F(K)-0.5
      GO TO 48
   83 F(K)=-F(K)-1.0
   48 F(K)=-F(K)
      IF(NPRINT) 175,175,174
  174 NPAGE = NPAGE + 1
      WRITE (6,142) NRUN,NPAGE
      WRITE (6,242) NPRINT
  175 CALL FOURCS (F,F,N,NOUT,NPRINT)
      DO 49 K=1,NOUT
   49 AS(J,K)=F(K)
   50 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CNCOEF (GV,XC)
      DIMENSION A(6),B(6),AS(6),BS(6),CN1(6),CN2(6),Q(6,7),P(6,6),XC(6)
      DIMENSION CVG(6)
      COMMON MZZZ,CD,R0,R1,R2,R3,PI,B,BS,A,AS,P
      VG=1.0/GV
      C=2.*CD
C
C        CALCULATE COEFFICIENT MATRIX
C
      TR0=2.*R0
   35 C4D=C/8.0
   36 CLN=ALOG(32.0/C)-1.0
      Q(1,1)=1.0-P(1,1)-TR0*C4D*CLN-R1*C4D
      Q(1,2)=R2/4.0*C4D-TR0*C4D*0.5*CLN
      Q(1,3)=-P(1,3)-R1*0.5*C4D+R3*C4D/6.0
      Q(1,4)=-R2*C4D/4.0
      Q(1,5)=-P(1,5)-R3*C4D/6.0
      Q(1,6)=0.0
      Q(2,1)=P(2,1)-TR0*C4D-2.*R1*C4D*CLN-R2*C4D
      Q(2,2)=P(2,2)-1.0-R1*C4D*CLN+(R1+R3)*C4D/4.0
      Q(2,3)=C4D*(-R0-R2/3.)
      Q(2,4)=P(2,4)-C4D*(R3+R1+R1)/8.0
      Q(2,5)=-R2*C4D/6.0
      Q(2,6)=P(2,6)-R3*C4D/8.0
      Q(3,1)=P(3,1)-C4D*(R1+R3)-2.*R2*C4D*CLN
      Q(3,2)=R0*C4D/2.0-R2*C4D*CLN
      Q(3,3)=P(3,3)-1.0-C4D*(R1/3.+R3/2.)
      Q(3,4)=C4D*(R2/8.-R0/2.)
      Q(3,5)=P(3,5)+C4D*(R3/10.-R1/6.)
      Q(3,6)=-R2*C4D/8.0
      Q(4,1)=P(4,1)-C4D*(R2+2.*R3*CLN)
      Q(4,2)=P(4,2)+C4D*(R1/4.-R3*CLN)
      Q(4,3)=C4D*(R0/3.-R2/2.)
      Q(4,4)=P(4,4)-1.0-R1*C4D/8.0
      Q(4,5)=C4D*(R2/10.-R0/3.)
      Q(4,6)=P(4,6)-R1*C4D/8.0
      Q(5,1)=P(5,1)-R3*C4D
      Q(5,2)=R2*C4D/4.0
      Q(5,3)=P(5,3)+C4D*(R1/6.-R3/2.)
      Q(5,4)=C4D*(R0/4.-R2/4.)
      Q(5,5)=P(5,5)-1.0-C4D*R1/15.
      Q(5,6)=-R0*C4D/4.0
      Q(6,1)=P(6,1)
      Q(6,2)=P(6,2)+R3*C4D/4.
      Q(6,3)=C4D*R2/6.0
      Q(6,4)=P(6,4)+C4D*(R1/8.-R3/4.)
      Q(6,5)=C4D*(R0/5.-R2/6.)
      Q(6,6)=P(6,6)-1.0-C4D*R1/8.
C
C        CALCULATE CONSTANTS
C
      DO 40 I=1,6
      CN1(I)=TR0*BS(I)-(2.*B(I))
      GO TO (41,42,43,44,45,46),I
   41 CN1(I)=CN1(I)+(R1*BS(2))+(R2*BS(3))+(R3*BS(4))
      GO TO 40
   42 CN1(I)=CN1(I)+R1*(BS(3)+(2.*BS(1)))+R2*(BS(2)+BS(4))+R3*(BS(3)+BS
     1(5))
      GO TO 40
```

```
43 CN1(I)=CN1(I)+R1*(BS(2)+BS(4))+R2*(BS(5)+(2.*BS(1)))+R3*(BS(2)+BS
  1(6))
   GO TO 40
44 CN1(I)=CN1(I)+R1*(BS(3)+BS(5))+R2*(BS(2)+BS(6))+2.*R3*BS(1)
   GO TO 40
45 CN1(I)=CN1(I)+R1*(BS(4)+BS(6))+R2*BS(3)+R3*BS(2)
   GO TO 40
46 CN1(I)=CN1(I)+R1*BS(5)+R2*BS(4)+R3*BS(3)
40 CONTINUE
   AS(1)=AS(1)+1.0
   DO 50 I=1,6
   CN2(I)=-2.*(A(I))+(TRO*(AS(I)))
   GO TO (51,52,53,54,55,56),I
51 CN2(I)=CN2(I)+R1*(AS(2))+R2*(AS(3))+R3*(AS(4))
   GO TO 50
52 CN2(I)=CN2(I)+R1*(2.*(AS(1))+AS(3))+R2*(AS(2)+AS(4))+R3*(AS(3)+AS(
  15))
   GO TO 50
53 CN2(I)=CN2(I)+R1*(AS(2)+AS(4))+R2*(2.*(AS(1))+AS(5))+R3*(AS(2)+AS(
  16))
   GO TO 50
54 CN2(I)=CN2(I)+R1*(AS(3)+AS(5))+R2*(AS(2)+AS(6))+R3*2.*(AS(1))
   GO TO 50
55 CN2(I)=CN2(I)+R1*(AS(4)+AS(6))+R2*(AS(3))+R3*(AS(2))
   GO TO 50
56 CN2(I)=CN2(I)+R1*(AS(5))+R2*(AS(4))+R3*(AS(3))
50 CONTINUE
   AS(1)=AS(1)-1.0
C
   CALL MATRIX(Q,6)
59 DO 60 I=1,6
   XC(I)=0.0
60 CVG(I)=CN1(I)+CN2(I)/GV
   DO 61 I=1,6
   DO 61 J=1,6
61 XC(I)=XC(I)+Q(I,J)*CVG(J)
   RETURN
   END
```

```
      SUBROUTINE PRESS
C
      DIMENSION TCRAT(7),XV(19),DVA(8,7),F(8,19),UP(25),UM(25),SUMU(25)
      DIMENSION ST(5),UCBP(25),VCBP(25),CST(5),SAVU(25),FX(25)
C
      DIMENSION B(6),BS(6),SA(6),SAS(6),P(6,6)
      DIMENSION C(6),A(25,6),AS(25,6),D(6),DS(6),SC(6),GS(6),H(6)
      DIMENSION RB(25),BR(25),BTA(25),TCBLD(25),RA(25),XPRES(25),PHI(5)
      DIMENSION UG(25),UQD(25),UGD(25),UV(25),UCB(25),VCB(25),
     1          UGP(25,25),UGA(25),GV(25),GRV(25),CPP(5,25),CPM(5,25)
      DIMENSION ALPHA(25),STALL(25),JSTL(25),TALK(20)
C
      COMMON MZZZ,CD,R0,R1,R2,R3,PI,B,BS,SA,SAS,P
      COMMON/NEAR1/ NRUN,NBLD,NZ,MZ,NPRES,IR,NTIME,NERR,NPAG,NPHI,NPRINT
      COMMON/NEAR2/ C,A,AS,D,DS,SC,GS,H
      COMMON/NEAR3/ RRP,XP,Z,BLD,RB,BR,BTA,TCBLD,TC,RCBRP,APA,ALF,XPRES,
     1              RA,XCB,XR,ELCBC,RMAX,PHI,CORJ,CORCB
      COMMON/NEAR4/ UG,UQD,UGD,UV,UCB,VCB,UGP,UGA,GV,GAM,GRV,CPP,CPM
      COMMON/NEAR5/ ARJ,ARJP,EPS,RAD,CL,ALPHA,STALL,JSTL,TALK
C
      DATA TCRAT/0.06,0.08,0.12,0.15,0.18,0.21,0.24/
      DATA XV/0.0,0.005,0.0125,0.025,0.05,0.075,0.1,0.15,0.2,0.25,0.3,
     1       0.4,0.5,0.6,0.7,0.8,0.9,0.95,1.0/
      DATA (DVA(1,I),I=1,7)/3.992,2.015,1.364,0.984,0.696,0.462,0.478/
      DATA (DVA(2,I),I=1,7)/2.90,1.795,1.31,0.971,0.694,0.561,0.478/
      DATA (DVA(3,I),I=1,7)/1.988,1.475,1.199,0.934,0.685,0.558,0.478/
      DATA (DVA(4,I),I=1,7)/1.60,1.312,1.112,0.90,0.675,0.557,0.478/
      DATA (DVA(5,I),I=1,7)/1.342,1.178,1.028,0.861,0.662,0.555,0.478/
      DATA (DVA(6,I),I=1,7)/1.167,1.065,0.946,0.818,0.648,0.550,0.478/
      DATA (DVA(7,I),I=1,7)/1.050,0.964,0.870,0.771,0.632,0.542,0.478/
      DATA (F(1,I),I=1,19)/0.,0.938,1.057,1.089,1.103,1.107,1.101,1.098,
     1   1.091,1.086,1.078,1.066,1.053,1.042,1.028,1.013,0.990,0.974,0./
      DATA (F(2,I),I=1,19)/0.,0.890,1.050,1.105,1.128,1.133,1.130,1.128,
     1   1.122,1.114,1.106,1.089,1.072,1.054,1.039,1.017,0.984,0.969,0./
      DATA (F(3,I),I=1,19)/0.,0.800,1.005,1.114,1.174,1.184,1.188,1.188,
     1   1.183,1.174,1.162,1.135,1.108,1.080,1.053,1.022,0.978,0.952,0./
      DATA (F(4,I),I=1,19)/0.,0.739,0.966,1.112,1.204,1.224,1.233,1.233,
     1   1.229,1.218,1.204,1.170,1.131,1.098,1.064,1.024,0.972,0.934,0./
      DATA (F(5,I),I=1,19)/0.,0.682,0.926,1.103,1.228,1.264,1.276,1.278,
     1   1.275,1.262,1.247,1.205,1.154,1.116,1.074,1.025,0.966,0.914,0./
      DATA (F(6,I),I=1,19)/0.,0.630,0.887,1.087,1.242,1.297,1.317,1.325,
     1   1.320,1.306,1.290,1.240,1.178,1.133,1.085,1.027,0.957,0.895,0./
      DATA (F(7,I),I=1,19)/0.,0.579,0.848,1.063,1.244,1.322,1.354,1.374,
     1   1.368,1.350,1.333,1.277,1.204,1.151,1.097,1.032,0.944,0.879,0./
C
  700 FORMAT (///10X,53HDUCT PRESSURE DISTRIBUTION CALCULATION ASSUMES T
     1/C = ,F5.3//)
C
      IF (MZZZ) 2,1,2
    1 CONTINUE
C
C       SET UP A TABLE OF CONTINUOUS VELOCITY CORRECTION FACTORS
C
      DO 31 J=1,7
      TCP=TCRAT(J)
      JTC=J
      IF (TC-TCRAT(J)) 32,33,31
   32 IF (J-1) 132,132,34
   31 CONTINUE
```

```
  132 WRITE (6,700) TCP
   33 DO 38 N=1,19
      IF (N-7) 37,37,38
   37 DVA(8,N)=DVA(JTC,N)
   38 F(8,N)=F(JTC,N)
      GO TO 39
   34 DELTC=(TCRAT(JTC)-TCRAT(JTC-1))/(TCRAT(JTC)-TC)
      J=JTC
      DO 35 N=1,19
   35 F(8,N)=(F(J-1,N)-F(J,N))/DELTC + F(J,N)
C
C       SET UP A TABLE OF THE DISCONTINUOUS VELOCITY CORRECTION FACTORS
C
      DO 36 N=1,7
   36 DVA(8,N)=(DVA(J-1,N)-DVA(J,N))/DELTC + DVA(J,N)
   39 CONTINUE
      RETURN
C
    2 CONTINUE
C
C       LOOK UP CONTINUOUS VELOCITY CORRECTION FACTOR
C
      DO 49 N=1,IR
      DO 42 J=1,19
      JK=J
      IF (XPRES(N)-XV(J)) 41,47,42
   42 CONTINUE
   41 DELX=(XV(JK)-XV(JK-1))/(XV(JK)-XPRES(N))
      FX(N)=(F(8,JK-1)-F(8,JK))/DELX + F(8,JK)
      GO TO 49
   47 FX(N)=F(8,JK)
   49 CONTINUE
      SNALF=SIN(ALF/RAD)
      CSALF=COS(ALF/RAD)
C
C       COMPUTE VELOCITY INDUCED BY TRAILING VORTEX CYLINDERS
C
      CALL GAMCYL (CD,XP,NZ,RB,UG,IR,XPRES,UGP,RRP,RA)
      DO 10 J=1,IR
   10 SUMU(J)=1.0
      DO 12 J=1,IR
      DO 12 N=1,NZ
   12 SUMU(J)=SUMU(J) + UGP(N,J)*GV(N)
C
C       COMPUTE VELOCITY INDUCED BY DUCT-BOUND VORTICITY
C
      IRC=-IR
      CALL VTXRNG (CD,XP,IRC,RB,C,UGD,XPRES,P)
      DO 14 J=1,IR
   14 SUMU(J)=SUMU(J) + UGD(J)*GV(NZ)
C
C       COMPUTE VELOCITY INDUCED BY THE CENTERBODY
C
      IRC=-IR
      CALL HUB (CD,XR,XCB,ELCBC,RMAX,IRC,XPRES,RB,RRP,UCBP,VCBP)
      DO 16 J=1,IR
      SUMU(J)=SUMU(J) + UCBP(J)*CORCB
      SUMU(J)=SUMU(J)*CSALF
   16 SAVU(J)=SUMU(J)
```

