Electromagnetic energy harvesting using magnetic levitation architectures: a review

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Abstract

Motion-driven electromagnetic energy harvesters have the ability to provide low-cost and customizable electric powering. They are a well-suited technological solution to autonomously supply a broad range of high-sophisticated devices. This paper presents a detailed review focused on major breakthroughs in the scope of electromagnetic energy harvesting using magnetic levitation architectures. A rigorous analysis of twenty-one design configurations was made to compare their geometric and constructive parameters, optimization methodologies and energy harvesting performances. This review also explores the most relevant models (analytical, semi-analytical, empirical and finite element method) already developed to make intelligible the physical phenomena of their transduction mechanisms. The most relevant approaches to model each physical phenomenon of these transduction mechanisms are highlighted in this paper. Very good agreements were found between experimental and simulation tests with deviations lower than 15%. Moreover, the external motion excitations and electric energy harvesting outputs were also comprehensively compared and critically discussed. Electric power densities up to 8 mW/cm^3 (8 kW/m^3) have already been achieved; for resistive loads, the maximum voltage and current were 43.4 V and 150 mA, respectively, for volumes up to 235 cm^3. Results highlight the potential of these harvesters to convert mechanical energy into electric energy both for large-scale and small-scale applications. Moreover, this paper proposes future research directions towards efficiency maximization and minimization of energy production costs.

Key words: Energy harvesting, Self-powering, Electromagnetic harvesting, Magnetic levitation, Modelling, Design optimization

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Terminology

a_c, b_c, c_c	Half of depth, half of length and half of height of a block magnet at the origin, respectively [m]
A	External excitation amplitude [m/s^2]
A_a	External excitation amplitude [m]
A_{ct}	Area of the coil turn [m^2]
A_c,B_c, C_c	Half of depth, half of length and half of height of a block magnet centered in $(\alpha_c, \beta_c, \gamma_c)$, respectively [m]
A_s	Surface area common to magnets [m^2]
B	Magnetic field density [T]
B_m	Flux density [T]
B_r	Residual magnetic flux density [T]
c	Total damping coefficient [Ns/m]
c_{air}	Air damping [Ns/m]
c_e	Electrical damping [Ns/m]
c_{fr}	Friction damping [Ns/m]
c_m	Mechanical damping [Ns/m]
c_p	Parasitic viscous damping [Ns/m]
f	Frequency [Hz]
f_{bw_n}, f_{bw_p}	Break-away forces for negative and positive $\dot{y}$, respectively [N]
f_{CO_n}, f_{CO_p}	Coulomb forces for negative and positive $\dot{y}$, respectively [N]
F_{bx}	Kelvin-Voight elasto-dissipative force due to the elastic bumpers used as strokes [N]
F_{fr}	Friction force [N]
F_{lz}	Lorentz force [N]
F_m	Repulsive magnetic force [N]
F_{m_d}, F_{m_u}	Repulsive magnetic force between the moving magnet and the bottom/top magnet, respectively [N]
g	Acceleration due to gravity [m/s^2]
H	Magnetic field strength [A/m]
I	Induced electric current [A]
J	Equivalent linear current density [A/m^2]
J_1	First order Bessel function
k_{v_n}, k_{v_p}	Viscous friction coefficients for negative and positive $\dot{y}$, respectively [Ns/m]
l	Coil length [m]
l_c	Axial coil height [m]
l_{ct}	Axial top coil height [m]
l_d, l_m, l_u, l_s	Height of the bottom, middle and top magnet and of the spacer, respectively [m]
l_h	Distance between the upper surface of the top fixed magnet to the below surface of the bottom fixed magnet [m]
L	Coil impedance [H]
m	Mass of the levitating stack [kg]
$\mathbb{m}_1, \mathbb{m}_2$, $\mathbb{m}_3$	Magnetic dipole moment of the bottom and top fixed magnet and the moving magnet, respectively [A.m^2]
M_d, M_u, M_m	Saturation magnetization of the bottom and top fixed magnet and the moving magnet, respectively [A/m]
N	Number of coil turns
N_i	Number of independent multi-turn coils
N_r, N_y	Number of radial and axial turns in a coil, respectively
N_t	Number of turns of the top coil
p	Magnetic spatial period [m]
P	Electric Power [W]
P_ρ	Electric Power density [W/m^3]
Q	Quality factor
r_c	Coil inner radius [m]
r_{ct}	Top coil inner radius [m]
r_d, r_m, r_u, r_s	Radius of the bottom, middle and top magnet and of the spacer, respectively [m]
r_{di}, r_{mi}	Hole radius of the annuli base and levitating magnet, respectively [m]
r_h	Casing inner radius [m]
r_{i_d}, r_{i_u}, r_{e_d}, r_{e_u}	Internal and external radii, respectively, for the bottom/top coil [m]
r_{mc}	Mean coil radius [m]
R_i	Coil internal eletric resistance [Ω]
R_l	Load electric resistance [Ω]
s_d, s_u	Distance between the moving magnet and the fixed magnet at the bottom/top, respectively [m]
U	Electromotive force (voltage) [V]
v_{min}	Low speed region $-v_{min} < \dot{y} < v_{min}$ where $\dot{y} = 0$ [m/s]
V	Harvester volume [m^3]
$y, \dot{y}, \ddot{y}$	Position, velocity and acceleration of the inertial mass relative to the device, respectively [m], [m/s], [m/s^2]
$\dot{y}_t$	Velocity of the top magnet relative to the device [m/s]
$y_0, ..., y_n$	Peaks of impulse response [m]
y_m	Axial distance between the geometrical centre and related bottom surface of the levitating magnet [m]
y_{d_b}, y_{d_t}, y_{u_b}, y_{u_t}	Inferior and superior limits of the bottom/top coil, respectively [m]
$\ddot{z}$	Acceleration of the harvester imposed by external power source dynamics [m/s^2]
α	Electromechanical coefficient [Vs/m]
$\alpha_c, \beta_c, \gamma_c$	x,y and z Coordinate of the block magnet centre, respectively
α_p	Parity phase equal to 0 or $\pi/2$ for an even or uneven number of alternate magnets, respectively
μ	Coulomb's friction coefficient
μ_0	Free space magnetic permeability [H/m]
μ_r	Relative magnetic permeability [H/m]
ξ	Damping factor
ϕ	Magnetic flux [Wb]
$\varnothing_c$	Coil diameter [m]

1 Introduction

Scientific breakthroughs in electric energy harvesting are of utmost importance for technological sophistication in many societal domains [1–7]. Energy harvesting from environment is becoming increasingly a promising methodology to power both large-scale and small-scale devices. These require efficient and customizable self-powering technologies to operate throughout long periods of time with reduced intermittence and high transduction rate, ensuring low production and maintenance costs, as well as performance adaptability for circumstantial variations in the dynamics of primary power sources [8–10].

Common renewable sources for large-scale electric powering, such as wind and sun, are intermittent, which result in complex grid managements and high energy production costs. The latter occurs mainly due to the backup costs to support the fluctuating electrical energy production, such as costs related to conversion processes and energy storage [11–15]. However, current non-intermittent renewable energy systems, including those harvesting electric energy from the ocean, require complex mechanical systems for energy transduction, such as turbines, oleo-hydraulic systems, transmission systems, etc., which bear high maintenance costs and significant performance losses. Besides, they are not able to ensure performance adaptation to varying motion-driven vibrations [16,17]. Therefore, there is a need to develop advanced technological solutions to highly improve the performance of non-intermittent clean energy harvesting systems. Research in this area is mandatory as the electricity consumption is estimated to increase around 55% worldwide by 2040, and the European Union already established goals to reduce conventional non-renewable energy sources by 80-95% up to 2050 [18,19]. Significant growth has also been observed in the development of self-powering small-scale power systems. Such an ability has been recognized as critical for emerging technologies to develop high-performance autonomous remote sensors [20,21], wearable devices [22–24], implantable biomedical devices [22,23,25] and mobile applications [26], among many others [27]. Most of these technologies must have their own incorporated power supply to ensure that enough energy is available, but also to reduce problems related to interconnection, electronic noise and control system complexity [13,28]. Nonetheless, at the moment, current harvesting solutions to power these multifunctional devices electrically do not fulfil existing requirements for effective self-powering, including enough electric current magnitudes, long-term operation, low maintenance costs, reduced performance losses and ability to adapt to excitation variations [25,29,30]. The use of batteries is far behind the power requirements of innovative stand-alone technologies, as they have limited capacity to store energy or their replacement is impractical or inconvenient [31]. In the scope of implantable medical devices, the limited service time of batteries exposes patients to surgical procedures and other potential risks that must be avoided. Besides, around 20% of bioelectronic medical devices failures, with possible catastrophic impacts on patients, are directly related to their electric power supply systems [32]. In contrast, advanced multifunctional medical devices need the incorporation of intracorporeal energy harvesting systems to ensure: (i) continuous and long-lasting acquisition of multiple monitoring bio-signals [33]; (ii) continuous and long-lasting therapy based on electromechanical actuation and/or biophysical stimulation [34–36]; (iii) processing capability to run intensive closed-loop dynamic control algorithms and complex artificial intelligence procedures [37,38]; (iv) frequent communication operations to extracorporeal systems, allowing clinicians to control and monitor medical devices [39–41].

As mechanical energy surrounding us is available [42–44], transduction mechanisms based on electromagnetic [45–47], piezoelectric [48–50], electrostatic [51–53] and triboelectric [54–56] principles have been extensively studied to convert mechanical energy into electric energy. This paper is focused on electromagnetic energy harvesting systems using magnetic levitation architectures. These are recent and promising self-powering technologies because they have the potential to implement high-performance energy harvesting for a wide range of devices, both for small-scale and large-scale self-powering [33,57–59], since they present distinctive properties like their non-complex design (which avoids the use of complex mechanical systems for energy conversion), low maintenance requirements and ability to operate autonomously with stable performance for long periods of time [25,33,60]. Several studies report that the magnitude of harvested energy is strongly dependent on geometric optimization prior to fabrication [25,61–63]. Moreover, their effectiveness have been investigated for applications involving severe dimensional constraints and time-varying power source dynamics. Although several architectures using magnetic levitation have already been proposed, research has been mainly conducted in the scope from mono-stable to multi-stable architectures (bi-stable, tri-stable and quad-stable harvesters) [25,45,58]. Multi-stable approaches require wider structures and additional magnets. Their nonlinearities are more prominent than the highly nonlinear behaviors observed in mono-stable architectures. As a consequence, multi-stable architectures demand much more complex optimization methods, which in turn demand much greater computational costs. Despite all scientific findings in this field, the highest performances that mono-stable and multi-stable harvesters are able to achieve are currently unknown. To date, no exhaustive and systematic effort has been done to compare harvester designs, optimization methods, harvested electric power, and modelling and validation of the transduction mechanisms of electromagnetic energy harvesting systems comprising mono-stable magnetic levitation architectures. This review presents all relevant studies that report the major achievements in this scientific area. The ultimate goal is to contribute towards the implementation of highly-sophisticated electromagnetic energy harvesters with ability to supply energy to a wide range of stand-alone devices.

2 Methods

2.1 Selection criteria

In this paper we present a rigorous analysis of electromagnetic harvesters focused on magnetic levitation architectures that fulfil four major requirements:

(1) Architectures must comprise two or more magnets, and one or more coils;

(2) Architectures including at least one fixed hard-magnetic element, and one or more hard-magnetic elements experiencing magnetic levitation;

(3) Architectures designed for axial motions of the levitating magnet(s) within the container;

(4) Architectures with mono-stable electromagnetic-induction configurations. Multi-stable configurations were considered outside the scope of this review.

The search was completed in September 2019. Twenty-nine relevant papers were selected according to these criteria.

2.2 Literature search strategy

The following data were extracted and analysed from the selected papers: (1) architectures proposed by each author; (2) the most relevant geometrical and construction parameters used to characterize each harvester, namely the geometry of the hollow container, coil(s) design, specifications of the hard-magnetic elements, including the levitating magnet(s); (3) different approaches to model each physical phenomenon of the transduction mechanisms of energy harvesters: the magnetic field produced by the hard magnetic elements, repulsive magnetic forces, induced voltages, electric currents, electromechanical coupling coefficients and damping forces; (4) agreement between the simulation and experimental tests; (5) electric output, namely power, voltage and current, as well as the excitation patterns and resistive loads used for model validation purposes; (6) approaches for design optimization.

2.3 Assessment of other literature reviews

To our knowledge, four review papers were published on the scope of electromagnetic energy harvesting from vibration sources that refer to research findings obtained using magnetic levitation architectures. Harb [3] only reports the working principle of a single magnetic spring generator (proposed by Saha *et al.* [64]) and its ability to harvest energy during human-induced motion. Wei and Jing [65] presented a review that includes theory, modelling methods and validation of piezoelectric, electromagnetic and electrostatic harvesters, but only mentioned the research findings of Mann and Sims [66] and the ability of magnetic levitation harvesters to operate in a wide range of vibration frequencies. The review paper published by Yildirim *et al.* [67] describes only the study of Mann and Sims [66], without carrying out an exhaustive and comprehensive analysis to the overall architectures already proposed in the literature. Siddique *et al.* [5] only refer a study whose architecture comprises magnetic levitation [68], but the energy transduction mechanism includes the piezoelectric principle. Therefore, no review that presents major breakthroughs achieved on the scope of magnetic levitation architectures has been published so far. Neither comparative nor critical analyses have been done highlighting the design configurations, construction parameters, energy outputs and modelling of transduction mechanisms.

