A study on a synthetic aperture sonar

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A STUDY ON A SYNTHETIC APERTURE SONAR

BY

ZAIQING MENG, BSc

A doctoral thesis
submitted in partial fulfilment of the requirement for the award of
doctor of philosophy of the Loughborough University of Technology

January 1995

Supervisor: Professor J. W. R. Griffiths, BSc (Eng), PhD, FEng, FIEE, FIOA

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ABSTRACT

Aperture synthesis, as its name implies, synthesises an aperture by storing successive echoes obtained from a moving platform and by processing the results as if they had been obtained from a multi-element array enables a high azimuth resolution to be obtained from a physically small array. The technique has been highly successful in radio astronomy, and in both satellite and aircraft borne radar. However the use of this technique has been very limited in the sonar environment mainly because of difficulties of maintaining a stable track under water and problems of under-sampling of the aperture arising from the relatively slow velocity of acoustic waves in water.

The thesis describes a study of the application of the synthetic aperture technique to sonar, highlighting some of these difficulties and possible means of overcoming them. A study has also been made the application of the bathymetric technique, a technique for measuring the height of objects on the sea bed, to synthetic aperture sonar.

In addition to the theoretical work and computer simulation, an experimental system has been built in a water tank measuring some 9m by 5m by 2m deep in order to test a number of the algorithms and some good results have been obtained.
ACKNOWLEDGEMENTS

I would like to thank my supervisor, Professor J.W.R. Griffiths, for his help, encouragement and financial support throughout the research. His great enthusiasm on the work will be remembered for ever.

I would also like to thank my director, professor C.F.N Cowan, for providing the convenience and the financial support.

I am deeply indebted to my colleagues in the sonar group, Mr. W.J. Wood, Dr. T.A. Rafik, Mr. A.D. Goodson, Mr. P. Lepper and Mrs. S. Clarson, for their help and friendship.

I must thank my parents for fostering my interest in science and technology in my childhood, for their love and support throughout so many years.

Finally, I would like to thank my wife, Liqin, for her endless love, support and understanding.

To Liqin, I dedicate this thesis.
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<td>A</td>
<td>amplitude (Chapter Two), or area (Chapter Four)</td>
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<tr>
<td>a</td>
<td>acceleration</td>
</tr>
<tr>
<td>B</td>
<td>frequency bandwidth</td>
</tr>
<tr>
<td>$B_s$</td>
<td>spatial frequency bandwidth</td>
</tr>
<tr>
<td>c</td>
<td>velocity of sound in water</td>
</tr>
<tr>
<td>CTFM</td>
<td>continuous transmission frequency modulation</td>
</tr>
<tr>
<td>CW</td>
<td>continuous waveform</td>
</tr>
<tr>
<td>D</td>
<td>horizontal dimension of transducer</td>
</tr>
<tr>
<td>d</td>
<td>spacing of the receiver pair</td>
</tr>
<tr>
<td>DI</td>
<td>directivity index</td>
</tr>
<tr>
<td>$D(x_n,R)$</td>
<td>demodulated sonar returns</td>
</tr>
<tr>
<td>E</td>
<td>transformation efficiency</td>
</tr>
<tr>
<td>exp</td>
<td>exponential function</td>
</tr>
<tr>
<td>$E(x_n,R)$</td>
<td>sonar returns</td>
</tr>
<tr>
<td>e(t)</td>
<td>echo signal</td>
</tr>
<tr>
<td>f</td>
<td>frequency</td>
</tr>
<tr>
<td>$f_d$</td>
<td>Doppler frequency</td>
</tr>
<tr>
<td>FM</td>
<td>frequency modulation</td>
</tr>
<tr>
<td>g</td>
<td>complex conjunction of the Doppler phase history</td>
</tr>
<tr>
<td>h</td>
<td>correlator output</td>
</tr>
<tr>
<td>l</td>
<td>acoustic intensity</td>
</tr>
<tr>
<td>i</td>
<td>imaginary part</td>
</tr>
<tr>
<td>j</td>
<td>imaginary part</td>
</tr>
<tr>
<td>I-Q</td>
<td>in-phase and quadratic</td>
</tr>
<tr>
<td>K</td>
<td>reflection factor</td>
</tr>
<tr>
<td>L</td>
<td>the length of synthetic aperture</td>
</tr>
<tr>
<td>l</td>
<td>length of cylinder</td>
</tr>
<tr>
<td>LB</td>
<td>time bandwidth product</td>
</tr>
<tr>
<td>ML</td>
<td>multi path signal level</td>
</tr>
<tr>
<td>NL</td>
<td>noise level</td>
</tr>
<tr>
<td>NM</td>
<td>nautical mile (=1852m)</td>
</tr>
<tr>
<td>PRF</td>
<td>pulse repetition frequency</td>
</tr>
</tbody>
</table>
\[ p \] sound pressure
\[ Q \] quality factor
\[ R \] range
\[ r \] distance
\[ \text{ref} \] reference
\[ R_{max} \] maximum range
\[ R_0 \] the shortest range between the sonar platform and the target
\[ RL \] reverberation level
\[ S \] along-track sampling spacing
\[ sa \] synthetic aperture
\[ S_{\text{ABASS}} \] synthetic aperture bathymetric sidescan sonar
\[ SAR \] synthetic aperture radar
\[ SAS \] synthetic aperture sonar
\[ S_b \] bottom reverberation strength
\[ SL \] source level
\[ SN \] system noise level
\[ SNR \] signal to noise ratio
\[ S_v \] volume reverberation strength
\[ s(t) \] transmitting signal
\[ T \] a time period
\[ TL \] transmission loss
\[ TS \] target strength
\[ TW \] transmitted signal waveform
\[ t \] time
\[ V \] voltage
\[ v \] towing speed
\[ VM \] volume
\[ W \] power
\[ WL \] wanted signal level
\[ x \] along-track co-ordinate
\[ x_0 \] along-track position closest to the target
\[ XR \] echo position matrix
\[ XY \] target matrix
\[ x(t) \] demodulated echo
\( \alpha \) angle
\( \beta \) angle
\( \Lambda \) waveform duration
\( \lambda \) wavelength
\( \tau \) pulse length
\( \gamma \) azimuth
\( \eta \) the declination angle between the transducer beam and the floor
\( \theta \) beam width (3 dB angle)
\( \rho \) fluid density
\( \sigma \) incidence angle to normal in a reflecting plane
\( \chi \) time modulated frequency
\( \Gamma \) Doppler phase
\( \phi \) phase
\( \Phi \) equivalent ideal beam angle
\( \delta \) finite difference
\( \delta R \) range resolution
\( \delta s \) depth of along-track sampling spacing
\( \delta v \) depth of along-track speed
\( \delta x \) along-track resolution
\( \Delta \) finite difference
\( \omega \) angular velocity
\( \xi \) depression angle of transducer beam
\( \psi \) stereo beam angle
\( \otimes \) convolution
CHAPTER ONE

INTRODUCTION

1.1 Introduction to the Sidescan Synthetic Aperture Technique

In recent years the sea bed has increasingly become the subject of attention and exploration in many branches of science and technology. It is a platform for engineering structures, a source of raw materials, a repository for unwanted materials, a route for communications, a potential battleground, a laboratory where physical, chemical and biological processes of great importance can be studied and many more things besides. All of these activities depend for reliable operation on a bank of information, i.e., the existence of images, virtual or real, of the appropriate areas.

Until fairly recently, the only images of the sea floor in common use were hydrographic charts comprising soundings and pilotage information, including tides, currents, etc., and descriptions of the sea floor derived from sampling.

Acoustics' provide the only known reasonable form of radiant energy to explore the sea bed, as electromagnetic radiation is severely absorbed by sea water and, the range of the visible wavelengths is only a few metres or at best tens of metres depending on the transparency of the sea water.

Sonar, which stands for "SOund NAvigation and Ranging", has been in practical use since World War I where it was used for submarine detection. Since then, many applications, civilian and military, have been developed. One of these applications is the sidescan sonar which can be traced back to World War II.
Sidescan sonar images are a visible representation of the strength of the acoustic backscatter from the sea floor onto a two dimensional image medium. Scanning takes place in two directions, namely along the survey track and perpendicular to it. Perpendicular or across-track scanning is achieved by the passage of the sound wave through the water, the reception of echoes from successively greater ranges occurring at later and later times from the instant of the transmission. Along-track scanning is achieved by physical translation of the transducer. In this direction, scanning is not continuous, but the field is sampled by a sequence of discrete pulse transmissions. The range resolution of a sonar is related to the bandwidth of transmitted pulse and is of the order of $c/2B$, where $c$ is the velocity of sound in water and $B$ is the signal bandwidth. The along-track resolution, on the other hand, is the horizontal width of the footprint except at close ranges where the along-track resolution is the distance travelled by the transducer during the reception interval. As the footprint on the sea bed gets wider with increasing range, the along-track resolution degrades at far ranges. The along-track resolution can be improved by using either a longer aperture or a higher operating frequency. However, a large array is costly and more difficult to operate and a high frequency severely restricts the maximum range of operation. The use of the synthetic aperture technique helps to circumvent these problems.

