# Helmholtz-Hodge Decomposition of vector fields on 2-manifolds

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## Why ?

**A Morse-like Decomposition ?**

- Morse-Smale decomposition for gradient (of scalar) fields is an interesting way of decomposing the domain into regions of unidirectional flow (from a source to a sink ).
- But works for gradient fields, which are conservative (irrotational), only.
- Can such a decomposition and analysis be extended to generic (consisting rotational component) vector fields ?
- Can we extract the rotational component out from generic vector fields ?

**Feature Identification ?**

- Analysis on the decomposed components of fields is simpler. eg Identification of critical points in the potentials of the two components is easy.

**Topological Consistency ?**

- Is there any relation between the topology of the components and the topology of the original field ?

**Limitation**

- So far, HH Decomposition exists only for piece-wise constant vector fields. Such a decomposition for piece-wise linear fields is desirable.

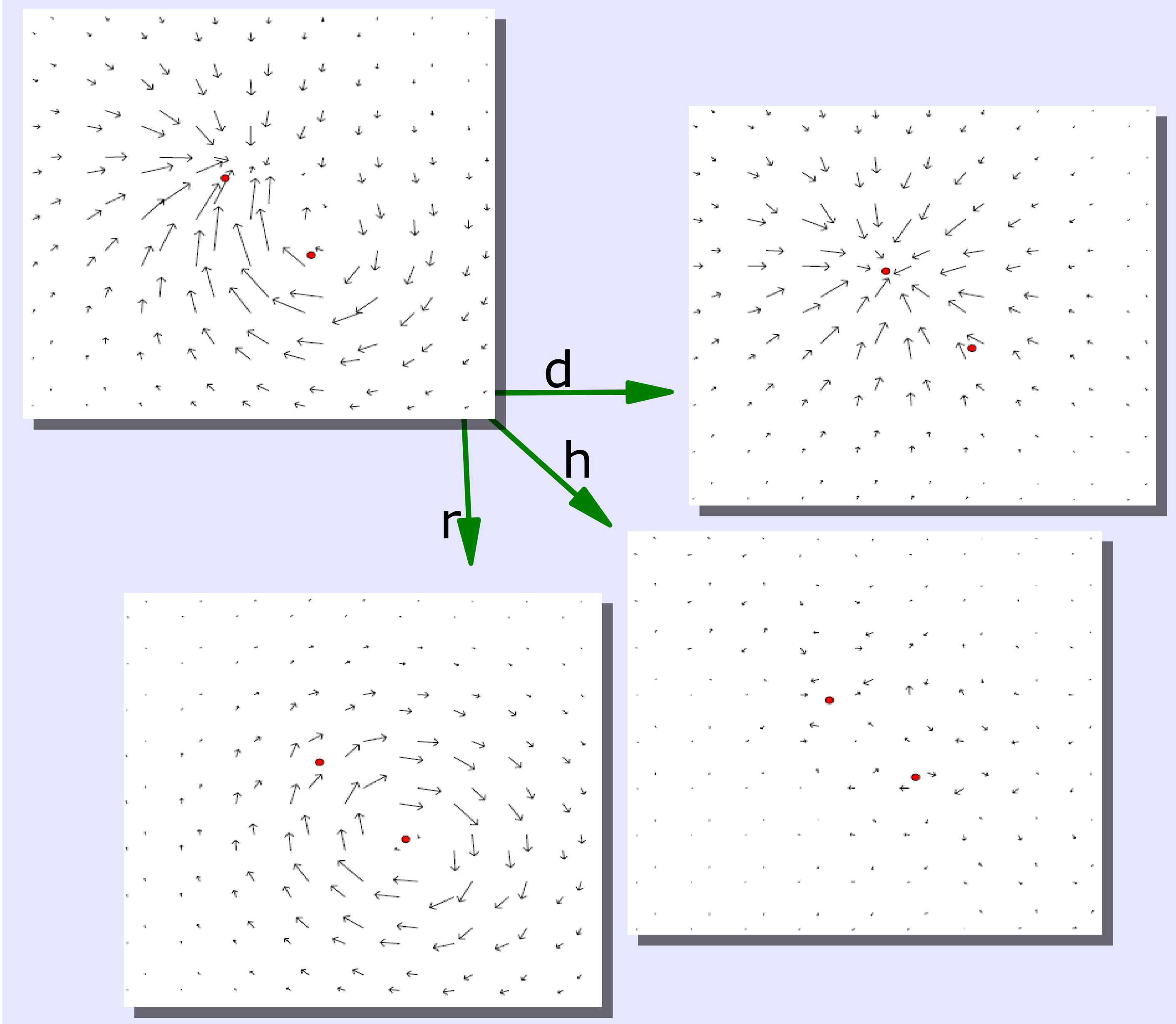
## What ?

