Matching Shapes Using The Current Distance

Sarang Joshi, Raj Varma Kommaraju, Jeff M. Philips, Suresh Venkatasubramanian

THE UNIVERSITY OF UTAH

Introduction

Current Distance: It was introduced by Vaillant and Glaunès as a way of comparing shapes (point sets, curves, surfaces). This distance measure is defined by viewing a shape as a linear operator on a k-form field, and constructing a (dual) norm on the space of shapes.

Shape Matching: Given two shapes P, Q, a distance measure d on shapes, and a transformation group $\mathcal{T}$, the problem of *shape matching* is to determine a transformation T that minimizes $d(P, T \circ Q)$.

Current Norm: For a point set P, current norm is

$$\|P\|^2 = \sum_i \sum_j K(p_i, p_j))\eta(p)\mu(q)$$

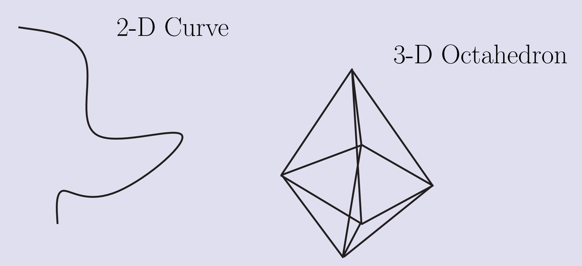
Current Distance: Distance between two point sets P and Q is

$$\begin{aligned} D^2(P, Q) &= \|P + (-1)Q\|^2 \\ &= \|P\|^2 + \|Q\|^2 - 2\sum_i \sum_j K(p_i, q_j))\eta(p)\mu(q) \end{aligned}$$

It takes $O(n^2)$ time to compute the current distance between two shapes of size n. Also current distance between 2 surfaces or curves can be reduced to set of distance computations on appropriately weighted point sets.

Motivation

- Attractive properties of current distance
  - Global in nature
  - Can be expressed as a direct computation
  - Defined in terms of norm which acts as signature of shape
  - Can be generalized to higher dimensional structures



Contributions

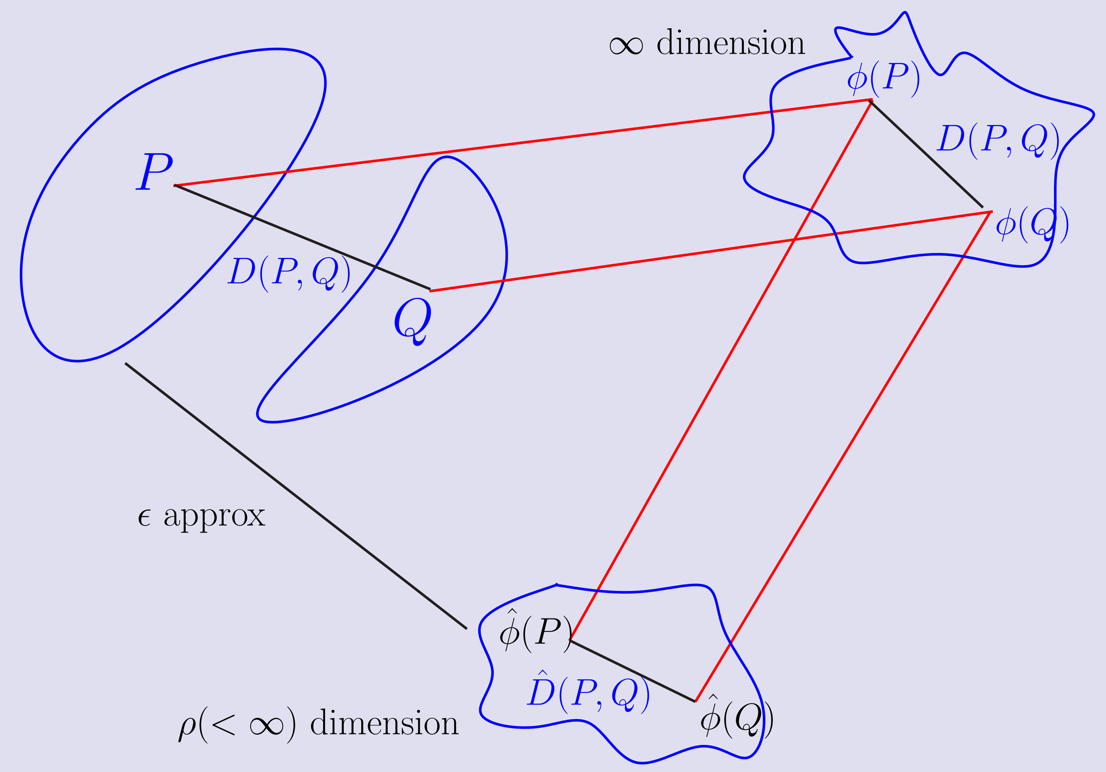
We provide the first algorithmic analysis of the current distance. The main results of our work are