3 Design configurations

3.1 Magnetic levitation architectures

The basic architecture of these energy harvesters comprises a hollow cylindrical container, three permanent magnets and a coil, as illustrated in Fig. 1a. The polarity of the magnets is arranged in a way that the levitating magnet experiences a repulsive force due to the fixed magnets, which are attached to the end extremities of the container. A portion of the container is wrapped in a multilayer coil around its outer surface [25]. Twenty-one design configurations were already tested using: (i) circular or rectangular containers [66,69]; (ii) cylindrical, ring or block magnets [70–72]; (iii) number of levitating magnets up to six [7]; (iv) number of coil windings up to five [73]; (v) planar or helicoidal coils [74,75]; (vi) levitating magnets with and without guidance systems [6,57]; (vii) levitating magnets in stack arrangements with or without spacers [61,64], and experiencing repulsive or attractive forces [76,77]; (viii) end extremities of containers with fixed permanent magnets or, differently, with springs or bumpers [78,79]; (ix) nonlinear FR4 planar spring anchored to the casing and glued to the top magnet, guiding its motion (use of dual-mass) [80]. According to the number of coils and permanent magnets, these twenty-one designs can be categorized as: (i) single coil and single levitating magnet: 3 configurations (Fig. 1a-c) [25,70,72]; (ii) single coil and multiple levitating magnets: 8 configurations (Fig. 2a-h) [7,13,57,64,76,77,81]; (iii) multiple coils and single levitating magnet: 5 configurations (Fig. 3a-e) [66,78,79,82,80]; (iv) multiple coils and multiple levitating magnets: 5 configurations (Fig. 4a-e) [6,73,75,77,83].

The first category is presented in twelve studies [25,61,69–72,74,84–88]. They designed circular (Fig. 1a,b) and rectangular containers (Fig. 1c) with a helicoidal coil around each outer surface; cylindrical (Fig. 1a), ring (Fig. 1b) and block magnets (Fig. 1c) were used; not only the basic positioning of the fixed magnets was studied (Fig. 1a,b), but also a configuration in which two fixed magnets were mounted in only one end cap of the device (Fig. 1c); guidance systems were also considered (Fig. 1b). When a single coil and multiple levitating magnets are incorporated (second category), seven studies [7,13,57,64,76,77,81] present only circular containers, used cylindrical (Fig. 2a-g) and ring magnets (Fig. 2h), counting up to six levitating magnets (Fig. 2g), with (Fig. 2b-e,h) and without spacers (Fig. 2a,f,g), in which the inertial magnets experienced both repulsive (Fig. 2c-f,h) or attractive forces (Fig. 2a-c,f,g). Two architectures utilize the behaviour of levitating magnets only using a fixed magnet (Fig. 2c,d); and another one was engineered with a guidance system (Fig. 2h). For harvesters embedding a single levitating magnet inside the container and attaching multiple coils (third category), six studies [66,78,79,82,89,80] propose cylindrical containers that include cylindrical (Fig. 3a-c,e) and ring magnets arranged along a shaft (Fig. 3d). These were arranged according to the basic architecture (Fig. 3a,d), or only with a fixed permanent magnet (Fig. 3b,c) or with a design that uses dual inertial mass (Fig. 3e). Two helicoidal coil windings were wrapped in all configurations, included one whose wire is wounded both inside and outside the annular permanent magnet to engineer both the inner and outer coils (Fig. 3d). Some harvesters were designed using the inertial magnet coupled to a spring (Fig. 3b) or two rubber bumpers bonded to the upper and lower lids (Fig. 3c). Besides, new designs were proposed in which the top magnet is freed and a FR4 mechanical spring is used to guide its motion (Fig. 3e). Finally, five studies [6,73,75,77,83] included up to five helicoidal (Fig. 4b) and sixteen planar coils (Fig. 4e), up to four stack magnets (Fig. 4c), and levitating magnets shaped as cylindrical (Fig. 4a-c), ring (Fig. 4d) and planar (2x5 block array) (Fig. 4e). Besides, the analysis was extended to architectures composed by guidance systems (Fig. 4d), and those in which the inertial mass is disposed so that the same poles are facing each other (Fig. 4a-d) with (Fig. 4a-c) and without spacers (Fig. 4d).

3.2 Construction parameters of magnetic levitation harvesters

The most relevant construction parameters that characterize each harvester are shown in Table 1 and illustrated in Fig. 5. Comparison includes geometric and volumetric data from the container and coil properties. The analysed harvesters have container lengths from 20 to 254 mm and respective inner radii between 2.2 and 28.6 mm. The harvesters' stroke has lengths from 13 to 184 mm, although most of them did not exceed 50 mm. Such dimensions correlate with volumes ranging from 0.5 to 235 cm^3, even though most harvesters were designed to exhibit volumes smaller than 17 cm^3. Hence, it is noteworthy to state that no harvesters were already designed for large-scale electric powering. In a significant number of studies (6 out of 29) [25,72,76,79,81,89], the authors selected PTFE polytetrafluorethylene (TEFLON®) to manufacture the container, as this material features low friction coefficients.

Energy harvesting was observed considering coils designed to take up approximately from 10 to 50% of the harvester lengths. As expected, higher induced voltages in the coil terminals are obtained for smaller distances between the inner coil radius and the levitating stack radius; nevertheless, researchers defined these distances in the range from 0.5 to 9 mm, most likely to avoid problems related to the mechanical integrity of the harvesters, or to minimize the complexity of the manufacturing process. A significant range of coil resistances (2.4 - 6191 Ω) and inductances (2.9×10^{-3} - 1.5 H) were also imposed due to the wide range in the number of coil turns (240 - 15000) and copper wire diameters (40 - 635 μm), as shown in Table 1. Notice that 23 out of 29 authors did not report the coil(s) inductance, disregarding their influence on the electric dynamics of the transduction mechanisms. Importantly, several authors (9) did not mention the coil(s) resistance, although it must be indicated as it is required to maximize the self-powering ability according to the maximum power transfer theorem.

The features related to levitating and fixed magnets strongly influence both mechanical and electric dynamics of energy harvesters (as highlighted in section 4.4) [71]. Therefore, detailed data concerning properties of the magnets chosen by each author, namely their mass, geometry (Fig. 5) and magnetization are also reported in Table 2. The inertial mass always includes levitating magnets that vary from 1.2 to 1.54×10^{3} g. Neodymium magnets were chosen by all authors, since they are able to provide strong magnetic fluxes through the coils (up to N45 grade) and ensure a stable magnetic moment during long periods of time. A remarkable amount of studies (22 out of 29) included configurations with cylindrical hard magnets, but ring (4 out of 29) and block (3 out of 29) geometries were also explored.

Table 1

Geometrical and constructive parameters[(a)]

	References	Container				Coil						Figure[(b)]
		l_h [mm]	r_h [mm]	$s_d + s_u$ [mm]	V [cm^3]	l_c [mm]	r_c [mm]	N [turns]	$\varnothing_c$ [μm]	R_i [Ω]	L [H]	
Category 1	Constantinou, Mellor, Wilcox[84]	65.4	ND	27.3	≈40	ND	18.9	476	360	ND	0.01	Fig.1(b)
	Constantinou, Mellor, Wilcox[70]	70.4	ND	32.3	ND	ND	18.9	240	600	2.4	0.0029	Fig.1(b)
	Bernal, García[74]	ND	ND	ND	ND	1000	ND	ND	ND	ND	ND	Fig.1(a)
	Foisal, Hong, Chung [71][(c)]	44/46	3.5	26-30	ND	5	4	1500	ND	ND	ND	Fig.1(a)
	Foisal, Lee, Chung [85]	46	3.5	28	≈7.4	5	4	1500	100	96.5	ND	Fig.1(a)
	Foisal, Chung [86]	48	3.5	32	≈7.4	5	4	1500	100	96.1	ND	Fig.1(a)
	Berdy, Valentino, Peroulis [69]	35	NA[(d)]	≈20.7	7.7	4	NA[(e)]	1000	635	450	ND	Fig.1(c)
	Berdy, Valentino, Peroulis [72]	35	NA[(d)]	≈20.7	7.7	4	NA[(e)]	1000	635	450	ND	Fig.1(c)
	Liu *et al.* [87]	46-66	3.5	36-56	4.4-6.3	10-30	5.5	ND	200	10-20	ND	Fig.1(a)
	Santos *et al.* [25]	58	3.1	50	≈12	20	4.1	15000	68	3630	1	Fig.1(a)
	Kecik *et al.* [61]	ND	ND	ND	ND	ND	ND	ND	ND	2300[(f)]	1.5	Fig.1(a)
	Kęcik [88]	ND	ND	ND	ND	ND	ND	ND	ND	1200	1.5	Fig.1(a)
Category 2	Saha *et al.* [64]	≈55	ND	≈34/ 35	12.7	6	8.5	1000	40	800	ND	Fig.2(b,c)-A
	Dallago, Marchesi, Venchi [76]	≈56	7.5	≈38	≈13	ND	ND	500	110	60	ND	Fig.2(a)-A
	Munaz, Lee, Chung [13]	≈80	5.5	≈35	9	5	6	1000	ND	115	ND	Fig.2(f)-R
	Masoumi, Wang [57]	254	28.6	101.6	≈235	101.6	31.2	10186	143	6191	ND	Fig.2(h)-R
	Wang *et al.* [7]	≈144/ 204	10.3	≈112 -184	≈71 /100	60	12.5	480	500	5	ND	Fig.2(f,g)-A
	Pancharoen, Zhu, Beeby [81]	≈20	2.2	≈13	≈0.5	ND	2.5	1100	50	236	ND	Fig.2(e)-R
	Struwig, Wolhuter, Niesler [77]	111.8 -121	5.5	93.8- 95	15.1- 15.7	6-13	6.5	280- 607	127	14-31	ND	Fig.2(c,d)-R
Category 3	Mann, Sims [66]	ND	ND	72.6	ND	ND	ND	ND	ND	188	ND	Fig.3(a)
	Bonisoli *et al.* [78]	ND	ND	ND	ND	ND	ND	ND	ND	ND	ND	Fig.3(c)
	Morais *et al.* [79]	ND	3.3	ND	3.8	24	ND	9600	63	ND	ND	Fig.3(b)
	Morgado *et al.*[89]	78.5	ND	49.5	≈16	26	5	10000	ND	ND	ND	Fig.3(a)
	Yang *et al.* [82]	28	ND	16	ND	ND	15[(g)]	2×600	ND	ND	ND	Fig.3(d)
	Aldawood,[(h)] Nguyen, Bardaweel [80]	100.8	6.6	72.2	≈220	13.7	8.7	450	80	≈93	ND	Fig.3(e)
Category 4	Saravia, Ramírez, Gatti [6]	≈110	8	≈65	≈74	ND	10.3	1850	240	ND	ND	Fig.4(c)-R
	Geisler *et al.* [73][(i)]	50	4.3	32	9	5×5	4.9	5×2800	57	5×950	ND	Fig.4(b)-R
	Apo, Priya [83]	ND	5.3	ND	6.2	9.6	5.5	5600	44	2800	ND	Fig.4(d)-R
	Zhang, Wang, Kim [75]	200	ND	ND	120	NA	NA	16[(j)]	ND	ND	ND	Fig.4(e)
	Struwig, Wolhuter, Niesler [77]	120.7	5.5	102.7	16.8	19.4	6.5	910	127	46	ND	Fig.4(a)-R

[(a)] Terminology: NA - not applicable; ND - not defined.
[(b)] Magnet interaction of the inertial stack: A-attracting; R-repelling.
[(c)] This paper studied 2 configurations, each with an array of four different generators. Their parameters are presented individually.
[(d)] $\approx 3.5 \times 25.7$.
[(e)] $\approx 6.8 \times 29$.
[(f)] $R_i + R_l$
[(g)] Wires were wounded both inside and outside of the annular permanent magnet. The radii of the inner coils (inside and outside) are 3.5 and 4.5 mm.
[(h)] Top coil parameters: l_{ct}=12.7 mm; r_{ct}=26.7 mm; N_t=1500 turns; $R_i \approx 890\ \Omega$.
[(i)] Coils winding are considered as independent.
[(j)] Number of planar coils wounded over the boundaries between the magnets of the 2×5 magnet array.