```
      IF (SNALF) 20,21,20
C
C      COMPUTE VELOCITY INDUCED BY ALPHA VORTEX RINGS
C
   20 CALL ALFRNG (CD,IR,SC,UGA,XPRES)
      GO TO 23
C
   21 DO 22 J=1,IR
   22 UGA(J)=0.0
   23 DO 99 M=1,NPHI
      CSPHI=COS(PHI(M)/RAD)
      SNPHI=SIN(PHI(M)/RAD)
      DO 90 J=1,IR
   90 SUMU(J)=SAVU(J)
      DO 25 J=1,IR
   25 SUMU(J)=SUMU(J) + UGA(J)*SNALF*CSPHI
   30 CONTINUE
C
C      CORRECT CONTINUOUS INDUCED VELOCITY
C
      DO 48 N=1,IR
   48 SUMU(N)=SUMU(N)*FX(N)
C
C      COMPUTE DISCONTINUOUS PORTION OF SURFACE VELOCITY
C
      DO 59 N=1,IR
      CT=1.-2.*XPRES(N)
      ST(1)=SQRT(1.-CT*CT)
      ST(2)=2.*ST(1)*CT
      DO 50 K=3,5
      KM=K-1
      KMM=K-2
   50 ST(K)=2.*ST(KM)*CT-ST(KMM)
      IF (XPRES(N)-0.10) 51,57,57
   51 DO 52 J=1,7
      JK=J
      IF (XPRES(N)-XV(J)) 54,53,52
   52 CONTINUE
   53 DVAX=DVA(8,JK)
      GO TO 55
   54 DELX=(XV(JK)-XV(JK-1))/(XV(JK)-XPRES(N))
      DVAX=(DVA(8,JK-1)-DVA(8,JK))/DELX + DVA(8,JK)
   55 CTN=DVAX*2.0*PI
      GO TO 56
   57 CT2=SQRT((1.+CT)/2.)
      CTN=CT2/SQRT(1.-CT2*CT2)
   56 GD=C(1)*CTN
      GA=SC(1)*CTN
      DO 58 J=1,5
      GD=GD+C(J+1)*ST(J)
   58 GA=GA+SC(J+1)*ST(J)
      GD=GD*GV(NZ)*CSALF
      GA=GA*SNALF*CSPHI
      UM(N)=SUMU(N)-GD/2.-GA/2.
   59 UP(N)=SUMU(N)+GD/2.+GA/2.
      UT=SNALF*SNPHI
C
C      COMPUTE PRESSURE COEFFICIENT INSIDE AND OUTSIDE DUCT SURFACE
C
```

```
      DO 60 N=1,IR
      UTT=UT*FX(N)
      UTT=UTT*UTT
      CPP(M,N)=1.-UP(N)*UP(N) + UTT
   60 CPM(M,N)=1.-UM(N)*UM(N) + UTT
   99 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE OUTPUT
C
      DIMENSION B(6),BS(6),SA(6),SAS(6),P(6,6)
      DIMENSION C(6),A(25,6),AS(25,6),D(6),DS(6),SC(6),GS(6),H(6)
      DIMENSION RB(25),BR(25),BTA(25),TCBLD(25),RA(25),XPRES(25),PHI(5)
      DIMENSION UG(25),UQD(25),UGD(25),UV(25),UCB(25),VCB(25),
     1          UGP(25,25),UGA(25),GV(25),GRV(25),CPP(5,25),CPM(5,25)
      DIMENSION ALPHA(25),STALL(25),JSTL(25),TALK(20)
      DIMENSION CTDP(10),CNDP(5),CMDP(5),E(6),ES(6),F(6),FS(6)
C
      COMMON MZZZ,CD,R0,R1,R2,R3,PI,B,BS,SA,SAS,P
      COMMON/NEAR1/ NRUN,NBLD,NZ,MZ,NPRES,IR,NTIME,NERR,NPAG,NPHI,NPRINT
      COMMON/NEAR2/ C,A,AS,D,DS,SC,GS,H
      COMMON/NEAR3/ RRP,XP,Z,BLD,RB,BR,BTA,TCBLD,TC,RCBRP,APA,ALF,XPRES,
     1              RA,XCB,XR,ELCBC,RMAX,PHI,CORJ,CORCB
      COMMON/NEAR4/ UG,UQD,UGD,UV,UCB,VCB,UGP,UGA,GV,GAM,GRV,CPP,CPM
      COMMON/NEAR5/ ARJ,ARJP,EPS,RAD,CL,ALPHA,STALL,JSTL,TALK
C
  101 FORMAT(15H1     RUN NUMBER,I5,49X,4HPAGE,I3//)
  701 FORMAT(15H1     RUN NUMBER,I5,17X15HOPTIONAL OUTPUT,17X,4HPAGEI3//)
  702 FORMAT (/17X1H(,F5.2,1H),29X1H(,F5.2,1H))
  703 FORMAT (1H1,5X,10HRUN NUMBER,I5,10X34HDUCT SURFACE PRESSURE DISTRI
     1BUTION,10X,4HPAGE,I3//)
  102 FORMAT ( 5X,20A4//)
  113 FORMAT (9X4HH(N))
  114 FORMAT (9X5HSC(N))
  115 FORMAT (9X4HC(N))
  116 FORMAT (9X4HB(N))
  117 FORMAT (9X5HB(N)*)
  118 FORMAT (9X8HSUM A(N))
  119 FORMAT (9X9HSUM A(N)*)
  120 FORMAT (9X4HD(N))
  121 FORMAT (9X5HD(N)*)
  122 FORMAT (9X6HA(M,N))
  123 FORMAT (9X7HA(M,N)*)
  124 FORMAT (14X,16HEFFECTIVE CAMBER,4F10.6//)
  143 FORMAT(/10X34HFOURIER COSINE SERIES COEFFICIENTS/15X5HN=1,65X4HM=1
     1,I2//)
  148 FORMAT (7X12HDUCT...  C/D6X4HXP/C6X3HT/C6X6HRTE/RP4X6HRCB/RP5X4HAP
     1/A/10X6F10.6//)
  149 FORMAT (15X5HALPHA,5X1HJ,9X2HJ',6X8HJ COS(A),2X8HJ'COS(A)/
     110X,F10.3,4F10.5)
  150 FORMAT(5X1HN4X4HR/RP4X5HUQD/V7X5HUGD/V7X4HUG/V8X5HUCB/V6X8HGAMMA/R
     1V)
  151 FORMAT(///15XI5,2X,24HITERATIONS,    EPSILON =,F9.6)
  152 FORMAT (/17X6HINFLOW,19X5HBLADE/5X1HN4X4HR/RP5X3HU/V8X5HGAM/V7X5HA
     1LPHA6X9HDELTA P/Q)
  153 FORMAT (/9X6HCTP(D)5X6HCTD(P)4X7HCTD(P)'5X4HCTDP7X5HCTDP'6X4HCNDP,
     1 7X4HCMDP)
  154 FORMAT (2X4H(A) ,7(1PE11.4))
  155 FORMAT (2X4H(B) ,7(1PE11.4))
  156 FORMAT (///5X8HNOTES...//10X55H(A)  COEFFICIENTS BASED ON FREE STR
     1EAM DYNAMIC PRESSURE//10X46H(B)  COEFFICIENTS BASED ON PROPELLER T
     1IP SPEED)
  157 FORMAT (/11X,53H*   BLADE SECTION LIFT COEFFICIENT HAS EXCEEDED CL
     1MAX/20X,14HIN ANNULI NOS.,25I3)
  244 FORMAT (5X,6(1PE13.6))
  245 FORMAT (F9.4,5(1X2(1PE10.3)))
  250 FORMAT (//2X7HAZIMUTH/3X8HPHI =  /,5(3H---3X,F7.2,4X4H---/))
```

```
  251 FORMAT (/5X4HX/C ,1(3X6HCP(IN),4X7HCP(OUT),1X))
  252 FORMAT (/5X4HX/C ,2(3X6HCP(IN),4X7HCP(OUT),1X))
  253 FORMAT (/5X4HX/C ,3(3X6HCP(IN),4X7HCP(OUT),1X))
  254 FORMAT (/5X4HX/C ,4(3X6HCP(IN),4X7HCP(OUT),1X))
  255 FORMAT (/5X4HX/C ,5(3X6HCP(IN),4X7HCP(OUT),1X))
  256 FORMAT(///9X50HPRESSURE COEFFICIENTS BASED ON PROPELLER TIP SPEED)
  257 FORMAT(//9X51HPRESSURE COEFFICIENTS BASED ON FREE STREAM VELOCITY)
  260 FORMAT (I6,F9.5,5(1PE12.5))
  261 FORMAT (2X1H*I3,F9.5,5(1PE12.5))
C
      IF (NTIME-50) 930,930,931
  931 NERR=1
      NPRINT=NPRINT+1
  930 SNALF=SIN(ALF/RAD)
      CSALF=COS(ALF/RAD)
      ARJV=ARJ/CSALF
      ARJVP=ARJP/CSALF
      NCYL=MZ
C
      IF (NPRINT.EQ.1.OR.NPRINT.GE.11) GO TO 900
      GO TO 901
  900 NPAG=NPAG+1
      WRITE (6,701) NRUN,NPAG
      WRITE (6,102) TALK
      WRITE (6,148) CD,XP,TC,RRP,RCBRP,APA
      WRITE (6,124) R0,R1,R2,R3
      WRITE (6,149) ALF,ARJV,ARJVP,ARJ,ARJP
      WRITE (6,702) CORJ,CORCB
      WRITE (6,150)
      DO 61 J=1,NZ
      X=UG(J)*GAM
      Y=UGD(J)*GAM
      R=RB(J)*RRP
      XQ=UQD(J)*CORJ
      XY=UCB(J)*CORCB
   61 WRITE (6,260) J,R,XQ,Y,X,XY,GRV(J)
      WRITE (6,151) NTIME,EPS
  901 IF (NPRINT.GE.10) GO TO 902
      GO TO 903
  902 NPAG=NPAG+1
      WRITE (6,701) NRUN,NPAG
      WRITE (6,102) TALK
      WRITE (6,148) CD,XP,TC,RRP,RCBRP,APA
      WRITE (6,149) ALF,ARJV,ARJVP,ARJ,ARJP
      WRITE (6,143) NCYL
      WRITE (6,115)
      WRITE (6,244) (C(J),J=1,6)
      IF (ALF) 904,905,904
  904 WRITE (6,114)
      WRITE (6,244) (SC(J),J=1,6)
  905 CONTINUE
      WRITE (6,116)
      WRITE (6,244) (B(K),K=1,6)
      WRITE (6,117)
      WRITE (6,244) (BS(K),K=1,6)
      WRITE (6,118)
      WRITE (6,244) (SA(J),J=1,6)
      WRITE (6,119)
      WRITE (6,244) (SAS(J),J=1,6)
```

```
      WRITE (6,120)
      WRITE (6,244) (D(N),N=1,6)
      WRITE (6,121)
      WRITE (6,244) (DS(N),N=1,6)
      WRITE (6,113)
      WRITE (6,244) (H(N),N=1,6)
      WRITE (6,122)
      DO 924 M=1,NCYL
  924 WRITE (6,244) (A(M,N),N=1,6)
      WRITE (6,123)
      DO 925 M=1,NCYL
  925 WRITE (6,244) (AS(M,N),N=1,6)
C
  903 NPAG=NPAG+1
      WRITE (6,101) NRUN,NPAG
      WRITE (6,102) TALK
      WRITE (6,148) CD,XP,TC,RRP,RCBRP,APA
      WRITE (6,149) ALF,ARJV,ARJVP,ARJ,ARJP
      WRITE (6,152)
      X=0.
      Y=0.
      DO 63 J=1,NZ
      X=GRV(J)/PI/ARJP*BLD/Z
      Y=Y+X
      R=RB(J)*RRP
      DELP=GRV(J)*BLD/PI/ARJP
      IF (STALL(J)) 62,64,62
   64 WRITE (6,260) J,R,UV(J),GV(J),ALPHA(J),DELP
      GO TO 63
   62 WRITE (6,261) J,R,UV(J),GV(J),ALPHA(J),DELP
   63 CONTINUE
      X=0.0
C
C       COMPUTE DUCTED PROPELLER THRUST COEFFICIENTS
C
      CTCF=ARJ*ARJ*PI/APA/8.0/CSALF/CSALF
      CTDP(1)=Y*APA*CSALF*CSALF
      DO 20 J=1,6
   20 E(J)=SA(J) + D(J)*CORCB + B(J)*GAM
      CON=-CD*PI*GAM*CSALF*CSALF
      CTDP(2)=C(1)*(4.*E(1)+2.*E(2)) + 2.*C(2)*E(1)
      CTALF=SC(1)*(4.*H(1)+2.*H(2)) + 2.*SC(2)*H(1)
      DO 21 N=2,5
      CTALF=CTALF + SC(N+1)*H(N) - SC(N)*H(N+1)
   21 CTDP(2)=CTDP(2) + C(N+1)*E(N) - C(N)*E(N+1)
      CTDP(2)=CTDP(2)*CON + PI*CD*(2.*SC(1)+SC(2))*SNALF*SNALF
      CTALF=-PI*CD/2.*SNALF*SNALF*CTALF
      CTDP(2)=CTDP(2) + CTALF
      X=RRP*RRP
      X=1.0-(1.0/X)
      X=X*DELP*CSALF*CSALF
      CTDP(3)=CTDP(2) + X
      CTDP(4)=CTDP(1) + CTDP(2)
      CTDP(5)=CTDP(1) + CTDP(3)
      DO 22 J=1,5
   22 CTDP(J+5)=CTDP(J)*CTCF
C
C       COMPUTE DUCT NORMAL FORCE COEFFICIENTS
C
```

