# Assessment of Temperature–Dependent Regression Model Terms of a RUAG Six–Component Block–Type Balance

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**A metric called the percent contribution was applied to regression models of temperature–dependent calibration data of a RUAG six–component block–type balance in order to assess the influence of temperature–dependent regression model terms on the balance load prediction. Regression models were examined that are needed if either the Iterative or the Non–Iterative Method is used for the load prediction. Computed values of the percent contribution confirmed that the cross–product term defined by a primary load and the temperature difference is the most influential temperature–dependent term of the regression model of a primary output that the Iterative Method needs. Similarly, the analysis showed that the cross–product term defined by a primary output and the temperature difference is the most influential temperature–dependent term of the regression model of a primary load that the Non–Iterative Method needs. Computed results support conclusions that were reported in an earlier theoretical study. This study asserted that the cross–product term defined by a primary load or output and the temperature difference models the temperature–dependent shift of the gage sensitivity. The influence of other temperature–dependent terms used in the regression models of the calibration data of RUAG's balance was negligible. This observation may be explained by the fact that RUAG's block–type balances have highly linear characteristics. Overall, the percent contribution has proven itself to be a reliable and easy–to–implement metric that may also be used for the assessment of the influence of temperature–dependent regression model terms on the load prediction of a six–component strain–gage balance.**

## Nomenclature

| | |
|---|---|
| $a_{\circ,k},\ a_{1,k},\ \ldots$ | = regression coefficients of the $k$–th transformed output (*Iterative Method*) |
| $b_{\circ,k},\ b_{1,k},\ \ldots$ | = regression coefficients of the $k$–th load component (*Non–Iterative Method*) |
| $k$ | = index of either a load component or a transformed output |
| $\mathbf{F}$ | = load vector |
| $F_1,\ \ldots,\ F_6$ | = generic names of the individual components of load vector $\mathbf{F}$ |
| $F_x$ | = axial force of a strain–gage balance |
| $F_x{}^{\star}$ | = maximum applied axial force during calibration –or– axial force capacity |
| $F_y$ | = side force of a strain–gage balance |
| $F_y{}^{\star}$ | = maximum applied side force during calibration –or– side force capacity |
| $F_z$ | = normal force of a strain–gage balance |
| $F_z{}^{\star}$ | = maximum applied normal force during calibration –or– normal force capacity |
| $M_x$ | = rolling moment of a strain–gage balance |
| $M_x{}^{\star}$ | = maximum applied rolling moment during calibration –or– rolling moment capacity |

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$M_y$ = pitching moment of a strain–gage balance
$M_y{}^\star$ = maximum applied pitching moment during calibration –or– pitching moment capacity
$M_z$ = yawing moment of a strain–gage balance
$M_z{}^\star$ = maximum applied yawing moment during calibration –or– yawing moment capacity
$P$ = percent contribution of the regression model term of an output (*Iterative Method*)
$Q$ = percent contribution of the regression model term of a load (*Non–Iterative Method*)
$U_1, \ldots, U_7$ = individual electrical outputs of a RUAG six–component block–type balance
$\mathbf{W}$ = output vector
$W_1$ = transformed output of a RUAG block–type balance that is proportional to $F_x$
$W_1{}^\star$ = maximum output at load capacity of first transformed output of a RUAG balance
$W_2$ = transformed output of a RUAG block–type balance that is proportional to $F_y$
$W_2{}^\star$ = maximum output at load capacity of second transformed output of a RUAG balance
$W_3$ = transformed output of a RUAG block–type balance that is proportional to $F_z$
$W_3{}^\star$ = maximum output at load capacity of third transformed output of a RUAG balance
$W_4$ = transformed output of a RUAG block–type balance that is proportional to $M_x$
$W_4{}^\star$ = maximum output at load capacity of fourth transformed output of a RUAG balance
$W_5$ = transformed output of a RUAG block–type balance that is proportional to $M_y$
$W_5{}^\star$ = maximum output at load capacity of fifth transformed output of a RUAG balance
$W_6$ = transformed output of a RUAG block–type balance that is proportional to $M_z$
$W_6{}^\star$ = maximum output at load capacity of sixth transformed output of a RUAG balance

$\Delta T$ = temperature difference
$\Delta T^\star$ = capacity of the temperature difference
$\eta$ = index of a regression coefficient
$\xi(F_k)$ = term of the regression model of the fitted load component $F_k$
$\xi(W_k)$ = term of the regression model of the fitted transformed output $W_k$

## I. Introduction

Temperature–dependent calibration data of RUAG Aviation's 788–6A six–component block–type wind tunnel balance was recently analyzed using a new approach. This approach predicted balance loads by using (i) the measured electrical outputs of the balance and (ii) the temperature difference as input (Ref. [1]). The analysis confirmed that the most influential temperature–dependent term in the regression model of the calibration data is the cross–product term that models the shift of the gage sensitivity (see, e.g., Ref. [2] for a detailed discussion of this phenomenon).

In general, the required temperature–dependent cross–product term is a function of the temperature difference and a primary load or a primary output of the balance depending on the chosen load prediction method. For clarity, precise definitions of primary load, primary output, and prime sensitivity are given in the box below. Let us assume, for example, that an analyst chooses the *Iterative Method* for the balance

**Primary Load – Primary Output – Prime Sensitivity**

A primary load of a strain–gage balance is defined to be a load component that is responsible for more than 90 % of the electrical signal of the related primary output whenever it is exclusively applied to the balance. For example, the axial force is a typical primary load of a six–component balance. The axial gage output is the related primary output because more than 90 % of its signal is caused by the axial force whenever it is exclusively applied to the balance. The prime sensitivity of a balance gage is the first derivative of a primary output with respect to the related primary load.

load prediction (see, e.g., Refs. [3], [4] for details about this load prediction approach). Then, the temperature–dependent cross–product term equals the product of the primary load of the fitted output with the temperature difference. Alternatively, an analyst may choose the *Non–Iterative Method* for the balance load prediction (see Ref. [4] for details about the *Non–Iterative Method*). Then, the temperature–

dependent cross–product term equals the product of the primary output of the fitted load component with the temperature difference.

RUAG's six–component block–type balances are designed to be highly linear in their behavior. This characteristic may explain the fact that only six cross–product terms, i.e., one term for each primary output or load component, out of the forty–two possible temperature–dependent terms were needed to develop temperature–dependent load prediction equations for the RUAG 788–6A balance. The forty–two possible terms consist of one pure temperature term plus six temperature–dependent cross–product terms for each one of the six primary load components or primary outputs of the balance. Other types of six–component balances may need more than six temperature–dependent terms for an accurate load prediction. Therefore, the authors concluded that is would be useful to have a reliable and easy–to–implement metric available so that the influence of different temperature–dependent regression model terms on the balance load prediction can objectively be assessed.

The so–called "percent contribution" is often used in the aerospace testing community to evaluate the influence of a regression model term of balance calibration data whenever the *Iterative Method* is used for the balance load prediction (a description of this metric is given in the appendix of Ref. [5]). Therefore, the authors decided to modify the original definition of the percent contribution so that it would work with both the *Iterative* and the *Non–Iterative Method*. Afterwards, they applied the metric to the recently analyzed temperature–dependent calibration data of RUAG's 788–6A balance in order to investigate if the metric would be useful for the assessment of temperature–dependent regression model terms.

Basic characteristics of the calibration data of RUAG's 788–6A balance are reviewed in the next section. Afterwards, results of the application of the percent contribution to the regression models of its calibration data using both the *Iterative* and *Non–Iterative Method* are discussed in detail.