Table 2

Features of levitating and fixed magnets(a)

	References	Nr. magnets(b)	m [g]	Dimensions [mm] $r_m \times l_m$	$r_u \times l_u$	$r_d \times l_d$	$r_s \times l_s$	Sat. magnetization [kA/m] M_m	M_u	M_d
Category 1	Constantinou, Mellor, Wilcox[84]		38	12.7/6 ×12.7	12.7/ND×12.7		NA		(N38)	
	Constantinou, Mellor, Wilcox[70]		38		12.7/6×12.7		NA		(N38)	
	Bernal, García[74]		ND	6.5×5	ND	ND	NA	1.15×10^3	ND	ND
	Foisal, Hong, Chung [71]	1	2.5-3.4	3×16 3×12 3×14 3×16	1.5×1 1×2 1×2 1×2	0.5×1 1×2 1×2 1.5×2	NA		(N35)	
	Foisal, Lee, Chung [85]		2.9	3×14	1×2		NA		(N35)	
	Foisal, Chung [86]		2.93	3×14	0.5×1	1×2	NA		(N35)	
	Berdy, Valentino, Peroulis [69]		7.7	NA(c)	NA	NA(d)	NA		(N42)	
	Berdy, Valentino, Peroulis [72]									
	Liu *et al.* [87]		1.2	3×6	1.5×2		NA		860 (N35)	
	Santos *et al.* [25]		1.2	3×6	3×1		NA	800	761	761
	Kecik *et al.* [61]		98	ND	ND	ND	NA		ND	
	Kęcik [88]		90	ND	ND	ND	NA		ND	
Category 2	Saha *et al.* [64]	2	27	7.5×8	5×1		7.5×3		ND	
	Dallago, Marchesi, Venchi [76]	2	20.9	7.5×8	5×1		NA		ND	
	Munaz, Lee, Chung [13]	3	11.5	5×10	5×5	5×10	NA		ND	
	Masoumi, Wang [57]	3	1539		25.4/3.2×25.4		25.4/3.2 ×12.7		1.03×10^3 (N42)	
	Wang *et al.* [7]	3/6	31.5/ 63		10×4		NA		890 (N35)	
	Pancharoen, Zhu, Beeby [81]	2	ND		2×1		2×ND[i]		868 (N35)	
	Struwig, Wolhuter, Niesler [77]	2/3	ND	5×5	NA	5×5	5×3		(N35)	
Category 3	Mann, Sims [66]		19.5	ND	ND	ND	NA		ND	
	Bonisoli *et al.* [78]		ND	ND	NA	ND	NA		ND	
	Morais *et al.* [79]	1	4.4	3×24	NA	2×2	NA		(N42)	
	Morgado *et al.*[89]		9.5	4×24	2×2	3×3	NA		ND	
	Yang *et al.* [82]		29	12/5 ×10	12/5×1		NA		ND	
	Aldawood, Nguyen, Bardaweel [80]		18.1	6.4 ×19.1	6.4×4.8		NA		(N42)	
Category 4	Saravia, Ramírez, Gatti [6]	4	48		7.5×5				(N37)	
	Geisler *et al.* [73]	3	5.7	4×4	1×1	2×1	3.5×2		(N45)	
	Apo, Priya [83]	3	22.4	4.8/1.6 ×4.8	3.2×0.8		NA		(N42)	
	Zhang, Wang, Kim [75]	10	40	6.4× ND×3.2	NA	ND	NA		ND	

(a) Terminology: NA - not applicable; ND - not defined.
(b) Number of inertial magnets.
(c) 25.4×3.18×12.7.
(d) 3.18×3.18×1.6.

4 Review of transduction mechanisms models

4.1 Overall analysis of modelling approaches

The interest in modelling transduction mechanisms of electromagnetic harvesters with magnetic levitation architectures emerges from the need and complexity to design them highly efficient. It is true that the amount of mechanical energy surrounding us, which is ready to be transduced into electricity, is significant [8,9]. Nevertheless, their performance optimization is problematic because: (i) geometric optimization prior to fabrication is hard to carry out as these harvesters present highly non-linear behaviour [25]; (ii) architecture adaptability is required to effectively address the issue of unknown, broadband and time-varying behaviour of mechanical power sources [70]; (iii) dimensional constraints must be considered for practical usability [69,77]. Modelling the energy transduction is then mandatory to ensure their superior performance over other motion-driven harvesters [25].

Twenty-nine different models were developed to intelligibly describe the most relevant physical phenomena occurring on the twenty-one proposed design configurations (Table 3). Most authors focused on modelling the magnetic field from levitating magnets, repulsive magnetic forces between permanent magnets, induced electromotive force from the relative motion between coil and levitating magnet(s), electric current, electromechanical coupling coefficient, mechanical friction and damping forces, as presented in Table 3. The overall dynamic behaviour of the energy transduction systems, which includes mechanical and electric dynamics, was always modelled by time-dependent ordinary differential equations. Four modelling approaches were distinguished to model each physical phenomenon: (i) empirical, which is based on parametric equations that fit experimental data; (ii) analytical, focused on physical laws, comprising parameters with physical meaning, to explain physical mechanisms under experimental observation; (iii) semi-analytical, established as a set of analytic approximations to describe the same physical mechanisms; (iv) and finite element method (FEM), a numerical technique used to perform finite element analysis (FEA) of a physical phenomenon. Almost all authors conducted hybridizations; differently, Bonisoli *et al.* [78] and Foisal *et al.* [71] developed purely analytical models. Six hybrid patterns were established, all of these including some type of analytical modelling, three comprising semi-analytical modelling, three with FEM modelling, and, surprisingly, three studies used empirical models (Table 3). The most common encloses three modelling approaches: the analytical-empirical-FEM pattern, counting with eight models [6,7,57,64,76,77,81,83], was the most considered one; however, the analytical-semi-analytical-empirical [69,70,72,89] and analytical-semi-analytical-FEM [13,73,75,82] hybridizations were also frequently used. Besides, three hybrid patterns were proposed using two modelling approaches: analytical-empirical [61,66,79,87,88], analytical-FEM [74,85,86] and analytical-semi-analytical [25,84]. These findings reveal that numerical analysis is more often used than analytical analysis and emphasize: (i) lower computational efficiency for optimization purposes, since it requires suitable identification of many parameters related to the design of coil(s), container and magnet(s); (ii) overall energy transduction mechanisms remain partially unclear. Nevertheless, even though only a qualitative evaluation of the accuracy of the models was carried out, very good approximations were achieved both for electric and mechanical dynamics (Table 3). It is noteworthy to recognize the results achieved by Mann and Sims [66], Constantinoub *et al.* [70], Berdy *et al.* [72], Soares dos Santos *et al.* [25] and Wang *et al.* [7]. Although few studies reported quantitative analyses, simulation-experiment agreements with deviations lower than 15% and 16% for voltage and power harvesting, respectively, as well as 5.2% for the inertial motion of levitating magnets have already been obtained (Table 3) [7,72,81]. About half of the studies (14 out of 29) did not report an overall electric or mechanical model validation. Concerning model validation related to the overall mechanical dynamics, Mann and Sims [66] were the only authors that investigated the inertial magnet dynamics; all the remaining authors studied the harvester resonant frequency under open circuit conditions [64,69,72,81,84]. The experimental validation of the magnetic field was only observed in three studies; more significantly, the repulsive magnetic forces and induced voltage were validated by eleven and nine studies, respectively [66,73,72]. Of all studies assessed, only Constantinou *et al.* [70] and Kecik *et al.* [61] obtained a good match between measured and calculated electromagnetic coupling coefficients. So far, the electric current, damping coefficients and forces were not validated. Furthermore, no modelling approaches under tri-dimensional motions were validated up to date. Only two-dimensional analyses were carried out on the scope of magnetic levitation architectures. In the next sections we describe how authors modelled the most relevant physical phenomena of the transduction mechanisms of these harvesters. Due to the significant number of models analysed and considering the impossibility to present all simulation results, we provide the ones of one of the most significant studies that reported a very good simulation-experiment agreement (Fig. 6) [25].

Table 3

Overall analysis of the modulation approaches$^{(a)}$

	References	Magnetic field	Repulsive magnetic force	Induced voltage	Electric current	Coupling coeff.	Damping coeff./forces	Overall model validation	Model validation related to each phenomenon
Category 1	Constantinou, Mellor, Wilcox[84]	ND	SA/A	A	A	SA	ND	Electric: ND; mechanical: resonant frequencies with average error of 1.6% for 3 different l_h.	MF: ND; RMF: ND; V: good approximation (OC); I: ND: CC: ND; DF: ND.
	Constantinou, Mellor, Wilcox[70]	SA	SA/ A/E	A	A	SA	A/E	Electric: power (2.1g, 37 Hz, ΔR_l) presents slight differences; mechanical: ND.	MF: ND; RMF: good approximation; V: ND; I: ND; CC good approximation: ND; DF: ND.
	Bernal, García[74]	A	A/ FEM	A	A	NA	A	ND	MF: good approximation; RMF: ND; V: ND; I: ND; DF: ND.
	Foisal, Hong, Chung [71]	A	A	A	ND	A	A	ND	ND
	Foisal, Lee, Chung [85]	FEM	ND	A	ND	NA	ND	ND	ND
	Foisal, Chung [86]	FEM	A	ND	ND	NA	ND	ND	MF: ND; RMF: ND; V: considerable differences (OC); I: ND; DF: ND.
	Berdy, Valentino, Peroulis [69]	A	A	A	A	SA	A/E	Electric: power with very good agreement (0.1g/0.075g, ΔR_l); mechanical: resonant frequency (0.05g) presents an error of ≈3%.	MF: ND; RMF: ND; V: ringdown test accurately correlated (OC); I: ND; CC: ND; DF: ND.
	Berdy, Valentino, Peroulis [72]	A	A	A	A	SA	A/E	Electric: average power was predicted within 14% accuracy errors; mechanical: resonant frequency with errors of 4.48%, 4.54% and 5.17% for vertical apparatuses using rotations of 15° and 30°.	MF: ND; RMF: ND; V: ringdown tests (OC) for the 3 developed devices under different apparatus, as well as under customized excitation (R_l=1000 Ω, Δf) exhibit very good agreement; I: ND; CC: ND; DF: ND.
	Liu *et al.* [87]	A	A	A	A	A	A/E	ND	MF: ND; RMF: ND; V: errors between 3.7% and 20% (0.5g-0.65g, 4.2-4.8 Hz); I: ND; CC: ND; DF: ND.
	Santos *et al.* [25]	SA	SA	SA	A	NA	A/SA	Electric: mean absolute percentage error of 6.02% and cross-correlations higher than 86%; mechanical: ND.	ND

Table 3

Continuation

	References	Magnetic field	Repulsive magnetic force	Induced voltage	Electric current	Coupling coeff.	Damping coeff./forces	Overall model validation	Model validation related to each phenomenon
Category 1	Kecik *et al.* [61]	ND	E/A	A	A	E/A	E	ND	MF: ND; RMF: good approximation; V: ND; I: ND; CC: very good approximation; DF: ND.
	Kęcik [88]	ND	E/A	A	A	E	E	ND	MF: ND; RMF: good approximation; V: ND; I: ND; CC: ND; DF: ND.
Category 2	Saha *et al.* [64]	FEM	FEM/A	A	ND	ND	A/E	Electric: ND; mechanical: resonant frequency (OC, 0.039g) presents an 5% error.	MF: ND; RMF: very good approximation; V: ND; I: ND; CC: ND; DF: ND.
	Dallago, Marchesi, Venchi [76]	FEM	FEM/A	A	A	A	A/E	Electric: the induced voltage (1g, 9/10.4 Hz, ΔR_l) presents maximum error: 20/80% without Lorentz force and (7/6%) considering the Lorentz force; mechanical: ND.	MF: ND; RMF: good approximation; V: peak voltage with an average error of 10.4% (1g, 7-12 Hz); I: ND; CC: ND; DF: ND.
	Munaz, Lee, Chung [13]	SA/FEM	ND	A	ND	NA	ND	ND	MF: ND; RMF: ND; V: RMS OC voltage (0.5g, 6 Hz) error is 5.7%; I: ND: DF: ND.
	Masoumi, Wang [57]	FEM	E/A	A	ND	A	A/E	ND	MF: ND; RMF: very good approximation; V: ND; I: ND; CC: ND; DF: ND.
	Wang *et al.* [7]	FEM	E/A	A	A	ND	A/E	Electric: voltage response for forward freq. sweeps point out maximum relative errors lower than 15% while the response for reverse freq. sweeps shows slight differences compared to simulations; mechanical: ND.	MF: ND; RMF: good approximation; V: ND; I: ND; CC: ND; DF: ND.
	Pancharoen, Zhu, Beeby [81]	A/FEM	A/FEM	A	ND	A	A/E	Electric: power error (0.5g) lower than 16%; mechanical: resonant frequency (0.5g) presents errors lower than 2.3%.	MF: errors between 0.5% and 14.6%; RMF: good approximation; V: OC voltage with errors lower than 8.5%; I: ND; CC: ND; DF: ND. Height of the quiescent position with errors in the range 10.3 - 21.1%.
	Struwig, Wolhuter, Niesler [77]	ND	A/FEM	A/FEM	A	NA	E	ND	MF: ND; RMF: very good approximation; V: ND; I: ND; DF: ND.
Category 3	Mann, Sims [66]	ND	E/A	A	A	A	A/E	Electric: ND; mechanical: for excitation that caused hysteresis, the highest velocities of the middle magnet were not achievable (the remaining experimental data matched the analytical solution).	MF: ND; RMF: very good approximation; V: ND; I: ND; CC: ND; DF: ND.
	Bonisoli *et al.* [78]	ND	ND	A	A	A	A	Electric: power and induced voltage were well approximated; mechanical: ND.	ND

Table 3

Continuation

	References	Magnetic field	Repulsive magnetic force	Induced voltage	Electric current	Coupling coeff.	Damping coeff./forces	Overall model validation	Model validation related to each phenomenon
Category 3	Morais *et al.* [79]	E	A	A	ND	A	A/E	Electric: power and induced voltage (0.09g-0.55g, 0.75-1.85 Hz, ΔR_l) exhibit slight differences; mechanical: ND.	ND
	Morgado *et al.*[89]	SA	SA	A	ND	NA	E	ND	MF: ND; RMF: ND; V: OC free fall test from a height of 15 mm shows good agreement with simulation results; I: ND; DF: ND.
	Yang *et al.* [82]	FEM	A	SA	ND	NA	ND	ND	ND
	Aldawood, Nguyen, Bardaweel [80]	A	SA FEM	SA	ND	NA	ND	Electric: ND; mechanical: forward and backward frequency responses (OC, 0.3g/0.5g, 5-15 Hz) were well approximated.	MF: ND; RMF: good approximation; V: very good approximation; I: ND; DF: ND.
Category 4	Saravia, Ramírez Gatti [6]	FEM	E/A	A	A	A	A/E	Electric: power and induced voltage (1g, R_l=100 Ω, Δf) were well approximated; mechanical: ND.	MF: ND; RMF: very good approximation; V: ND; I: ND; CC: ND; DF: ND.
	Geisler *et al.* [73]	FEM	SA	A	A	NA	A	Electric: power (1g, R_l=950 Ω, Δf) was well approximated; mechanical: ND.	MF: good approximation; RMF: ND; V: ND; I: ND; DF: ND.
	Apo, Priya [83]	FEM	A	A	A	A	A/E	ND	ND
	Zhang, Wang, Kim [75]	SA/ FEM	A	A	ND	NA	A	ND	ND

[(a)] Terminology: NA - not applicable; ND - not defined; NV - non validated; A - analytical; SA - semi-analytical; E - empirical; FEM - Finite Element Mfethod; MF - magnetic field; RMF - repulsive magnetic force; V - induced voltage;
I - electric current; CC - coupling coefficient; DF - damping coefficients/forces; Δvar - variable defined in a specific range.