```
      DO 30 J=1,5
      CMDP(J)=0.0
   30 CNDP(J)=0.0
      SACA=SNALF*CSALF
      IF (SACA) 31,23,31
   31 MZZZ=-1
C
C      COMPUTE F(N) AND F-STAR(N) FOURIER COEFFICIENTS
C
      NF=50
      CALL VTXRNG (CD,XP,NF,RB,C,UGD,XPRES,P)
      MZZZ=1
      DO 32 J=1,6
      FS(J)=UGD(J)
   32 F(J)=UGD(J+6)
      DO 33 J=1,6
   33 ES(J)=SAS(J) + DS(J)*CORCB + (BS(J)+FS(J))*GAM
      CNDP(2)=SC(1)*(4.*ES(1)+2.*ES(2)) + 2.*SC(2)*ES(1)
      DUM=0.0
      DO 34 J=2,5
   34 DUM=DUM + SC(J+1)*ES(J) - SC(J)*ES(J+1)
      CNDP(2)=(CNDP(2)+DUM)*PI*CD*SACA/2.
      CNDP(3)=C(1)*(4.*GS(1)+2.*GS(2)) + 2.*C(2)*GS(1)
      DUM=0.0
      DO 35 J=2,5
   35 DUM=DUM + C(J+1)*GS(J) - C(J)*GS(J+1)
      CNDP(3)=(CNDP(3)+DUM)*PI*CD*SACA/2.*GAM
      CNDP(4)=PI*CD*SACA*(2.*SC(1)+SC(2))
      CNDP(1)=CNDP(2) + CNDP(3) + CNDP(4)
C
C      COMPUTE DUCT MOMENT COEFFICIENTS
C
      CMDP(2)=C(1)*(4.*GS(1)+4.*GS(2)+2.*GS(3)) + C(2)*(GS(2)-GS(4))
     1         + C(3)*(2.*GS(1)-GS(5)) + C(4)*(GS(2)-GS(6)) + C(5)*GS(3)
     2         + C(6)*GS(4)
      CMDP(2)=CMDP(2)*PI/4.*GAM*SACA*CD*CD
      CMDP(3)=PI/2.*CD*CD*SACA*(2.*SC(1)+SC(3))
      CMDP(4)=SC(1)*(4.*ES(1)+4.*ES(2)+2.*ES(3)) + SC(2)*(ES(2)-ES(4))
     1         + SC(3)*(2.*ES(1)-ES(5)) + SC(4)*(ES(2)-ES(6))
     2         + SC(5)*ES(3) + SC(6)*ES(4)
      CMDP(4)=CMDP(4)*PI/4.*SACA*CD*CD
      DO 36 J=1,6
   36 E(J)=E(J) + F(J)*GAM
      CMDP(5)=SC(1)*(4.*E(1)+2.*E(2)) + 2.*SC(2)*E(1)
      DUM=0.0
      DO 37 J=2,5
   37 DUM=DUM + SC(J+1)*E(J) - SC(J)*E(J+1)
      CMDP(5)=-(CMDP(5)+DUM)*PI/2.*SACA*CD
      DO 38 J=2,5
   38 CMDP(1)=CMDP(1) + CMDP(J)
   23 CONTINUE
      WRITE (6,153)
      WRITE (6,154) (CTDP(N),N=1,5) ,CNDP(1),CMDP(1)
      CNDP(1)=CNDP(1)*CTCF
      CMDP(1)=CMDP(1)*CTCF*RRP/2.0
      WRITE (6,155) (CTDP(N),N=6,10),CNDP(1),CMDP(1)
      WRITE (6,156)
      N=0
      NTOT=0
```