## II. Balance Description and Temperature Calibration

Block–type balances have been under development for many years at RUAG's Aerodynamics Department (see Refs. [6] and [7]). These balances may be used as internal balances when installed close to the model's aerodynamic center of pressure. Similarly, they may be used as external balances in vehicle aerodynamic testing when mounted on a turntable below the test section floor. Figure 1 below shows RUAG's family of block–type balances. The family currently consists of six balances that are scaled to meet different

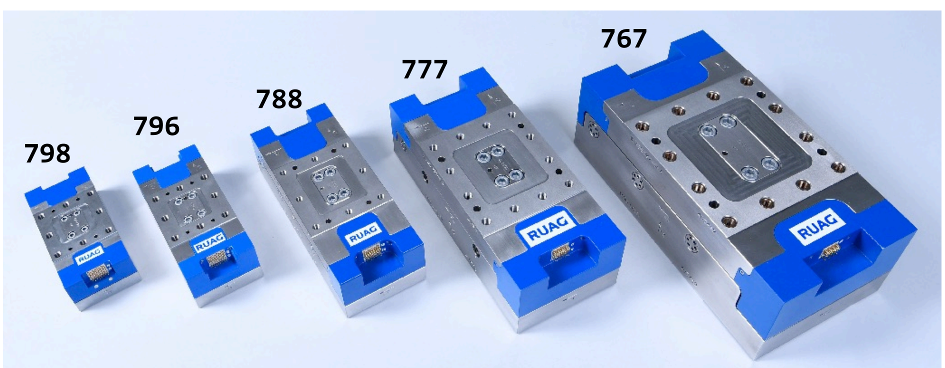


**Fig. 1** Family of RUAG's six–component block–type balances (Models 798, 796, 788, 777, 767).

applications (only five of the six balances are shown in Fig. 1). Table 1 below lists, for example, the "design loads" of the RUAG 788–6A balance. These upper load limits were defined assuming that all loads act simultaneously on the balance.

**Table 1:** Design loads† of RUAG's 788–6A block–type balance.

| $F_x,\ N$ | $F_y,\ N$ | $F_z,\ N$ | $M_x,\ Nm$ | $M_y,\ Nm$ | $M_z,\ Nm$ |
|---|---|---|---|---|---|
| 4 000 | 600 | 8 000 | 300 | 1 100 | 1 000 |

†Design load ≡ maximum load assuming that all loads act simultaneously on the balance.

Similarly, Table 2 below lists the "limit loads" of the RUAG 788–6A balance. These alternate upper load limits are defined assuming that only a single load acts on the balance at a time. The balance itself has

**Table 2:** Limit loads† of RUAG's 788–6A block–type balance.

| $F_x,\ N$ | $F_y,\ N$ | $F_z,\ N$ | $M_x,\ Nm$ | $M_y,\ Nm$ | $M_z,\ Nm$ |
|---|---|---|---|---|---|
| 4 000 | 10 000 | 25 000 | 1 000 | 3 000 | 1 100 |

†Limit load ≡ maximum load assuming that a single load acts on the balance.

a non–metric base plate with seven trapezoidal beams that are connected by joint rods to their counterparts on the metric side. Each beam is instrumented with a strain–gage bridge that experiences an electrical signal change whenever the beam elastically deforms.

RUAG's block–type balances measure six load components, i.e., $F_x$, $F_y$, $F_z$, $M_x$, $M_y$, $M_z$, that are described in "direct–read format." They are predicted using seven electrical output measurements, i.e., $U_1$, $U_2$, ..., $U_7$, as input. These seven outputs can be transformed to a set of six independent outputs, i.e., $W_1$, $W_2$, ..., $W_6$, so that (i) approximate linear relationships between one load component and one transformed output are established and (ii) the mathematical analysis of the balance data does not lead to an overdetermined linear system of equations. The output transformations for a RUAG block–type balance can be summarized as follows:

$$W_1 = +U_7 \iff \textit{primary load} \equiv F_x \tag{1a}$$

$$W_2 = +U_5 + U_6 \iff \textit{primary load} \equiv F_y \tag{1b}$$

$$W_3 = +U_1 + U_2 + U_3 + U_4 \iff \textit{primary load} \equiv F_z \tag{1c}$$

$$W_4 = +U_1 + U_2 - U_3 - U_4 \iff \textit{primary load} \equiv M_x \tag{1d}$$

$$W_5 = -U_1 + U_2 - U_3 + U_4 \iff \textit{primary load} \equiv M_y \tag{1e}$$

$$W_6 = +U_5 - U_6 \iff \textit{primary load} \equiv M_z \tag{1f}$$

The use of the six output transformations above allows for the description of both loads and outputs of the calibration data of a RUAG block–type balance in "direct–read format" as each load component, i.e., each primary load, becomes directly related to a single transformed output, i.e., a single primary output. This characteristic makes it possible to define the percent contribution of a regression model term of calibration data from a RUAG block–type balance if either the *Iterative Method* or the *Non–Iterative Method* is used for the balance load prediction.

Recently, RUAG performed a calibration of its 788–6A balance at two temperatures to better characterize the physical behavior of the balance at different temperatures. Details of this calibration can be found in Ref. [1]. The analysis of the temperature–dependent calibration data of the RUAG 788–6A balance was done with NASA's BALFIT software tool (Ref. [8]). The calibration data supported a total of twenty–seven terms for each one of the six regression models of the outputs (*Iterative Method*) and for each one of the six regression models of the loads (*Non–Iterative Method*). The calculation of the percent contribution of the temperature–dependent terms of these twelve regression models is discussed in the next section.

## III. Percent Contribution of Temperature–Dependent Terms

### A. Definition of the Percent Contribution

It is possible to use the percent contribution for the assessment of the significance of temperature–dependent terms of regression models that were used for the analysis of the RUAG 788–6A balance calibration data. First, the definition of the percent contribution for the *Iterative Method* is discussed (see also the appendix of Ref. [5] for more details). This definition can be summarized as follows:

**Iterative Method – Percent Contribution of a Regression Model Term**

The percent contribution $P$ of a term $\xi(W_k)$ of the regression model of a balance output $W_k$ is a fraction that is multiplied by 100 %. The numerator of this fraction equals the product of the term's coefficient and the capacities of all variables, i.e, loads and/or the temperature difference, that define the term. The denominator of the fraction equals the product of the coefficient of the primary load of the fitted output $W_k$ and the related load capacity. The load capacity is defined as the maximum applied load of the component during calibration.

It is useful to express this verbal description of the percent contribution for the *Iterative Method* by using the following formula:

$$P\Big[\xi(W_k)\Big] \equiv \frac{Coefficient\Big[\xi(W_k)\Big] \times Capacity\Big[\xi(W_k)\Big] \times 100\ \%}{Coefficient\Big[Primary\ Load(W_k)\Big] \times Capacity\Big[Primary\ Load(W_k)\Big]} \tag{2}$$

$$where$$

$$W_k \equiv k-th\ output\ of\ the\ balance$$

$$\xi(W_k) \equiv term\ of\ the\ regression\ model\ of\ the\ fitted\ output\ W_k$$

The exact meaning of "capacity" still needs to be defined in this context so that the percent contribution can be computed. The authors define "capacity" to be the "maximum applied load" of a load component that acts on the balance during its calibration. In other words, "capacity" is simply a reference value that is needed to make the percent contribution a dimensionless quantity. It is not necessarily identical with the "design load" or the "limit load" of a load component that may be needed to monitor stresses of the balance during use in the wind tunnel.