4.2 Modelling the inertial mass dynamics

The overall dynamic behaviour of energy transduction systems, that include mechanical and electric dynamics, has always been modelled by time-dependent ordinary differential equations. Table 4 summarizes the governing equations that express the mechanical dynamics of the motion-driven inertial mass, which can comprise levitating magnets and spacers in between. All authors modelled this dynamics using the Newton's second law as a result of a sum of forces along one degree of freedom, usually the vertical direction, although some inclinations were also imposed, and horizontal tests were also carried out [7,72]. Most models include the effects due to the inertia force of the moving magnet(s)-spacer(s), the damping coefficient, the repulsive (levitation) force between the moving mass and the fixed magnets, and the gravitational force. A couple of studies detailed the damping coefficient as a combined effect of both electrical and mechanical damping [7,57,66,79,87]. Some authors modelled the electrical damping with an electromechanical coupling coefficient (as described in Table 9), but more complex models considered the Lorentz force to include the magnetic force opposing the inertial motion of the levitating magnet(s) caused by the electric current flowing through the coil(s) [25,74]. A system reduction to a Duffing oscillator under both static and dynamic loads was also proposed by modelling the repulsive forces between magnets as a polynomial function (as explored in Table 6) [66]. Although the Runge-Kutta method was used by some authors to numerically compute the inertial mass dynamics [6,80], most studies did not clarify which numerical method were used to compute the proposed models.

Table 4

Mechanical dynamics of inertial masses

Equations	References
$-m\ddot{z} = m\ddot{y} + c\dot{y} + F_m$	[7,13,57,64,71,73,77,79,84,85,87]
$-m\ddot{z} = m\ddot{y} + c\dot{y} + F_m + mg$	[6,61,66,70,75,76,81,83,86,88,89,80]
$-m\ddot{z} = m\ddot{y} + (c_e + c_p)\dot{y} + F_{fr} - F_m + mg$	[69,72]
$m\ddot{z} = -m\ddot{y} + F_{lz} - F_{fr} + F_m$	[74]
$-m\ddot{z} = m\ddot{y} + F_{lz} + F_{fr} + F_m + mg$	[25]
$-F = -m\ddot{y} + F_{fr} + F_{bx} + F_m$	[78]

4.3 Modelling the magnetic field of permanent magnets

The magnetic field (B) variation is required to drive the mechanical-electric transduction mechanism. It was modelled by analytical, semi-analytical and FEM approaches. Three different methods have been proposed to model this phenomenon (Table 5): (i) B is constant along the tri-dimensional space enclosed by the harvester, not computed using the magnet characteristics (equation Q1) [66,79] or computed by analytic approximations (equation Q2) [71,87]; (ii) B as a single variable function defined on a set of axial distances to the centre of the levitating hard-magnet (equations Q3 and Q5) [69,72,81]; and (iii) B described as a two-variable function of radial and axial distances to the centre of the levitating magnet (equations Q6 to Q9) [25,70,74,89].

Researchers who simulate the behaviour of this physical phenomenon by FEM used ANSIS 2D [13,83,85,86], COMSOL Multiphysics [73,75,81], Finite Element Method Magnetics [6,57], Flux 2D [76] and ANSOFT Maxwell [7]. Two studies did not specify the software [64,82] and some did not describe how the models were designed, namely their domains, electric and magnetic properties of materials, 3D mesh characterization and convergence test analyses. Apo and Priya [83] reported the use of 96 mesh elements.

Foisal *et al.* [71] studied this phenomenon based on the vector potential model, which in turn is based on the molecular current approach as the magnetization vector is assumed to be in line with the axial direction. They characterized the model by subdividing the magnetic field in four domains and used them to compute an overall mathematical expression that relates the coil inner radius with the levitating magnet radius and length (equation Q2). The model proposed by Liu *et al.* [87] is quite similar to the previously mentioned study (Q2), but it was obtained by the line-charge model. Additionally, Pancharoen *et al.* [81] estimated the magnetic flux density produced by a cylindrical magnet using a technique that calculates the magnetic field of the whole magnet integrating, over its volume, the contributions from the infinitesimal dipoles, and relates the radius and length of the levitating magnet as presented by equation Q3. Identically to equation Q3, Aldawood *et al.* [80] were the only researchers that approached this phenomenon along the longitudinal axis by adopting the magnetic dipole moment of the levitating magnet (equation Q4). Berdy *et al.* [69,72] studied the interaction between two parallelepiped magnets using analytical calculations based on the Coulombian or equivalent charge model. They considered two charged parallel surfaces carrying two distinct densities to infer the interaction energy and integrated the result into four spatial domains (equation Q5). Constantinou *et al.* [70] modelled this phenomenon as the superposition of the magnetic field from two equivalent current sheets representing the outer and inner surfaces of the ring magnet, as considered in equation Q6. Morgado *et al.* [89] modelled the magnetic field based on the principle that a cylindrical permanent magnet (axially polarized) creates a similar magnetic field when an electric current flows through a thin wall solenoid of same diameter and height with constant current density. Equation Q7 is presented in cylindrical coordinates and exposes the sum of the magnetic field through every cylindrical current sheets of each coil (the number of turns, their length and thickness are considered). Bernal and García [74] considered the magnetostatic problems with cylindrical symmetry and modelled the magnetic potential using Bessel functions (equation Q8). Finally, Soares dos Santos *et al.* [25] used the equivalent surface current model that discretizes the magnet into a finite set of current loop elements and then superimposed the resultant magnetic field of each layer (equation Q9). Fig. 6a illustrates how the magnetic field behaves in the tri-dimensional Cartesian space according to the model proposed by Soares dos Santos *et al.* [25].

Table 5

Equations of magnetic field

	Equations	References
Q1	$B = \mathrm{c}^{\mathrm{te}}$	[66,79]
Q2	$B = \frac{B_r}{2}\left[\frac{r_c - r_m + l_m}{\sqrt{{r_m}^2 + (r_c - r_m)^2}} - \frac{r_c - r_m}{\sqrt{{r_m}^2 + (r_c - r_m)^2}}\right]$	[71,87]
Q3	$B(y) = \frac{B_r}{2}\left[\frac{y + l_m}{\sqrt{(y + l_m)^2 + {r_m}^2}} - \frac{y}{\sqrt{y^2 + {r_m}^2}}\right]$ $B_r \approx \mu_0 H$	[81]
Q4	$B(y) = \frac{\mathrm{m}_1 \mu_0}{2\pi {r_m}^2 l_m}\left(\frac{l_m - 2y}{\sqrt{4{r_m}^2 + (l_m - 2y)^2}} + \frac{l_m + 2y}{\sqrt{4{r_m}^2 + (l_m + 2y)^2}}\right)$	[80]
Q5	$B = \sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{k=0}^{1}(-1)^{i+j+k}\frac{M_m}{4\pi\mu_0}\arctan\left[\frac{(x - x_i)(y - y_i)}{(z - z_k)\Gamma_{ijk}}\right]$ $\Gamma_{ijk} = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}$	[69,72]
Q6	$B = \nabla \times A_m$ $A_{m-p}(r_p, y_p) = \int_{y_p - l_m}^{y_p + l_m}(A_{out-t}(r_p, y) + A_{in-t}(r_p, y))dy$	[70]
Q7	$B(y, r) = 2\pi \int \left(\frac{N_u}{l_u(r_{e_u} - r_{i_u})}\int_{r_{i_u}}^{r_{e_u}}\int_{y_{u_b}}^{y_{u_t}} rA_\varphi(y, r)dydr - \frac{N_d}{l_d(r_{e_d} - r_{i_d})}\int_{r_{i_d}}^{r_{e_d}}\int_{y_{d_b}}^{y_{d_t}} rA_\varphi(y, r)dydr\right)rdr$ $A_\varphi(y, r) = \frac{M_m}{2}\sqrt{\frac{r_m}{r}}\left[\sigma k\left(\frac{k^2 + h^2 - h^2 k^2}{h^2 k^2}K(k^2) - \frac{1}{k^2}E(k^2) + \frac{h^2 - 1}{h^2}\Pi(h^2 \vert k^2)\right)_{\sigma-}^{\sigma+}\right]$ $h^2 = \frac{4r_m r}{(r_m + r)^2}; \quad k^2 = \frac{4r_m r}{(r_m + r)^2 + \sigma^2}; \quad \sigma\pm = y \mp \frac{l_m}{2}; \quad [F(\sigma)]_{\sigma-}^{\sigma+} = F(\sigma+) - F(\sigma-)$ $K(s) = \int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{1 - s\sin^2(\theta)}}\mathrm{d}\theta; \quad E(s) = \int_0^{\frac{\pi}{2}}\sqrt{1 - s\sin^2(\theta)}\mathrm{d}\theta; \quad \Pi(s \vert t) = \int_0^{\frac{\pi}{2}}\frac{1}{(1 - s\sin^2\theta)\sqrt{1 - t\sin^2\theta}}\mathrm{d}\theta$	[89]
Q8	$B(r, y) = \mu M_y(r, y - y_m) + \frac{\mu M_m r_m}{2}\int_0^\infty(e^{-k\vert y - y_m - l_m\vert} - \mathrm{sign}(y - y_m)\mathrm{sign}(y - y_m - l_m)e^{-k\vert y - y_m\vert})J_1(kr_m)J_0(kr)dk$ $M(z) = M_y(z)\hat{y} = M_m\prod(r; r_m)\prod(y, l_m)\hat{y}$	[74]
Q9	$B(y, r) = \mu_0\frac{M_m}{2\pi}\int_{z+y-l_m/2}^{z+y+l_m/2} f(y, \dot{y})[Z_t(y, \dot{y})E(k) + K(k)]\mathrm{d}\dot{y}$ $E(k) = \int_0^{\frac{\pi}{2}}\sqrt{1 - k^2 sin^2(\phi)}\mathrm{d}\phi; \quad K(k) = \int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{1 - k^2 sin^2(\phi)}}\mathrm{d}\phi; \quad f(y, \dot{y}) = \frac{1}{\sqrt{(r_h + r)^2 + (y - \dot{y})^2}}$ $k = \sqrt{\frac{r_h 4r}{(r_h + r)^2 + (y - \dot{y})^2}}; \quad Z_t = \frac{r_h^2 - r^2 - (y - \dot{y})^2}{(r_h - r)^2 + (y - \dot{y})^2}$	[25]

4.4 Modelling the repulsive magnetic force

The total repulsive magnetic force is given by the sum of the repulsive magnitude forces that keep the inertial magnetic mass in levitation. It can be computed by the sum of the repulsive forces of the top and bottom fixed magnets (Fig. 1a,b; Fig. 2a,b,e-h; Fig. 3a,d; Fig. 4b-d). All developed models are illustrated in Table 6. The restoring force and, therefore, the resonance frequency of the system, can be adjusted by changing the inertial mass, the stroke, the grade of the permanent magnets or by incorporating fixed magnets only in one end cap of the container (instead of using magnets at both ends). Most studies (19 out of 29) computed the force as a function of the positioning of the inertial magnet(s) by searching the polynomial coefficients of power series by curve fitting (equations Q10 to Q13). This phenomenon was analytically analysed [13,71,79,80,82,83,85–87] or combining: (i) empirical-analytical techniques fitting with experimental measurements [6,7,57,61,66,88]; (ii) FEM-analytical techniques fitting with numeric results [64,76,81]; and (iii) semi-analytical-analytical techniques, using the Ampère's force law [70,84]. Equation Q14 analytically describes the magnetic force relating the saturation magnetization of the top, moving and bottom magnets, and the distances between the moving and fixed magnets [71,87]. Zhang *et al.* [75] approached this phenomenon for small vibrations near to the initial position of the inertial mass, as described by equation Q15, which was obtained by fitting with values obtained by numerical simulation. The analytical-FEM approach proposed by Struwig *et al.* [77] (equation Q16) was established by creating a modified version of Coulomb's law by introducing a parameter to better approximate the curve for small motions of the levitating magnet(s). A constant parameter showing a second order relationship with magnetization was formulated by Pancharoen *et al.* [81] to model the bottom repulsive force (equation Q17). Yang *et al.* [82] conducted a study of this phenomenon taking into consideration the air-gap flux, the permeance between the moving and fixed magnets, and the principle of continuity (equation Q18). The described formulation proposed in equation Q19 by Apo and Priya [83] relates magnetization with height of the magnets, surface area common to both magnets and distance between top and levitating hard-magnetic elements. The authors that proposed equations Q14, Q17, Q18 and Q19 did not provide the deduction method for their mathematical formulations. Constantinou *et al.* [70,84] estimated this parameter between two annular magnets by determining the forces between equivalent current sheets using the

Ampère's law of force (equation Q20). The force between two permanent rectangular magnets was also modelled using the Coulomb model, which consider an equivalent magnetic charge density on their surfaces (equation Q21) [69,72]. Morgado *et al.* [89] modelled this phenomenon combining the Coulomb approach and the Amperian current model (equation Q22). Equation Q23 was deduced by Aldawood *et al.* [80] by approximating the levitated magnet as three dipoles. The total magnetic force acting on the levitated magnet was computed as the gradient of the dot product between the surrounding magnets' magnetic field and the levitated magnet's dipole moment. The semi-analytical approaches using the first order Bessel function (equations Q24 and Q25), illustrated by Fig. 6b, were deduced from the derivation of the interaction energy between magnets, assuming their relative positions and considering the magnetization, radius, height and distance separating them [25,73,74].