1.2 The Use of a Synthetic Aperture in a SONAR Environment

The concept of a synthetic aperture is to synthesise an aperture by sampling as an array or element moves along a given path. This relies on the fact that in a stationary system it does not make any difference whether a set of samples at different positions in space is observed at the same time or in sequence. Thus the performance of the synthetic aperture system will be the same as a normal array that has the same length as the synthetic aperture. By processing a variable aperture, fine along-track resolution, independent of range and frequency, can be generated.

Difficulties facing an acoustic attempt at forming a successful synthetic aperture were apparent from the start and have restricted the application of the synthetic aperture technique to underwater applications. Motion irregularities and media turbulence, causing phase errors greater than $\lambda/4$, must be corrected to generate a synthetic aperture [13]. The relatively slow acoustic propagation velocity in water implies a low pulse
repetition frequency leading to the consequences that large amount of irregular and unknown motion in the transducer path can occur between pulses, the multipath pattern of propagation through the ocean can exhibit significant instability in this period and the coherence between the pulses is not easy to maintain.

The above problems, which has limited the spreading use of the synthetic aperture technique in the sonar environment, have been studied by many researchers. In the following, a brief review of these studies is presented.

1.3 Historical Review

Nearly 40 years have passed since it was observed that a side-looking radar could improve its azimuth resolution by utilising the Doppler spread of the echo signal. This landmark observation signified the birth of the technology of synthetic aperture. This concept was applied to radio astronomy and radar in the late the 50's and early 60's [1, 2, 3, 4, 5]. Since then, synthetic aperture radar has become well established and has given excellent results for three decades [6, 7, 8, 9].

The application of the technique in acoustics started with the ultrasonic imaging systems in the early 70's [10, 11]. An experimental short pulsed system built in Japan showed that the synthetic aperture worked fundamentally, under laboratory conditions. As these experiments were intended for medical use, they were done in the ultrasonic frequency range, i.e. 1-2 MHz, within a well controlled and small area.

The technique was first applied to underwater sonar in the middle 70's. In 1975, L. J. Cutrona [12] proposed a design procedure for a synthetic aperture sonar to achieve ambiguity avoidance without giving up the resolution. A multi-beam system was suggested to increase the along track sampling rate. About the same time, R. E. Williams [13] conducted a synthetic aperture experiment using a ship-towed source and midwater receiving hydrophones. The tows were conducted along straight lines, and a CW signal at 400Hz and a swept FM between 350 and 450 Hz were transmitted continuously during the tows. He reported that although major difficulties were encountered in maintaining straight tows in the presence of surface swells, it was possible to construct a number of synthetic apertures for the 400Hz transmissions on an
intermittent basis and, the length of these apertures extended to 1/2 NM and more, corresponding to coherence time interval of up to 7.5 min in the ocean. A few years later, D. G. Checketts and B. V. Smith [14] analysed the effects of platform motion errors upon synthetic aperture sonar. They concluded that aperture synthesis relies heavily on maintaining coherence across the length of the aperture being synthesised and the sway, i.e. lateral towfish movement, is the most critical motion affecting aperture synthesis. Meanwhile, some research has been carried out on analysis of effects of medium turbulence [15, 16]. Several processing schemes have been developed to overcome the difficulty of monitoring the trajectory of a sonar platform adequately and effects of medium turbulence. A synthetic aperture sonar system capable of operating at high speed and in turbulent media was developed by P.T. Gough in 1983 [17, 18, 20, 23]. This system, based on continuous transmission with some form of frequency modulation (CTFM), enables the towbody to transverse at a high speed by trading off the range resolution. In addition, a phase-differential synthetic algorithm was suggested to combine with CTFM to reduce the effect of lateral towfish movement and the effects of medium turbulence. There also have been some other investigations, including robust processing schemes and broad band systems, to overcome the restrictions on the use of synthetic aperture in sonar. In 1984, P. Heering [19] proposed alternate schemes in SAS processing. It was suggested that broad-band mapping with low-Q transmission could circumvent azimuthal ambiguities, and the envelope-only processing could ease the platform trajectory compensation significantly. An analysis on the broad-band and narrow-band approaches of synthetic aperture has been studied by M. E. Zakharia, J. Chatillon and M. E. Bouhier [21, 22, 27] since 1990. They reported that broad-band processing provided better resolution in both perfect motion and disturbed motion cases. They pointed out that even in the wide-band case, the image quality is sensitive to unwanted movement of the towed body with an order of magnitude of wavelength.

1.4 Motivation of Study

As has been showed above, the practical use of synthetic aperture technique in sonar environment is very limited. As far as the author is aware there are no published results available of seabed mapping of a practically sensible large area.
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The first objective of this research is to investigate the performance of broad band signal on reducing the ambiguities caused by the along track under sampling aimed to increase the towing speed.

Secondly, some study is to be made aimed at overcoming the effects caused by the track errors. Two processing schemes to implement the synthetic aperture, which are the coherent addition and the I-Q processing, are to be compared. Two other algorithms, the phase differential synthetic aperture and the envelope only processing, suggested to reduce the effect of track errors will be studied by comparing their performances with the conventional I-Q processing with the presence of various track error. Besides the algorithms tolerating the track errors, a motion compensation processing, i.e. contrast optimisation autofocusing, will be examined under various track errors.

Finally, a new approach for producing a 3-D image of the sea bed is to be investigated by applying the Synthetic Aperture processing to the BAthymetric Sidescan Sonar (SABASS) which could produce a 3-D sea bed image with a constant azimuth resolution.

The work presented in this study involved many computer simulations as well as implementation in practice.

1.5 Study Synopsis

Chapter Two outlines the synthetic-aperture sidescan sonar, and then reviews the theory which is the basis of synthetic aperture processing. The restrictions on the underwater applications of the synthetic aperture technique are introduced followed by a discussion of some algorithms suggested to overcome them.

Chapter Three concentrates on resolving one of the most serious problems restricting the SAS to the practical applications, i.e., the transducer's motion error. An autofocus technique that compensates the motion errors with the information extracted from the echo signals is proposed in sonar applications.
Also other synthetic aperture processing algorithms which are not sensitive to the motion error of the transducer are discussed.

Chapter Four contains the details of the hardware and software design of the experimental synthetic aperture sonar system built in the department's tank.

The performance of the processing schemes introduced in Chapter Two and motion compensation algorithms introduced in Chapter Three are studied by the computational simulations in Chapter Five. The system simulations are also presented, in which the system parameters are selected to match the ones in the experimental system (where applicable) in order to compare the simulated results with the experimental ones.

The results obtained using the experimental system in the department's tank are presented in Chapter Six. The various synthetic aperture processing algorithms have been tested and, their results are analysed and compared with the simulation results in Chapter Five where applicable.

A new application for sidescan synthetic aperture sonar, Synthetic Aperture BAthymetric Sidescan Sonar (SABASS), is proposed in Chapter Seven. Some simulation and experimental results are presented.

Chapter Eight contains the conclusions of this entire research and the considerations for the further study.

References


CHAPTER ONE


CHAPTER ONE


CHAPTER TWO

SYNTHETIC APERTURE SONAR PROCESSING

2.1 Introduction

The basic principle of the synthetic aperture technique is introduced in this chapter. After outlining the restrictions on the underwater acoustic applications, some algorithms to overcome these difficulties are discussed.

2.2 Outline of Side-scan Sonar Theory

Fig. 2.1 shows a simplified geometry of a side-looking real-aperture sonar. The sonar is carried on a platform (vessel or towfish) moving at speed \( v \) in a straight line at constant depth. It is assumed that the sonar beam is directed perpendicular to the moving path of the vessel and downwards towards the seabed.

The resolution of the sonar in slant range is defined as the minimum range separation of two points that can be distinguished as separate by the system. If the arrival time of the leading edge of the pulse echo from the more distant point is later than the arrival time of the trailing edge of the echo from the nearer point, each point can be distinguished in the time history of the sonar echo. If the time extent of the sonar pulse is \( \tau \), the minimum separation of two resolvable points is then

\[
\delta R = \frac{c\tau}{2} = \frac{c}{2B}
\]  

(2.1)

where \( \delta R \) is the resolution in slant range, \( B \) is the bandwidth of transmitted signal and \( c \)
is the speed of sound in water. The ground range resolution can be easily derived from the geometry shown in Fig. 2.1 as

$$\delta R_{\text{ground}} = \frac{\delta R}{\cos \eta}$$  \hspace{1cm} (2.2)$$

where $\eta$ is the range dependent declination angle from which the range dependence of ground resolution results (referring to Fig. 2.1 target A).