The general definition of the percent contribution can also be extended to the *Non–Iterative Method*. Then, the definition of the percent contribution can be summarized as follows:

**Non–Iterative Method – Percent Contribution of a Regression Model Term**

The percent contribution $Q$ of a term $\xi(F_k)$ of the regression model of a balance load $F_k$ is a fraction that is mulitplied by 100 %. The numerator of this fraction equals the product of the term's coefficient and the capacities of all variables, i.e., outputs and/or the temperature difference, that define the term. The denominator of the fraction equals the product of the coefficient of the primary output of the fitted load $F_k$ and the related output capacity. The output capacity is defined in this context as the maximum output at load capacity.

Again, it is helpful to express this verbal description of the percent contribution for the *Non–Iterative Method* by the following formula:

$$Q\Big[\xi(F_k)\Big] \;\equiv\; \frac{Coefficient\,\Big[\xi(F_k)\Big] \;\times\; Capacity\,\Big[\xi(F_k)\Big] \;\times\; 100\;\%}{Coefficient\,\Big[Primary\,Output(F_k)\Big] \;\times\; Capacity\,\Big[Primary\,Output(F_k)\Big]} \tag{3}$$

$$where$$

$$F_k \;\equiv\; k-th\; load\; component\; of\; the\; balance$$

$$\xi(F_k) \;\equiv\; term\; of\; the\; regression\; model\; of\; the\; fitted\; load\; component\; F_k$$

It will be demonstrated in the next section how the percent contribution was computed for the temperature–dependent terms of the six regression models of the outputs that were used during the application of the *Iterative Method* to the calibration data of the RUAG 788–6A balance.

### B. Results for the Iterative Method

The *Iterative Method* first fits electrical outputs of a balance as a function of the loads and the temperature difference. Afterwards, an iteration equation is constructed from the result so that loads can be predicted from outputs and the temperature during a wind tunnel test. The percent contribution of the terms of the regression model of the outputs can be defined by using the temperature–dependent regression model that was used for the analysis of the calibration data of the RUAG 788–6A balance. BALFIT's analysis of the calibration data showed that twenty–seven regression model terms for each transformed output were supported (intercept, seven linear terms, six quadratic terms, and thirteen cross–product terms). The corresponding regression model of a transformed output has the following general form ...

$$\begin{aligned} W_k \;=\; a_{\circ,k} \;&+\; \underbrace{a_{1,k}\cdot F_x \;+\; a_{2,k}\cdot F_y \;+\; \ldots \;+\; a_{6,k}\cdot M_z \;+\; a_{7,k}\cdot \Delta T}_{linear\;terms} \\ &+\; \underbrace{a_{8,k}\cdot F_x^2 \;+\; a_{9,k}\cdot F_y^2 \;+\; \ldots \;+\; a_{13,k}\cdot M_z^2}_{quadratic\;terms} \\ &+\; \underbrace{a_{14,k}\cdot F_x\cdot M_z \;+\; a_{15,k}\cdot F_y\cdot M_x \;+\; \ldots \;+\; a_{20,k}\cdot M_x\cdot M_z}_{cross-product\;terms} \\ &+\; \underbrace{a_{21,k}\cdot F_x\cdot \Delta T \;+\; a_{22,k}\cdot F_y\cdot \Delta T \;+\; \ldots \;+\; a_{26,k}\cdot M_z\cdot \Delta T}_{temperature-dependent\;cross-product\;terms} \end{aligned} \tag{4a}$$

assuming that "$W_k$" is the $k$–th component of output vector $\mathbf{W}$. This vector is defined as follows:

$$\mathbf{W} \;=\; \begin{bmatrix} W_1 \\ \vdots \\ W_k \\ \vdots \\ W_6 \end{bmatrix} \tag{4b}$$

Table 3 below shows the subset of temperature–dependent regression model terms of the six transformed outputs that were supported by the load schedule of the balance calibration data (for more detail see Ref. [1]).

**Table 3:** Temperature–dependent regression model terms of the outputs of the calibration data.

| *Iterative Method* $\Longrightarrow$ List of Temperature–Dependent Terms for $W_1$, $W_2$, ..., $W_6$ |
| --- |
| $\Delta T$ , $(F_x\cdot\Delta T)$ , $(F_y\cdot\Delta T)$ , $(F_z\cdot\Delta T)$ , $(M_x\cdot\Delta T)$ , $(M_y\cdot\Delta T)$ , $(M_z\cdot\Delta T)$ |

Then, using the general definition of the percent contribution that is provided in Eq. (2) above and the coefficient nomenclature given in Eq. ($4a$), percent contributions of the temperature–dependent terms of the three primary outputs $W_1$, $W_2$, and $W_3$ of the RUAG 788–6A balance are defined as follows:

**Table 4a:** Percent contribution definition of the temperature terms of the gage outputs $W_1$ to $W_3$.

| $\xi(W_k)$ | $W_1$ (primary load $= F_x$) | $W_2$ (primary load $= F_y$) | $W_3$ (primary load $= F_z$) |
|---|---|---|---|
| $\Delta T$ | $\dfrac{a_{7,1} \cdot \Delta T^\star}{a_{1,1} \cdot F_x{}^\star} \cdot 100\%$ | $\dfrac{a_{7,2} \cdot \Delta T^\star}{a_{2,2} \cdot F_y{}^\star} \cdot 100\%$ | $\dfrac{a_{7,3} \cdot \Delta T^\star}{a_{3,3} \cdot F_z{}^\star} \cdot 100\%$ |
| $F_x \cdot \Delta T$ | $\dfrac{a_{21,1} \cdot F_x{}^\star \cdot \Delta T^\star}{a_{1,1} \cdot F_x{}^\star} \cdot 100\%$ | $\dfrac{a_{21,2} \cdot F_x{}^\star \cdot \Delta T^\star}{a_{2,2} \cdot F_y{}^\star} \cdot 100\%$ | $\dfrac{a_{21,3} \cdot F_x{}^\star \cdot \Delta T^\star}{a_{3,3} \cdot F_z{}^\star} \cdot 100\%$ |
| $F_y \cdot \Delta T$ | $\dfrac{a_{22,1} \cdot F_y{}^\star \cdot \Delta T^\star}{a_{1,1} \cdot F_x{}^\star} \cdot 100\%$ | $\dfrac{a_{22,2} \cdot F_y{}^\star \cdot \Delta T^\star}{a_{2,2} \cdot F_y{}^\star} \cdot 100\%$ | $\dfrac{a_{22,3} \cdot F_y{}^\star \cdot \Delta T^\star}{a_{3,3} \cdot F_z{}^\star} \cdot 100\%$ |
| $F_z \cdot \Delta T$ | $\dfrac{a_{23,1} \cdot F_z{}^\star \cdot \Delta T^\star}{a_{1,1} \cdot F_x{}^\star} \cdot 100\%$ | $\dfrac{a_{23,2} \cdot F_z{}^\star \cdot \Delta T^\star}{a_{2,2} \cdot F_y{}^\star} \cdot 100\%$ | $\dfrac{a_{23,3} \cdot F_z{}^\star \cdot \Delta T^\star}{a_{3,3} \cdot F_z{}^\star} \cdot 100\%$ |
| $M_x \cdot \Delta T$ | $\dfrac{a_{24,1} \cdot M_x{}^\star \cdot \Delta T^\star}{a_{1,1} \cdot F_x{}^\star} \cdot 100\%$ | $\dfrac{a_{24,2} \cdot M_x{}^\star \cdot \Delta T^\star}{a_{2,2} \cdot F_y{}^\star} \cdot 100\%$ | $\dfrac{a_{24,3} \cdot M_x{}^\star \cdot \Delta T^\star}{a_{3,3} \cdot F_z{}^\star} \cdot 100\%$ |
| $M_y \cdot \Delta T$ | $\dfrac{a_{25,1} \cdot M_y{}^\star \cdot \Delta T^\star}{a_{1,1} \cdot F_x{}^\star} \cdot 100\%$ | $\dfrac{a_{25,2} \cdot M_y{}^\star \cdot \Delta T^\star}{a_{2,2} \cdot F_y{}^\star} \cdot 100\%$ | $\dfrac{a_{25,3} \cdot M_y{}^\star \cdot \Delta T^\star}{a_{3,3} \cdot F_z{}^\star} \cdot 100\%$ |
| $M_z \cdot \Delta T$ | $\dfrac{a_{26,1} \cdot M_z{}^\star \cdot \Delta T^\star}{a_{1,1} \cdot F_x{}^\star} \cdot 100\%$ | $\dfrac{a_{26,2} \cdot M_z{}^\star \cdot \Delta T^\star}{a_{2,2} \cdot F_y{}^\star} \cdot 100\%$ | $\dfrac{a_{26,3} \cdot M_z{}^\star \cdot \Delta T^\star}{a_{3,3} \cdot F_z{}^\star} \cdot 100\%$ |