Similarly to the previous section, most researchers who used FEM did not describe how the models were designed. They only refer that the following softwares were used: COMSOL Multiphysics [74,81], Flux 2D [76] and ANSIS 2D [83] (two studies did not report which softwares were used [64,77]). The exceptions were Bernal and García [74], who used 214992 mesh elements and obtained convergence errors around 50%, as well as Apo and Priya [83] who developed computational models only using 236 mesh elements.

4.5 Modelling the electromotive force

The mechanical energy is converted into electrical energy when the magnetic field changes through the coil, inducing an electromotive force (EMF). Most authors focused their analyses on the analytical methodology to model this physical phenomenon. Hereupon, the equations to model the EMF are detailed in Table 7. The EMF is mainly described by the induction Faraday's law, as modelled by equation Q26 [6,7,13,64,69], or as a function of the electromechanical coupling coefficient and velocity of the levitating magnet(s), as highlighted by the equation Q27 [61,66,70]. The mathematical formulation Q28, proposed by Zhang *et al.* [75], is based on the Faraday's law and depends on the acceleration amplitude, mass of the levitating arrangement, total damping coefficient and time-rate change of the magnetic flux. The open-circuit output voltage was modelled by Pancharoen *et al.* [81] (equation Q29) as

Table 6

Equations of repulsive magnetic force

	Equations	References
Q10	$F_m = ky$	[13,64,70,71,79,82,84–86]
Q11	$F_m = ky + k_3 y^3$	[7,57,61,66,81,83,87,88]
Q12	$F_m = ky + k_3 y^3 + k_5 y^5$	[6]
Q13	$F_m = k_0 + ky + k_2 y^2 + k_3 y^3 + k_4 y^4 + k_5 y^5 + k_6 y^6$	[76]
Q14	$F_m = \frac{\mu_0 M_m}{4\pi} \left[\frac{M_u}{{s_u}^2} - \frac{M_d}{{s_d}^2} \right]$	[64,71,86,87]
Q15	$F_m = \frac{\mu_0 M_u M_d}{4\pi} \left(\frac{1}{{r_0}^2} + \frac{2}{{r_0}^3} y \right)$ $r_0 = \sqrt{\frac{\mu_0 M_u M_d}{4\pi m g}}$	[75]
Q16	$F_m = \frac{\mu_0 M_m M_d}{4\pi y^2 + G}$ **(i)**	[77]
Q17	$F_{m_d} = \frac{3(\pi M_m {r_m}^2 l_m)^2}{8} [\eta(s_d) - \eta(s_d + l_d)]$ $\eta(y) = \frac{1}{y^4} - \frac{{r_d}^2 + 3y^2}{3({r_d}^2 + y^2)^3}$	[81]
Q18	$F_m = \frac{1}{2\mu_0} \left[\frac{B_r (l_m - l_u)}{\mu_0 \mu_r / \Lambda_l + (l_m + l_u)/[\pi({r_d}^2 - {r_u}^2)]} \right]^2 \pi({r_d}^2 - {r_u}^2)$ $\left\{ \frac{1}{\left[\pi({r_d}^2 - {r_u}^2)/s_d + 1.632(r_d - r_u)\right]^2 {s_d}^2} - \frac{1}{\left[\pi({r_d}^2 - {r_u}^2)/s_u + 1.632(r_d - r_u)\right]^2 {s_u}^2} \right\}$ $\Lambda_l = \frac{\pi {r_m}^2 \mu}{(l_m + l_u)(M_m/B_m - 1)}$	[82]
Q19	$F_{m_u} = \frac{\mu_0 {M_m}^2 {A_s}^2 (l_m + {r_u}^2)}{4\pi {l_m}^2} \left[\frac{1}{Su^2} + \frac{1}{(s_u + 2l_m)^2} + \frac{2}{(s_u + l_m)^2} \right]$	[83]
Q20	$F_{m_d} = 2\pi r_{mi} \int_{s_d}^{s_d + l_m} \frac{J dA_{ct_{r_m, r_{mi}}}(y)}{dy} dy + 2\pi r_m \int_{s_d}^{s_d + l_m} \frac{J dA_{ct_{r_m, r_m}}(y)}{dy} dy +$ $+ 2\pi r_m \int_{s_d}^{s_d + l_m} \frac{J dA_{ct_{r_{di}, r_m}}(y)}{dy} dy + 2\pi r_{mi} \int_{s_d}^{s_d + l_m} \frac{J dA_{ct_{r_{di}, r_{mi}}}(y)}{dy} dy$ $\vec{A} = \frac{\vec{a_\phi} \mu_0 J r}{2\pi} \int_{z_1 - \frac{l_m}{2}}^{z_1 + \frac{l_m}{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin(\dot{\phi})}{r_h} d\dot{\phi} d\dot{z}; \quad J = \frac{B_r}{\mu_r \mu_0}$ $\frac{1}{r_h} = \frac{1}{R\sqrt{1 + \frac{r^2}{R^2} - \frac{2r}{R} \sin\theta \sin\dot{\phi}}}$	[70,84]
Q21	$F_m = \frac{M_m M_d}{4\pi \mu_0} \sum_{i=0}^{1} \sum_{j=0}^{1} \sum_{k=0}^{1} \sum_{l=0}^{1} \sum_{p=0}^{1} \sum_{q=0}^{1} (-1)^{i+j+k+l+p+q} \phi_y$ $\phi_y = -uw \ln(r - u) - vw \ln(r - v) + uv \tan^{-1} \frac{uv}{rw} - rw$ $u = u_{ij} = \alpha_c + (-1)^j A_c - (-1)^i a_c; \quad v = v_{kl} = \beta_c + (-1)^l B_c - (-1)^k b_c$ $w = w_{pq} = \gamma_c + (-1)^q C_c - (-1)^p c_c; \quad r = \sqrt{u_{ij}^2 + v_{kl}^2 + w_{pq}^2}$	[69,72]
Q22	$F_{m_u} = -\frac{M_u M_m}{2\mu_0} \sum_{i=1}^{2} \sum_{j=3}^{4} a_{i,j}^{(1)} a_{i,j}^{(2)} a_{i,j}^{(3)} (-1)^{i+j} f_{i,j}^u$ $a_{i,j}^{(1)} = y_i^u - y_j^u; \quad a_{i,j}^{(2)} = \frac{(r_m - r_u)^2}{(a_{i,j}^{(1)})^2} + 1$ $a_{i,j}^{(3)} = \sqrt{(r_m + r_u)^2 + (a_{i,j}^{(1)})^2}; \quad a_{i,j}^{(4)} = \frac{4 r_m r_u}{(r_m + r_u)^2 + (a_{i,j}^{(1)})^2}$ $f_{i,j}^u = K(a_{i,j}^{(4)}) - \frac{1}{a_{i,j}^{(2)}} E(a_{i,j}^{(4)}) + \left(\frac{(a_{i,j}^{(1)})^2}{(a_{i,j}^{(3)})^2} - 1 \right) \Pi \left(\frac{a_{i,j}^{(4)}}{1 - a_{i,j}^{(2)}} \vert a_{i,j}^{(4)} \right)$	[89]
Q23	$F_m = \frac{\mathrm{m}_1 4 {r_m}^2 \mu_0}{3} \sum_{i=1}^{3} \left\{ \frac{\mathrm{m}_2}{\pi {r_u}^2 l_u} \left\{ \frac{1}{\left(4{r_m}^2 + \left[l_u - 2\left(S_u + l_m/2 + l_u/2 + \frac{(i-2) l_m}{4}\right)\right]^2\right)^{3/2}} \right. \right.$ $\left. - \frac{1}{\left(4{r_m}^2 + \left[l_u + 2\left(S_u + l_m/2 + l_u/2 + \frac{(i-2) l_m}{4}\right)\right]^2\right)^{3/2}} \right\}$ $+ \frac{\mathrm{m}_3}{\pi {r_d}^2 l_d} \left\{ \frac{1}{\left(4{r_m}^2 + \left[l_d + 2\left(S_d + l_m/2 + l_d/2 - \frac{(i-2) l_m}{4}\right)\right]^2\right)^{3/2}} \right.$ $\left. \left. - \frac{1}{\left(4{r_m}^2 + \left[l_d - 2\left(S_d + l_m/2 + l_d/2 - \frac{(i-2) l_m}{4}\right)\right]^2\right)^{3/2}} \right\} \right\}$	[80]

(i) As the Coulomb's law provides a poor approximation when the magnets are close to each other (good accuracy is only obtained when they are far enough), a much better fitting to the numerical simulation data is achieved for small distances by including the additional parameter G in the dominator of Coulomb's law [77].

Table 6

Continuation

	Equations	References
Q24	$F_{m_u} = \pi\mu_0 M_m M_u r_m r_u \left[\int\limits_0^\infty (e^{l_m k} - 1) e^{k(y_m - S_u)} \frac{J_1(kr_m)J_1(kr_u)}{k} dk - \right.$ $\left. - \int\limits_0^\infty (e^{l_m k} - 1) e^{k(y_m - S_u - l_u)} \frac{J_1(kr_m)J_1(kr_u)}{k} dk \right]$	[74]
Q25	$F_{m_u} = \mu_0 \pi r_u r_m M_u M_m \int\limits_0^\infty J_1(\varepsilon r_u) J_1(\varepsilon r_m)[e^{-\varepsilon(s_u + l_u)} + e^{-\varepsilon(s_u - l_m)} -$ $- e^{-\varepsilon(s_u)} - e^{-\varepsilon(s_u + l_u - l_m)}]\varepsilon^{-1} \mathrm{d}\varepsilon$ $F_{m_u} = \mu_0 \pi r_u r_m M_u M_m (I_1 + I_2 + I_3 + I_4)$ $I_1 = \int_0^\infty x^m e^{-(s_u + l_m)x} J_1(r_m x) J_1(r_u x) \mathrm{d}x$ $I_2 = \int_0^\infty x^m e^{-(s_u + l_u)x} J_1(r_m x) J_1(r_u x) \mathrm{d}x$ $I_3 = \int_0^\infty x^m e^{-s_u x} J_1(r_m x) J_1(r_u x) \mathrm{d}x$ $I_4 = \int_0^\infty x^m e^{-(s_u + l_m + l_u)x} J_1(r_m x) J_1(r_u x) \mathrm{d}x$	[25,73]

a function of the square of the magnetic flux density and also relates the distances between fixed and levitating magnet(s), radius and height of the magnet(s) experiencing levitation, height of the bottom fixed magnet and mass of the stack. Morais *et al.* [79] proposed model Q30, which is a result of a frequency domain analysis considering the dynamic behaviour of harvesters modelled by linear differential equations. As a result, the inertial mass dynamics is related with the amplitude and frequency of external excitations, the natural frequency of the system, the damping factor and the coupling coefficient. Bernal and García [74] carried out an analysis by considering three distinct space regions, as presented in equation Q31. Assuming the magnet as only comprising seven dipoles, the voltage generated on the middle and top coil, due to the motion of two moving magnets (the levitating one and the FR4 planar spring-guided top magnet) was found by Aldawood *et al.* [80] (equation Q32). The three piecewise-defined cases are dependent on the position of the levitating magnet with respect to the upper and lower region of the coil, and were deduced admitting the magnetic potential in terms of Bessel functions. Yang *et al.* [82] computed the total EMF for the inner and outer coils as the sum of the EMF produced in each single turn, according to model Q33 (induced and motional EMF). Using the Kelvin-Stokes theorem, Soares dos Santos *et al.* [25] obtained an approximate solution by considering the coil as a set of single circular turns and a tri-dimensional surface surrounded by a closed contour defined by each of these turns (equation Q34; Figs. 6d-f). The EMF phenomenon was studied by Geisler *et al.* [73] considering multiple identical and independent coils interacting with the moving magnet (equation Q35). Their mathematical modelling differs from the remaining approaches because they use the magnetic spatial period (distance between the centres of two consecutive coils), the parity phase (distinct value for an even or odd number of alternate stack magnets) and the average flux over the coil thickness.