Fig. 2.1 Simplified geometry of a sidescan real aperture sonar

To obtain a reasonable resolution $\delta R$ and a sufficient echo signal to noise ratio (SNR) for reliable detection, a pulse compression technique is commonly employed to achieve high resolution with a longer pulse and a high SNR. With appropriate processing of the received pulse, i.e., matched filtering, the range resolution obtainable depends on the frequency bandwidth of the transmitted pulse (Equ. 2.1). This resolution can be made arbitrarily fine within practical limits by increasing the pulse bandwidth.
As shown in Fig. 2.1, the sonar transducer has a length $D$ in the dimension along track. Then the sonar beam, which is the angular direction in space to which the transmitted acoustic energy is confined and from which the system can respond to a received signal, has an angular spread in that dimension of $\theta = \frac{\lambda}{D}$, where $\lambda$ is the wavelength of the transmitted energy.

Two targets on the sea floor, separated by an amount $\delta x$ in azimuth direction and at the same slant range $R$, can be resolved only if they are not both in the sonar beam at the same time (Fig. 2.1, targets B, C). Thus azimuth resolution is the two-way (transmission and reception) 3dB dimension of the footprint in the azimuth direction

$$\delta x = R\theta / 2 = \frac{R\lambda}{2D}$$ (2.3)

This quantity is the resolution limit of a conventional side-scan sonar, in the azimuth coordinate.

To improve the along-track resolution $\delta x$ at some specified slant range $R$ and wavelength $\lambda$, it is necessary to increase the transducer array length in the along-track dimension. The mechanical problem involved in constructing an array with a surface precision accurate to within a fraction of a wavelength, and the difficulty in maintaining that level of precision in an operational environment, make it quite difficult to attain values of $D/\lambda$ greater than a few hundreds. It is also costly to make an extremely long array.

Without increasing the physical array size, synthetic aperture technology can improve the azimuth resolution significantly. A synthetic aperture sonar is a coherent system in that it retains both phase and magnitude of the backscattered echo signal. The high resolution is achieved by synthesising in a signal processor an extremely long array aperture. This is typically performed digitally in a computer by compensating for the phase changing associated with what is effectively near field imaging by the long synthetic array. The net effect is that the SAS system is capable of achieving a resolution independent of the range.

It is the azimuth resolution that distinguishes a SAS from conventional sonar, whereas for both conventional or SAS sonar systems, the range resolution is determined by the
type of pulse coding and the way in which the return from each pulse is processed. Therefore, the overview of SAS theory will concentrate on the processing relevant to the azimuth resolution.

2.3 Overview of SAS Theory

The arrival time of echoes due to a single target at a sequence of along-track positions is suggested by Fig. 2.2. As the sonar footprint passes over the target, the time of arrival over the two-way path from the sonar to the target is

$$t = \frac{2R}{c}$$  \hspace{1cm} (2.4)

where

$$R = [R_0^2 + (x - x_0)^2]^{\frac{1}{2}}$$ \hspace{1cm} (2.5)

For some purposes it is useful to use an approximation to this relationship which is possible at range much greater than the synthetic aperture length.
and $R_0$ is the range at the point of closest approach.

Fig. 2.3 Data array containing returning signal

Fig. 2.3 shows the two dimensional array containing the sonar returns recorded by the receiver at a sequence of track positions where the slant range $R$ equals to $ct$.

Next, the SAS processing will be approached from the view of equivalent stationary array theory and Doppler point of view.

2.3.1 Array Theory Approach

This approach is based on the assumption that if the whole observing system is stationary, and the samples taken from a set of positions simultaneously should be the same as the ones taken from the same positions sequentially.
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Assume a sonar platform carrying a transmitter and a receiver moves along a straight line, and stopping at a set of equally spaced track positions. During each stop, a pulse is transmitted and the echoes are captured. Whereas, imagine a physically existing array having the same length as the track of the sonar platform traversed through, the element spacing being the same as the track position spacing, and the observed scene the same as the one having been observed by the moving sonar platform. It will be found that the only difference between the samples taken sequentially by the sonar platform and the ones recorded by a stationary array simultaneously is that the time of arrival change along the aperture. This is because in a stationary array system, all the returns due to a particular target appearing to the different elements result from a single transmission made by a transducer at a fixed position. The echoes appearing on each receiver have the same outgoing paths. It is the differences in their return path that makes the time of arrival different. However, for a moving sonar platform, the echoes received at different track positions correspond to the different transmissions made at different track positions. Besides the difference in signal returning path, there is also the same difference in its outgoing path. Therefore, the time of arrival changes along a synthetic aperture twice as fast as it changes along a physical array with the same length (Fig. 2.4).

From this point of view, the aperture synthesising is just a process to focus a physical existing array. Although it was assumed that the sonar platform stops when the transmission and recording occur, the equivalent is still appropriate for a continuously moving system where the platform's motion can be ignored during the time period of the transmitting pulse.
Consequently, the processing procedure should be composed of phase compensation according to the time of arrival of echoes along the aperture (equations 2.4, 2.5 and 2.6) and an addition over all the data collected over the interval, \( L = 8R \), for which the target is in the sonar beam. Due to the two-way effect, the 3dB angle of this synthetic array is

\[
\theta_{\text{sa}} = \frac{\lambda}{2L} = \frac{\lambda}{2 \cdot 8 \cdot R} = \frac{\lambda}{2 \cdot 2 \cdot R} = \frac{D}{2 \cdot R},
\]

where \( \theta = \lambda / D \) is the one way 3dB angle of the transducer beampattern, \( D \) is the horizontal dimension of the transducer. Consequently the along-track resolution is

\[
\delta x_{\text{sa}} = \theta_{\text{sa}} \cdot R = \frac{D}{2}
\]

where along-track resolution is range independent, providing the range dependent length of synthetic aperture.

The alternative derivation for the along-track resolution is also available from the point of view of the Doppler frequency.

### 2.3.2 Doppler Frequency Approach

When the sonar footprint sweeps over a target, the Doppler phenomenon will be observed in the echo signal due to the relative motion between the target and the sonar platform. For a point target at along-track position \( x_0 \) and slant range at closest approach point \( R_0 \), with the sonar at some arbitrary position along the track, the phase difference between transmitted and received wave forms due to two-way travel over the range \( R \) is

\[
\varphi = -4\pi R / \lambda
\]

Referring to Fig. 2.2, the phase change over the two-way path is

\[
\Delta \varphi = -4\pi \Delta R / \lambda
\]

where
\[ \Delta R = [R_0^2 + (x - x_0)^2]^{1/2} - R_0 \]  
(2.11)

or

\[ \Delta R = \frac{(x - x_0)^2}{2R_0}; \quad |x - x_0| < R_0 \]  
(2.12)

and \( R_0 \) is the range at the point of closest approach.

The returning waveform received by the transducer at track position \( x \) is described by the complex waveform

\[ f(x) = \exp[-j\varphi(x)] = \exp[-j4\pi R(x) / \lambda] = \exp[-j(4\pi / \lambda)[R_0 + (x - x_0)^2 / (2R_0)]] \]  
(2.13)

This signal has the instantaneous spatial frequency of

\[ f_D(x) = \frac{1}{2\pi} \frac{d\varphi}{dx} = -\frac{2(x - x_0)}{\lambda R_0} \]  
(2.14)

and a spatial frequency bandwidth \( B = 2\Lambda / (\lambda R_0) \), where the frequency changing during the time period of transmitting pulse length is ignored.

For full resolution, the processing must use all the data collected over the interval, \( \Lambda = \Theta R_0 \), for which the target is in the sonar beam. If this quadratic phase is compensated so that the returns from each pulse due to the target at \( x_0 \) can be added coherently, targets at \( x \neq x_0 \) will correspond to improperly compensated returns so they will cancel. The processed returns from the target at \( x_0 \) will then dominate returns from other targets at the same range.

By processing \( f_D(x) \), it is wished to determine the position of the target. The compensation due to \( x_0 \) is in the form as

\[ \exp[(j4\pi / \lambda)\Delta R] = \exp[j(\frac{2\pi}{\lambda R_0})(x - x_0)^2] \]  
(2.15)
Lacking the knowledge of the target position, it must be processed with a variety of compensations matched to trial values of $x_0 = x'$ and the peak response picked in order to measure $x_0'$.