Similarly, percent contributions of the temperature–dependent terms of the three primary outputs $W_4$, $W_5$, and $W_6$ are defined as follows:

**Table 4b:** Percent contribution definition of the temperature terms of the gage outputs $W_4$ to $W_6$.

| $\xi(W_k)$ | $W_4$ (primary load $= M_x$) | $W_5$ (primary load $= M_y$) | $W_6$ (primary load $= M_z$) |
|---|---|---|---|
| $\Delta T$ | $\dfrac{a_{7,4} \cdot \Delta T^\star}{a_{4,4} \cdot M_x{}^\star} \cdot 100\%$ | $\dfrac{a_{7,5} \cdot \Delta T^\star}{a_{5,5} \cdot M_y{}^\star} \cdot 100\%$ | $\dfrac{a_{7,6} \cdot \Delta T^\star}{a_{6,6} \cdot M_z{}^\star} \cdot 100\%$ |
| $F_x \cdot \Delta T$ | $\dfrac{a_{21,4} \cdot F_x{}^\star \cdot \Delta T^\star}{a_{4,4} \cdot M_x{}^\star} \cdot 100\%$ | $\dfrac{a_{21,5} \cdot F_x{}^\star \cdot \Delta T^\star}{a_{5,5} \cdot M_y{}^\star} \cdot 100\%$ | $\dfrac{a_{21,6} \cdot F_x{}^\star \cdot \Delta T^\star}{a_{6,6} \cdot M_z{}^\star} \cdot 100\%$ |
| $F_y \cdot \Delta T$ | $\dfrac{a_{22,4} \cdot F_y{}^\star \cdot \Delta T^\star}{a_{4,4} \cdot M_x{}^\star} \cdot 100\%$ | $\dfrac{a_{22,5} \cdot F_y{}^\star \cdot \Delta T^\star}{a_{5,5} \cdot M_y{}^\star} \cdot 100\%$ | $\dfrac{a_{22,6} \cdot F_y{}^\star \cdot \Delta T^\star}{a_{6,6} \cdot M_z{}^\star} \cdot 100\%$ |
| $F_z \cdot \Delta T$ | $\dfrac{a_{23,4} \cdot F_z{}^\star \cdot \Delta T^\star}{a_{4,4} \cdot M_x{}^\star} \cdot 100\%$ | $\dfrac{a_{23,5} \cdot F_z{}^\star \cdot \Delta T^\star}{a_{5,5} \cdot M_y{}^\star} \cdot 100\%$ | $\dfrac{a_{23,6} \cdot F_z{}^\star \cdot \Delta T^\star}{a_{6,6} \cdot M_z{}^\star} \cdot 100\%$ |
| $M_x \cdot \Delta T$ | $\dfrac{a_{24,4} \cdot M_x{}^\star \cdot \Delta T^\star}{a_{4,4} \cdot M_x{}^\star} \cdot 100\%$ | $\dfrac{a_{24,5} \cdot M_x{}^\star \cdot \Delta T^\star}{a_{5,5} \cdot M_y{}^\star} \cdot 100\%$ | $\dfrac{a_{24,6} \cdot M_x{}^\star \cdot \Delta T^\star}{a_{6,6} \cdot M_z{}^\star} \cdot 100\%$ |
| $M_y \cdot \Delta T$ | $\dfrac{a_{25,4} \cdot M_y{}^\star \cdot \Delta T^\star}{a_{4,4} \cdot M_x{}^\star} \cdot 100\%$ | $\dfrac{a_{25,5} \cdot M_y{}^\star \cdot \Delta T^\star}{a_{5,5} \cdot M_y{}^\star} \cdot 100\%$ | $\dfrac{a_{25,6} \cdot M_y{}^\star \cdot \Delta T^\star}{a_{6,6} \cdot M_z{}^\star} \cdot 100\%$ |
| $M_z \cdot \Delta T$ | $\dfrac{a_{26,4} \cdot M_z{}^\star \cdot \Delta T^\star}{a_{4,4} \cdot M_x{}^\star} \cdot 100\%$ | $\dfrac{a_{26,5} \cdot M_z{}^\star \cdot \Delta T^\star}{a_{5,5} \cdot M_y{}^\star} \cdot 100\%$ | $\dfrac{a_{26,6} \cdot M_z{}^\star \cdot \Delta T^\star}{a_{6,6} \cdot M_z{}^\star} \cdot 100\%$ |

The regression coefficients listed in Tables 4a and 4b above were determined during the regression analysis that is described in Ref. [1]. Therefore, only the capacities of the seven independent variables are missing in order to compute the percent contribution of the terms. They are listed in Table 5a below.

**Table 5a:** Capacities of the temperature difference and the load components.†

| $\Delta T^\star,\ degK$ | $F_x{}^\star,\ N$ | $F_y{}^\star,\ N$ | $F_z{}^\star,\ N$ | $M_x{}^\star,\ Nm$ | $M_y{}^\star,\ Nm$ | $M_z{}^\star,\ Nm$ |
|---|---|---|---|---|---|---|
| 21 | 4300 | 5300 | 5300 | 500 | 1200 | 800 |

†capacity of a load component ≡ maximum applied load of the component that acted during the calibration.

Finally, the percent contributions of the temperature–dependent terms were computed. Table 5b below lists results of this calculation. It is observed that the six temperature–dependent cross–product terms associated with the primary load component of each output, i.e., the terms that model the sensitivity shift of a gage, are the dominant temperature–dependent terms. Corresponding values are printed in boldface in Table 5b above. All other temperature–dependent terms make negligible contributions.