Table 7

Equations of induced electromotive force

	Equations	References
Q26	$U = -N\frac{d\phi}{dt}$	[6,7,13,57,64,69,71,72,76,77,85,89]
Q27	$U = \alpha\dot{y}$	[61,66,70,78,81,83,84,87,88]
Q28	$U = \lvert\frac{d\phi}{dy}\rvert\frac{m(2\pi f)^2 A_a}{c}$	[75]
Q29	$U = \frac{-NB^2 l r_m{}^2 l_m \pi}{\mu_0}\sqrt{\frac{3A_a(\eta(s_d)-\eta(s_d+l_d)-\eta(s_u)-\eta(s_u+l_d))}{4m\xi}}$ (i)	[81]
Q30	$U = \frac{\alpha A_a(2\pi f)^3}{\sqrt{(k/m-(2\pi f)^2)^2+(2\xi\sqrt{k/m}(2\pi f))^2}}$	[79]
Q31	$U = \begin{cases} -\iota\frac{N}{l_c}\dot{y}\int\limits_0^\infty (e^{l_m k}-1)\frac{e^{k(y_m+l_c/2)}-e^{k(y_m-l_c/2)}}{k} \\ J_1(r_m k)J_1(r_{mc}k)dk & \text{if } y \geq y_m + l_m \\ -\iota\frac{N}{l_c}\dot{y}\int\limits_0^\infty \frac{e^{-k(y_m+l_m)}(e^{kl_c/2}-e^{-kl_c/2})+e^{k(y_m-l_c/2)}-e^{(y_m+l_c/2)}}{k} \\ J_1(r_m k)J_1(r_{mc}k)dk & \text{if } y_m < y < y_m + l_m \\ -\iota\frac{N}{l_c}\dot{y}\int\limits_0^\infty (e^{-l_m k}-1)\frac{e^{-ky_m}(-e^{-kl_c/2})+e^{kl_c/2})}{k} \\ J_1(r_m k)J_1(r_{mc}k)dk & \text{if } y \leq y_m \end{cases}$ $\iota = \pi\mu_0\frac{N}{l_c}M_m r_m r_{mc}$	[74]
Q32	$U = \frac{2Nr_c\mu_0}{l_c}\left\{\frac{\mathrm{m}_1\dot{y}}{7}\sum_{i=1}^{7}\left\{\frac{1}{\left\{\left[l_c-2\left(S_d+l_m/2+l_d/2-\frac{y_{d_t}+y_{d_b}}{2}-\frac{(i-4)l_m}{8}\right)\right]^2+4r_c{}^2\right\}^{3/2}} - \frac{1}{\left\{\left[l_c+2\left(S_d+l_m/2+l_d/2-\frac{y_{d_t}+y_{d_b}}{2}-\frac{(i-4)l_m}{8}\right)\right]^2+4r_c{}^2\right\}^{3/2}}\right\} + \mathrm{m}_2\dot{y}_t\left\{\frac{1}{\left\{\left[l_c-2\left(l_h-l_u/2-l_d/2-\frac{y_{d_t}+y_{d_b}}{2}\right)\right]^2+4r_c{}^2\right\}^{3/2}} - \frac{1}{\left\{\left[l_c+2\left(l_h-l_u/2-l_d/2-\frac{y_{d_t}+y_{d_b}}{2}\right)\right]^2+4r_c{}^2\right\}^{3/2}}\right\}\right\}$ $U_{topcoil} = \frac{2N_t r_{ct}\mu_0}{l_{ct}}\left\{\frac{\mathrm{m}_1\dot{y}}{7}\sum_{i=1}^{7}\left\{\frac{1}{\left\{\left[l_{ct}-2\left(S_d+l_m/2+l_d/2-\frac{y_{u_t}+y_{u_b}}{2}-\frac{(i-4)l_m}{8}\right)\right]^2+4r_{ct}{}^2\right\}^{3/2}} - \frac{1}{\left\{\left[l_{ct}+2\left(S_d+l_m/2+l_d/2-\frac{y_{u_t}+y_{u_b}}{2}-\frac{(i-4)l_m}{8}\right)\right]^2+4r_{ct}{}^2\right\}^{3/2}}\right\} + \mathrm{m}_2\dot{y}_t\left\{\frac{1}{\left\{\left[l_{ct}-2\left(l_h-l_u/2-l_d/2-\frac{y_{u_t}+y_{u_b}}{2}\right)\right]^2+4r_{ct}{}^2\right\}^{3/2}} - \frac{1}{\left\{\left[l_{ct}+2\left(l_h-l_u/2-l_d/2-\frac{y_{u_t}+y_{u_b}}{2}\right)\right]^2+4r_{ct}{}^2\right\}^{3/2}}\right\}\right\}$	[80]
Q33	$U = \sum_{i=1}^{N}\dot{y}\left(-\frac{\partial B_{y_i}}{\partial h}\frac{\pi\varnothing_c{}^2}{4}+B_{r_i}l\right)$	[82]
Q34	$U = 2\pi\frac{\partial}{\partial t}\left(\sum_{k=1}^{N_y}\sum_{j=1}^{N_r}\int\limits_0^{r_j}B(r,y_k)r\mathrm{d}r\right)$	[25]
Q35	$U = -2\frac{N}{l_c}\phi_{rmc}\cos\left(\frac{2\pi}{p}y+\alpha_p\right)\times\sin\left(\frac{\pi l_c}{p}\right)\dot{y}$ $\phi_{rmc} = \frac{1}{r_{mc}}\int\limits_{r_c}^{2r_{mc}-r_c}\Phi_{\max}(r)dr$	[73]

(i) η defined in Table 6 (equation Q17).

4.6 Modelling the electric current in the coil

The electric current flowing through the electric circuit is usually deduced using the Ohm's law, as presented in equations Q36 to Q38 (Table 8). Some modelling approaches (Q36 and Q39) disregarded the effects of the coil inductance on the current dynamics. By means of the multiple scales perturbation technique and after introducing a polar form, Mann and Sims [66] were the only authors who provided an analytical solution taken into account the non-linear system dynamics under harmonic excitation. The current that flows through the resistive load was modelled as a relation of proportionality between a sinusoidal function and a variable named 'a' by the authors (equation Q39), which is a parameter that results from an analytical solution.

Table 8

Equations of coil induced current

	Equations	References
Q36	$I = \frac{U}{R_i+R_l}$	[7,69,72,76,77,83,87]
Q37	$\frac{dI}{dt} = \frac{U-I(R_i+R_l)}{L}$	[6,25,61,70,74,84,88]
Q38	$\frac{dI}{dt} = \frac{U-R_iI-u_i}{L}$ (i)	[78]
Q39	$I(t) = -\left(\frac{\alpha}{R_l+R_i}\right) 2\pi f a \sin(2\pi f t - \gamma)^{(\mathrm{ii})}$	[66]

(i) There is a conditioning circuit between R_i and R_l.
(ii) More information about the parameter "a" can be found in the study of Mann and Sims [66].

4.7 Modelling the electromechanical coupling coefficient

The electromechanical coupling coefficient relates mechanical and electrical input and output energies. It was mainly studied using the analytical approach, as highlighted in Table 9. This parameter was usually considered as a constant (equations Q40 to Q41), as a result of an analysis significantly simplified which considers a constant magnetic field throughout the tri-dimensional space enclosed by the harvester. Equations Q40 and Q41 consider the magnetic field and the total coil length. Models Q41 and Q42 introduce the number of coil turns, while Dallago *et al.* [76] use the mean length of the coil turns (equation Q42). Nevertheless, Kecik *et al.* [61] proposed a method to experimentally determinate this coefficient as a function of the non-linear position of the levitating magnet. The model is described as an odd polynomial function of thirteen degree (equation Q43) and is parameterized by the least squares curve-fitting technique. This coefficient was also considered as non-linear by several other authors who obtained it as the sum of the electromagnetic coupling coefficient of each coil turn (equation Q44). This formulation merges with the vector magnetic potential at the coordinates of the coil turn due to all the magnets in the system [69,70,72,84]. Some researchers did not directly model this coefficient and preferred to approach the mechanical-electrical interaction with the Lorentz force [25,74].

Table 9

Electromechanical coupling coefficient

	Equations	References
Q40	$\alpha = Bl$	[71,79,83]
Q41	$\alpha = NBl$	[6,57,66,81,87]
Q42	$\alpha = NB2\pi r_{mc}$	[76]
Q43	$\alpha = a_1 y + a_3 y^3 + a_5 y^5 + a_7 y^7 + a_9 y^9 + a_{11} y^{11} + a_{13} y^{13}$	[61]
Q44	$\alpha = \sum_{i=1}^{N_r} \sum_{j=1}^{N_y} k_{e\_ij}$ $k_{e\_ij} = -2\pi r_{ij} \frac{dA_{ij}}{dz}$, with A defined in Table 6 (Q20)	[69,70,72,84]
Q45	$\alpha = \left(-\frac{d\lambda_1}{dy} + \frac{d\lambda_2}{dy}\right)$ (i)	[78]

(i) λ_1 and λ_2 were not defined.

4.8 Modelling the damping coefficients and forces

Damping forces have significant impact on the overall dynamics (amplitude and frequency) of the motion-driven levitating magnet(s). Table 10 presents the most relevant analytical and semi-analytical models developed so far. The mechanical damping was usually modelled as a function of the mechanical friction due to the physical contact between the moving magnet and the container [25], but other damping sources were also employed, such as air damping [6]. To avoid air compression, the harvesters were commonly designed with tiny holes on the extremities so that the air flux can leave the inner container when the stack moves in it. Model Q46, proposed by Pancharoen *et al.* [81] to determine the damping factor, was experimentally taken by stopping the shaker abruptly in order to record the attenuated response of the harvester. They related the variation of the peak responses over time due to impulse excitations. The damping coefficient in equation Q47 is dependent on the damping factor, the mass of the levitating stack and the natural frequency of the system [71,79]. Saravia *et al.* [6] modelled the damping coefficient introducing the effects of mechanical friction, air damping and electric damping as presented in equation Q48. Differently, the electric damping force is generated only if a current flows through the coil, creating a magnetic field that opposes to motion of the levitating magnet(s). When the coil inductance is neglected, the electrical damping coefficient is mainly described by equation Q49 [7,57,66,69], which relates the square of the electromechanical coupling coefficient with the total resistance of the electric circuit. Similarly to equation Q49, but considering the coil inductance, equation Q50 introduces the approach taken by Constantinou *et al.* [70] and Foisal *et al.* [86]. A more complex model was implemented by Geisler *et al.* [73]. In fact, these authors developed a location-dependent approach in which some coils are equally affected by the induction, depending on the position of the inertial levitating magnet(s). This approach was considered because an effective number of independent multi-turn coils was included, which correspond to the coils close to the moving magnets for the periodic model (equation Q51). As the mechanical damping factor is harvester-dependent, Dallago *et al.* [76] estimated its value using a relation between the natural frequency of the harvester and a quality factor, indirectly including the non-linearities of the system as described by equation Q52. It is noteworthy to emphasize that, although the electric damping was usually modelled as a function of the coupling coefficient, Soares dos Santos *et al.* [25] and Bernal and

García [74] proposed to analyse this phenomenon using the Lorentz force (equations Q53 and Q54). The model proposed by Berdy *et al.* [69,72] is simplified because it only provides the effects due to the Coulomb force (equation Q55). Soares dos Santos *et al.* [25] modelled the frictional force using the Karnopp friction model that considers, for both negative and positive velocities, the effect of different viscous friction coefficients, different break-away forces and different Coulomb forces (equation Q56). They also considered a low velocity region where no relative displacement occurs (Fig. 6c). The force due to the dry friction proposed by Bonisoli *et al.* [78] is simpler than the Karnopp friction model developed by Soares dos Santos *et al.* [25], as they only considered the Coulomb force and a break-away region (equation Q57). Finally, a generally observed trend was the computation of the electric damping disregarding the influence of the mechanical damping. Moreover, the validation of models concerning the damping factor was usually performed in an empirical basis.

5 Experimental energy outcomes

Experimental results concerning each harvester design, namely electric parameters (electric power, voltage, current and load) and the excitation patterns are summarized in Table 11 and Fig. 8. Low excitation magnitudes drive a linear behaviour of the motion experienced by the levitating magnet, resulting in a response with a single periodic attractor (unique solution associated with any initial condition) as depicted in Fig. 7a. However, increasing the external acceleration magnitudes will cause a high nonlinear behaviour characterized by multiple periodic attractors and hysteresis (Fig. 7b) [66,90,91]. Hence, if the excitation is enough to exhibit different solutions for the same initial condition, the analysis was carried out registering the range up to the highest achievable experimental data of electric quantities in any trajectory (ascending and descending), as illustrated in Fig. 7b [7,66]. The analysis of simulation results was carried out by selecting the reported minimum and maximum peak values [61,88].