Decomposition of a vector field into **conservative and rotational** components

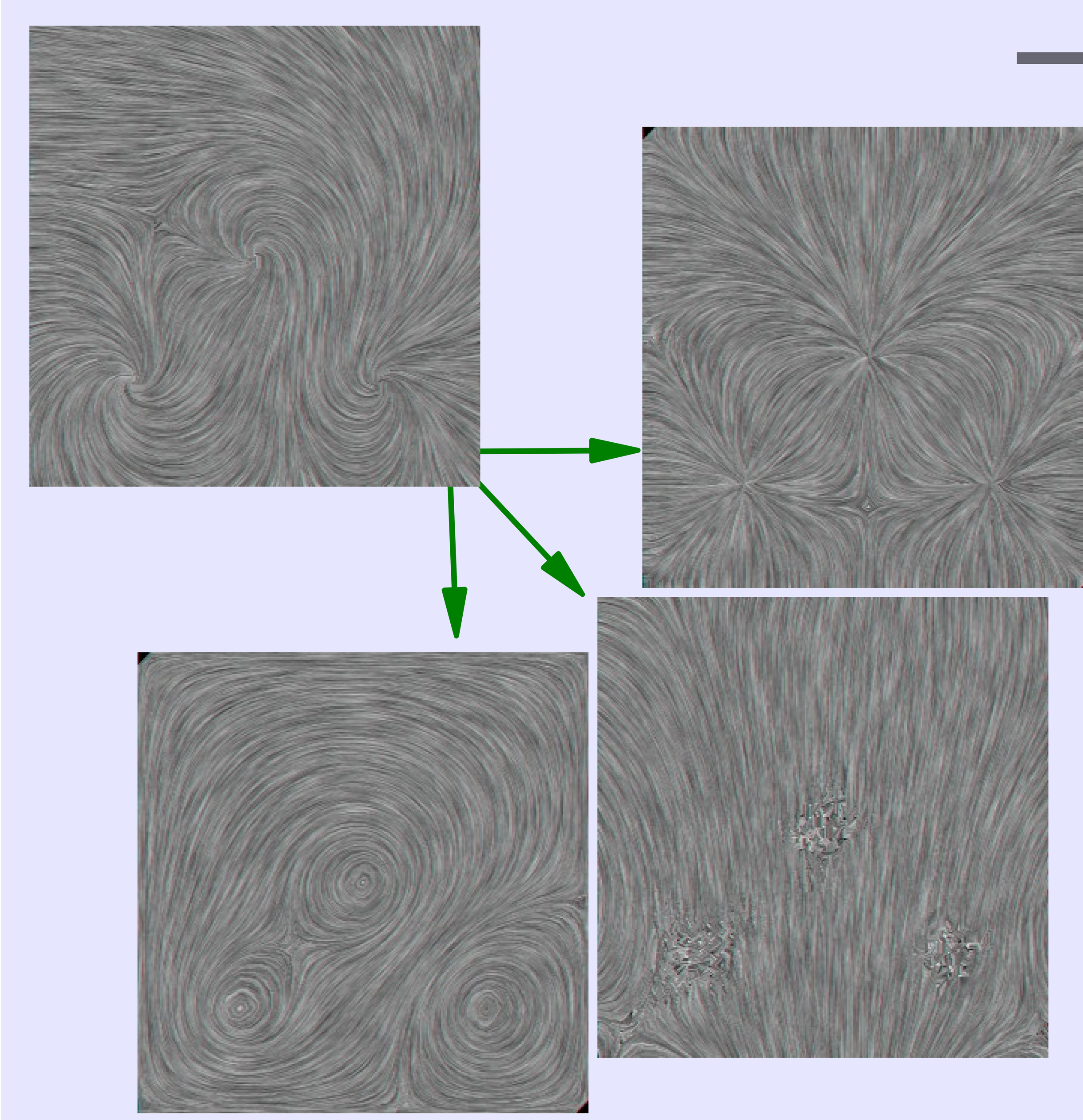
$$\begin{aligned} \xi &= (\nabla u) + (\nabla \times w) + h \\ &= d + r + h \end{aligned}$$