```
   70 DO 71 J=1,NZ
      IF (STALL(J)) 71, 71, 72
   72 N=N+1
      JSTL(N)=J
      NTOT=N
   71 CONTINUE
   73 IF (NTOT) 75, 75, 74
   74 WRITE (6,157) (JSTL(N),N=1,NTOT)
   75 CONTINUE
      IF (NPHI) 99,99,80
C
C      COMPUTE DUCT SURFACE PRESSURE COEFFICIENTS
C
   80 CALL PRESS
      DO 81 I=1,NPHI
      DO 81 J=1,IR
      IF (XPRES(J)-XP) 81,82,82
   82 CPP(I,J)=CPP(I,J) + DELP*CSALF*CSALF
   81 CONTINUE
      IF (NPRES.LE.1) GO TO 85
      DO 83 I=1,NPHI
      DO 83 J=1,IR
      CPP(I,J)=CPP(I,J)*ARJV*ARJV/2.0
   83 CPM(I,J)=CPM(I,J)*ARJV*ARJV/2.0
   85 NPAG=NPAG+1
      WRITE (6,703) NRUN,NPAG
      WRITE (6,102) TALK
      WRITE (6,148) CD,XP,TC,RRP,RCBRP,APA
      WRITE (6,149) ALF,ARJV,ARJVP,ARJ,ARJP
      WRITE (6,250) (PHI(J),J=1,NPHI)
      GO TO (1,2,3,4,5),NPHI
    1 WRITE (6,251)
      GO TO 6
    2 WRITE (6,252)
      GO TO 6
    3 WRITE (6,253)
      GO TO 6
    4 WRITE (6,254)
      GO TO 6
    5 WRITE (6,255)
    6 CONTINUE
      DO 86 J=1,IR
   86 WRITE (6,245) XPRES(J), (CPP(I,J),CPM(I,J) ,I=1,NPHI)
      IF (NPRES.LE.1) GO TO 8
      WRITE (6,256)
      GO TO 99
    8 WRITE (6,257)
   99 RETURN
      END
```

8. SAMPLE CASES

In this section, the input and output for several sample cases are described. The input decks are shown in figure 6 and illustrate both methods of stacking runs discussed in section 4.1. The first case considers the ducted propeller on the Bell X-22A aircraft (ref. 13). This input deck for this case is followed by a blank card, since a different configuration follows. The second and succeeding cases consider the ducted fan used on the Doak VZ-4DA aircraft (refs. 12 and 14). There are four cases illustrated in figure 6. The first of these four, which follows the blank card, has a complete input deck. The remaining three cases, which have run numbers of 2010, 2020, and 2030, use only a single card, since only the advance ratio or the angle of attack is changed. The quantities required as input were obtained from references 12-14. The only areas in which questions might arise are the geometric camber coefficients and the centerbody configuration, both of which are discussed below.

The Bell duct has an unusual camberline shape, and the coefficients resulting from equation (48) do not yield a shape that matches the actual camberline well. A better fit to the camberline was obtained by solving equation (47) at $x/c = 0$, 0.15, 0.45, and 0.7. These are the coefficients shown on the third card of the input deck. The centerbody model is determined by placing the maximum centerbody radius at its true location aft of the fan. The Rankine body results in a centerbody which is larger and more blunt than the true centerbody. The shapes are compared in figure 3 of reference 1.

The camberline of the Doak duct was best fit by solving equation (47) at $x/c = 0$, 0.25, 0.55, and 0.8; and these are the coefficients on the third card of the input deck for the second case. The centerbody model is determined by assuming the maximum centerbody radius to be at the propeller station. The resulting shape fits the centerbody nose very well but it is shorter in length than the true centerbody as is shown in figure 4 of reference 1.

Two runs making up the sample case output are shown in figure 7. They are the first two runs obtained from the stacked input decks shown in figure 6. Since the output was described in detail in section 5, no further comments will be given here.

9. DATA COMPARISONS

In order to give the program user some indication of the nature of the results obtained from the computer program, some comparisons with experimental data are presented. The comparisons included herein are brief because of the similarity between the current results and those included in reference 4.

Comparison between the measured and predicted total thrust coefficient on the Bell X-22A ducted propeller in axial flow (ref. 13) is shown in figure 8. These results illustrate the effect of advance ratio and fan blade pitch angle. The agreement is good until comparisons are made in regions of possible blade stall; that is, low advance ratio and high blade pitch angles. Note that the program predicts the correct trend due to blade stall when $\beta_{3/4} = 49^\circ$ even though the total thrust is overpredicted.

The total thrust coefficients presented in this report include the pressure thrust due to the increased pressure aft of the fan acting on the duct. The pressure thrust is included to give better agreement between measured and predicted duct thrust as is shown in reference 5. Thus the overprediction of the total thrust coefficient is due primarily to the predicted fan thrust as discussed in the above reference. The fan thrust is very dependent on the fan blade characteristics, and an error of 2° in the blade pitch angle can cause large differences in fan thrust.

In figure 9, the effect of angle of attack and advance ratio on the thrust, normal force, and pitching moment on the Doak ducted fan (ref. 14) are shown. The thrust and normal force agree generally with the results in reference 5; that is, the thrust is well predicted at high advance ratios and at low angles of attack, and the normal force is well predicted at high advance ratios. The major difference in results occurs with pitching moments. Reference 5 showed good prediction of pitching moments, but figure 9(c) shows poor agreement with experiment. In the early work described in references 2 and 4, some simplifying assumptions were necessary in order to compute duct pitching moments. The analysis of reference 1 removed these assumptions and showed some of them to be incorrect. Unexplainably, the more exact analysis results in poor agreement with experiment.

Comparison of predicted and measured duct surface pressure coefficients on the Doak ducted fan are shown in figures 10, 11, and 12. The present results are nearly the same as those shown in reference 4 where a larger range of conditions was investigated. Although $\alpha = 20^\circ$ is the highest angle of attack shown in figures 11 and 12, the nature of the agreement on duct pressure coefficients is the same for angles of attack as large as $\alpha = 80^\circ$.

Nielsen Engineering & Research, Inc.
Palo Alto, Calif.
June 1969

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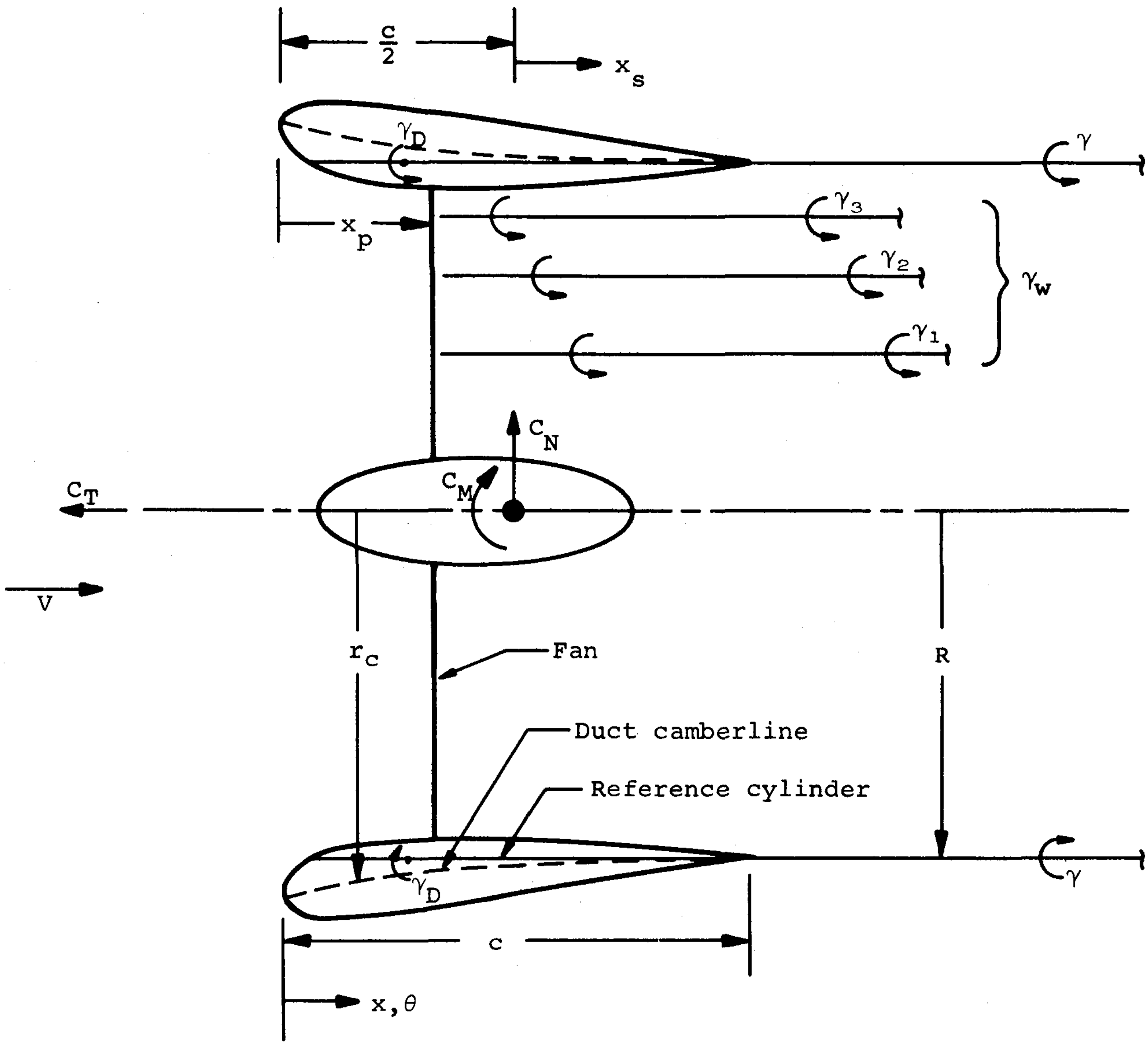


Figure 1.- Theoretical model of ducted fan in axial flow.

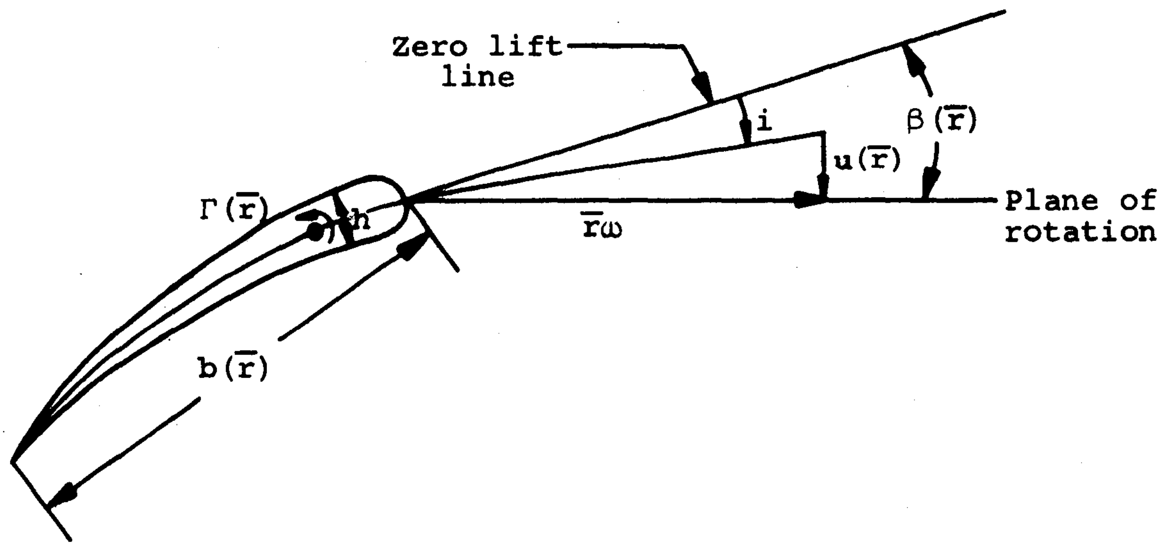
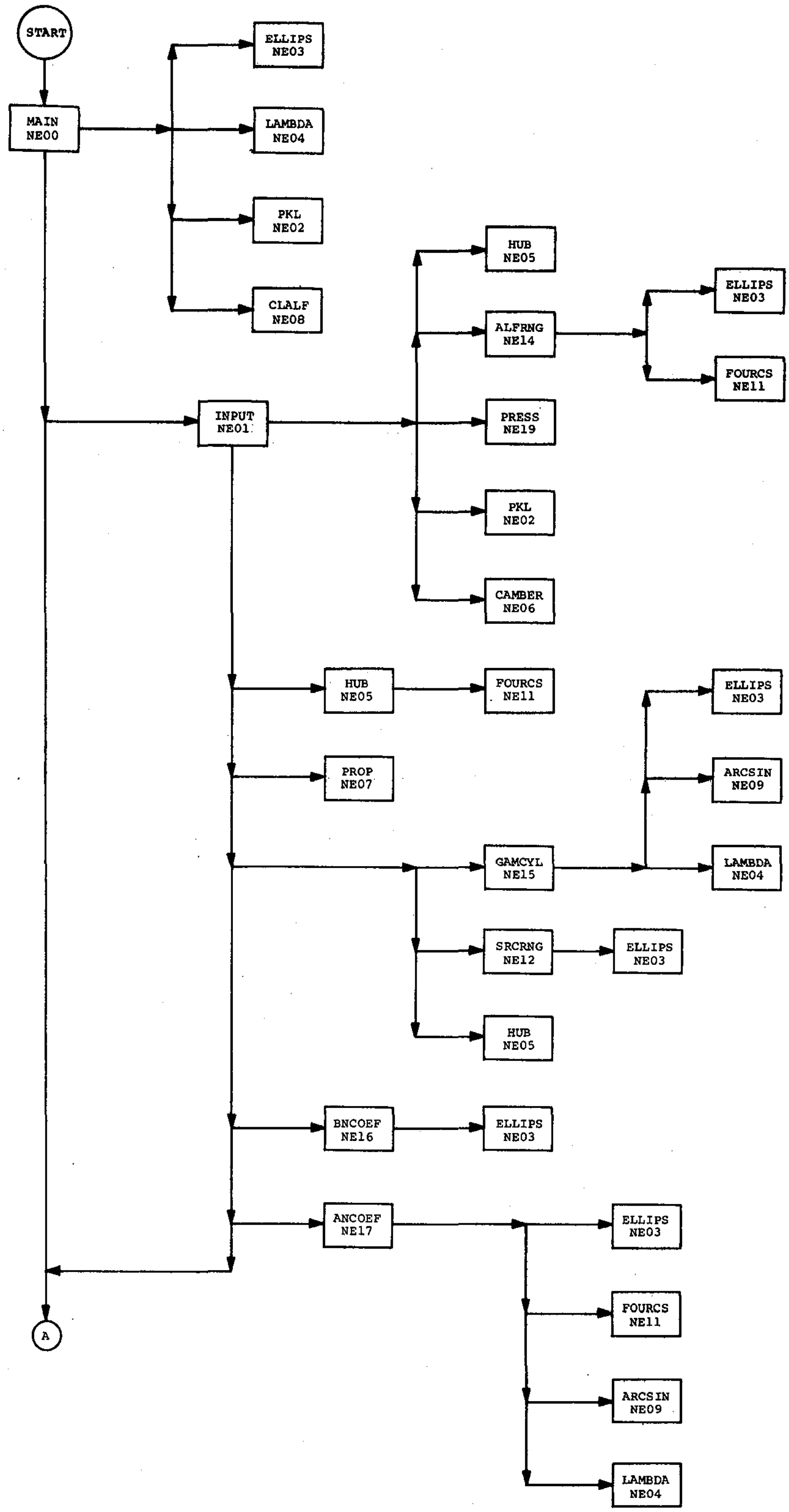
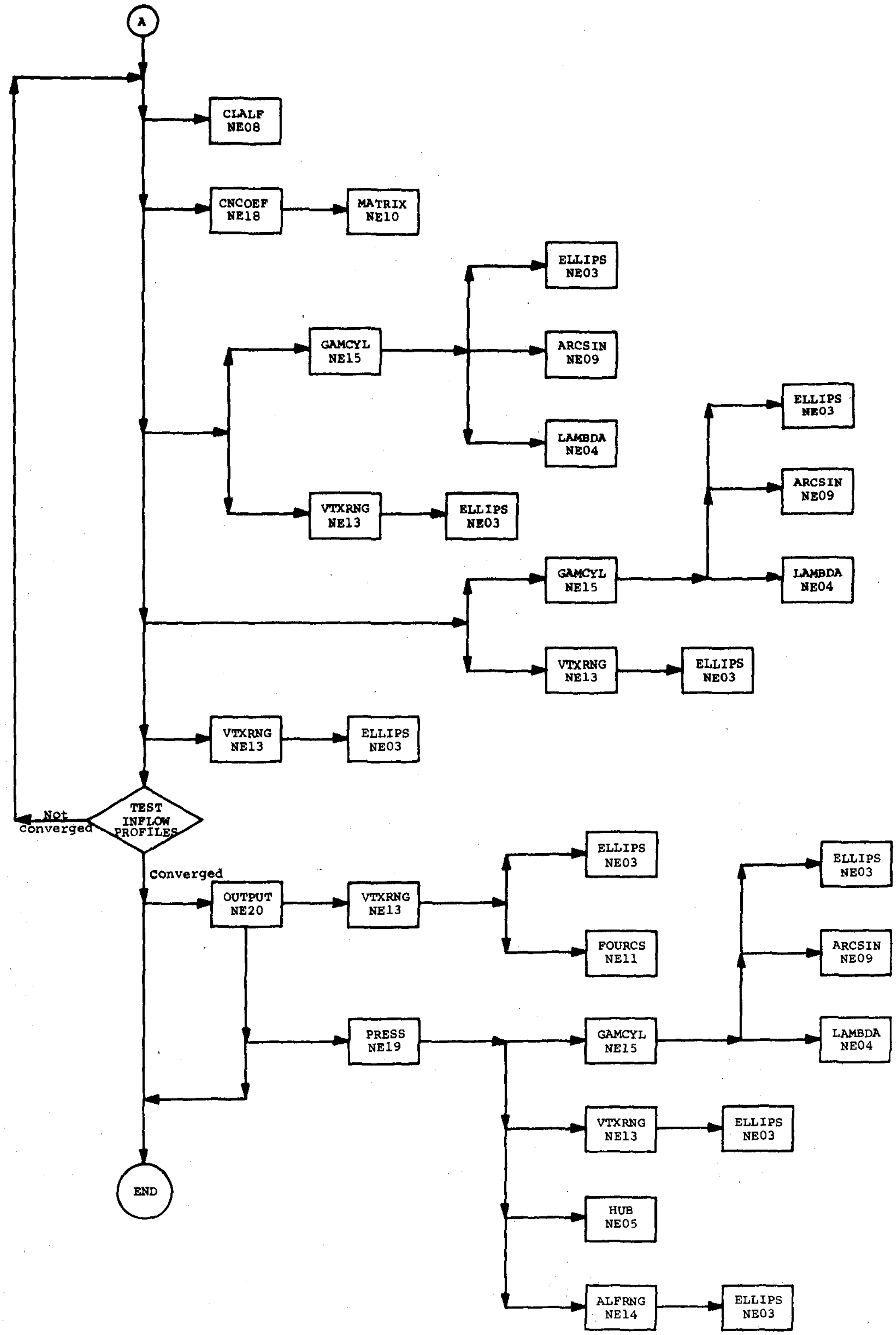


Figure 2.- Blade section flow characteristics.



(a) Initialization section.

Figure 3.- Relationship between subprograms.



(b) Iteration and conclusion section.

Figure 3.- Concluded.

Card No. 1

Variable	Title
Description	Any alphabetic or numeric identification information
Units	None
Card Column	1-80
Format	None
Value	

Card No. 2

Variable	c/D	x_p/c	t/c	R/R_p	R_{CB}/R_p	ϵ
Description	Duct chord-to-diameter ratio	Propeller location in fraction of duct chord	Duct maximum thickness-to-chord ratio	Ratio of duct trailing edge radius to propeller radius	Ratio of centerbody radius to propeller radius	Convergence criterion
Units	None	None	None	None	None	None
Card Column	5	15	25	35	45	55
Format	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx
Value						

Card No. 3

Variable	R_o	R_1	R_2	R_3
Description	Fourier coefficients of the duct geometric camberline			
Units	None	None	None	None
Card Column	5	15	25	35
Format	±x.xxxx	±x.xxxx	±x.xxxx	±x.xxxx
Value				

Figure 4.- Format of input data.

Card No. 4

Variable	ℓ_{CB}/c	x_{CB}/c	r_{max}/ℓ_{CB}	$(x/c)_{r_{max}}$
Description	Ratio of centerbody length to duct chord	Location of centerbody nose in fraction of duct chord	Ratio of maximum radius to length of centerbody	Location of maximum centerbody radius
Units	None	None	None	None
Card Column	5	15	25	35
Format	±x.xxxx	±x.xxxx	±x.xxxx	±x.xxxx
Value				

Card No. 5

Variable	NBLD	NZ	NZP	IR	NPRES	NPRINT
Description	Number of propeller blades	Number of equal area annuli on propeller	Number of stations in propeller table	Number of x-stations on duct surface	Output option for pressure coefficient	Output index
Units	None	None	None	None	None	None
Card Column	4	9	14	19	25	29
Format	xx	xx	xx	xx	x	xx
Value						

Card No. 6[a]

Variable	r/R_p							
Description	Propeller radii at which blade characteristics are to be input, specified as a fraction of propeller radius							
Units	None	None	None	None	None	None	None	None
Card Column	5	15	25	35	45	55	65	75
Format	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx
Value								

[a] The number of 10 column fields will be equal to NZP (Card No. 5). Each card will accomodate 8 fields; therefore, if NZP > 8, the remaining fields are included on following cards having the same format as Card No. 6.

Figure 4.- Continued.

Card No. 7[b]

Variable	b/R_p							
Description	The propeller blade chord at stations corresponding to radii on Card No. 6, specified as a fraction of propeller radius.							
Units	None	None	None	None	None	None	None	None
Card Column	5	15	25	35	45	55	65	75
Format	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx
Value								

Card No. 8[b]

Variable	β							
Description	Propeller blade pitch angle at stations corresponding to radii on Card No. 6.							
Units	Degrees	Degrees	Degrees	Degrees	Degrees	Degrees	Degrees	Degrees
Card Column	5	15	25	35	45	55	65	75
Format	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx
Value								

Card No. 9[b]

Variable	t/b							
Description	Propeller thickness-to-chord ratio at stations corresponding to radii on Card No. 6.							
Units	None	None	None	None	None	None	None	None
Card Column	5	15	25	35	45	55	65	75
Format	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx
Value								

[b] The same number of cards are required for b/R_p, β, and t/b as are required for r/R_p.

Figure 4.- Continued.

Card No. 10[c]

Variable	x/c							
Description	Location on duct at which pressure coefficients will be calculated, specified as a fraction of duct chord.							
Units	None	None	None	None	None	None	None	None
Card Column	5	15	25	35	45	55	65	75
Format	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx
Value								

Card No. 11

Variable	NRUN	NPHI	J	α	ϕ				
Description	Run number $\neq 0$	Number of azimuth angles ≤ 5	Advance ratio ($J \neq 0$)	Angle of attack ($\lvert\alpha\rvert < 90^{\circ}$)	Azimuth angles at which duct pressure distributions will be calculated.				
Units	None	None	None	Degrees	Degrees	Degrees	Degrees	Degrees	Degrees
Card Column	2	10	15	25	35	45	55	65	75
Format	xxxx	x	x.xxxx	x.xxxx	xxx.xx	xxx.xx	xxx.xx	xxx.xx	xxx.xx
Value									

[c]The number of 10 column fields will be equal to IR (Card No. 5).

Figure 4.- Concluded.

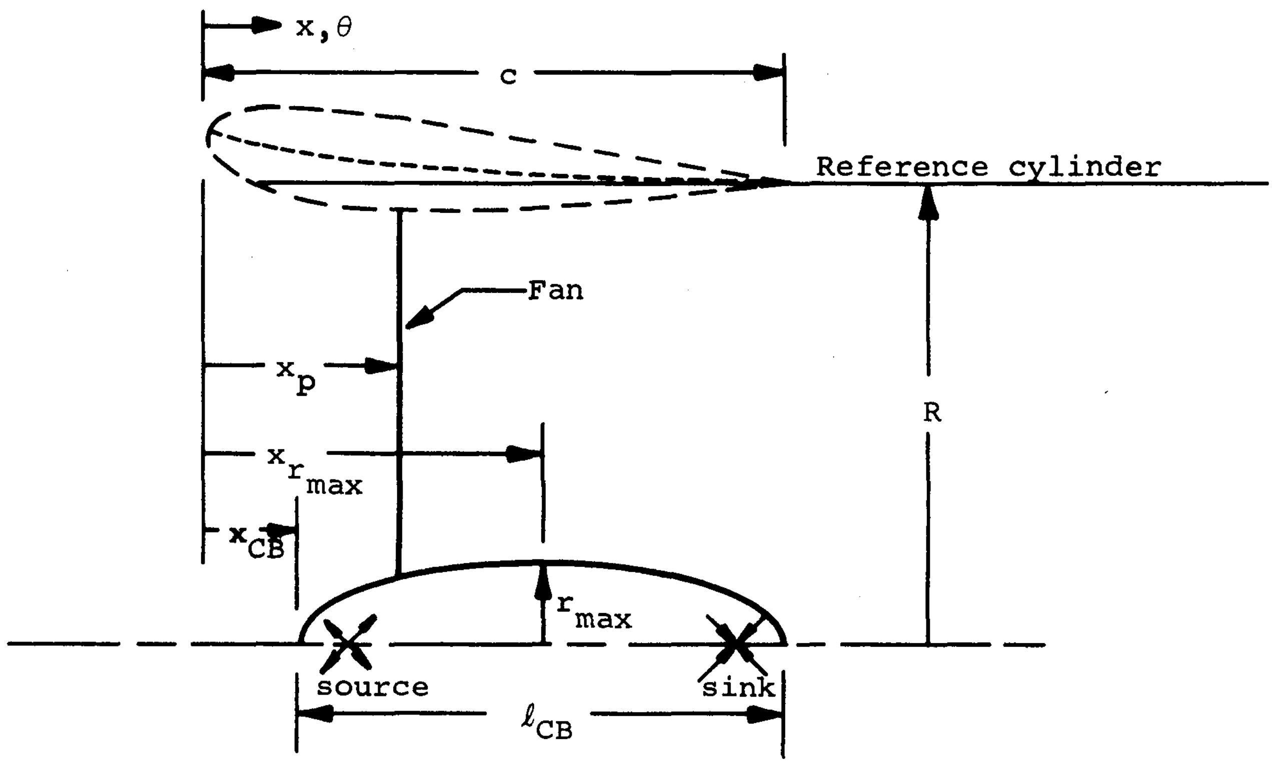


Figure 5.- Centerbody geometry.