The experimental validation of transduction mechanisms was performed by means of two approaches: either by attaching the harvesters to a shaker (with previously known sinusoidal accelerations) or by coupling them to vibrational energy sources, which usually excite the

Table 10

Damping coefficients and forces

	Equations	References
Q46	$\xi = \frac{\psi}{\sqrt{4\pi^2+\psi^2}}$ $\psi = \frac{1}{n}\ln\frac{y_1}{y_{n+1}}$	[81]
Q47	$c = 2\xi m\omega_n$	[64,71,75,79] [81,83]
Q48	$c = c_{air} + \frac{c_{fr}}{\dot{y}} + \frac{U^2}{R_l+R_i}$	[6]
Q49	$c_e = \frac{\alpha^2}{R_l+R_i}$	[7,57,66,69,72] [79,83,87]
Q50	$c_e = \frac{\alpha^2}{\sqrt{(R_l+R_i)^2+(\omega L)^2}}$	[70,71]
Q51	$c_e(y) = \frac{N_i}{R_l+R_i}\left(2\frac{N}{l_c}\phi_{r_{mc}}\sin\left(\frac{\pi l_c}{p}\right)\right)^2\cos^2\left(\frac{2\pi}{p}y+\alpha_p\right)$	[73]
Q52	$c_m = \frac{\omega_n}{Q}$	[76]
Q53	$F_{lz} = 2\pi I\sum\limits_{k=1}^{N_y}\sum\limits_{j=1}^{N_r}B_r(r_j,y_k)r_j$	[25]
Q54	$F_{lz} = 2\pi M_m r_m A_\phi(y_m+l_m,r_m,l_c,r_{mc}) - A_\phi(y_m,r_m,l_c,r_{mc})$ $A_\phi(y,r,l_c,Rmc) = \frac{\mu_0}{2\pi}\frac{l}{N}Ir_{mc}\int_0^\pi\log\left(\frac{f^+(\phi',r,y;r_{mc},l_c)}{f^-(\phi',r,y;r_{mc},l_c)}\right)d\phi'$ $f^\pm(\phi',r,y;r_{mc},l_c) = 2y \pm l_c + \sqrt{{l_c}^2 \pm 4l_c y + 4({r_{mc}}^2+y^2+r^2) - 8r_{mc}r\cos\phi'}$	[74]
Q55	$F_{fr} = \mathrm{sign}(\dot{y})F_d$	[69,72]
Q56	$F_{fr} = \begin{cases} f_{re} & \text{if } -f_{bw_n} < f_{re} < f_{bw_p} \\ f_{co_p} + k_{v_p}\frac{dy}{dt} & \text{if } \frac{dy}{dt} > v_{min} \\ -f_{co_n} + k_{v_n}\frac{dy}{dt} & \text{if } \frac{dy}{dt} < -v_{min} \end{cases}$	[25]
Q57	$F_{fr} = \begin{cases} -f(\dot{y})\mathrm{sign}(\dot{y}) & \text{if } z = \pm z_{lim}, x = \pm x_{lim} \\ 0 & \text{if } \lvert z\rvert < z_{lim}, \lvert x\rvert < x_{lim} \end{cases}$	[78]

harvesters with irregular accelerations. Most authors used excitations with defined amplitudes and frequencies varying from 0.039 g to 8 g (0.38 to 78.5 m/s^2) and from 0.75 to 46 Hz, respectively. Interestingly, accelerations and dominant frequencies lower than 1 g and 15 Hz, respectively, were driven in most of the studies that used sinusoidal excitations (Fig. 8). Several customized excitations were applied to the harvesters, namely by tyre moving (harvesters housed radially into the inner layer of a tyre as shown in Fig. 3f) [78], hand shaking (Fig. 1d) [73,87] and other human body motions at various speeds (harvesters attached to upper arm, hip, lower-limb, chest's side, back, among other locations as illustrated in Figs. 1e, 2i,j and 4f,g) [7,64,72,73,75].

More than two thirds of the studies were conducted using resistive loads similar to the coil resistance of the harvester, ensuring maximum electric power transfer. The load resistance applied by the authors was between 4 Ω and 1 MΩ. The electric power outcome is of paramount importance when evaluating the harvester efficiency. Instantaneous power levels up to 1.9 W were achieved, although most of the studies did not exceed 6 mW. These are not impressive magnitudes, mainly if we consider the ability of these harvesters for large-scale electric powering. Nevertheless, they are related to small-scale harvesters that were prototyped so far (volumes up to 235 cm^3, as highlighted in Table 1). Although most studies report power densities in the 20 - 70 $\mu W/cm^3$ range, densities up to 8015 $\mu W/cm^3$ ($\approx$8 kW/m^3) were already achieved, which allow to predict their effective application in large-scale devices. Load voltages in the 0.3 – 7 V range (maximum of 43.4 V) and electric currents up to 21.5 mA (maximum of 150 mA) were measured.

Open load conditions were discarded from the analysis between categories. The harvesters from the first category provided electric power in the range from 15 μW to 180 mW [84,87] and power densities in the range from 3.5 to 7229 $\mu W/cm^3$ [86,87]. The maximum voltage and current achieved were 6 V (for 3.5 kΩ of resistive load) and 150 mA (for 4 Ω load), respectively [25,70]. When considering the developed harvesters from the second category, electric power was monitored between 15 μW to 1.9 W [57,64] with power densities from 1.1 to 8015 $\mu W/cm^3$ [7,57]. The maximum voltage and current achieved were 43.4 V and 43.4 mA (under 1 kΩ load), respectively [57]. In the third category, power levels from 300 μW to 69.3 mW [79,82] and power densities from 79.8 to 1710 $\mu W/cm^3$ were achieved [79]. Maximum voltage and current were measured up to 8 V and 21.5 mA, respectively [78,82]. Finally, the maximum power and power density associated with the fourth category ranged from 40 μW to 85 mW [6,75] and 0.3 to

2080 μW/cm^3, respectively [75,83]. Up to 7 V was measured for the maximum load voltage (for 3.8 kΩ load) and up to 29.2 mA for the electric current (under 100 Ω load) [6,83].

Table 11

Harvesters performance$^{(a)}$

	References	$\ddot{z}$ [m/s^2]	R_l [Ω]	P [mW]	P$_\rho$ [μW/cm^3]	U [V]	I [mA]
Category 1	Constantinou, Mellor, Wilcox[84]	$4.3g\sin(2\pi 46t)$	9	180 (RMS)	4500	1.3 (RMS)	140 (RMS)
	Constantinou, Mellor, Wilcox[70]	$2g\sin(2\pi 37t)$	$\approx$4	90 (AVR)	600	0.6 (AVR)	150 (AVR)
	Bernal, García[74]	ND	ND	ND	ND	ND	ND
	Foisal, Hong, Chung [71]	$0.5g\sin(2\pi ft)$ $7<f<10$	ND	2.37 (MAX)	21.92	ND	ND
				2.09 (MAX)	52.02	ND	ND
	Foisal, Lee, Chung [85]	$0.16g\sin(2\pi 9t)$	97	1.18 (MAX)	$\approx$159.7	0.3 (MAX)	3.5 (MAX)
	Foisal, Chung [86]	$0.13g\sin(2\pi 8.1t)$	97	53.5 (MAX)	$\approx$7229	2.3 (MAX)	23.4 (MAX)
	Berdy, Valentino, Peroulis [69]	$0.1g\sin(2\pi 6.7t)$	1×10^3	0.41 (MAX)	53.2	0.6 (MAX)	0.6 (MAX)
	Berdy, Valentino, Peroulis [72]	$0.075g\sin(2\pi 6.7t)$	ND	0.33 (MAX)	42.9	ND	ND
		cust. 4.8-9.7 km/h	1×10^3	0.071-0.34 (RMS)	9.2-44.4	0.3-0.6 (RMS)	0.3-0.6 (RMS)
	Liu *et al.* [87]	$A\sin(2\pi ft)$ $0.45g<A<0.6g$ $4.4<f<4.8$	10	0.015-0.28 (RMS)	3.5-45.3	0.012-0.053 (RMS)	1.2-5.3 (RMS)
		cust. f=4.5;A=0.01m	6 < 93	$\approx$0.1-0.31 (RMS)	$\approx$15.9-49.4	$\approx$0.035-0.094 (RMS)	$\approx$1-5.6 (RMS)
		cust. f=6.7;A=0.01/0.02m	10	0.57/0.65 (RMS)	90.7/104	$\approx$0.075/0.081 (RMS)	7.5/8.1 (RMS)
	Santos *et al.* [25]	$A\sin(2\pi ft)$ $0.84g<A<1.96g$ $3.5<f<9$	3.5×10^3	$\approx$0.8-10.3 (MAX)	$\approx$66.7 -858.3	$\approx$1.7-6 (MAX)	$\approx$0.5-1.7 (MAX)
		$A\sin(2\pi ft)$ $0.84g<A<1.8g$ $3.5<f<8$	8.9×10^4	$\approx$0.1-1.1 (MAX)	$\approx$11.7-93.3	$\approx$3.5-10 (MAX)	$\approx$0.039-0.1 (MAX)
	Kecik *et al.* [61]	$A\sin(2\pi ft)$ $A<10.2g$ $f<15.9$	ND	ND	ND	ND	$\approx$0-38 (MAX)
	Kęcik [88]	$A\sin(2\pi ft)$ $0.05g<A<14.3g$ $1.6<f<15.9$	1.1×10^3	ND	ND	ND	$\approx$0-56 (MAX)
Category 2	Saha *et al.* [64]	$0.039g\sin(2\pi 7.6t)$	7300	0.015 (MAX)	1.18	0.3 (MAX)	0.045 (MAX)
		cust. $2<f<2.75$;$0.5g<A<1g$	800	0.3-2.46 (AVR)	74.8-193.7	0.9-1.4 (AVR)	1-1.8 (AVR)
	Dallago, Marchesi, Venchi [76]	$1g\sin(2\pi ft)$ f=9/10.4	10-1×10^4	$\approx$0.48-24	$\approx$36.9-1846	$\approx$0.25-3.2	$\approx$0.2-25
	Munaz, Lee, Chung [13]	$0.5g\cos(2\pi 6t)$	1×10^3	4.84 (RMS)	535	2.2 (RMS)	2.2 (RMS)
	Masoumi, Wang [57]	$3.4g\sin(2\pi ft)$ $6<f<12$	1×10^3	190-1.8×10^3 (MAX)	$\approx$808.5-7831	13.8-42.9 (MAX)	13.8-42.9 (MAX)
		$A\sin(2\pi 9t)$ $1.22g<A<3.41g$	1×10^3	3.8-1.9×10^3 (MAX)	$\approx$16.2-8015	1.9-43.4 (MAX)	1.9-43.4 (MAX)
	Wang *et al.* [7]	$A\sin(2\pi ft)$ $0.35g<A<0.85g$ $4<f<10$	1×10^4	0-0.081 (MAX)	$\approx$0-1.1	0-0.9 (MAX)	0-0.09 (MAX)
		cust. 5-9 km/h	5	0.4-6 (AVR)	$\approx$5.6-60	0.2-0.7 (MAX)	8.9-34.6 (AVR)
	Pancharoen, Zhu, Beeby [81]	$0.5g\sin(2\pi 14t)$	247	8.1×10^{-3}	$\approx$16.2	0.04	0.2
	Struwig, Wolhuter, Niesler [77]	cust. A=2.2g	40	1.7-1.9 (AVR)	106.4-122.8	0.26-0.27 (AVR)	6.5-6.8 (AVR)

Table 11

Continuation

	References	$\ddot{z}$ [m/s^2]	R_l [Ω]	P [mW]	P_ρ [μW/cm^3]	U [V]	I [mA]
Category 3	Mann, Sims [66]	$A\sin(2\pi ft)$ $0.21g<A<0.95g$ $6<f<13$	1×10^6	$\approx1.6\times10^{-4}$-0.13 (MAX)	ND	≈0.4-11.6 (MAX)	≈ 4×10^{-4}-1.2×10^{-2} (MAX)
	Bonisoli *et al.* [78]	cust. 40-100 km/h	ND	2-11 (AVR)	ND	2-8 (MAX)	ND
	Morais *et al.* [79]	$A\sin(2\pi ft)$ $0.09g<A<0.55g$ $0.75<f<1.85$	5×10^3	≈0.3-6.5 (RMS)	≈79.8 1710	≈1.2-5.6 (RMS)	≈0.2-1.1 (RMS)
	Morgado *et al.*[89]	cust.$^{(b)}$	-	-	-	-	-
	Yang *et al.* [82]	$8g\sin(2\pi 20t)$	ND	28.3 (MAX)	ND	2.67 (MAX)	10.59 (MAX)
	Aldawood, Nguyen, Bardaweel [80]	$0.04g\sin(2\pi 11t)$	150	≈69.3	≈315.2	≈3.2	≈21.5
Category 4	Saravia, Ramírez, Gatti [6]	$1g\sin(2\pi ft)$ $4<f<10$	100	≈5-85 (MAX)	≈67.6-1148	≈0.8-2.9 (MAX)	≈7.1-29.2 (MAX)
	Geisler *et al.* [73]	$1g\sin(2\pi 9.5t)$	950	≈2.25 (MAX)	≈250	≈1.5 (MAX)	≈1.5 (MAX)
		cust. 6.4/8 km/h	950	3.9-5 (AVR)	438-550	1.9-2.2 (AVR)	2-2.3 (AVR)
		cust. $f=6;A=2g$	950	6.6 (AVR)	730	2.5 (AVR)	2.6 (AVR)
	Apo, Priya [83]	$A\sin(2\pi ft)$ $0.25g<A<1g$ $13<f<16$	3.8×10^3	5.9-12.9 (RMS)	958-2080	4.7-7 (RMS)	1.2-1.8 (RMS)
	Zhang, Wang, Kim [75]	$A\sin(2\pi 4t)$ $0.03g<A<0.11g$	96	0.04-1.23 (RMS)	0.3-10.3	0.1-0.7 (RMS)	0.6-3.5 (RMS)
		cust. 1.6-12.9 km/h	96	≈0-32	≈0-266.7	≈0-1.8	≈0-18.3
	Struwig, Wolhuter, Niesler [77]	cust. $A=2.2g$	40	3 (AVR)	179.4	0.35 (AVR)	8.7 (AVR)

$^{(a)}$ Terminology: ND - not defined; cust. - customized; AVR - average; MAX - maximum; RMS - root mean square.
$^{(b)}$ OC free fall test from a height of 15mm.