This is to say that the signal processing should correlate the signal $f_D(x)$ in equation (2.14) with the known waveform

$$g^*(x - x') = \exp[j(4\pi / \lambda)(x - x')^2 / (2R_0)] , \quad |x - x'| < \Lambda / 2$$

(2.16)

a normalised correlator output is

$$h(x') = (1 / \Lambda) \int f_D(x) g^*(x - x') dx$$

(2.17)

whose magnitude is

$$|h(x')| = |\sin[2\pi(x' - x_0)(\Lambda - |x' - x_0|) / (\lambda R_0)] / [2\pi(x' - x_0)\Lambda / (\lambda R_0)]| , \quad |x' - x_0| < \Lambda$$

(2.18)

taking careful account of limits of integration and the sign of $x'$. If the time bandwidth product of this signal,

$$\Lambda B_x = 2\Lambda^2 / (\lambda R_0)$$

(2.19)

is sensibly large, say > 10, over regions where $|h(x')|$ is not small, $|h(x')|$ is

$$|h(x')| = |\sin[u(x' - x_0)] / [u(x' - x_0)]| , \quad u = 2\pi\Lambda / (\lambda R_0)$$

(2.20)

This function peaks at $x' = x_0$, the target location, and has a width of the order of

$$\delta x = \lambda R_0 / (2\Lambda) = 1 / B_x$$

(2.21)

Replica correlation of the quadratic phase waveform in equation (2.13) with itself results in a correlator output with a width which is independent of waveform duration $\Lambda$, under reasonable assumptions. The same result can be generated by matched filtering of the return waveform and the two approaches can be shown to be equivalent.
Such replica correlation, or matched filtering, is the heart of high resolution SAS image formation algorithms. In the specific context at hand, from equation (2.21) the correlator output is seen to resolve targets to within

\[ \delta x = \frac{\lambda}{2R_0} - \frac{1}{2} \left( \frac{\lambda}{D/2} \right) = D/2 . \]  

(2.22)
which is same as the result derived from the array theory in equation (2.8). Fig 2.5(a) shows a typical quadratic phase waveform, and Fig. 2.5(b) is its auto-correlation function.

From the point of view of implementation, the two approaches for synthetic aperture processing have their own advantages and disadvantages. For the array approach which is widely accepted in sonar signal processing, the analysis is more straightforward, and standard array processing can be easily applied to synthetic aperture data. On the other hand, to form a range-dependent parabola for quadratic phase compensation could be computationally expensive. However, for practical processing, this expense could be avoided by using a pre-calculated look-up table providing an effectively large size of memory. With the Doppler approach, which is very popular in SAR imaging, frequency domain processing, e.g. FFT, can be employed to speed up the processing. When a linear Doppler history is assumed, the synthetic aperture data has to be collected along the same range cell from the sequential track position which is leading to the range migration problem. To correct this range migration, additional calculation is needed. If a wide beam is used for aperture synthesis, the linear Doppler history assumption will not be appropriate for the part of aperture near the edge. The non linear Doppler has to be counted in.

With the relatively lower carrier frequency of the sonar signal and today's electronic techniques, it is not difficult to implement a sampling frequency several times higher than the signal carrier. Thus, several samples can be taken within a single range resolution cell. The phase information carried by the sampled signal is more precise than the one carried by the single sample per range cell. It is possible to obtain a reasonably good result by compensating and adding the originally sampled data of carrier frequency. This implementation is relatively simple and computationally efficient, having no complex number computation involved. Obviously, the array theory approach is the better way to look at this particular algorithm.

2.4 The Restrictions on the Underwater Applications of Synthetic Aperture

The obvious value of a synthetic aperture arises from the ability to achieve good azimuthal resolution without the need to deploy a very long receiving array. Such a
system has been prevented from the dramatic success mainly by the slow propagation speed of sound in water, the platform irregular motion and media turbulence.

2.4.1 Limitations Due to the Low Propagation Speed of Sound in Water

In a sidescan sonar system, the choice of the PRF (Pulse Repetition Frequency) has to be compatible with both range and angular sampling [5]. In order to avoid range ambiguities, a signal can only be transmitted after the arrival of all the echoes from the previously transmitted signals. Hence, it follows that

\[ PRF < \frac{c}{2R_{\text{max}}}, \]  

(2.23)

where \( c \) is speed of sound in water and \( R \) is slant range.

In order to avoid azimuthal ambiguities from bearing -90° to 90°, the aperture must be sampled every half of the wavelength. If only to achieve no ambiguities within the transducer -3dB angle, the aperture can be sampled at a spacing equals to half of the transducer dimension.

\[ PRF > \frac{2v}{\lambda} \text{ or } PRF > \frac{4v}{D}, \]  

(2.24)

where \( v \) is the along-track speed of the platform.

Thus combining equations (2.23) and (2.24) gives

\[ v < \frac{\lambda c}{4R_{\text{max}}} \text{ or } v < \frac{D \cdot c}{8R_{\text{max}}}, \]  

(2.25)

For example, if the width of measuring band \( (= R_{\text{max}}) \) is 100m, \( \lambda \) is 4cm and \( c \) is 1500m/sec, the maximum platform along-track velocity is 15cm/sec. This unrealistically low mapping rate is one of main reasons for the lack of acceptance of synthetic aperture techniques in sonar. Using a long transducer array can increase the PRF, however a
compromise has to be made between obtaining high PRF and leading the system back to a conventional sidescan sonar.

2.4.2 Transducer Motion Errors

The successful application of the synthetic aperture relies on the accurate knowledge of the transducer trajectory. If the transducer positions or motions are not known precisely, or some of them are unknown, the phase errors will be introduced which will seriously degrade the image [4, 5]. Due to the low tow speed of the SAS, severe motion errors could occur within the aperture, which make it difficult to maintain the pulse to pulse coherence. Therefore, motion compensation needs to be applied before an aperture can be successfully synthesised.

2.4.3 Media Turbulence

The turbulence of the propagation media is also a factor that has restricted the application of the synthetic aperture to underwater systems. The turbulence caused by inhomogeneities of salinity and temperature, for example, can lead to degradation of the reconstructed image by the introduction of phase errors [8,9]. These phase errors must be corrected somehow to value less than $\pi/2$ to obtain reasonably good results by synthetic aperture processing.

2.5 The Implementations of the Synthetic Aperture in the Sonar System

2.5.1 Coherent Addition

An x-y target plane (Fig. 2.6) can be transferred into a two-dimensional data array on x-R plane (Fig. 2.3) by sampling the returns at a sequence of sampling position, where $R$ is slant range.
Assuming a point target located at Target($x_o, y_o$) (Fig. 2.6) on target plane, its corresponding transformation on $x-R$ plane will be a parabola shaped bend (Fig. 2.3). The shape of the parabola is decided by the minimum distance between the target and the transducer, which is

$$R(x_o, y_o, x) = 2\sqrt{(x - x_o)^2 + y_o^2}, \quad \text{where } x \text{ is sampling position.} \quad (2.26)$$

Once the whole target plane has been swept by the transducer footprint, the image reconstruction can commence. For each trial position Position($x, y$) on the $x-y$ plane, a corresponding parabola will be fitted on the $x-R$ plane. If this parabola overlays on an in-phase wavefront in the data band such that the returns from each pulse due to the target at $x_o$ can be added coherently, targets at $x \neq x_o$ will correspond to out-of-phase added returns so they will cancel. The processed returns from the target at $x_o$ will then dominate returns from other targets at the same range.

The magnitude of the addition results in a pixel of the image. When every point on the $x-y$ plane has been processed, an image is reconstructed.

There is one restriction in this processing scheme which is that the raw data in the data array are sampled at carrier frequency so that the data collected along the parabola is in
phase. Because sonar returns are directly sampled at the carrier frequency, the range sampling rate must be high enough to obtain good phase information in the sampled echoes. The effect of the range sampling rate on this algorithm will be shown in later computer simulations. Because most sonar systems work on the frequency ranged from several kHz to several tens of kHz, it is not difficult to build a system with today's electronic techniques having a sufficiently high sampling rate. The coherent addition based on real signal sampling provides a simple, fast processing algorithm in which only simple real number addition is involved.

2.5.2 Broad-band Mapping with Low-Q Transmission

In 1984, de Heering proposed a processing scheme which takes advantage of broad-band low-Q transmission to increase the mapping rate [1]. The sampling constraint in Section 2.3.1 is a consequence of the narrow-band processing assumption which is typical of radar aperture synthesis. However, broad-band sources are available for underwater acoustic applications. Broad-band echoes, besides providing target classification data not otherwise available, can remove ambiguities associated with spatial under sampling which can increase the maximum mapping rate of the synthetic aperture sonar.

The ambiguous images characterising narrow band synthetic aperture processing with azimuth under sampling can be interpreted as being caused by the grating lobes of the synthetic array not being sufficiently far removed from the main lobe of the horizontal beam pattern of the physical array [10]. Since the angular position of these grating lobes is frequency dependent, wide-band operation and processing will result in smearing of the grating lobes. In particular, the situation where the second grating lobe of the synthetic array at the highest transmitted frequency, is at or beyond the first grating lobe at lowest transmitted frequency may be considered as equivalent to a total smearing of the ambiguous images. This situation corresponds to the quality factor $Q$ of the transmitted pulse satisfying

$$Q \leq 0.7$$  \hspace{1cm} (2.27)

In this situation, which is illustrated in Fig.2.7, azimuth ambiguities cannot occur because the transmitted pulse is essentially equivalent to a one-cycle pulse.
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From the array theory point of view, if the transmitted pulse is so short (wide band width in frequency domain) that the whole array can not be illuminated simultaneously, it is necessary to use a time delay function (i.e. the parabola in Fig. 2.3) to illuminate the whole array by the echo wavefront in order to obtain the output of the array. Since the time delay function is position dependent, in the situation shown in Fig. 2.7, there is a unique wave front which can be brought to the whole array length simultaneously by the delay function due to $(x_0, y_0)$. The ambiguities therefore can be reduced.