```
SAMPLE CASE I ... BELL X-22A DUCTED PROPELLER ... BETA(3/4)=29 DEGREES
   0.525      0.286      0.170      1.102      0.175      0.01
 -.039985   -.083845   -.062813   -.027351
   0.890     -0.067      0.202      0.467
   3   20    12   23     1    1
   0.175      0.250      0.30       0.40       0.45       0.50       0.55       0.60
   0.65       0.75       0.85       1.00
   0.333      0.309      0.293      0.260      0.246      0.235      0.228      0.224
   0.222      0.217      0.212      0.204
  64.0       58.5       55.0       48.0       44.4       41.0       37.7       34.8
  32.5       29.0       26.5       23.5
   0.320      0.280      0.255      0.210      0.190      0.180      0.160      0.145
   0.130      0.100      0.070      0.030
   0.0        0.0025     0.0050     0.010      0.020      0.030      0.040      0.050
   0.10       0.15       0.20       0.25       0.285      0.287      0.30       0.40
   0.50       0.60       0.70       0.80       0.90       0.95       1.00
 1000     1   0.10       0.0        0.0

SAMPLE CASE II ... DOAK VZ-4DA DUCTED FAN ... BETA(TIP)= 15 DEGREES
   0.608      0.293      0.160      1.131      0.332      0.01
 -.002748   -.029679   -.032916    .006403
   2.16      -0.561      0.115      0.293
   8   20     8   23     1
   0.332      0.40       0.50       0.60       0.70       0.80       0.90       1.00
   0.156      0.155      0.152      0.144      0.131      0.117      0.104      0.091
  48.0       43.0       36.0       30.0       25.0       20.8       17.5       15.0
   0.190      0.154      0.120      0.100      0.084      0.072      0.062      0.054
   0.0        0.0025     0.0050     0.01       0.02       0.03       0.04       0.05
   0.10       0.15       0.20       0.25       0.292      0.294      0.30       0.40
   0.50       0.60       0.70       0.80       0.90       0.95       1.00
 2000     5   0.178     20.0        0.0       45.0       90.0      135.0      180.0
 2010     5   0.178     10.0        0.0       45.0       90.0      135.0      180.0
 2020     1   0.178      0.0        0.0
 2030     5   0.342     20.0        0.0       45.0       90.0      135.0      180.0
```

Figure 6.- Sample input data.

RUN NUMBER 1000 PAGE 0

SAMPLE CASE I ... BELL X-22A DUCTED PROPELLER ... BETA(3/4)=29 DEGREES

INPUT

DUCT GEOMETRY...	C/D	XP/C	T/C	RTE/RP	RCB/RP
	0.525000	0.286000	0.170000	1.102000	0.175000

CAMBER COEFFICIENTS, -0.039985 -0.083845 -0.062813 -0.027351

PROPELLER GEOMETRY... 3 BLADES

R/RP	B/RP	BETA	TH/CHD
0.175000	0.333000	64.000	0.32000
0.250000	0.309000	58.500	0.28000
0.300000	0.293000	55.000	0.25500
0.400000	0.260000	48.000	0.21000
0.450000	0.246000	44.400	0.19000
0.500000	0.235000	41.000	0.18000
0.550000	0.228000	37.700	0.16000
0.600000	0.224000	34.800	0.14500
0.650000	0.222000	32.500	0.13000
0.750000	0.217000	29.000	0.10000
0.850000	0.212000	26.500	0.07000
1.000000	0.204000	23.500	0.03000

CENTERBODY GEOMETRY...	LCB/C	XCB/C	RMAX/LCB	X(RMAX)/C
	0.89000	-0.06700	0.20200	0.46700

CONVERGENCE CRITERION... EPSILON =0.01000

DEFINITION OF SYMBOLS USED IN TABULAR OUTPUT...

R/RP	RADIAL PROPELLER STATION IN FRACTION OF PROPELLER RADIUS
B/RP	PROPELLER CHORD IN FRACTION OF PROPELLER RADIUS
BETA	PROPELLER PITCH IN DEGREES
TH/CHD	PROPELLER BLADE THICKNESS-TO-CHORD RATIO
V	FREE STREAM VELOCITY
U	TOTAL INFLOW VELOCITY
J	ADVANCE RATIO
J'	RATIO OF V TO PROPELLER TIP SPEED
GAM/V	STRENGTH OF INTERNAL VORTEX CYLINDER N
ALPHA	ANGLE OF ATTACK, DEGREES
DELTA P/Q	RISE IN TOTAL PRESSURE ACROSS PROPELLER NORMALIZED ON FREE STREAM DYNAMIC PRESSURE
CTP(D)	THRUST COEFFICIENT ON PROPELLER IN THE DUCT
CTD(P)	THRUST COEFFICIENT ON THE DUCT
CTDP	TOTAL THRUST COEFFICIENT
CTD(P)'	THRUST COEFFICIENT ON DUCT INCLUDING PRESSURE THRUST ON THE DUCT AFT OF THE PROPELLER
CTDP'	TOTAL THRUST COEFFICIENT INCLUDING PRESSURE THRUST
CNDP	TOTAL NORMAL FORCE COEFFICIENT
CMDP	TOTAL PITCHING MOMENT COEFFICIENT

(a) Bell X-22A ducted propeller.

Figure 7.- Sample output.

```
RUN NUMBER 1000                    OPTIONAL OUTPUT                     PAGE  1

SAMPLE CASE I ... BELL X-22A DUCTED PROPELLER ... BETA(3/4)=29 DEGREES

  DUCT...  C/D       XP/C       T/C       RTE/RP    RCB/RP     AP/A
        0.525000  0.286000  0.170000  1.102000  0.175000  0.798231

        EFFECTIVE CAMBER -0.040554 -0.123546 -0.075738 -0.026365

             ALPHA       J          J'       J COS(A)  J'COS(A)
             0.000    0.10000   0.02888    0.10000   0.02888

               ( 4.08)                                    ( 6.37)
 N     R/RP     UQD/V          UGD/V         UG/V          UCB/V        GAMMA/RV
 1  0.22812 2.08706E-01 5.23663E 00 1.42872E 00 5.40376E-01 1.82966E 00
 2  0.31920 2.24891E-01 5.33044E 00 1.23518E 00 3.75017E-01 2.09227E 00
 3  0.38836 2.42306E-01 5.42540E 00 1.20587E 00 2.86758E-01 2.29096E 00
 4  0.44669 2.61317E-01 5.52278E 00 1.17636E 00 2.31453E-01 2.45454E 00
 5  0.49814 2.82151E-01 5.62282E 00 1.14719E 00 1.93476E-01 2.50681E 00
 6  0.54471 3.05032E-01 5.72554E 00 1.11808E 00 1.65756E-01 2.40893E 00
 7  0.58758 3.30190E-01 5.83089E 00 1.08937E 00 1.44616E-01 2.32899E 00
 8  0.62751 3.57869E-01 5.93876E 00 1.06077E 00 1.27958E-01 2.27892E 00
 9  0.66504 3.88313E-01 6.04889E 00 1.03272E 00 1.14493E-01 2.24979E 00
10  0.70056 4.21754E-01 6.16098E 00 1.00454E 00 1.03385E-01 2.23391E 00
11  0.73435 4.58395E-01 6.27481E 00 9.77391E-01 9.40685E-02 2.19695E 00
12  0.76666 4.98375E-01 6.39017E 00 9.50348E-01 8.61464E-02 2.17241E 00
13  0.79766 5.41744E-01 6.50594E 00 9.23912E-01 7.93312E-02 2.16097E 00
14  0.82749 5.88411E-01 6.62078E 00 8.97986E-01 7.34095E-02 2.13735E 00
15  0.85628 6.38125E-01 6.73757E 00 8.72429E-01 6.82195E-02 2.10810E 00
16  0.88414 6.90406E-01 6.85087E 00 8.47518E-01 6.36364E-02 2.09190E 00
17  0.91114 7.44598E-01 6.95638E 00 8.23243E-01 5.95621E-02 2.07090E 00
18  0.93737 7.99848E-01 7.05912E 00 7.99407E-01 5.59186E-02 2.04197E 00
19  0.96288 8.55179E-01 7.15662E 00 7.76403E-01 5.26428E-02 2.00963E 00
20  0.98773 9.09743E-01 7.24777E 00 7.53845E-01 4.96836E-02 1.97441E 00