**Table 5b:** *Iterative Method* – Percent contribution of the temperature–dependent terms of the regression models of the six transformed outputs of the RUAG 788–6A block–type balance.

| $\xi(W_k)$ | $W_1$ | $W_2$ | $W_3$ | $W_4$ | $W_5$ | $W_6$ |
|---|---|---|---|---|---|---|
| $\Delta T$ | +0.01 % | −0.01 % | ±0.00 % | +0.02 % | +0.01 % | −0.02 % |
| $F_x \cdot \Delta T$ | **+0.52** % | −0.04 % | −0.04 % | ± 0.00 % | −0.02 % | −0.04 % |
| $F_y \cdot \Delta T$ | +0.08 % | **+0.54** % | +0.07 % | +0.04 % | +0.01 % | +0.01 % |
| $F_z \cdot \Delta T$ | +0.16 % | +0.02 % | **+0.54** % | +0.11 % | −0.03 % | −0.01 % |
| $M_x \cdot \Delta T$ | +0.03 % | −0.03 % | −0.02 % | **+0.58** % | −0.12 % | −0.08 % |
| $M_y \cdot \Delta T$ | −0.03 % | +0.02 % | −0.04 % | +0.01 % | **+0.60** % | −0.02 % |
| $M_z \cdot \Delta T$ | ±0.00 % | +0.01 % | +0.02 % | +0.04 % | −0.02 % | **+0.62** % |

The percent contributions of the temperature–dependent terms of the six regression models of the loads are discussed in the next section of the paper. Those regression models were obtained by applying the *Non–Iterative Method* to the calibration data.

## C. Results for the Non–Iterative Method

The *Non–Iterative Method* was also used for the analysis of the temperature–dependent calibration data of the RUAG 788–6A balance. In that case, the six load components were directly fitted as a function of the transformed outputs. Afterwards, the percent contribution of individual terms of the chosen regression models of the loads was computed. Again, as it was the case for the regression models of the electrical outputs, the temperature–dependent calibration data supported a total of twenty–seven terms for each load component (intercept, seven linear terms, six quadratic terms, and thirteen cross–product terms). The resulting regression model for each one of the six load components can be defined as follows …

$$
\begin{aligned}
F_k \;=\; b_{\circ,k} \;&+\; \underbrace{b_{1,k} \cdot W_1 \;+\; \ldots \;+\; b_{6,k} \cdot W_6 \;+\; b_{7,k} \cdot \Delta T}_{linear\ terms} \\
&+\; \underbrace{b_{8,k} \cdot W_1^2 \;+\; \ldots \;+\; b_{13,k} \cdot W_6^2}_{quadratic\ terms} \\
&+\; \underbrace{b_{14,k} \cdot W_1 \cdot W_6 \;+\; b_{15,k} \cdot W_2 \cdot W_4 \;+\; \ldots \;+\; b_{20,k} \cdot W_4 \cdot W_6}_{cross-product\ terms} \\
&+\; \underbrace{b_{21,k} \cdot W_1 \cdot \Delta T \;+\; b_{22,k} \cdot W_2 \cdot \Delta T \;+\; \ldots \;+\; b_{26,k} \cdot W_6 \cdot \Delta T}_{temperature-dependent\ cross-product\ terms}
\end{aligned}
\tag{5a}
$$

assuming that "$F_k$" is the $k$–th component of a load vector $\mathbf{F}$. This vector is defined as follows:

$$\mathbf{F} \quad = \quad \begin{bmatrix} F_1 \\ \vdots \\ F_k \\ \vdots \\ F_6 \end{bmatrix} \quad \equiv \quad \begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} \tag{5b}$$

Table 6 below shows the subset of temperature–dependent regression model terms of the six loads that were supported by the load schedule of the balance calibration data (see also Ref. [1]).

**Table 6:** Temperature–dependent regression model terms of the loads of the calibration data.

| *Non–Iterative Method* $\Longrightarrow$ List of Temperature–Dependent Terms for $F_x$, $F_y$, ..., $M_z$ |
|---|
| $\Delta T$ , $(W_1 \cdot \Delta T)$ , $(W_2 \cdot \Delta T)$ , $(W_3 \cdot \Delta T)$ , $(W_4 \cdot \Delta T)$ , $(W_5 \cdot \Delta T)$ , $(W_6 \cdot \Delta T)$ |

Now, after combining the general definition of the percent contribution given in Eq. (3) above with the coefficient nomenclature given in Eq. (5$a$), the percent contributions of the temperature–dependent terms of the three forces $F_x$, $F_y$, and $F_z$ can be defined as follows:

**Table 7a:** Percent contribution definition of the temperature terms of the forces $F_x$, $F_y$, and $F_z$.

| $\xi(F_k)$ | $F_1 \equiv F_x$ (primary output $= W_1$) | $F_2 \equiv F_y$ (primary output $= W_2$) | $F_3 \equiv F_z$ (primary output $= W_3$) |
|---|---|---|---|
| $\Delta T$ | $\frac{b_{7,1} \cdot \Delta T^\star}{b_{1,1} \cdot W_1^\star} \cdot 100\%$ | $\frac{b_{7,2} \cdot \Delta T^\star}{b_{2,2} \cdot W_2^\star} \cdot 100\%$ | $\frac{b_{7,3} \cdot \Delta T^\star}{b_{3,3} \cdot W_3^\star} \cdot 100\%$ |
| $W_1 \cdot \Delta T$ | $\frac{b_{21,1} \cdot W_1^\star \cdot \Delta T^\star}{b_{1,1} \cdot W_1^\star} \cdot 100\%$ | $\frac{b_{21,2} \cdot W_1^\star \cdot \Delta T^\star}{b_{2,2} \cdot W_2^\star} \cdot 100\%$ | $\frac{b_{21,3} \cdot W_1^\star \cdot \Delta T^\star}{b_{3,3} \cdot W_3^\star} \cdot 100\%$ |
| $W_2 \cdot \Delta T$ | $\frac{b_{22,1} \cdot W_2^\star \cdot \Delta T^\star}{b_{1,1} \cdot W_1^\star} \cdot 100\%$ | $\frac{b_{22,2} \cdot W_2^\star \cdot \Delta T^\star}{b_{2,2} \cdot W_2^\star} \cdot 100\%$ | $\frac{b_{22,3} \cdot W_2^\star \cdot \Delta T^\star}{b_{3,3} \cdot W_3^\star} \cdot 100\%$ |
| $W_3 \cdot \Delta T$ | $\frac{b_{23,1} \cdot W_3^\star \cdot \Delta T^\star}{b_{1,1} \cdot W_1^\star} \cdot 100\%$ | $\frac{b_{23,2} \cdot W_3^\star \cdot \Delta T^\star}{b_{2,2} \cdot W_2^\star} \cdot 100\%$ | $\frac{b_{23,3} \cdot W_3^\star \cdot \Delta T^\star}{b_{3,3} \cdot W_3^\star} \cdot 100\%$ |
| $W_4 \cdot \Delta T$ | $\frac{b_{24,1} \cdot W_4^\star \cdot \Delta T^\star}{b_{1,1} \cdot W_1^\star} \cdot 100\%$ | $\frac{b_{24,2} \cdot W_4^\star \cdot \Delta T^\star}{b_{2,2} \cdot W_2^\star} \cdot 100\%$ | $\frac{b_{24,3} \cdot W_4^\star \cdot \Delta T^\star}{b_{3,3} \cdot W_3^\star} \cdot 100\%$ |
| $W_5 \cdot \Delta T$ | $\frac{b_{25,1} \cdot W_5^\star \cdot \Delta T^\star}{b_{1,1} \cdot W_1^\star} \cdot 100\%$ | $\frac{b_{25,2} \cdot W_5^\star \cdot \Delta T^\star}{b_{2,2} \cdot W_2^\star} \cdot 100\%$ | $\frac{b_{25,3} \cdot W_5^\star \cdot \Delta T^\star}{b_{3,3} \cdot W_3^\star} \cdot 100\%$ |
| $W_6 \cdot \Delta T$ | $\frac{b_{26,1} \cdot W_6^\star \cdot \Delta T^\star}{b_{1,1} \cdot W_1^\star} \cdot 100\%$ | $\frac{b_{26,2} \cdot W_6^\star \cdot \Delta T^\star}{b_{2,2} \cdot W_2^\star} \cdot 100\%$ | $\frac{b_{26,3} \cdot W_6^\star \cdot \Delta T^\star}{b_{3,3} \cdot W_3^\star} \cdot 100\%$ |