The simulation to investigate the limitations of under sampling will be presented in a later chapter.

2.5.3 I-Q Processing

An I-Q demodulation scheme can be employed to ease the range sampling rate requirement. Also, sometimes after the sonar return having been sampled on carrier frequency, however, for some reason, i.e., bathymetric measurement, the phase information of the return is required, the demodulation can be used to resolve the phase value from the returns. For SAS imaging, removing carrier frequency from echoes can smooth the final image. If the demodulation is done by analogue hardware, the requirement of the A/D sampling rate and the size of relevant storage can be reduced significantly. Under the near field condition (It is true for most SAS processing), a complex echo set used to form an image pixel is still collected along the parabola in the
manner employed by coherent addition (Section 2.5.1). Since the phase difference due to different arrival time, has been introduced by the demodulation, the echoes have to be compensated in phase before being added. The amplitude of the result of complex summation is taken for each image pixel. Alternatively, the image can be formed by use of a replica correlator (Section 2.3.2).

2.5.4 The Frequency Domain Processing Using CTFM

A synthetic aperture system using CTFM (Continuous Transmission Frequency Modulation) was suggested by P. T. Gough [11]. In this type of synthetic aperture sonar, the along-track velocity of the vessel is not determined by the pulse repetition rate (refer Section 2.4.1), but it is determined by the frequency resolution of the spectrum analyser that converts the sonar's output into a target-strength-versus-range display. A property of this type of sonar is that the range resolution can be traded off to enable the sonar to move with a higher velocity covering the wanted synthetic aperture in a shorter time. The increase in velocity alone will produce an improvement in the performance of a synthetic aperture sonar as the surrounding medium has less time to alter its character significantly.

Consider a convenient swept-frequency signal described by

\[ s(t) = \cos(\omega t - \omega t^2 / 4T), \quad 0 < t < T \]  \hspace{1cm} (2.28)

where \( \omega = 2\pi f \), which is a signal with a decreasing swept frequency from \( f \) to \( f/2 \) in a period \( T \). A CTFM sonar can cover any convenient bandwidth, but transducers with bandwidth greater than one octave are difficult to construct.

The signal radiates out from the transmitting transducer at the velocity of propagation \( c \), reflects off a single point target at range \( R \), and its echo, a delayed replica of the radiated wave form is detected by the receiver. The echo is described by

\[ e(t) = A \cos(\omega(t - \Delta t) - \omega(t - \Delta t)^2 / 4T) \]  \hspace{1cm} (2.29)

where \( \Delta t = 2R / c \) and represents the time taken for the sound to travel from sonar to the target at a range \( R \) and return to the sonar. The processing required by a CTFM
sonar is to multiply $e(t)$ with a replica of $s(t)$ and to low-pass the resultant signal to eliminate the sum frequency at $2\omega$. Let this resultant signal be

$$q(t) = [e(t) \cdot s(t)] \otimes i(t)$$

(2.30)

where $i(t)$ is the impulse response of the low-pass filter and $\otimes$ denotes convolution. Consequently

$$q(t) = A \cos[\chi t + \Gamma]$$

(2.31)

where

$$\chi = \omega(\Delta t / 2T) = \omega R / Tc$$

(2.32)

and

$$\Gamma = \omega\Delta t(1 + \Delta t / 4T)$$

(2.33)

so that $\chi$ is a frequency proportional to $\Delta t$ and also proportional to $R$.

When there is more than one reflecting object, the combined return echo is a complex mixture of replicas of the transmitted waveform, all delayed by different amounts. Thus a collection of targets at different ranges produces a demodulated signal $x(t)$ that is now a collection of different frequencies described by

$$x(t) = \sum_{i=1}^{M} A_i \cos[\chi_i t + \Gamma_i]$$

(2.34)

where $A_i$ is amplitude of the $i$th sinusoid, $\chi_i$ is the radian frequency, $\Gamma_i$ is the phase and $M$ is an integer equal to the number of reflecting objects. This collection of frequencies is used as input to a spectrum analyser for decomposition into individual components and subsequent display as target strength versus range. Each spectral component that is displayed as an output from the analyser represents a group of frequencies surrounding the spectral line, and so the amplitude of the spectral line is actually the combined effect of all the frequencies that go into that particular spectral component. The synthetic
aperture processing for CTFM sonar is to bring all the frequency components along the synthetic aperture due to a certain point in phase and add them together.

The rate at which the CTFM sonar can produce a new estimate of target strength versus range (the refresh rate) is determined solely by the frequency resolution of the spectrum analyser. The mixture of the sinusoids produced by the CTFM sonar used as input to a spectrum analyser decomposes into a series of discrete spectral components. The value of any single spectral component is proportional to the power contained in a narrow band of frequencies surrounding the spectral line. The bandwidth of the frequencies contributing to a single spectral component is often loosely termed the frequency resolution of the analyser. The broader this band contributing frequencies, the coarser the frequency resolution and more rapidly the spectral component may be calculated. Thus an interesting compromise now exists. The coarser the range resolution required the coarser the frequency resolution, which enables the spectral analyser to use a shorter time window as the input so that the analyser can be refreshed more quickly. Therefore, the sonar platform can traverse from sampling position to sampling position faster without disobeying the spatial sampling theory.

2.5.5 Envelope Processing

When the received sonar echo returns lack phase coherence due to transmission, propagation, reception, recording or sonar platform effects, aperture synthesis can still be based on the incoherent information contained in the range history of the returns.

A simple expression of the azimuth resolution attainable can be derived by considering as an approximation (Fig. 2.8) that the range history of the returns from a point target located at \((x_o, R_0)\) is described by equation (2.5), and that each return has an extent in range of \(2\delta R\) with a constant amplitude, where \(\delta R\) is the range resolution.

The azimuth resolution of the system can be defined in terms of the correlation of the envelope of the echo return with a replica of the envelope of transmitted signal. This resolution is taken to be twice azimuth increment \(\Delta\) that causes the correlation to drop to one-half of its maximum value. In the simple signal model assumed for return (Fig. 2.8), the correlation is proportional to the fraction of the target return overlapping with the replica (shadowed area in Fig. 2.8).
The four parabolas in this picture are described by the functions

\[ z_1 = kx^2 + 2R_0 + 2\delta R \]  \hspace{1cm} (2.35)
\[ z_2 = kx^2 + 2R_0 \]  \hspace{1cm} (2.36)
\[ z_3 = k(x - \Delta)^2 + 2R_0 + 2\delta R \]  \hspace{1cm} (2.37)
\[ z_4 = k(x - \Delta)^2 + 2R_0 \]  \hspace{1cm} (2.38)

where \( k = l/R_0 \), \( D \) is the horizontal dimension of the transducer and \( L = R_0\sqrt{\lambda/(2D)} \) is the length of synthetic aperture. The area surrounded by a pair of parabola, e.g. \( R_1 \) and \( R_2 \), is given by

\[ \text{Area}_{\text{band}} = \int_{-L/2}^{L/2} (z_1 - z_2)\,dx = \int_{-L/2}^{L/2} (2\delta R)\,dx = 2\delta RL \]  \hspace{1cm} (2.39)

and the area covered by the shadow is
Area_{\text{shadow}} = \frac{1}{2} Area_{\text{band}} \quad (2.41)

Substituting equations (2.39) and (2.42) into Equ. (2.41), the \( \Delta \) can be obtained from

\[ L \delta R = \frac{R_0}{\Delta} \cdot 2 \delta R^2 \quad (2.43) \]

as

\[ \Delta = 2 \frac{R_0}{L} \delta R = 2 \frac{\delta R}{\theta} = 2 \frac{D}{\lambda} \delta R \quad (2.44) \]

Thus

\[ \delta x = 4 \frac{R_0}{L} \delta R = 4 \frac{D}{\lambda} \delta R = 4 \frac{D}{\lambda} \cdot \frac{c}{2B} = 2D \frac{f}{B} = 2DQ \quad (2.45) \]
where $B$ is the bandwidth of the transmitted pulse, and $Q$ is the quality factor. A wideband pulse can therefore be useful for aperture synthesis, even in the absence of phase information. Fig. 2.9(a) shows a typical auto-correlation function of an envelope of echoes due to a point target, and Fig. 2.9(b) is the central slice of the auto-correlation peak along the track direction.