            8  ITERATIONS,     EPSILON = 0.010000
```

(a) Continued.

Figure 7.- Continued.

RUN NUMBER 1000 PAGE 2

SAMPLE CASE I ... BELL X-22A DUCTED PROPELLER ... BETA(3/4)=29 DEGREES

DUCT...	C/D	XP/C	T/C	RTE/RP	RCB/RP	AP/A
	0.525000	0.286000	0.170000	1.102000	0.175000	0.798231

ALPHA	J	J'	J COS(A)	J'COS(A)
0.000	0.10000	0.02888	0.10000	0.02888

	N	R/RP	INFLOW U/V	GAM/V	BLADE ALPHA	DELTA P/Q
*	1	0.22812	8.22806E 00	-5.35298E-01	1.12870E 01	6.04887E 01
*	2	0.31920	8.25299E 00	-3.83307E-01	1.43056E 01	6.91703E 01
*	3	0.38836	8.44199E 00	-3.03430E-01	1.42232E 01	7.57390E 01
*	4	0.44669	8.62658E 00	-9.48361E-02	1.31372E 01	8.11472E 01
	5	0.49814	8.74799E 00	1.78412E-01	1.19394E 01	8.28753E 01
	6	0.54471	8.73259E 00	1.48369E-01	1.10174E 01	7.96392E 01
	7	0.58758	8.73975E 00	9.42183E-02	1.01865E 01	7.69965E 01
	8	0.62751	8.78313E 00	5.52901E-02	9.52116E 00	7.53412E 01
	9	0.66504	8.85469E 00	3.02945E-02	9.00640E 00	7.43781E 01
	10	0.70056	8.94585E 00	7.09063E-02	8.61084E 00	7.38530E 01
	11	0.73435	9.02441E 00	4.74073E-02	8.18429E 00	7.26311E 01
	12	0.76666	9.12106E 00	2.21891E-02	7.84249E 00	7.18197E 01
	13	0.79766	9.23580E 00	4.59868E-02	7.57733E 00	7.14415E 01
	14	0.82749	9.34240E 00	5.73186E-02	7.29640E 00	7.06608E 01
	15	0.85628	9.44943E 00	3.18934E-02	7.02046E 00	6.96937E 01
	16	0.88414	9.56927E 00	4.33428E-02	6.80902E 00	6.91584E 01
	17	0.91114	9.67902E 00	5.57921E-02	6.59597E 00	6.84342E 01
	18	0.93737	9.78178E 00	6.48346E-02	6.37873E 00	6.75075E 01
	19	0.96288	9.87603E 00	7.12116E-02	6.16215E 00	6.64384E 01
	20	0.98773	9.96066E 00	7.14088E 00	5.94959E 00	6.52739E 01

	CTP(D)	CTD(P)	CTD(P)'	CTDP	CTDP'	CNDP	CMDP
(A)	5.7580E 01	5.1231E 01	6.2755E 01	1.0881E 02	1.2034E 02	0.0000E-39	0.0000E-39
(B)	2.8327E-01	2.5204E-01	3.0873E-01	5.3531E-01	5.9200E-01	0.0000E-39	0.0000E-39

NOTES...

(A) COEFFICIENTS BASED ON FREE STREAM DYNAMIC PRESSURE

(B) COEFFICIENTS BASED ON PROPELLER TIP SPEED

* BLADE SECTION LIFT COEFFICIENT HAS EXCEEDED CLMAX
IN ANNULI NOS. 1 2 3 4

(a) Continued.

Figure 7.- Continued.

RUN NUMBER 1000 DUCT SURFACE PRESSURE DISTRIBUTION PAGE 3

SAMPLE CASE I ... BELL X-22A DUCTED PROPELLER ... BETA(3/4)=29 DEGREES

DUCT...	C/D	XP/C	T/C	RTE/RP	RCB/RP	AP/A
	0.525000	0.286000	0.170000	1.102000	0.175000	0.798231

ALPHA	J	J'	J COS(A)	J'COS(A)
0.000	0.10000	0.02888	0.10000	0.02888

AZIMUTH
PHI = /--- 0.00 ---/---

X/C	CP(IN)	CP(OUT)
0.0000	-2.536E 02	-2.536E 02
0.0025	-2.735E 02	-1.905E 02
0.0050	-2.861E 02	-1.307E 02
0.0100	-2.724E 02	-9.522E 01
0.0200	-2.444E 02	-5.532E 01
0.0300	-2.214E 02	-3.528E 01
0.0400	-2.033E 02	-2.414E 01
0.0500	-1.851E 02	-1.480E 01
0.1000	-1.425E 02	-2.476E 00
0.1500	-1.218E 02	2.554E-01
0.2000	-1.117E 02	8.872E-01
0.2500	-1.059E 02	9.914E-01
0.2850	-1.031E 02	9.998E-01
0.2870	-3.768E 01	9.999E-01
0.3000	-3.685E 01	9.998E-01
0.4000	-2.998E 01	9.861E-01
0.5000	-1.931E 01	9.005E-01
0.6000	-7.510E 00	5.112E-01
0.7000	1.453E 00	1.531E-01
0.8000	2.062E 00	5.952E-01
0.9000	-2.995E 00	9.926E-01
0.9500	3.679E 00	9.944E-01
1.0000	6.627E 01	1.000E 00

PRESSURE COEFFICIENTS BASED ON FREE STREAM VELOCITY

(a) Concluded.

Figure 7.- Continued.

SAMPLE CASE II ... DOAK VZ-4DA DUCTED FAN ... BETA(TIP)= 15 DEGREES

INPUT

DUCT GEOMETRY...

C/D	XP/C	T/C	RTE/RP	RCB/RP
0.608000	0.293000	0.160000	1.131000	0.332000

CAMBER COEFFICIENTS, -0.002748 -0.029679 -0.032916 0.006403

PROPELLER GEOMETRY... 8 BLADES

R/RP	B/RP	BETA	TH/CHD
0.332000	0.156000	48.000	0.19000
0.400000	0.155000	43.000	0.15400
0.500000	0.152000	36.000	0.12000
0.600000	0.144000	30.000	0.10000
0.700000	0.131000	25.000	0.08400
0.800000	0.117000	20.800	0.07200
0.900000	0.104000	17.500	0.06200