Similarly, the percent contributions of the temperature–dependent terms of the regression models of the three moments $M_x$, $M_y$, and $M_z$ are defined as follows:

**Table 7b:** Percent contribution definition of the temperature terms of the moments $M_x$, $M_y$, and $M_z$.

| $\xi(F_k)$ | $F_4 \equiv M_x$ (primary output $= W_4$) | $F_5 \equiv M_y$ (primary output $= W_5$) | $F_6 \equiv M_z$ (primary output $= W_6$) |
|---|---|---|---|
| $\Delta T$ | $\frac{b_{7,4} \cdot \Delta T^\star}{b_{4,4} \cdot W_4{}^\star} \cdot 100\%$ | $\frac{b_{7,5} \cdot \Delta T^\star}{b_{5,5} \cdot W_5{}^\star} \cdot 100\%$ | $\frac{b_{7,6} \cdot \Delta T^\star}{b_{6,6} \cdot W_6{}^\star} \cdot 100\%$ |
| $W_1 \cdot \Delta T$ | $\frac{b_{21,4} \cdot W_1{}^\star \cdot \Delta T^\star}{b_{4,4} \cdot W_4{}^\star} \cdot 100\%$ | $\frac{b_{21,5} \cdot W_1{}^\star \cdot \Delta T^\star}{b_{5,5} \cdot W_5{}^\star} \cdot 100\%$ | $\frac{b_{21,6} \cdot W_1{}^\star \cdot \Delta T^\star}{b_{6,6} \cdot W_6{}^\star} \cdot 100\%$ |
| $W_2 \cdot \Delta T$ | $\frac{b_{22,4} \cdot W_2{}^\star \cdot \Delta T^\star}{b_{4,4} \cdot W_4{}^\star} \cdot 100\%$ | $\frac{b_{22,5} \cdot W_2{}^\star \cdot \Delta T^\star}{b_{5,5} \cdot W_5{}^\star} \cdot 100\%$ | $\frac{b_{22,6} \cdot W_2{}^\star \cdot \Delta T^\star}{b_{6,6} \cdot W_6{}^\star} \cdot 100\%$ |
| $W_3 \cdot \Delta T$ | $\frac{b_{23,4} \cdot W_3{}^\star \cdot \Delta T^\star}{b_{4,4} \cdot W_4{}^\star} \cdot 100\%$ | $\frac{b_{23,5} \cdot W_3{}^\star \cdot \Delta T^\star}{b_{5,5} \cdot W_5{}^\star} \cdot 100\%$ | $\frac{b_{23,6} \cdot W_3{}^\star \cdot \Delta T^\star}{b_{6,6} \cdot W_6{}^\star} \cdot 100\%$ |
| $W_4 \cdot \Delta T$ | $\frac{b_{24,4} \cdot W_4{}^\star \cdot \Delta T^\star}{b_{4,4} \cdot W_4{}^\star} \cdot 100\%$ | $\frac{b_{24,5} \cdot W_4{}^\star \cdot \Delta T^\star}{b_{5,5} \cdot W_5{}^\star} \cdot 100\%$ | $\frac{b_{24,6} \cdot W_4{}^\star \cdot \Delta T^\star}{b_{6,6} \cdot W_6{}^\star} \cdot 100\%$ |
| $W_5 \cdot \Delta T$ | $\frac{b_{25,4} \cdot W_5{}^\star \cdot \Delta T^\star}{b_{4,4} \cdot W_4{}^\star} \cdot 100\%$ | $\frac{b_{25,5} \cdot W_5{}^\star \cdot \Delta T^\star}{b_{5,5} \cdot W_5{}^\star} \cdot 100\%$ | $\frac{b_{25,6} \cdot W_5{}^\star \cdot \Delta T^\star}{b_{6,6} \cdot W_6{}^\star} \cdot 100\%$ |
| $W_6 \cdot \Delta T$ | $\frac{b_{26,4} \cdot W_6{}^\star \cdot \Delta T^\star}{b_{4,4} \cdot W_4{}^\star} \cdot 100\%$ | $\frac{b_{26,5} \cdot W_6{}^\star \cdot \Delta T^\star}{b_{5,5} \cdot W_5{}^\star} \cdot 100\%$ | $\frac{b_{26,6} \cdot W_6{}^\star \cdot \Delta T^\star}{b_{6,6} \cdot W_6{}^\star} \cdot 100\%$ |

The coefficients listed in Tables 7a and 7b above were determined during the regression analysis of the data that is described in Ref. [1] (only coefficients of the axial force $F_x$ were actually published in Ref. [1]). The "capacities" of the seven independent variables, i.e., of the six transformed outputs and the temperature difference, still need to be specified in order to compute the percent contributions. The capacity of the temperature difference equals 21 degK, i.e., the same value that is specified in Table 5a. The authors suggest to assign the maximum outputs at load capacity to be the "capacities" of the six outputs. These values are simply obtained by multiplying the prime sensitivity of each transformed output with the capacity of the related primary load. Then, the following expression is obtained for the capacities of the outputs:

$$\textit{definition of maximum output at load capacity} \quad \Longrightarrow \quad W_k{}^\star \quad \equiv \quad \underbrace{\frac{\partial \, W_k}{\partial \, F_k}}_{sensitivity} \quad \cdot \quad F_k{}^\star \qquad (6)$$

Table 8 below lists the prime sensitivity for each transformed output that was computed after applying the *Non–Iterative Method* to the calibration data of RUAG's 788–6A balance. Those values are the inverse values $(1/b_{1,1})$, $(1/b_{2,2})$, $\ldots$, $(1/b_{6,6})$ of the coefficients of the primary outputs $W_1$, $W_2$, $\ldots$, $W_6$ in the regression models of the related primary loads $F_x$, $F_y$, $\ldots$, $M_z$ that are defined in Eq. ($5a$).

**Table 8:** Prime sensitivities of the transformed outputs of the RUAG 788–6A balance.

| $\partial W_1/\partial F_x$ $(\mu V/V)/N$ | $\partial W_2/\partial F_y$ $(\mu V/V)/N$ | $\partial W_3/\partial F_z$ $(\mu V/V)/N$ | $\partial W_4/\partial M_x$ $(\mu V/V)/(Nm)$ | $\partial W_5/\partial M_y$ $(\mu V/V)/(Nm)$ | $\partial W_6/\partial M_z$ $(\mu V/V)/(Nm)$ |
|---|---|---|---|---|---|
| 0.4653 | 0.3639 | 0.3093 | 8.1179 | 2.6933 | 3.5753 |

Now, after using Eq. (6) above in combination with the computed sensitivities (Table 8) and the assigned load capacities (Table 5a), the capacities of (i) the temperature difference and (ii) the transformed outputs can be determined. The following values were obtained:

**Table 9a:** Capacities of the temperature difference and the transformed outputs.†

| $\Delta T^\star$ $degK$ | $W_1{}^\star$ $\mu V/V$ | $W_2{}^\star$ $\mu V/V$ | $W_3{}^\star$ $\mu V/V$ | $W_4{}^\star$ $\mu V/V$ | $W_5{}^\star$ $\mu V/V$ | $W_6{}^\star$ $\mu V/V$ |
|---|---|---|---|---|---|---|
| 21 | 2 001 | 1 929 | 1 639 | 4 059 | 3 232 | 2 860 |

†capacity of a transformed output $\equiv$ maximum output at load capacity; see also Eq. (6).