The algorithms introduced above all have their own advantages for certain applications. The coherence addition provides a good opportunity for the real time synthetic aperture processing by using simple real number addition. If the phase information needed, or
high sampling rate not available, the I-Q correlator is an alternative, however, more computation tasks are introduced by the complex calculation. The CTFM and Low-Q broad-band processing are both aimed to increase the SAS mapping rate which severely restricts the application of the synthetic aperture technique to sonar. The higher mapping rate gives the surrounding medium less time to alter its characters significantly, moreover the motion errors have less time to happen.

Some algorithms, i.e., phase differential processing, have been developed to be less sensitive to media turbulence and transducer motion errors, and the motion compensation technique, autofocus, which has been very successful in SAR, should be able to contribute to SAS. These processing algorithms aim to overcome the problems mentioned above and will be studied in the next chapter.

References


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CHAPTER THREE

MOTION COMPENSATION

3.1 Introduction

Platform motion error is one of the most common problems preventing the synthetic aperture from sonar applications. It is therefore a preliminary to resolve these problems before the implementation of a practical SAS [6, 7].

First, autofocus techniques, a technique to correct the motion error using the information included in the returns, is applied to SAS processing. This technique has been very successful in SAR motion compensation [2, 3, 4, 5], therefore it is expected to contribute to SAS processing.

Secondly, some efforts to ease the problems caused by the motion errors are made by using phase differential synthetic aperture algorithms and envelope-only processing which are not sensitive to these errors.

3.2 The Platform Motion Errors

In an SAS application, aperture synthesis relies heavily on maintaining coherence across the length of the aperture being synthesised. Coherence can be reduced by the phase errors due to unexpected signal path introduced by unknown motions between the target and the towed array. The distorted geometry can be divided into two categories: along-track and across-track. It has been shown [1, 2] that the across-track acceleration and along-track velocity errors are the most critical motions in synthetic aperture imagery as they cause image defocusing.
These error motions can be compensated by internal navigation information, but, such information is not always available or not precise enough to produce acceptable results. In such circumstances, autofocus is introduced for motion compensation.

### 3.3 Autofocusing

Autofocus means that, in the absence of complete knowledge of the platform path and scatterer geometry, the matched filter for processing is estimated from the raw data itself. This section is to quantify that the two error motions mentioned in the last section constitute significant deviation from uniform straight line motion. A simple criterion is used to assess the significance of error motion, which is that a total phase error across the synthetic aperture in excess of \( \pi \) radians will degrade image quality. Whatever autofocus is used it should not be expected to give results more accurate than the above phase error criterion suggests.

#### 3.3.1 Depth of Velocity

Depth of velocity is defined as the along-track velocity error that causes a total phase error of \( \pi \) radians across the synthetic aperture. The parabolic variation of the two-way distance of the scatterer was derived in equation (2.12). Consequently,

\[
2\Delta R = \frac{(x - x_0)^2}{R} \quad (3.1)
\]

\[
2\Delta R = \frac{v^2 \cdot t^2}{R} \quad (3.2)
\]

where \( t \) is the time taken for the platform traversing from \( x_0 \) to \( x \) (Fig. 2.2), and \( v \) is the along-track velocity of the platform. The phase error of \( \pi \) radians corresponds to a distance of \( \lambda/2 \), which implies an error of \( \lambda/4 \) at the edge of the beam. Therefore, from the definition of depth of velocity,

\[
2(\Delta R_{\text{vel}} - \Delta R_{\text{rel}}} = \frac{\lambda}{4} \quad (3.3)
\]
where \( L \) is the length of synthetic aperture. Thus

\[
\frac{(v + \delta v)^2 - v^2}{R} \times \left( \frac{L}{2v} \right)^2 = \frac{\lambda}{4} \quad (3.4)
\]

\[
\frac{\delta v \cdot L^2}{2vR} + \frac{\delta v^2 \cdot L^2}{4v^2R} = \frac{\lambda}{4} \quad (3.5)
\]

The second term may be neglected for small velocity changes,

\[
\frac{\delta v}{v} = \frac{R \cdot \lambda}{2L^2} = \frac{\delta x}{L} = \frac{2 \delta x^2}{R \lambda} \quad (3.6)
\]

where \( \delta x \) is along-track resolution. Rearrange terms as

\[
\frac{L}{v} = \frac{\delta x}{\delta v} \quad (3.7)
\]

This shows that the time taken to traverse the synthetic aperture is the same as the time taken to traverse a resolution cell at the depth of velocity and that lower resolution images formed from shorter apertures require greater along-track velocity errors to blur them. It should be noted that the depth of velocity derived is only one-sided velocity tolerance which is reasonable within a reasonably short autofocus interval.

### 3.3.2 Depth of Across-track Acceleration

Assuming a parabolic trajectory due to across-track acceleration of the sonar platform

\[
\delta R = \frac{1}{2} \cdot a \cdot t^2 \quad (3.8)
\]

where \( a \) is the across-track acceleration. Depth of across-track acceleration is defined as that acceleration which causes a total phase error of \( \pi \) radians across the synthetic aperture. Thus
where the factor of '2' is to allow for return path. Substituting equation (3.9) into (3.8), the depth of across-track acceleration is

\[ a = \frac{\lambda \cdot \nu^2}{L^2} = \frac{4\delta x^2 \nu^2}{R^2} \]  

(3.10)

where \( \nu \) is along-track velocity and \( \delta x \) is along-track resolution.

3.3.3 Along-track Velocity Error and Across-track Acceleration Equivalence

The across-track acceleration can be interpreted in terms of the along-track velocity error. The equivalence is found by equating the terms that give rise to a phase error of \( \pi \) radians across the aperture.

\[ \frac{\delta \nu \cdot L^2}{\nu \cdot 2R} = \frac{a \cdot L}{\nu \cdot 4} \]  

(3.11)

\[ a = \frac{2 \cdot \nu \cdot \delta \nu}{R} \]  

(3.12)

Because of the existence of this equivalence, autofocusing aimed to compensate for the error caused by the across-track acceleration can be carried out by autofocusing on an effective along-track velocity error.

3.3.4 Autofocus Interval

The autofocus estimate needs updating at intervals along track. This interval depends on two factors, the rate of change of autofocus parameters, and the length of the synthetic aperture, where the autofocus parameter is the factor affecting the focusing of the image, e.g., along-track velocity or across-track acceleration. The raw data used to estimate the autofocus must necessarily average in some sense the platform motions occurring during that interval, and no autofocus method will be capable of resolving
unknown motions with a structure much finer than the length of the raw data. On the other hand, the length of raw data used for autofocusing is the sum of the synthetic aperture length and the width of the azimuth strip that is autofocused. A minimum width of azimuth strip is required for estimating the along-track contrast. This minimum width in addition to the synthetic aperture imposes a maximum rate of change that may be followed by the autofocus method.

Obviously, the tolerance to changes in platform motion reduces with the finer resolution of processing while the effect of the motion averaging increases due to longer synthetic aperture.

### 3.4 Contrast Optimisation

Several methods have been developed for autofocusing [2, 3, 4, 5], one of them named 'contrast optimisation' is introduced in this section.

The whole method rests on the assumption that the maximum contrast image corresponds to the correctly focused image. Such an assumption is intuitively reasonable but is not entirely foolproof. If the image consists of single isolated scatterers then the maximum contrast occurs when the modulus of the processed scatterer is correctly focused. However, real SAS imagery does not consist of isolated point scatterers. A very common feature of the imagery is a sea bed that displays a speckled image. This is the very opposite of the ideal isolated scatterer and cannot be used for focusing. Instead, areas in the image of high structural content should be used. These are likely to consist of strong scatterers but they are most unlikely to be isolated and the maximum contrast image is not necessarily the optimum focus, due to interference effects between the scatterers. However, on average there is no reason to suppose that the interference effects should display any asymmetry, and it is tacitly assumed that the averaging process is sufficient to reduce interference effects.

Contrast optimisation is in principle a simple trial and error method of autofocusing. To simplify the description, a name ‘autofocus parameter’ is given to the variable being estimated for the autofocus. The trial consists of processing the raw data at a number of different values of effective autofocus parameter, and the particular processing
CHAPTER THREE

parameter that produces the image with the maximum contrast is taken as the optimum effective autofocus parameter.

Autofocusing is computed at fixed intervals along the track, the distance being a sensible compromise between spanning an excessive parameter change and having sufficient azimuth data to compute the contrast. Contrast is defined as the standard deviation of the processed image pixels, normalised by dividing by the mean of the image pixels. This is done individually for each range gate and then averaged over all range gates. Obviously, it is not practical to process all the range gates at all trial parameters. Therefore, an initial processing of all ranges at an estimated parameter is used to select a number of ranges for subsequent processing at remaining trial parameters. The ranges are selected on the basis of maximum contrast, which favours range gates of high structural content and discriminates against speckle.

To simplify the discussion, a specified autofocus parameter, velocity, is used although all the discussions are also true for the other autofocus parameters maintained in the early section.