1.000000	0.091000	15.000	0.05400

CENTERBODY GEOMETRY...

LCB/C	XCB/C	RMAX/LCB	X(RMAX)/C
2.16000	-0.56100	0.11500	0.29300

CONVERGENCE CRITERION... EPSILON =0.01000

DEFINITION OF SYMBOLS USED IN TABULAR OUTPUT...

R/RP	RADIAL PROPELLER STATION IN FRACTION OF PROPELLER RADIUS
B/RP	PROPELLER CHORD IN FRACTION OF PROPELLER RADIUS
BETA	PROPELLER PITCH IN DEGREES
TH/CHD	PROPELLER BLADE THICKNESS-TO-CHORD RATIO
V	FREE STREAM VELOCITY
U	TOTAL INFLOW VELOCITY
J	ADVANCE RATIO
J'	RATIO OF V TO PROPELLER TIP SPEED
GAM/V	STRENGTH OF INTERNAL VORTEX CYLINDER N
ALPHA	ANGLE OF ATTACK, DEGREES
DELTA P/Q	RISE IN TOTAL PRESSURE ACROSS PROPELLER NORMALIZED ON FREE STREAM DYNAMIC PRESSURE
CTP(D)	THRUST COEFFICIENT ON PROPELLER IN THE DUCT
CTD(P)	THRUST COEFFICIENT ON THE DUCT
CTDP	TOTAL THRUST COEFFICIENT
CTD(P)'	THRUST COEFFICIENT ON DUCT INCLUDING PRESSURE THRUST ON THE DUCT AFT OF THE PROPELLER
CTDP'	TOTAL THRUST COEFFICIENT INCLUDING PRESSURE THRUST
CNDP	TOTAL NORMAL FORCE COEFFICIENT
CMDP	TOTAL PITCHING MOMENT COEFFICIENT

(b) Doak VZ-4DA ducted fan.

Figure 7.- Continued.

RUN NUMBER 2000 PAGE 1

SAMPLE CASE II ... DOAK VZ-4DA DUCTED FAN ... BETA(TIP)= 15 DEGREES

DUCT...	C/D	XP/C	T/C	RTE/RP	RCB/RP	AP/A
	0.608000	0.293000	0.160000	1.131000	0.332000	0.695593

ALPHA	J	J'	J COS(A)	J'COS(A)
20.000	0.17800	0.05010	0.16727	0.04708

N	R/RP	INFLOW U/V	GAM/V	BLADE ALPHA	DELTA P/Q
1	0.36267	5.23679E 00	-1.35218E-01	8.25662E 00	2.89134E 01
2	0.41983	5.26777E 00	3.17357E-02	7.92511E 00	3.04108E 01
3	0.46999	5.27579E 00	9.98359E-02	7.29327E 00	3.00561E 01
4	0.51524	5.25198E 00	1.19916E-01	6.65377E 00	2.89533E 01
5	0.55680	5.22034E 00	1.72383E-01	6.12156E 00	2.76551E 01
6	0.59545	5.16455E 00	1.52069E-01	5.54300E 00	2.58393E 01
7	0.63174	5.12088E 00	1.76659E-01	5.12549E 00	2.42868E 01
8	0.66604	5.06680E 00	2.05082E-01	4.70483E 00	2.25413E 01
9	0.69866	5.00018E 00	1.67460E-01	4.26461E 00	2.05933E 01
10	0.72982	4.95392E 00	1.84996E-01	3.93421E 00	1.90650E 01
11	0.75970	4.90020E 00	2.03283E-01	3.59579E 00	1.74419E 01
12	0.78845	4.83857E 00	1.74513E-01	3.24839E 00	1.57372E 01
13	0.81618	4.79220E 00	1.56123E-01	2.96404E 00	1.43398E 01
14	0.84300	4.75555E 00	1.68195E-01	2.72148E 00	1.31412E 01
15	0.86899	4.71342E 00	1.79273E-01	2.47416E 00	1.19045E 01
16	0.89423	4.66602E 00	1.36541E-01	2.22443E 00	1.06487E 01
17	0.91877	4.63838E 00	1.27250E-01	2.04616E 00	9.73527E 00
18	0.94268	4.61378E 00	1.33504E-01	1.88825E 00	8.91761E 00
19	0.96599	4.58513E 00	1.38477E-01	1.72899E 00	8.09456E 00
20	0.98875	4.55222E 00	1.87724E 00	1.57029E 00	7.27852E 00

	CTP(D)	CTD(P)	CTD(P)'	CTDP	CTDP'	CNDP	CMDP
(A)	1.1534E 01	8.9945E 00	1.0397E 01	2.0528E 01	2.1931E 01	2.7529E 00	1.1474E 00
(B)	2.0631E-01	1.6089E-01	1.8598E-01	3.6719E-01	3.9228E-01	4.9241E-02	1.1606E-02

NOTES...

(A) COEFFICIENTS BASED ON FREE STREAM DYNAMIC PRESSURE

(B) COEFFICIENTS BASED ON PROPELLER TIP SPEED

(b) Continued.

Figure 7.- Continued.

RUN NUMBER 2000 DUCT SURFACE PRESSURE DISTRIBUTION PAGE 2

SAMPLE CASE II ... DOAK VZ-4DA DUCTED FAN ... BETA(TIP)= 15 DEGREES

DUCT... C/D	XP/C	T/C	RTE/RP	RCB/RP	AP/A
0.608000	0.293000	0.160000	1.131000	0.332000	0.695593

ALPHA	J	J'	J COS(A)	J'COS(A)
20.000	0.17800	0.05010	0.16727	0.04708

AZIMUTH PHI =	0.00		45.00		90.00		135.00		180.00	
X/C	CP(IN)	CP(OUT)	CP(IN)	CP(OUT)	CP(IN)	CP(OUT)	CP(IN)	CP(OUT)	CP(IN)	CP(OUT)
0.0000	-7.151E 01	-7.151E 01	-6.346E 01	-6.346E 01	-4.598E 01	-4.598E 01	-3.126E 01	-3.126E 01	-2.597E 01	-2.597E 01
0.0025	-7.547E 01	-4.980E 01	-6.775E 01	-4.375E 01	-5.081E 01	-3.072E 01	-3.628E 01	-1.994E 01	-3.096E 01	-1.613E 01
0.0050	-7.831E 01	-3.116E 01	-7.098E 01	-2.685E 01	-5.479E 01	-1.776E 01	-4.071E 01	-1.051E 01	-3.550E 01	-8.037E 00