Finally, the percent contributions of the temperature–dependent terms of the regression models of the six loads can be computed. Table 9b below lists results of this calculation. Again, it is observed that the temperature–dependent cross–product term of the primary output of the load component, i.e., the term directly associated with the sensitivity shift of a given output, is the dominant term. Corresponding values are printed in boldface in Table 9b below. All other temperature–dependent terms make negligible contributions to the load prediction.

**Table 9b:** *Non–Iterative Method* – Percent contribution of the temperature–dependent terms of the regression models of the forces and moments of the RUAG 788–6A block–type balance.

| $\xi(F_k)$ | $F_1 \equiv F_x$ | $F_2 \equiv F_y$ | $F_3 \equiv F_z$ | $F_4 \equiv M_x$ | $F_5 \equiv M_y$ | $F_6 \equiv M_z$ |
|---|---|---|---|---|---|---|
| $\Delta T$ | −0.02 % | +0.02 % | ±0.00 % | −0.02 % | ±0.00 % | +0.02 % |
| $W_1 \cdot \Delta T$ | **−0.51** % | +0.04 % | +0.04 % | ±0.00 % | +0.02 % | +0.04 % |
| $W_2 \cdot \Delta T$ | −0.08 % | **−0.54** % | −0.06 % | −0.04 % | −0.01 % | −0.02 % |
| $W_3 \cdot \Delta T$ | −0.16 % | −0.02 % | **−0.53** % | −0.11 % | +0.03 % | +0.01 % |
| $W_4 \cdot \Delta T$ | −0.03 % | +0.02 % | +0.01 % | **−0.57** % | +0.10 % | +0.08 % |
| $W_5 \cdot \Delta T$ | +0.03 % | −0.02 % | +0.04 % | ±0.00 % | **−0.59** % | +0.02 % |
| $W_6 \cdot \Delta T$ | ±0.00 % | −0.02 % | −0.02 % | −0.04 % | +0.02 % | **−0.61** % |

The authors observed during the analysis that the percent contributions shown in Table 5b for the *Iterative Method* and in Table 9b above for the *Non–Iterative Method* appear to be similar in magnitude but opposite in sign if values of related terms are compared (e.g., compare the value for $F_x \cdot \Delta T$ in Table 5b with the value for $W_1 \cdot \Delta T$ in Table 9b). This characteristic is no coincidence. In fact, it is rigorously proven in the appendix of the paper that the percent contribution obtained for a regression model term of a primary output must be similar in magnitude but opposite in sign to the percent contribution of the corresponding regression model term of the related primary load as long as (i) the given balance data is analyzed in its design format (i.e., direct–read balance data is analyzed in direct–read format, or, force balance data is analyzed in force balance format, or, moment balance data is analyzed in moment balance format), (ii) the type and number of terms of the regression model of a primary output matches the type and number of terms of the regression model of the related primary load, and (iii) the capacity of an output is assigned to be the maximum output at load capacity.

## IV. Summary and Conclusions

A metric called the "percent contribution of a regression model term" was successfully applied to temperature–dependent calibration data of a RUAG six–component block–type balance in order to assess the influence of different types of temperature–dependent regression model terms of balance calibration data on the load prediction. A detailed review of the computed percent contributions showed that the cross–product term directly associated with the modeling of the temperature–dependent nature of the gage sensitivity is by far the most influential temperature–dependent term. This result can be explained by the fact that RUAG's block–type balances are highly linear in their behavior. Additional temperature–dependent cross–product terms may become significant if the authors' approach is applied to other types of six–component strain–gage balances. Percent contributions obtained for the temperature–dependent terms of regression models of the *Iterative Method* are, as expected, similar in magnitude but opposite in sign when compared with values that are obtained for corresponding regression model terms of the *Non–Iterative Method*. The authors recommend the use of the percent contribution for the evaluation of temperature–dependent regression model terms of strain–gage balance data because the metric appears to be reliable and is easily implemented.

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## Appendix: Relationship between Percent Contributions

A comparison of the percent contributions of the temperature–dependent regression model terms of the outputs (Table 5b) with corresponding values of the loads (Table 9b) indicates that values of related terms appear to be similar in magnitude but opposite in sign. This observation is not limited to temperature–dependent terms. In fact, the characteristic can rigorously be proven for a subset of terms if the following five assumptions are made: (I) the balance data is analyzed in its "design" format (e.g., direct–read balance data is analyzed in direct–read format, or, force balance data is analyzed in force balance format, or, moment balance data is analyzed in moment balance format); (II) the type and number of terms of the regression model of a primary output matches the type and number of terms of the regression model of the related primary load; (III) the capacity of an output is assigned to be the maximum output at load capacity; (IV) the absolute value of the percent contribution of a term is on the order of -or- less than 10 %; (V) the electrical outputs of a gage are supplied as differences relative to its "natural zero", i.e, relative to the electrical output that the gage would have if the balance is in an assumed "weightless" condition.

The proof starts by comparing an approximation of the regression model of an output with the corresponding regression model of a load. First, the regression model of the output, i.e., Eq. ($4a$), is simplified after taking assumptions (I) and (V) into account. Then, it is reasonable to assume that (i) the intercept term is negligible and (ii) only the primary load and one additional regression model term make significant contributions on the right–hand side of Eq. ($4a$). Consequently, the following approximation of the regression model of an output can be made:

$$W_k \quad \approx \quad a_{k,k} \; \cdot \; F_k \;\; + \;\; a_{\eta,k} \; \cdot \; \xi(W_k) \tag{7a}$$

Then, after dividing both sides of Eq. ($7a$) above by the first term on the right–hand side, we get:

$$\frac{W_k}{a_{k,k} \cdot F_k} \quad \approx \quad 1 \;\; + \;\; \frac{a_{\eta,k} \; \cdot \; \xi(W_k)}{a_{k,k} \; \cdot \; F_k} \tag{7b}$$

We also know that the regression model term $a_{k,k}$ is the partial derivative of the primary output $W_k$ with respect to the primary load $F_k$. Then, we can write:

$$a_{k,k} \quad = \quad \frac{\partial \; W_k}{\partial \; F_k} \tag{7c}$$

Now, after using (i) the right–hand side of Eq. ($7c$) to replace $a_{k,k}$ on the left–hand side of Eq. ($7b$) and (ii) multiplying both sides of the resulting equation by 100 %, we get:

$$\frac{W_k}{(\partial W_k / \partial F_k) \; \cdot \; F_k} \; \cdot \; 100 \; \% \quad \approx \quad 100 \; \% \;\; + \;\; \frac{a_{\eta,k} \; \cdot \; \xi(W_k)}{a_{k,k} \; \cdot \; F_k} \; \cdot \; 100 \; \% \tag{7d}$$