6 Design optimizations

Some research efforts have been conducted so far to develop optimized motion-driven electromagnetic energy harvesters using magnetic levitation architectures. The addressed optimization methodology followed by each author is presented in Table 12. The appropriate selection of materials (with low friction coefficient and negligible magnetic permeability), as well as good surface finish (to minimize its roughness and non-linearities due to contact friction), are required to manufacture the container. Nevertheless, the harvester characteristics must be optimized prior to fabrication to ensure maximum efficiency for known and narrowband excitations, even though their application may impose hard dimensional constraints, unconstrained motion amplitudes and arbitrary orientations [25]. The design optimizations were carried out by analysing the effects by varying: (i) the distance between the fixed magnets [6,7,71,77,84]; (ii) the dimensions, mass and number of inertial magnet(s)

[6,13,57,69,73,77,79,83,85,86,88]; and (iii) the coil(s) properties (number of turns, width, height, position) [69,71,73,77,79,81,83,85–87]. The proper selection of the shaft material [70], container material [72] and spacer material (with different permeabilities) [57] was also analysed in some studies. An innovative optimization method was proposed by Diala *et al.* [92] using the Associated Linear Equations (ALEs) of the nonlinear system to obtain the Output Frequency Response Function. Noticeably, no models developed and validated so far (for two or tri-dimensional motions of the harvesters) were used to optimize (prior fabrication) the container, coil(s) and magnets. Consequently, the influence of the design parameters is currently unknown for multivariable performance optimization. This finding is noticeable when analysing Fig. 8, as no correlation between the external excitation and the power density was found. Moreover, as no adaptive mechanisms were already proposed to maximize the energy efficiency for time-varying patterns of external mechanical power sources, it is currently unknown how much the power density can be maximized.

Table 12

Optimization methodologies[(a)]

	References	Description
Category 1	Constantinou, Mellor, Wilcox[84]	(3) Distance between fixed magnets.
	Constantinou, Mellor, Wilcox[70]	(3) Shaft material and (3) shaft cross sections (bearing contact area).
	Bernal, García[74]	NO
	Foisal, Hong, Chung [71]	Number of coil turns, coil width, coil position and distance between fixed magnets.
	Foisal, Lee, Chung [85]	Permanent magnets dimensions, coil width and coil position.
	Foisal, Chung [86]	Inertial mass, size of fixed magnets, coil width and coil position.
	Berdy, Valentino, Peroulis [69]	Section for the container (rectangular, due to volume restrictions), dimensions of permanent magnets, number of coil turns and coil position.
	Berdy, Valentino, Peroulis [72]	Container cross section (with and without a guide rail) and container material.
	Liu *et al.* [87]	(3) Distance between fixed magnets and coil width.
	Santos *et al.* [25]	NO
	Kecik *et al.* [61]	NO
	Kęcik [88]	Repulsive magnetic force, coil inductance and coupling coefficient influence.

Table 12

Continuation

	References	Description
Category 2	Saha *et al.* [64]	NO
	Dallago, Marchesi, Venchi [76]	NO
	Munaz, Lee, Chung [13]	Length of fixed magnets and number of levitating magnets.
	Masoumi, Wang [57]	Magnet stack assembly configuration, spacers thickness and material (with different permeabilities).
	Wang *et al.* [7]	Magnet stack assembling configuration, (3/6) number of levitating magnets and (2) spacing between fixed magnets.
	Pancharoen, Zhu, Beeby [81]	Coil resistance.
	Struwig, Wolhuter, Niesler [77]	Number of coils, coil height, coil spacing, number of magnets, distance between levitating magnets and overall size of the device.
Category 3	Mann, Sims [66]	NO
	Bonisoli *et al.* [78]	NO
	Morais *et al.* [79]	Coil length, weight and height of the inertial mass.
	Morgado *et al.*[89]	NO
	Yang *et al.* [82]	NO
	Aldawood, Nguyen, Bardaweel [80]	NO
Category 4	Saravia, Ramírez Gatti [6]	Number of levitating magnets, influence of spacers and spacing between fixed magnets.
	Geisler *et al.* [73]	Permanent magnets dimensions, total inertial mass, inertial mass displacement, magnetic period, number of turns per coil, coil resistance, coil outer diameter, average magnetic flux and average electric damping.
	Apo, Priya [83]	Dimensions and number of levitating magnets and coil height.
	Zhang, Wang, Kim [75]	NO

[(a)] Terminology: NO - not optimized.

7 Discussion and conclusions

In this paper we present a systematic review of relevant literature reports that highlight major scientific achievements in the design of electromagnetic energy harvesters with mono-stable magnetic levitation architectures. It focuses their major features concerning design configurations, construction parameters, modelling and experimental validation of transduction mechanisms and energy outcomes. Recent research findings show the potential of these electromagnetic harvesters to electrically power a wide range of devices requiring self-powering, from small-scale to large-scale electric systems. The reduced intermittence exhibited by these energy harvesting systems is a significant advantage over many other electric power generators.

Besides, maintenance requirements are remarkably reduced in comparison to current large-scale energy systems because: (i) the number of components is lower, as no coupling, motion transmission or motion transformation systems are required; (ii) their components do not undergo critical wear, as the current performances of permanent magnets ensure a very long service time. For example, neodymium magnets only lose a very small fraction of their magnetic strength over the years under optimum working conditions, which are fulfilled in many large-scale applications if the harvesters are hermetically encapsulated; (iii) the non-levitating magnets do not allow that levitating magnet(s) can be damaged throughout energy harvesting; (iv) the transduction mechanism design is not complex, as the mechanical design does not demand turbines or oleo-hydraulic systems, among other complex systems, and the electrical design is usually based on coil(s) wrapping the outer surface of containers; (v) the mechanical friction of moving components is much lower since only the levitating magnet(s) experience(s) the friction on the magnet(s)-container interface. As a result, reduced maintenance and production costs are expected. These technologies are scalable and adaptable, enabling the implementation of harvesters with different sizes and for different external excitations. Hence, they can be optimized and customized for large-scale power generation, like the conversion of ocean energy into electric energy by generators using magnetic levitation [57,58,65]. Due to their non-intermittent operation, these generators hold potential to provide higher efficiency when compared with electric powering using the wind or sun as the primary power source. Besides, the simplicity of their mechanical and electrical designs ensures long-term operation with reduced performance losses. Nevertheless, the optimization of these harvesters is an important task leading to viable alternatives relatively to batteries and other conventional non-renewable energy sources.

In conclusion, the following core findings can be highlighted:

(1) Twenty-nine relevant studies focused on electromagnetic energy harvesting with mono-stable magnetic levitation architectures were found;

(2) Twenty-one design configurations were already studied and demonstrate promising results for future applications of energy harvesting both for small-scale and large-scale electric powering;

(3) Four categories of energy harvesters emerged according to the number of coils and levitating magnets. Architectures including levitating magnets coupled to springs and magnets moving according to guidance systems were also considered, as well as those using top magnets coupled to planar springs;

(4) Relevant approaches to model each physical phenomenon were explored. Most authors focused on modelling the magnetic field from levitating magnets, repulsive magnetic forces between permanent magnets, induced voltage in the coil terminals, electric current, electromechanical coupling coefficient and damping forces;

(5) Four modelling approaches were distinguished to model each physical phenomenon: empirical, analytical, semi-analytical, and finite element method. Almost all authors conducted a hybridization on their models, which resulted in the development of analytical-empirical-FEM, analytical-empirical and analytical-semi-analytical-empirical models;

(6) Experimental validations of many transduction mechanisms were successfully achieved;

(7) Experimental energy outcomes leave no doubt about the potential of these harvesters to supply energy to multifunctional micro-systems and to provide large-scale electric powering. Densities up to 8 mW/cm^3 (8 kW/m^3) have already been achieved;

(8) Voltages and currents up to 43.4 V and 150 mA were measured for volumes up to 235 cm^3;

(9) No multivariable performance optimization has already been performed. Consequently, the influence of the design parameters are currently unknown in multivariable approaches;

(10) Although prototypes have been developed for small-scale testing, electromagnetic energy generators using magnetic levitation can be scaled up and customized for large-scale power generation with reduced maintenance and production costs expected.

8 Future research prospects

The main limitation of electromagnetic energy harvesters is their highly non-linear behaviour, which makes performance maximization hard to achieve. The accurate modelling of their energy transduction mechanisms is mandatory. It is noteworthy to state that, even though current findings seem to highlight a strong societal impact of these harvesters in the near future, their designs ensuring competitive energy production costs still demand intensive research efforts to optimize harvesting performance. For future applications in both small-scale and large-scale power generation, a multivariable geometric optimization prior to fabrication must be conducted. Besides, design optimization must be performed using very accurate computational models to predict both the mechanical and electric dynamics for tri-dimensional motions of the harvester due to realistic excitation patterns of external mechanical power sources. New methodologies must be implemented to optimize harvester architectures so that their performance can be maximized for unknown and time-varying patterns of mechanical power sources exciting the harvesters. Large-scale prototypes must be developed to trigger detailed analyses to their harvesting performances, as well as to identify the main problems that emerge when these technologies are scaled up. Finally, realistic economic studies must be conducted to estimate their maintenance and production costs in order to evaluate their competitiveness in the energy market.

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Additional information

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Fig. 1. Architectures with a single coil and a single levitating magnet (category 1): (a) basic configuration; (b) ring permanent magnets with guidance system; (c) rectangular section, block magnets and two fixed magnets in one end cap. Customized experimental validation (reproduced with permission, all rights reserved): (d) in-plane handshaking test setup proposed by Liu *et al.* [87]; (e) vest with energy harvester and data acquisition hardware proposed by Berdy *et al.* [72].

Fig. 2. Architectures with a single coil and multiple levitating magnets (category 2): (a) two cylindrical levitating magnets; (b) two inertial magnets with spacer; (c) one fixed and two levitating magnets with spacer; (d) one fixed and three levitating magnets with spacers; (e) extremities of the moving mass are composed by two cylindrical tungsten pieces; (f) the moving mass consists of three magnets; (g) the inertial mass consists of six magnets; (h) three levitating ring magnets with spacers and guidance system. Customized experimental validation (reproduced with permission, all rights reserved): (i) generator in rucksack proposed by Saha *et al.* [64]; (j) setup with the generator vertically and horizontally attached to human lower-limb proposed by Wang *et al.* [7].

Fig. 3. Architectures with multiple coils and a single levitating magnet (category 3): (a) two helicoidal coils; (b) inertial magnet coupled to a spring; (c) two rubber bumpers bonded to the upper and lower lids; (d) ring magnets arranged along a shaft and the wire is wounded both inside and outside of the permanent central magnet; (e) top magnet is freed and the FR4 mechanical spring is used to guide its motion (dual inertial mass). Customized experimental validation (reproduced with permission, all rights reserved): (f) energy harvester for automotive wireless tire sensors proposed by Bonisoli *et al.* [78].

Fig. 4. Architectures with multiple coils and multiple levitating magnets (category 4): (a) two coils, one fixed and two levitating magnets with spacer; (b) five coils and three levitating magnets with spacers; (c) three coils and four levitating magnets with spacers; (d) two coils and three levitating ring magnets with shaft; (e) sixteen planar coils and 2x4 levitating magnet block array. Customized experimental validation (reproduced with permission, all rights reserved): (f) electromagnetic energy harvester, conditioning circuitry and accelerometer attached to a leg proposed by Struwig *et al.* [77]; (g) non-linear generator in a smartphone armband and tested locations proposed by Geisler *et al.* [73].

Fig. 5. (a) Trimetric-view and (b) section-view of a levitation-based harvester with the most relevant constructive parameters highlighted in Tables 1 and 2.

Fig. 6. Experimental and simulation results achieved by Soares dos Santos *et al.* [25]: (a) magnetic field from the equivalent surface current model; (b) magnetic force-displacement behaviour for the following magnetizations of the levitating magnet: 700 kA/m (red), 800 kA/m (black) and 1000 kA/m (blue); (c) friction force between the moving magnet and the container's inner surface using the Karnopp friction model; (d,e,f) experimental (red dots) and simulation (solid black lines) steady-state voltages for external excitation defined by $12.25\sin(10\pi t)$, $7.75\sin(15\pi t)$ and $6\sin(18\pi t)$ [mm].

Fig. 7. Relative velocity response with excitation amplitudes of (a) 0.5 m/s^2 and (b) 4 m/s^2. The model and related parameters used for the simulation were the same as the ones considered by Mann and Sims [66]. The theoretical behaviour due to the nonlinearity, observed for upward and downward frequency sweeps, are identified by the black and red solid lines (stable periodic solutions), while the blue dashed line illustrates unstable periodic solutions.

Fig. 8. Experimental energy harvesting achieved using sinusoidal excitations and resistive loads similar to the coil resistance. Data related to all customized excitations were not illustrated. The results from each category are identified with different dot colours: category 1 - black, category 2 - red; category 3 - green; category 4 - blue.