0.0100	-7.351E 01	-2.101E 01	-6.700E 01	-1.784E 01	-5.256E 01	-1.123E 01	-3.991E 01	-6.107E 00	-3.520E 01	-4.420E 00
0.0200	-6.440E 01	-1.039E 01	-5.916E 01	-8.491E 00	-4.747E 01	-4.655E 00	-3.715E 01	-1.889E 00	-3.328E 01	-1.057E 00
0.0300	-5.740E 01	-5.510E 00	-5.300E 01	-4.257E 00	-4.317E 01	-1.823E 00	-3.446E 01	-2.230E-01	-3.118E 01	1.957E-01
0.0400	-5.210E 01	-3.038E 00	-4.830E 01	-2.146E 00	-3.978E 01	-4.817E-01	-3.222E 01	4.923E-01	-2.936E 01	6.936E-01
0.0500	-4.693E 01	-1.128E 00	-4.369E 01	-5.382E-01	-3.642E 01	4.839E-01	-2.995E 01	9.381E-01	-2.750E 01	9.600E-01
0.1000	-3.484E 01	8.448E-01	-3.281E 01	1.017E 00	-2.826E 01	1.181E 00	-2.421E 01	9.874E-01	-2.267E 01	8.022E-01
0.1500	-2.883E 01	9.920E-01	-2.734E 01	1.059E 00	-2.401E 01	1.023E 00	-2.106E 01	7.083E-01	-1.995E 01	4.967E-01
0.2000	-2.525E 01	8.816E-01	-2.408E 01	9.170E-01	-2.147E 01	8.311E-01	-1.918E 01	5.029E-01	-1.832E 01	2.96[illegible]E-01
0.2500	-2.247E 01	7.867E-01	-2.152E 01	8.165E-01	-1.941E 01	7.329E-01	-1.758E 01	4.296E-01	-1.690E 01	2.396E-01
0.2920	-2.053E 01	7.407E-01	-1.972E 01	7.718E-01	-1.793E 01	7.008E-01	-1.639E 01	4.236E-01	-1.582E 01	2.484E-01
0.2940	-1.403E 01	7.372E-01	-1.323E 01	7.679E-01	-1.146E 01	6.962E-01	-9.936E 00	4.185E-01	-9.377E 00	2.431E-01
0.3000	-1.375E 01	7.353E-01	-1.297E 01	7.668E-01	-1.123E 01	6.987E-01	-9.742E 00	4.264E-01	-9.196E 00	2.538E-01
0.4000	-9.965E 00	6.813E-01	-9.407E 00	7.177E-01	-8.203E 00	6.777E-01	-7.200E 00	4.569E-01	-6.844E 00	3.124E-01
0.5000	-6.710E 00	5.929E-01	-6.312E 00	6.286E-01	-5.474E 00	5.996E-01	-4.810E 00	4.080E-01	-4.585E 00	2.809E-01
0.6000	-4.091E 00	4.139E-01	-3.800E 00	4.476E-01	-3.207E 00	4.232E-01	-2.769E 00	2.493E-01	-2.633E 00	1.336E-01
0.7000	-2.156E 00	2.850E-01	-1.943E 00	3.189E-01	-1.527E 00	3.034E-01	-1.252E 00	1.503E-01	-1.178E 00	4.663E-02
0.8000	-1.196E 00	3.668E-01	-1.039E 00	4.046E-01	-7.485E-01	4.072E-01	-5.840E-01	2.845E-01	-5.527E-01	1.970E-01
0.9000	-7.532E-01	5.950E-01	-6.440E-01	6.370E-01	-4.589E-01	6.595E-01	-3.849E-01	5.707E-01	-3.867E-01	5.014E-01
0.9500	2.597E-01	5.884E-01	3.413E-01	6.292E-01	4.671E-01	6.560E-01	4.920E-01	5.816E-01	4.727E-01	5.211E-01
1.0000	7.427E 00	1.000E 00	7.427E 00	1.000E 00	7.427E 00	1.000E 00	7.427E 00	1.000E 00	7.427E 00	1.000E 00

PRESSURE COEFFICIENTS BASED ON FREE STREAM VELOCITY

(b) Concluded.

Figure 7.- Concluded.

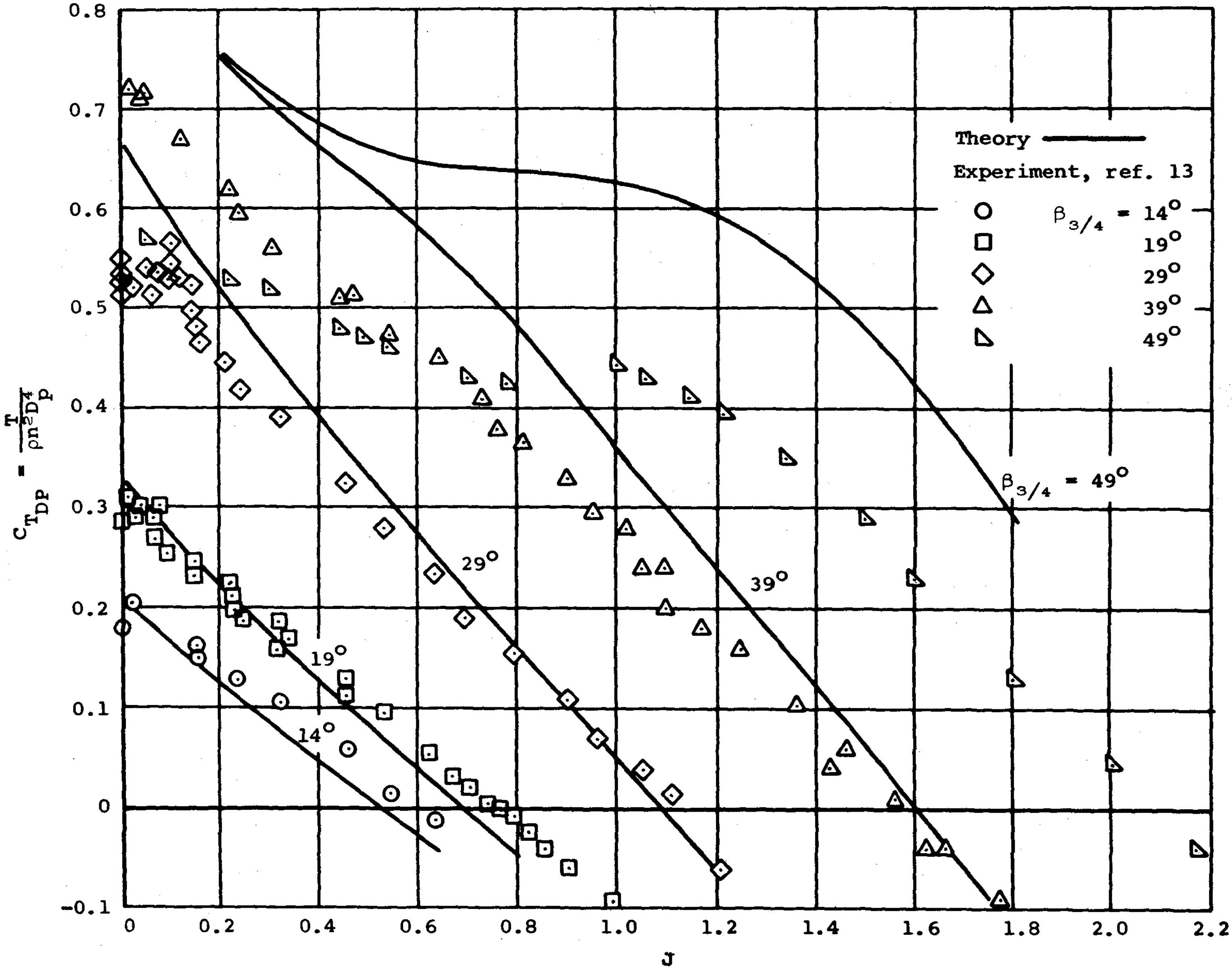
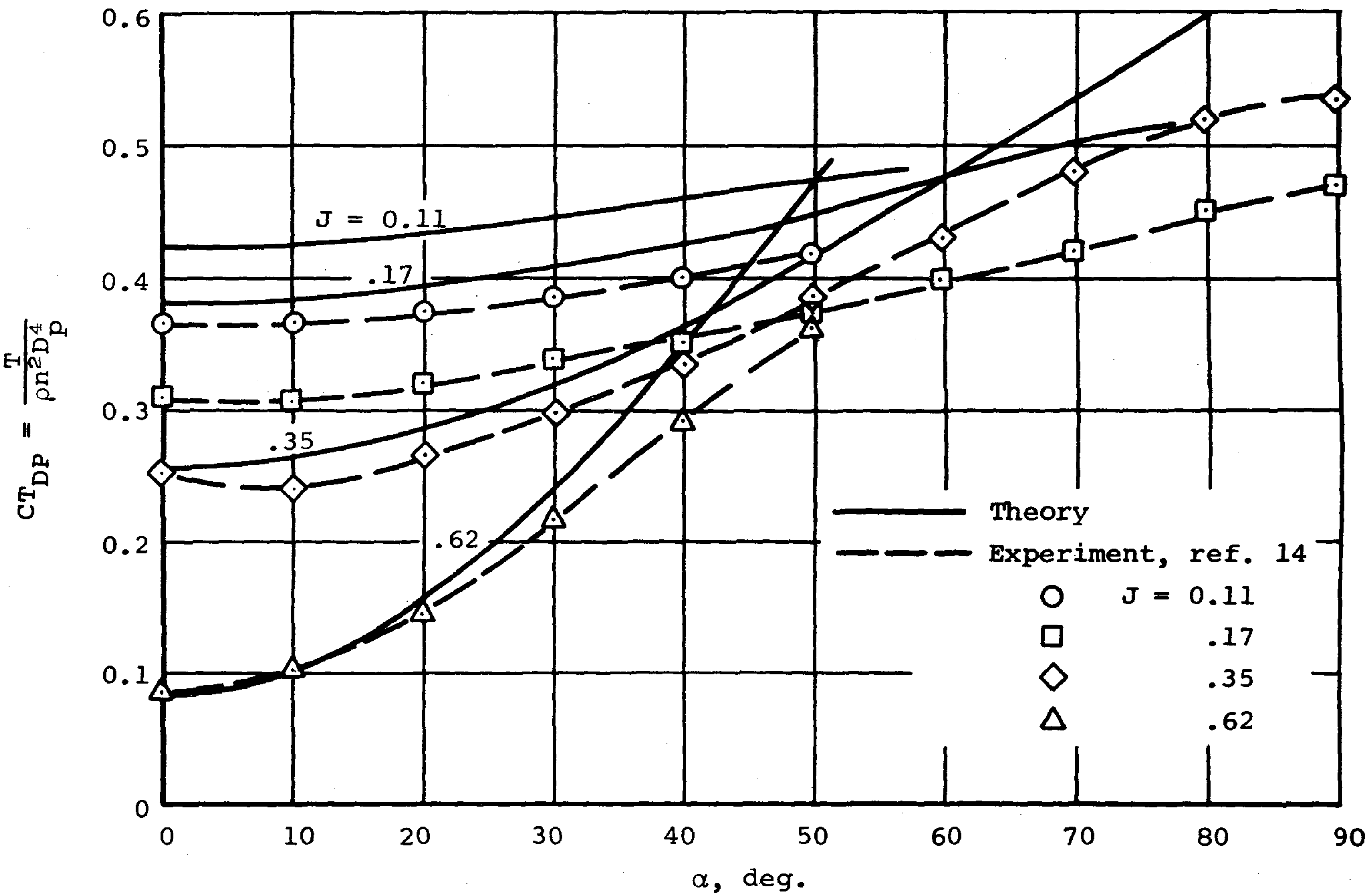
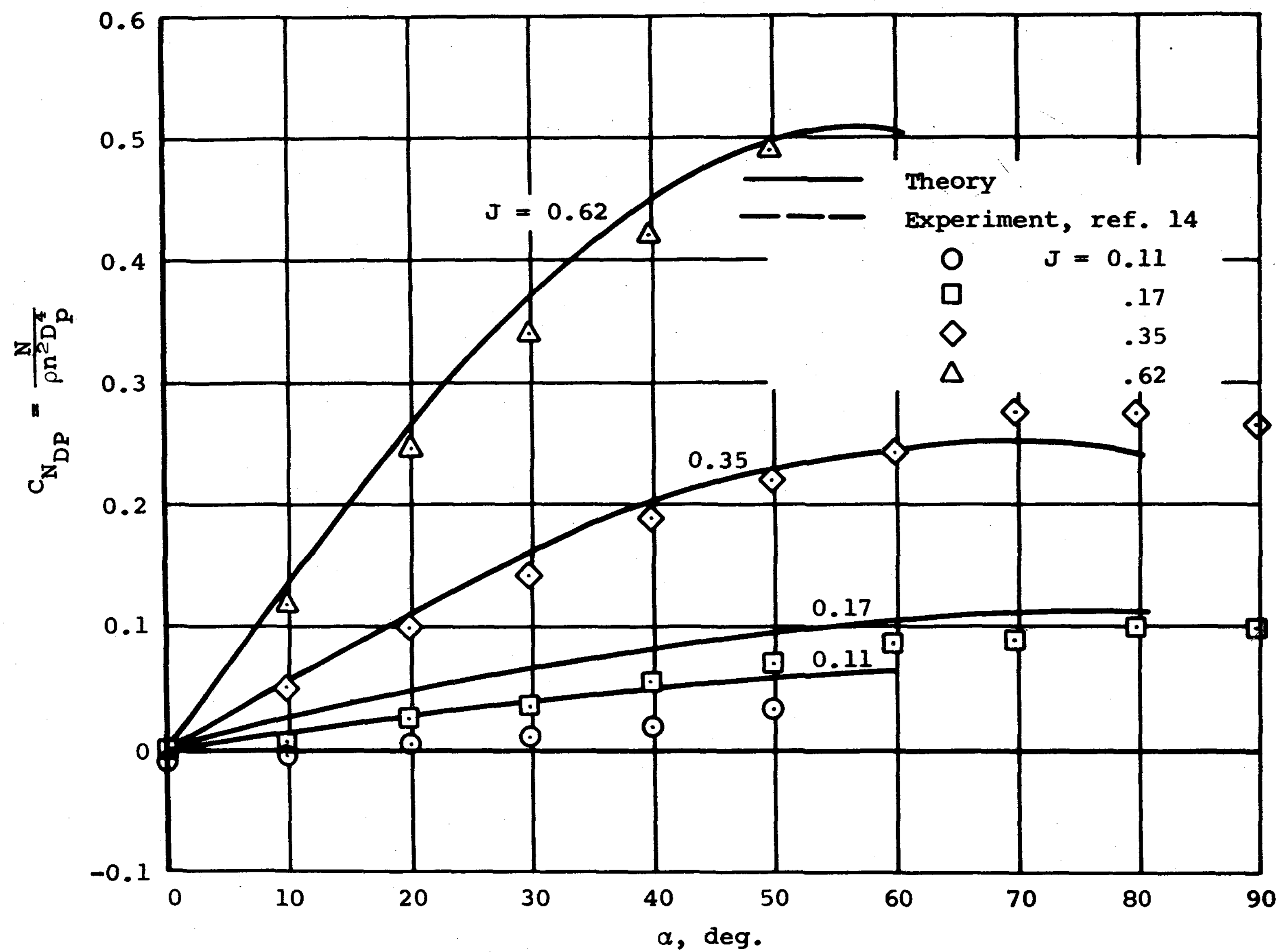


Figure 8.- Measured and predicted total thrust coefficient on the Bell X-22A ducted propeller in axial flow.



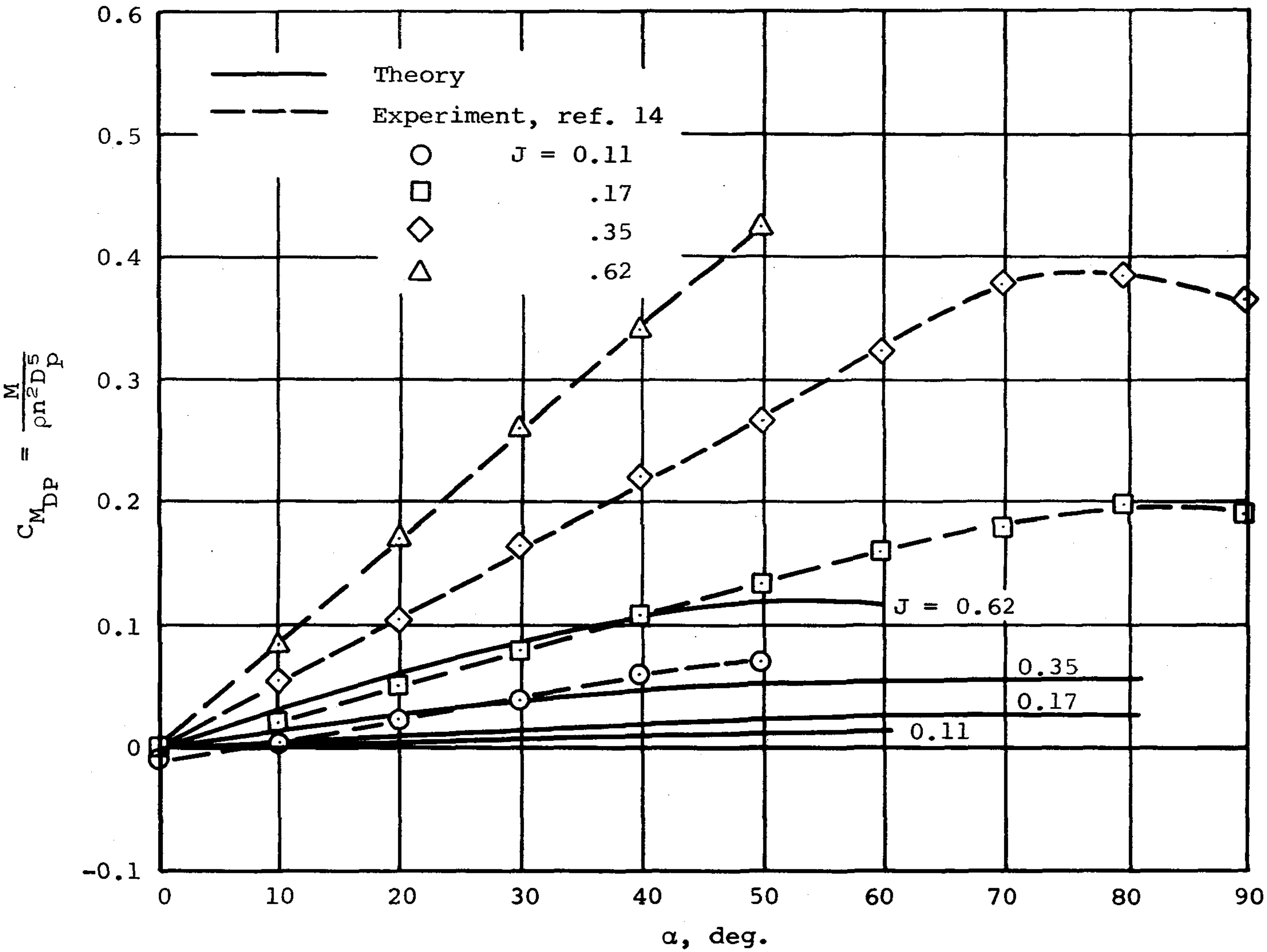
(a) Total thrust coefficient.

Figure 9.- Measured and predicted aerodynamic coefficients on the Doak VZ-4DA ducted fan, $\beta_{tip} = 15^\circ$.



(b) Normal force coefficient.

Figure 9.- Continued.



(c) Pitching moment coefficient.

Figure 9.- Concluded.

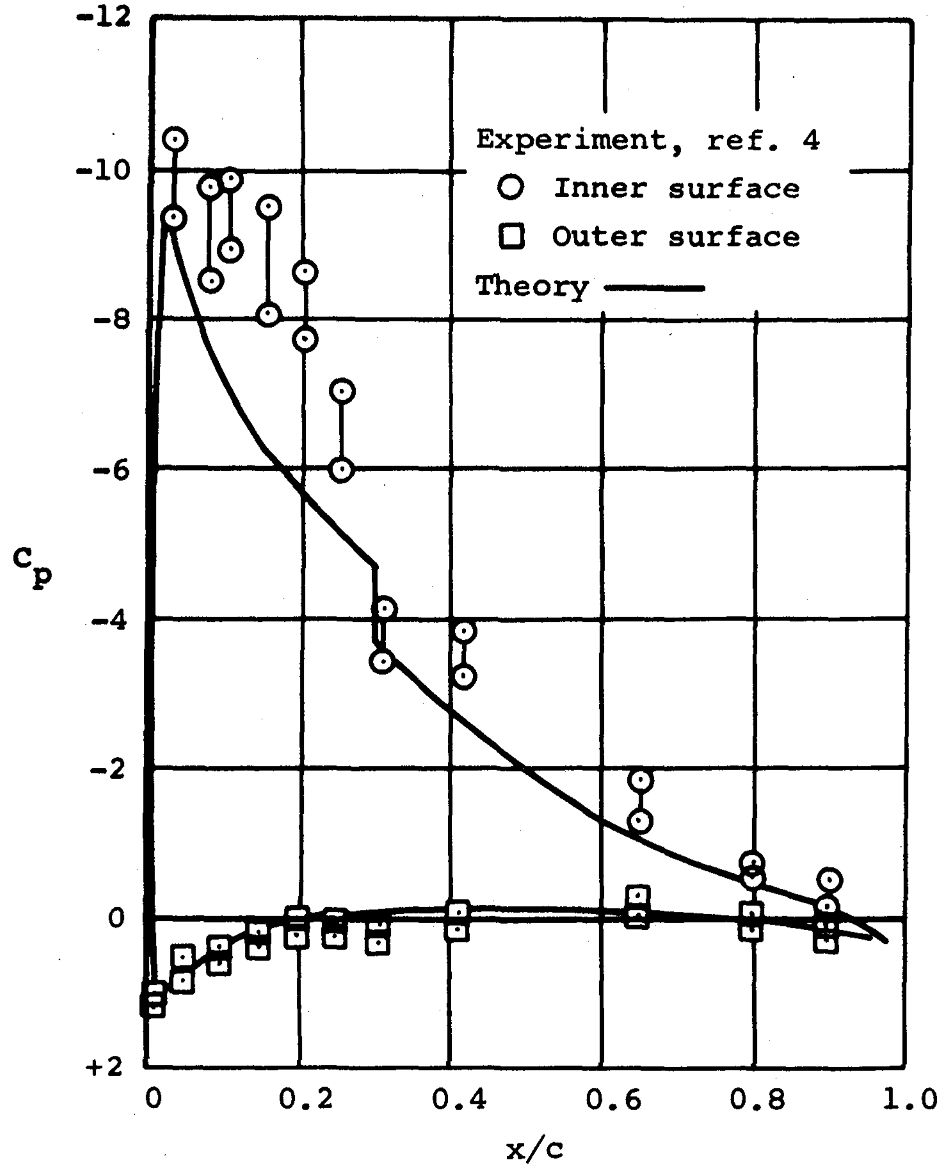
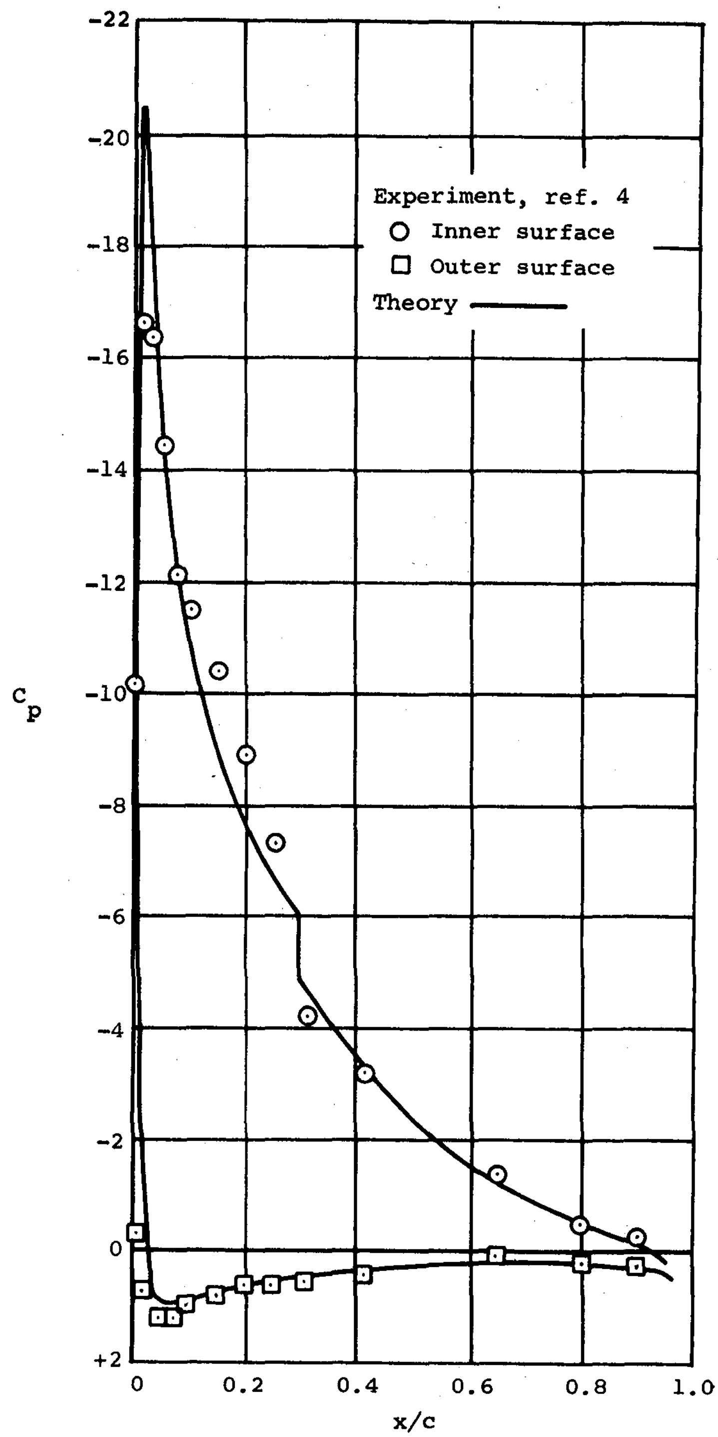
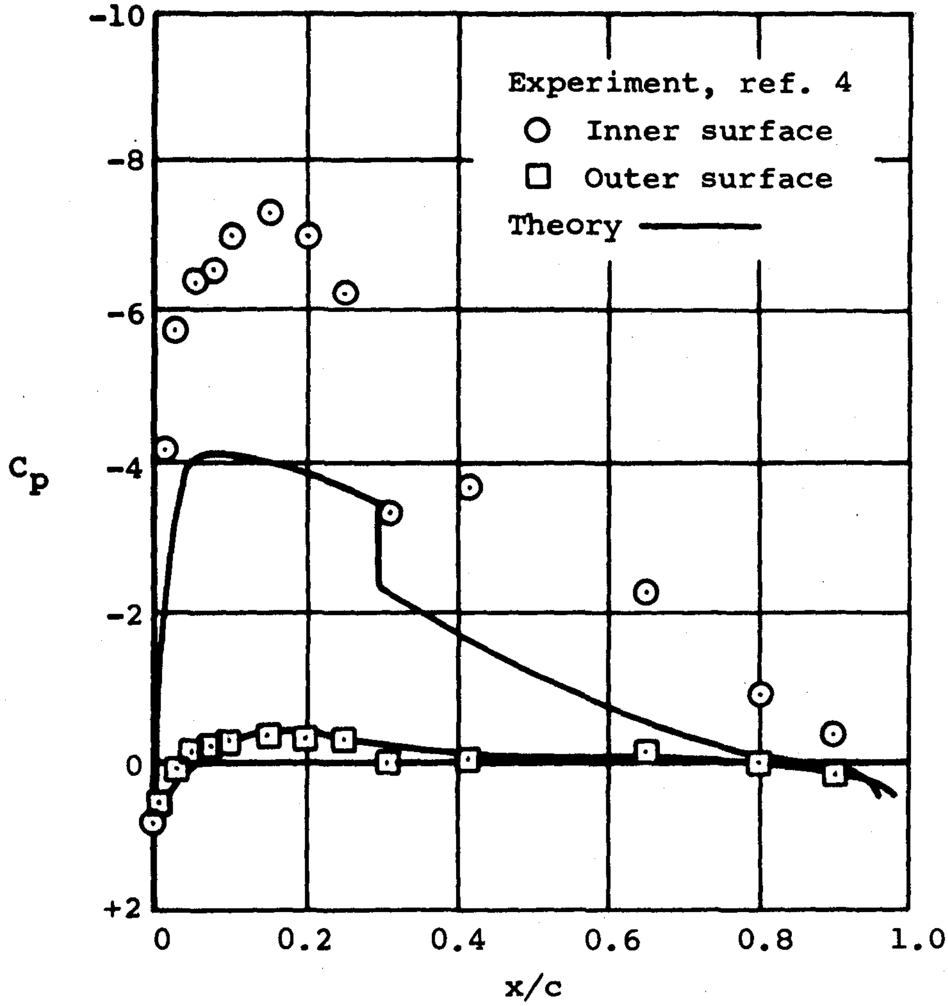


Figure 10.- Measured and predicted pressure distribution on the Doak duct at $\alpha = 0^{\circ}$, $J = 0.342$.



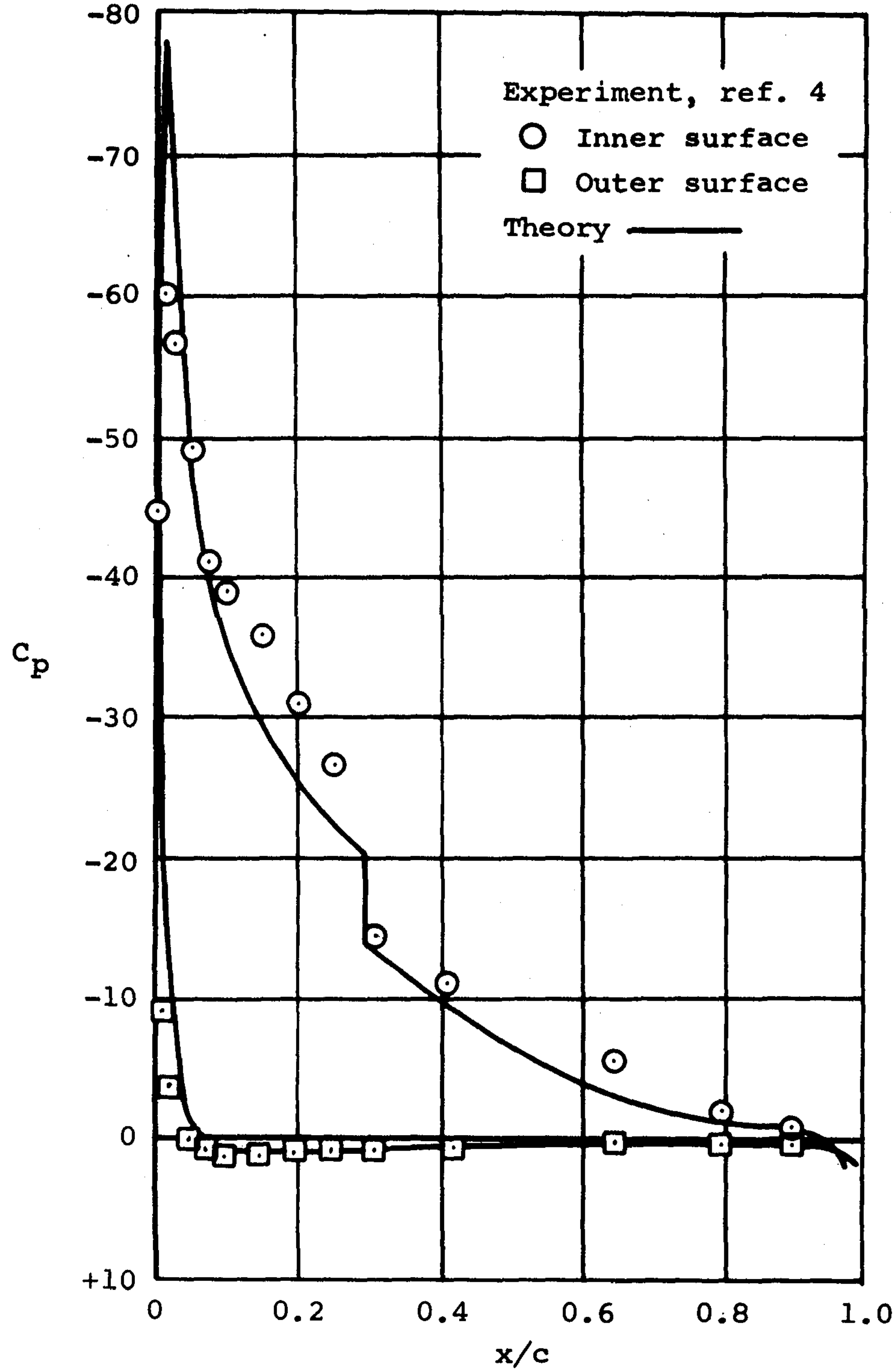
(a) $\phi = 0^{o}$.

Figure 11.- Measured and predicted pressure distribution on the Doak duct at $\alpha = 20^{o}$, $J = 0.342$.



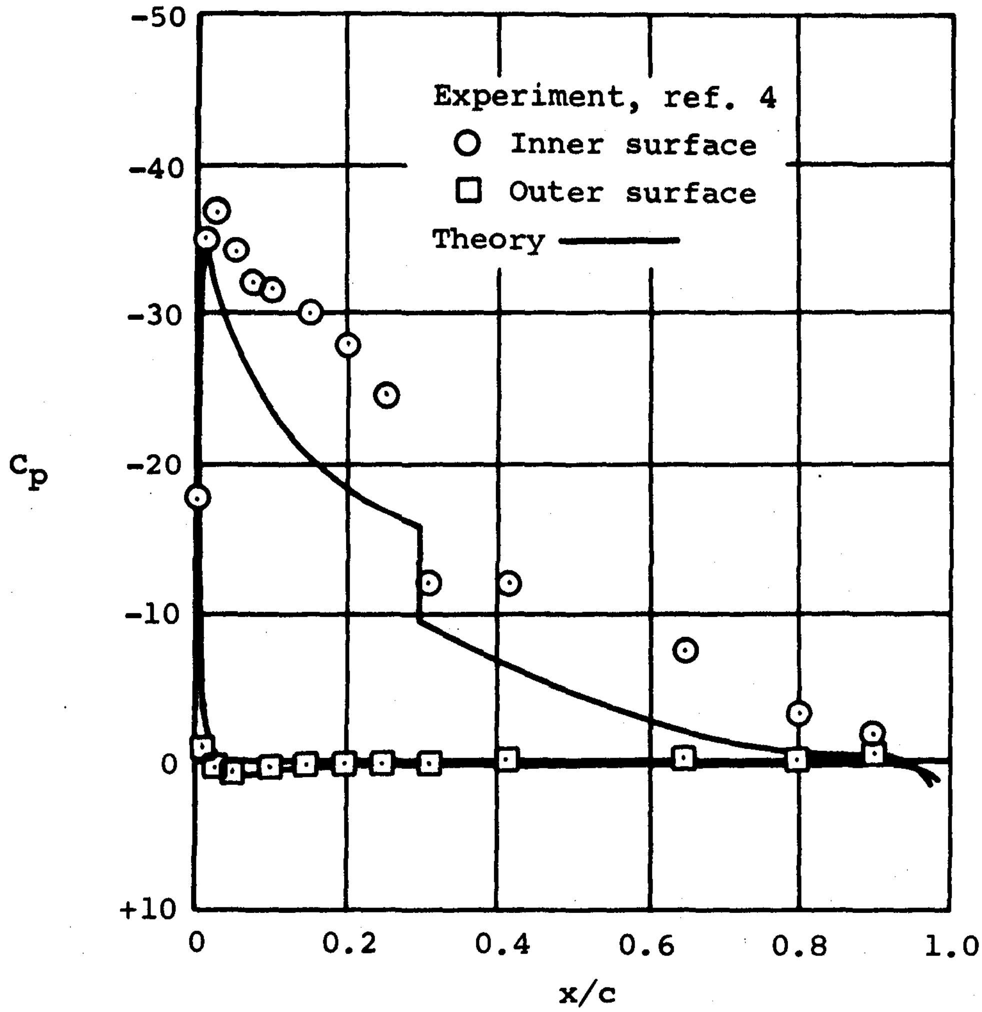
(b) $\phi = 180^\circ$.

Figure 11.- Concluded.



(a) $\phi = 0^{o}$.

Figure 12.- Measured and predicted pressure distribution on the Doak duct at $\alpha = 20^{o}$, J = 0.178.



(b) $\phi = 180^{\circ}$.

Figure 12.- Concluded.