3.4.1 Contrast Velocity Peak Detection

In order to select the maximum contrast velocity, the selected range gates need to be processed at different velocities. Clearly, it is not practical to process the range gates at all conceivable velocities. A compromise is necessary between effective averaging over many range gates and the time saved by focusing on fewer range gates.

The maximum contrast velocity is found in stages using a hierarchical algorithm. At each stage, three equally spaced velocities are considered.

The velocity spacing is specified in terms of the depth of velocity. This ensures that the spacing is scaled automatically to the SAS parameters, and in particular to the resolution of the SAS processing. Initially, the centre autofocus parameter is the best estimate of the processing velocity, which is usually the optimum velocity determined for the previous azimuth strip.
CHAPTER THREE

Having determined the triple of velocities, the selected range gates are then processed at these velocities. If the centre velocity possesses the highest contrast of the three velocities, control passes to the next stage of the maximum contrast algorithm. If not, the centre velocity is set equal to the maximum contrast velocity. In other words, there is a step in the direction of increasing contrast, and this may be to a greater or smaller velocity, depending on which velocity possesses the highest contrast. This is repeated as many times as is necessary. Clearly, if a finite contrast function possesses at least one local peak, the algorithm will terminate in a finite number of steps.

The maximum contrast algorithm is hierarchical because the same procedure can be repeated for three velocities with a smaller spacing, whose centre velocity is the optimum velocity found from the previous velocity spacing. Typically, two or three levels are used, with spacing of, say, four, two, and one depths of velocity.

Such a simple peak finding algorithm is effective because the general characteristics of the contrast velocity function are predictable. The function is typically dominated by a large peak whose width is of the order of a depth of velocity, although the main peak may exhibit subsidiary peaks of similar width on its flanks. Subsidiary peaks are eliminated from consideration by the hierarchical structure of the algorithm. Initially, the contrast velocity function is explored by samples more widely spaced than typical peak widths and, with an initial velocity spacing of four times depths of the velocity, say, the resulting the maximum contrast velocity of the estimate is normally in the region of the dominant peak. The sampling of the function at a reduced velocity spacing can then determine the location of the peak more precisely, with no fear of locking on to a subsidiary peak.

3.4.2 Velocity Hysteresis and Follow-Down Processing

Velocity hysteresis proved to be a problem when large changes of velocity occurred between successive azimuth strips. The maximum contrast velocity tended to remain at the previous velocity estimate rather than change to the new and significantly different velocity. The problem was easily detected by executing a forward and backward 'tow'. This is done by autofocusing azimuth strips in one order, and then using the same package to autofocus the azimuth strips in reverse order. Obviously, the autofocus results should be independent of the flight direction.
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The maximum contrast velocity of a particular azimuth strip is used as the initial estimate of the maximum contrast velocity of the succeeding azimuth strip. At a large change in velocity, the initial estimate of velocity depends on the tow direction. This provides the channel by which the information on the previous velocity estimate is communicated, but it is not the root cause of the problem, because some initial estimate is always needed, whatever strategy is used to generate it.

The basic cause of the velocity hysteresis is the short-cut taken by processing a selected number of range gates that were selected on the basis of highest contrast. However, the range gates are selected by initially processing the whole image at the initial velocity estimate that may be considerably different from the best focusing velocity. The reason for extracting those ranges of maximum contrast is to ensure a high structural content. However, if the initial processing is so much in error, then the whole image is badly blurred and any structural content tends to be lost. As a result, high contrast is no longer a reliable indicator of structural content. Instead, the range gates selected from the badly blurred image are those ranges whose interference effects of scatterers just happen to conspire to give a higher contrast at the initial processing velocity. It is also unlikely that the interference effects would conspire to give a maximum at the correct velocity. As a result, typical contrast velocity curves in these instances show a maximum at the initial velocity estimate that leads to the autofocus estimate being stuck on an error estimate.

The solution adopted is called follow-down processing. If the velocity change is too large in terms of the depth of velocity, then the velocity change itself cannot be reduced but the depth of velocity can be increased by processing at a coarser resolution. By processing at the coarser resolution, the image structural content is preserved, and although the number of independent measurements of structure is reduced by the coarser resolution, it can nevertheless distinguish between speckle and regions of relatively few scatterers. Alternatively, the reduced resolution processing may be regarded as detuning the matched filter by using only that length of raw data that may be matched adequately by the initial erroneous filter. The processing is then prevented from choosing those range gates that happen to be 'tuned' to the longer high resolution filter. As a result of processing at the lower resolution, no account can be taken of the interference effects of raw data and the extremes of the longer filter, (a form of
randomised additive noise), and hence the bias of the selected range gates towards the initial velocity estimate is eliminated.

The follow-down processing involves processing the whole image at the reduced resolution to select the range gates for contrast maximisation. The maximum contrast velocity is then determined using the peak detection algorithm that also processes the gates at reduced resolution. The follow-down processing continues by repeating the peak detection algorithm at a finer resolution by using the coarse resolution maximum contrast velocity as the initial velocity estimate. This process can be repeated through finer and finer resolution until the desired resolution is obtained.

The computer simulations and experimental results of contrast optimisation autofocusing are presented in Chapter five and Chapter six respectively.

### 3.5 Phase Differential SAS

Phase differential SAS was first suggested by P. T. Gough [8] in 1983. Like autofocus, this algorithm is concerned to reduce the effect of platform error motion on the image produced by a SAS.

#### 3.5.1 Phase Differential Monopulse Technique

Consider a sonar system having a single wide-beam transmitting transducer and a pair of closely spaced receiving transducers. A pulse of a few cycles is radiated, and some of this energy is reflected by a single point target somewhere in the irradiated volume. If the target is not on the line bisecting the two receiving transducers, there is a phase difference between the outputs signals from the two receivers, and this phase difference $\Delta \phi$ is related to the angle $\gamma$ by

$$\Delta \phi = \frac{2\pi \cdot \Delta R}{\lambda} = \frac{2\pi d \cdot \sin \gamma}{\lambda}$$  \hspace{1cm} (3.13)

where $d$ is the separation of the receiving transducers and $\Delta R$ is the radiation path difference as shown in Fig. 3.1.
As with all two-element interferometers, grating lobes give rise to angular ambiguities if $d$ is greater than $\lambda/2$. The ambiguities may be eliminated in a number of ways. These include phase unwrapping, shaping or restricting the individual beamwidth so that only the central lobe of the interferometer is illuminated.

### 3.5.2 SAS Using Monopulse Technique

The concept of phase-difference monopulse sonar can be combined with SAS in the following way. Having measured the range from delay of the echo and the angle of arrival from the phase difference between the two transducers, the transducers can be moved so that the new position of the first receiver is now at the previous position of the second receiver. A new estimate of range and angle can be made, and this process can be repeated indefinitely.

For every position of track a pulse is radiated and the echo detected so that there is a continuous record of echo amplitude against time delay. In addition to the amplitude data for each range sample, the monopulse configuration has measured the phase difference between adjacent channels for each range sample.

Using this method, a synthetic aperture can be formed by selecting a particular target point, combining the pulse amplitude and phase data for all the synthetic aperture
element positions and calculating a pixel of the image just like the coherent processing introduced earlier. However, the major difference of this method from the normal coherent processing is that the phase history of the echoes along the aperture is not directly measured from returning signal, but calculated from the phase difference between adjacent channels for each range sample. The calculation of the phase history relative to the transmitted pulse from the phase difference has to be done before the image forming. The calculation of the phase from a sequence of contiguous phase difference is a small part of something much larger known as the 'phase problem'. Briefly, this arises when a series of modulus and phase measurements are made where the modulus is far more accurately known than the phase. Often these measurements are made in the aperture plane of a radiation detecting system, and the image of the radiating object is estimated by processing the set of measurements taken across this aperture. In this research, a straightforward and relatively simple procedure is used. Recall that, for every target point, the pulse amplitude and the phase difference between contiguous positions of track are measured. A useful estimate of the phase can be made by integrating the phase difference from an arbitrary position somewhere in the aperture. Although the errors are cumulative, owing to the phase-difference measurements being insensitive to roll, pitch and displacement, the phase calculated by straight integration is also insensitive to these movements and so is far more accurate than a direct measurement of phase.

3.6 Envelope-only Processing

As described in Section 2.5.5, envelope-only processing is to form a pixel of an image by adding the echoes’ amplitude according to the return history. Since this processing is totally independent of the phase information, it is obviously insensitive to the phase error introduced by the motion error of the sonar platform.