- a method for computing the current distance between two shapes in near-linear time
- a *core-set* construction that allows us to approximate the current norm of a point set using a constant-sized sample
- an FPTAS for minizing the current distance between two d-dimensional point sets under translation and rotation

Approximate Current Distance

Approximate Feature Maps:

- ***Kernel Trick***: For a reproducing kernel K, there exists a map $\phi : X \to \underline{H}$ such that $K(x, y) = \langle \phi(x), \phi(y) \rangle$
- $D^2(P, Q) = \|C_{\phi(P)} - C_{\phi(Q)}\|^2$ where $C_{\phi(P)}$ is the centroid of $\phi(P)$



- We can approximate kernel function using a mapping $\tilde{\phi} : X \to \underline{R}^\rho$ such that $K(x, y) \approx \langle \tilde{\phi}(x), \tilde{\phi}(y) \rangle$
- IFGT and Random Projections gives a mapping to feature space such that we can approximate the current distance.

Coreset for Current Norm

We can approximate the current norm by extracting a small subset (a coreset) from the input.

- By constructing a subset S of some size $k = O(1/\epsilon^3)\log(n/\delta)\log((1/\epsilon\delta)\log n))$ we guarantee $D^2(\underline{P},) \leq \epsilon W^2$, with probability at least $1 - \delta$
- Now this subset acts as the signature of the shape

Translation and Rotation

- $T^* = \arg\min_{T \in \underline{R}^d} D^2(P, Q \circ T)$, be the optimal translation
- We find a $\hat{T}$ such that $D^2(P, Q \circ \hat{T}) - D^2(P, Q \circ T^*) \leq \epsilon W^2$, for any parameter $\epsilon > 0$

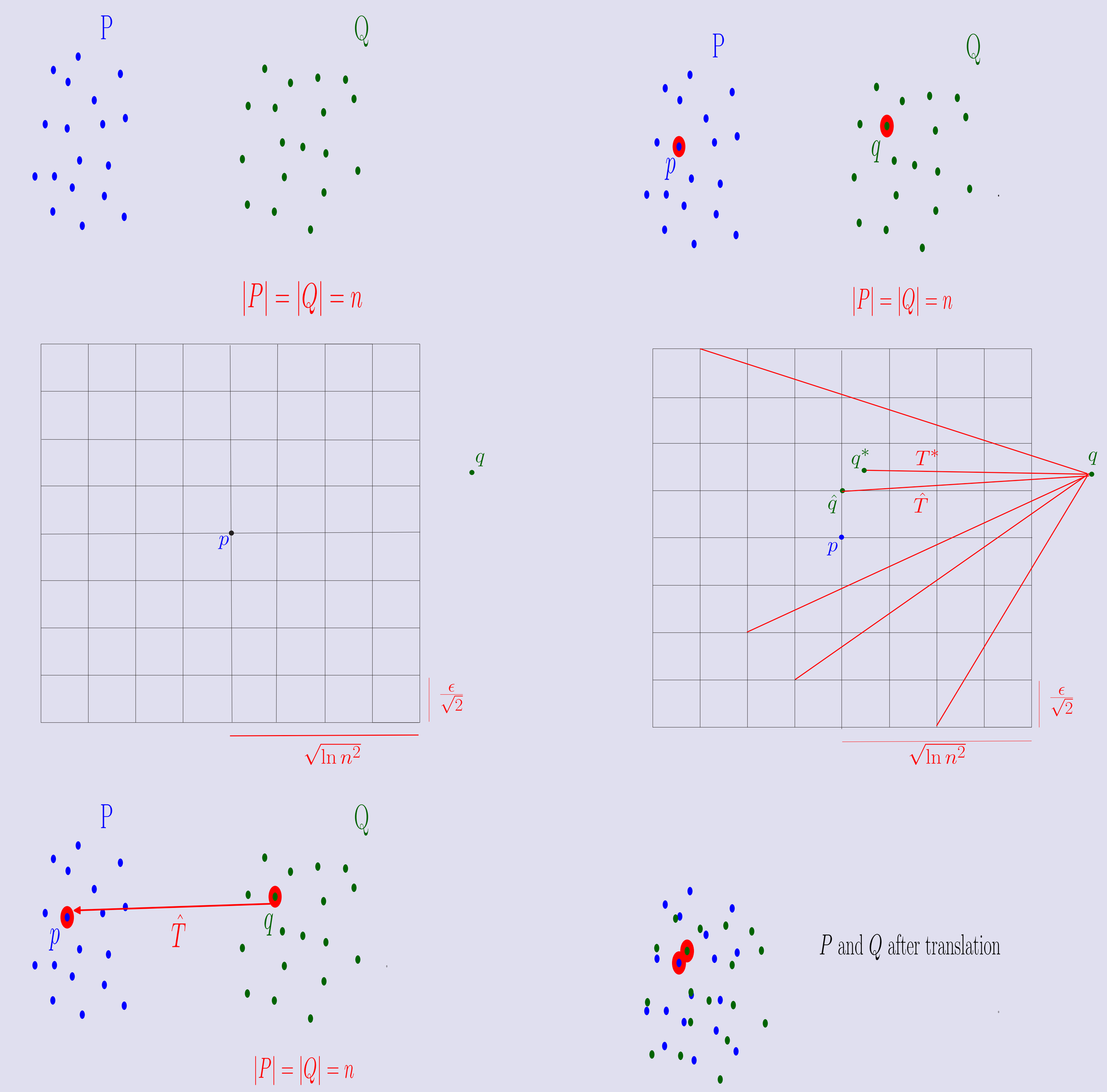
Algorithm for Translation:

- for each point $p \in P$
  - for each point $q \in Q$
    - for each $g \in$ grid around p
      - Apply translation $T_{p,q,g}$ such that $q + T_{p,q,g} = g$
      - if $D(P, Q + T_{p,q,g})$ is minimum then $\hat{T} = T_{p,q,g}$

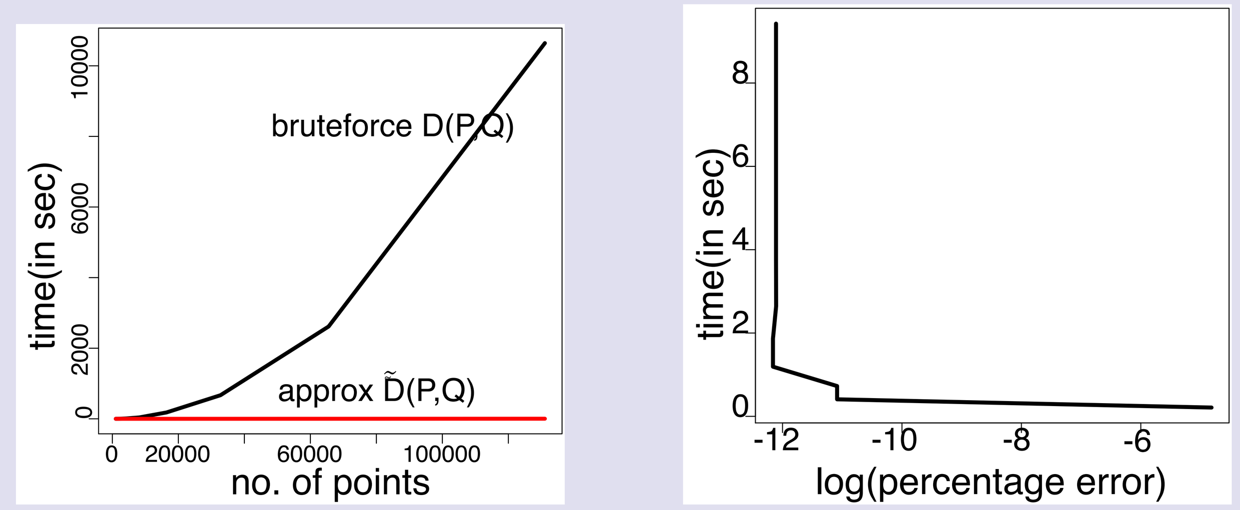
Similar algorithm works for Rotations.

Translation-How It Works

- There exists some pair $p \in P$ and $q \in Q$ such that $\|p - (q + T^*)\| \leq \sqrt{\ln(n^2)}$
- Translating a point by some distance less than ϵ will only change K by at most ϵ



Results



(a) (b)

(a)exact $(D(P, Q))$ vs approximate $(\tilde{D}(P, Q))$ time comparison (b) $\log_{10}$ % error for approximate distance measure on 131072 points: $|D(P, Q) - \tilde{D}(P, Q)|/D(P, Q)$.

References

1. Sarang Joshi, Raj Varma Kommaraju, Jeff M. Philips, Suresh Venkatasubramanian. *Shape Matching Using The Current Distance.* arXiv:1001.0591
2. Vaillant M. and Glaunes J. *Surface matching via currents.* In Proc. Information processing in medical imaging(January 2005), vol. 19 , pp. 381-92.