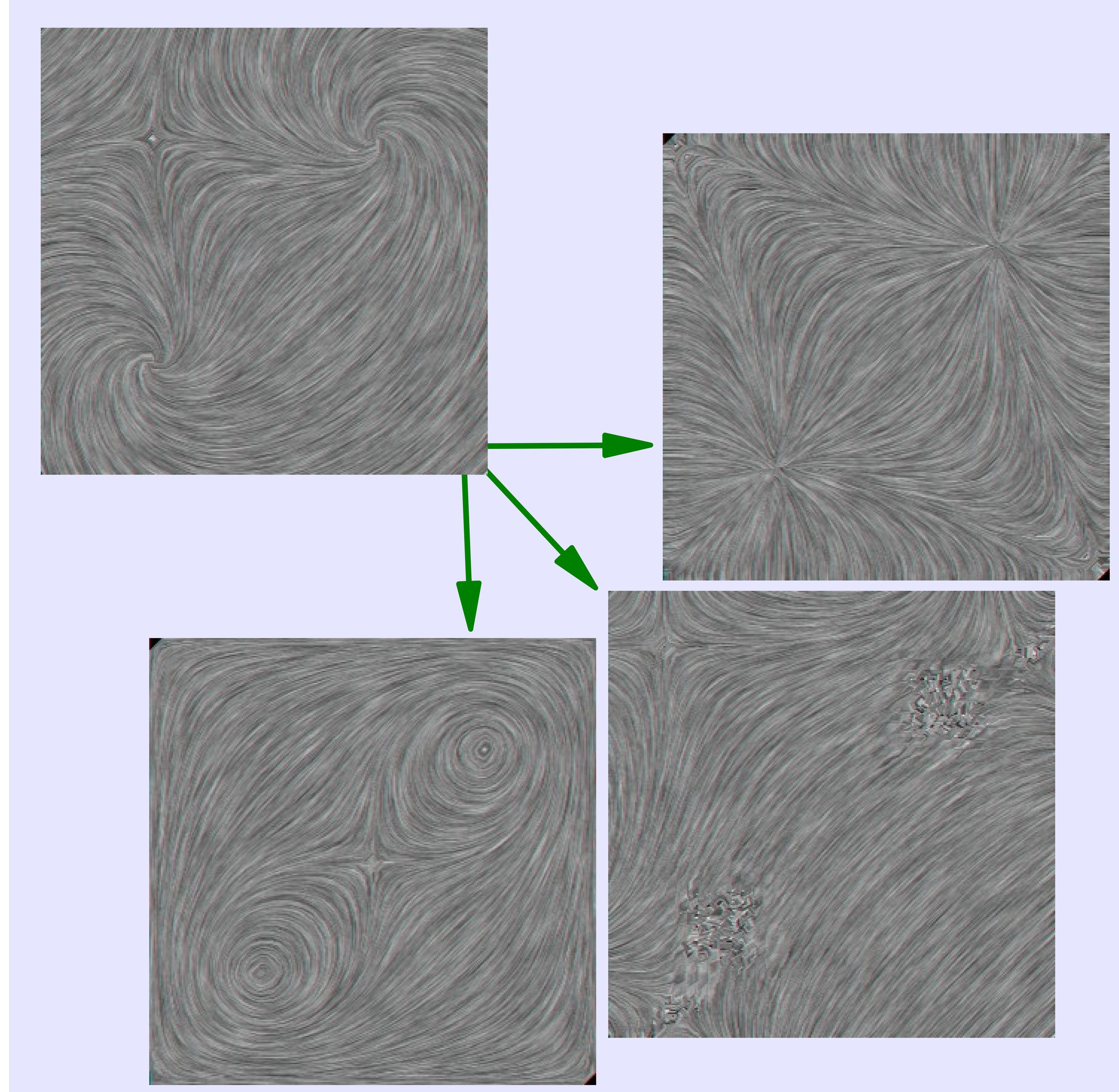
1. Conservative Component ( d ) has zero rotation.
2. Rotational Component ( r ) has zero divergence.
3. Harmonic Component ( h ) has zero divergence and zero rotation.

## Decomposition[1]




## Results





## How ?

Discrete HH decomposition can be calculated as a global energy minimization over the Domain[2]

$$\xi = \nabla u + J \, \nabla w + h$$

1. Quadratic energy functional to determine u

$$F(u) = \int_{M_l} \left( |\nabla u|^2 - \langle \nabla u, \xi \rangle \right)$$

2. Quadratic energy functional to determine w

$$G(w) = \int_{M_l} \left( |J\nabla w|^2 - \langle J\nabla w, \xi \rangle \right)$$

3. Harmonic component

$$h = \xi - \nabla u - J \, \nabla w$$

## References

1. K Polthier, E Preuβ : Variational Approach to Vector Field Decomposition
2. K Polthier, E Preuβ : Identifying Vector Field Singularities Using a Discrete Hodge Decomposition