The last term on the right–hand side of Eq. ($7d$) becomes the percent contribution $P[\xi(W_k)]$ of the term $\xi(W_k)$ of the regression model of the output $W_k$ if the capacities of related variables $(F_k{}^{\star}, W_k{}^{\star}, \Delta T^{\star})$ are used on both sides of the equation. Then, Eq. ($7d$) can be written as follows:

$$\frac{W_k{}^{\star}}{(\partial W_k / \partial F_k) \; \cdot \; F_k{}^{\star}} \; \cdot \; 100 \; \% \quad \approx \quad 100 \; \% \;\; + \;\; P[\xi(W_k)] \tag{7e}$$

The numerator and the denominator of the fraction on the left–hand side of Eq. ($7e$) above are identical because the numerator can be replaced by the right–hand side of Eq. (6) of the body of the text. Therefore, the fraction on the left–hand side of Eq. ($7e$) equals "one" and we get the following simplification of Eq. ($7e$):

$$100 \; \% \quad \approx \quad 100 \; \% \;\; + \;\; P[\xi(W_k)] \tag{7f}$$

Similarly, the regression model of a load, i.e., Eq. ($5a$), can be simplified. Again, it is reasonable to assume that (i) the intercept term is negligible and (ii) only the primary output and one additional regression model term make significant contributions on the right–hand side of Eq. ($5a$) if assumptions (I) and (V) listed in the first paragraph of the appendix are taken into account. Then, the following approximation of the regression model of a load is valid:

$$F_k \quad \approx \quad b_{k,k} \cdot W_k \;+\; b_{\eta,k} \;\cdot\; \xi(F_k) \tag{8a}$$

Now, after dividing both sides of Eq. ($8a$) above by the first term on the right–hand side, we get:

$$\frac{F_k}{b_{k,k} \;\cdot\; W_k} \quad \approx \quad 1 \;+\; \frac{b_{\eta,k} \;\cdot\; \xi(F_k)}{b_{k,k} \;\cdot\; W_k} \tag{8b}$$

We also know that the regression model term $b_{k,k}$ is the partial derivative of the primary load $F_k$ with respect to the primary output $W_k$. Then, we can write:

$$b_{k,k} \quad = \quad \frac{\partial\, F_k}{\partial\, W_k} \tag{8c}$$

Then, after using (i) the right–hand side of Eq. ($8c$) to replace $b_{k,k}$ on the left–hand side of Eq. ($8b$) and (ii) multiplying both sides of the resulting equation by 100 %, we get:

$$\frac{F_k}{(\partial F_k / \partial W_k) \;\cdot\; W_k} \cdot 100\ \% \quad \approx \quad 100\ \% \;+\; \frac{b_{\eta,k} \;\cdot\; \xi(F_k)}{b_{k,k} \cdot W_k} \cdot 100\ \% \tag{8d}$$

The last term on the right–hand side of Eq. ($8d$) becomes the percent contribution $Q[\xi(F_k)]$ of the term $\xi(F_k)$ of the regression model of the load $F_k$ if the capacities of related variables $({F_k}^\star, {W_k}^\star, \Delta T^\star)$ are used on both sides of the equation. Then, Eq. ($8d$) can be written as follows:

$$\frac{{F_k}^\star}{(\partial F_k / \partial W_k) \;\cdot\; {W_k}^\star} \cdot 100\ \% \quad \approx \quad 100\ \% \;+\; Q[\xi(F_k)] \tag{8e}$$

We also know that

$$(\partial F_k / \partial W_k) \quad = \quad 1 \;/\; (\partial W_k / \partial F_k) \tag{8f}$$

Then, after replacing the partial derivative on the left–hand–side of Eq. ($8e$) with the right–hand side of Eq. ($8f$), we get the following result for Eq. ($8e$):

$$\frac{(\partial W_k / \partial F_k) \;\cdot\; {F_k}^\star}{{W_k}^\star} \cdot 100\ \% \quad \approx \quad 100\ \% \;+\; Q[\xi(F_k)] \tag{8g}$$

The numerator and denominator of the fraction on the left–hand side of Eq. ($8g$) above are identical because the denominator can be replaced by the right–hand side of Eq. (6) of the body of the text. Therefore, the fraction on the left–hand side of Eq. ($8g$) equals one and we get the following simplification of Eq. ($8g$):

$$100\ \% \quad \approx \quad 100\ \% \;+\; Q[\xi(F_k)] \tag{8h}$$

In the next step, after multiplying the left– and right–hand sides of Eq. ($7f$) with the left– and right–hand sides of Eq. ($8h$), we get:

$$(100\ \%)^2 \quad \approx \quad \Big[\, 100\ \% \;+\; P[\xi(W_k)] \,\Big] \;\cdot\; \Big[\, 100\ \% \;+\; Q[\xi(F_k)] \,\Big] \tag{9}$$

In addition, after expanding the brackets on the right–hand side of Eq. (9), we get:

$$(100\ \%)^2 \quad \approx \quad (100\ \%)^2 \;+\; 100\ \% \;\cdot\; P[\xi(W_k)] \;+\; 100\ \% \;\cdot\; Q[\xi(F_k)] \;+\; P[\xi(W_k)] \;\cdot\; Q[\xi(F_k)] \tag{10}$$

Equation (10) can be simplified further after (i) the square of 100 % is subtracted from both sides of the equation and (ii) the result is divided by 100 %. Then, we get

$$0 \approx P[\xi(W_k)] + Q[\xi(F_k)] + \frac{P[\xi(W_k)] \cdot Q[\xi(F_k)]}{100\ \%} \tag{11}$$

Now, after (i) subtracting $P[\xi(W_k)]$ from both sides of the Eq. (11) and (ii) extracting the common multiplier $Q[\xi(F_k)]$ of the last two terms on the right–hand side of the Eq. (11), we get:

$$-P[\xi(W_k)] \approx Q[\xi(F_k)] \cdot \left[ 1 + \frac{P[\xi(W_k)]}{100\ \%} \right] \tag{12}$$

It is known from experience that the magnitude of the percent contribution for the vast majority of regression model terms of real–world balance data is <u>on the order of</u> -or- <u>less than</u> the conservative threshold of 10 % if a balance has highly linear behavior and the metric is not computed for the term that is defined by the <u>primary output</u> or the <u>primary load</u> itself (see also assumption (IV) that is listed in the first paragraph of the appendix). This observation can be described as follows:

$$|\ P[\xi(W_k)]\ | \leq 10\ \% \tag{13}$$

The absolute value of the percent contribution must be used on the left–hand side of the inequality above because the percent contribution is either a positive or negative quantity. Now, after using the above inequality in order to simplify the contents of the bracket on the right–hand side of Eq. (12), we get:

$$\left[ 1 + \frac{P[\xi(W_k)]}{100\ \%} \right] \approx 1 \pm \frac{10\ \%}{100\ \%} \approx 1 \pm 0.1 \approx 1 \tag{14}$$

Finally, after replacing the bracket on the right–hand side of Eq. (12) by the value of "1", i.e., by the approximation that is derived in Eq. (14), we get the following relationship between the percent contribution of a regression model term of the primary output and the percent contribution of the related regression model term of the primary load:

$$\boxed{-P[\xi(W_k)] \approx Q[\xi(F_k)] \qquad (15)}$$

The final result given in Eq. (15) above confirms the authors' observation that the percent contributions $P[\xi(W_k)]$ and $Q[\xi(F_k)]$ of related terms of the regression models of the outputs and loads are <u>similar in magnitude</u> but <u>opposite in sign</u>. This result is valid for the percent contributions of most regression model terms of balance data and not just for the percent contributions of temperature–dependent terms as long as the absolute value of the percent contribution of a term is on the order of 10 % or less.