After all, the phase-difference SAS imaging and envelope-only processing should be expected to be less sensitive to the platform error motion. The simulations and experimental results for these algorithms will be presented in the later chapters.
References


CHAPTER FOUR

AN EXPERIMENTAL SYNTHETIC APERTURE SONAR SYSTEM

4.1 Introduction

In order to test the various SAS algorithms in an underwater environment, an experimental synthetic aperture sonar system has been built in the department's tank. This chapter describes the system hardware and software, and outlines the alternatives of experimental configurations for the different purposes. The sonar equation parameters are calculated theoretically, as well as measured practically. The acoustic source level and the target strength are taken 1m away from the acoustic centre referring to 1μPa.

4.2 Overview of the Experimental Apparatus in the Tank Room

Fig. 4.1(a) The cut out view of the department's tank
The transducer carriage shown in Fig. 4.1(b) is driven by two stepper motors with 16 D.P., 15 T.P.I. steel gears which engage 1/2" by 1/2" nylon racking laid on each of the RSJ girders laid across one end of the tank. The transducers are fixed at the bottom of the tower which can be raised or lowered in the water to the desired depth, and the depression angle of the transducers is also adjustable. The motion of the platform is controlled by a PC through a parallel port and counter board.
4.3 Transducer

The transducer used for this experimental system comprises four one wavelength diameter elements working at a nominal frequency of 40 kHz. Those four elements are housed in a line (Fig. 4.2) with separate electrical connections so that each element can be used alone or joined with others. Another two 40 kHz single-element transducer blocks were used for certain experiments.

4.4 System Hardware

The whole system is based on a PC486 with several plug-in boards. Fig. 4.3 shows a block diagram of the system. The LSI DSP56001 system board which has dual channel A/D and D/A converters on board is used for the transmitting signal generation, data acquisition and some on-line pre-processing, e.g., I-Q demodulation or carrying out an FFT. As the size of available memory on the DSP56001 board is very limited, sampled data is transferred through a Transputer compatible link into a Microway i860 board. Besides acting like a data buffer, the i860 can do some real time synthetic aperture processing with its powerful computational ability. The PC is used for the displaying of the results and for data storage.

4.4.1 LSI DSP56001 System Board

The DSP56001 is a fourth generation digital signal processor, incorporating MCU-style on-chip peripherals, program and data memory, as well as a memory expansion port. The DSP56001 architecture has two independent expandable data memory spaces (up to 64k×24 bits each), two address arithmetic units, and a Data ALU which has two accumulators. The duality of the architecture facilitates writing software for DSP applications.

The DSP56001 board with a clock rate of 20 MHz can transfer data between itself and its host PC through the host interface using host command interrupts generated by the PC. A parallel-serial adapter connects the DSP56001 parallel port expansion and the i860 Transputer link to provide a direct data path between the DSP56001 and the i860.
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The dual A/D channels both have a maximum 240 kHz sampling rate, and the D/A converters can be operated on a maximum sampling rate 480 kHz or 240 kHz depending on whether one or two channels are being used. They can be driven by a software clock, a hardware clock interrupt or an external trigger. Each of the two input channels and each of the two output channels are provided with a 3rd order Butterworth active low-pass filters having a cut-off frequency of 70 kHz.

Fig. 4.3 Sketch of the system hardware

4.4.2 Microway's Number Smasher-860

Microway's Number Smasher-860 is a coprocessor board that runs in conjunction with the 80386 or 80486 processor in the PC AT bus system. The board comes with all the facilities needed to let it run independently of the host CPU, including memory, boot EPROM, timer and communications channel to the host. Its processor is an Intel i860 running at 20 MHz with 8 Mbytes of 64-bit DRAM. It includes two Transputer compatible link adapters for communication with Transputer compatible systems or a host computer.

4.4.3 Parallel Port and Counter Board - PC14AT

The PC14AT is a plug-in board which provides 48 programmable I/O lines organised
into six ports, and three independent programmable 16 bit counter/timers. For each stepper motor driver card, two I/O lines are needed for mode and direction control, and two counters for speed and distance control.

4.5 System Software

4.5.1 DSP56001 Assembly Language Program

The transmitting signal generation and data acquisition software is written in Motorola DSP56001 assembly language. This program also transfers the captured data from DSP56001 board onto the i860 board. For some processing schemes, on-line processing, e.g., I-Q demodulation or an FFT, can be done on this stage by an assembly language program.

4.5.2 NDP C Language Program

The data transaction, storage and display program is written in NDP C language running on the i860 board. An on-line processing facility also can be provided at this stage.

4.5.3 Microsoft Quick C Program

The Quick C program running on the PC486 is designed to control the stepper motor through the PC14AT board, communicate with the DSP56001 board and to load and start the compiled program onto the DSP56001 board.

4.6 Experimental Configuration

4.6.1 Midwater Setting

This arrangement was used for the early experiments to examine some of the parameters of the system. Targets and transducers were set at about half depth (Fig. 4.4). The four-element transducer block was used with all four elements joined together as an array for both transmission and reception, switching being carried out by a relay circuit. The use
of the relative narrow beam (one way 3 dB angle of approx. 15°) of this 4λ array reduced the effect of multipath from the surface and the bottom and provided a well controlled environment for the system examination.

4.6.2 Tank Floor Setting

Figure 4.5 shows the arrangement used to simulate the situation which the targets are on the sea bed or the target is the sea bed itself. The transducer array is tiled at an angle of 30° aiming its beam at the targets laid on the floor. Either one or two elements were used for transmission and reception to provide a wider swath. To minimise electrical noise and simplify the circuit, separate elements were used for transmission and reception.
4.6.3 Data Acquisition

Although the width of the tank is 5 m the useful length of the platform traversing track is only about 3.5 m. The platform was driven at a constant velocity across the aperture and every 2 cm a pulse was transmitted and a number of samples (usually 900) of the received signal stored. The transmitted signal used was a 120 μs pulse of 40 kHz and the acquisition system sampling frequency was 240 kHz.

4.7 Sonar Equation in the Experimental System

In the SAS as with any other type of sonar system, it is fundamental to be able to detect the signal against the unwanted background and the sonar equation plays a basic role [1]. The signal to be detected is the acoustic energy generated or reflected by the target, and unwanted background could be the media noise or system self noise together with the reverberation, although in some cases such as the studies of the bottom structure this is the wanted target. For this particular system, the signal is transmitted at a level of $SL$, transmission loss $-2TL$ happens due to return paths, the target reflection contributes the target strength $TS$, and the echo will be detected against reverberations $RL$, environmental acoustic noise $NL$ and system noise $SN$. The transducer directivity can depress the acoustic noise $NL$ by $DI$, thus the signal noise ratio at the input of the processing is

$$\text{SNR} = SL - 2TL + TS - RL - (NL - DI) \quad \text{where environmental noise dominates, or}$$

$$\text{SNR} = SL - 2TL + TS - RL - SN \quad \text{where system noise dominates.} \quad (4.1)$$

Some of these quantities will be discussed in more detail in the following sections.

4.7.1 The Source Level

The source level was first calculated from theory. The electrical power input to the transducer was 0.8 watts (input $V_{pp}=90$ V, element impedance $=1.2$ kΩ), the efficiency of the transducer was 60%, thus the power transmitted into water was 0.48 watts. The acoustic intensity $I$ at a distance of 1 m from an omnidirectional source radiating an acoustic power of $W$ is
CHAPTER FOUR

\[ I = \frac{W}{4\pi}. \quad (4.2) \]

For this case, \( W \) is 0.48 watts, the corresponding acoustic intensity was

\[ I_0 = 10 \cdot \log \frac{W}{4\pi} = 10 \cdot \log \frac{0.48}{4\pi} = -15 dB \quad \text{ref} \ 1W / m^2. \]

The directivity index of an element is defined as

\[ DI_1 = 10 \cdot \log \frac{4\pi A}{\lambda^2} \quad (4.3) \]

where \( A \) is the area of the source transmitting was composed of circular elements. When the diameter of the element is \( \lambda \), its directivity index \( DI_1 \) is given by

\[ DI_1 = 10 \cdot \log \frac{4\pi^2 (\frac{\lambda}{2})^2}{\lambda^2} = 10 dB \]

The acoustic intensity can also be determined by the sound pressure at the measuring point, fluid density and sound speed as

\[ I = \frac{p^2}{\rho c} \quad (4.4) \]

For the reference acoustic intensity \( I_{\text{ref}} \), \( p \) is the sound pressure with reference value 1\( \mu Pa \), \( \rho \) is the fluid density which is 1000kg/m\(^3\) for water, and \( c=1500m/sec \) is the sound speed in water.

\[ I_{\text{ref}} = 10 \cdot \log \frac{(1 \times 10^{-6})^2}{1.5 \times 10^6} = -181 dB \quad \text{ref} \ 1W / m^2 \quad (4.5) \]

Therefore, where a single element used for a transmitter, the source level was defined as
\[ SL_1 = 10 \cdot \log_{10} \left( \frac{I_0}{I_{\text{ref}}} \right) + DI_1 = -15dB + 181dB + 10dB = 176dB \]  

(4.6)

Where an array composed of two such elements spaced by \( \lambda \) is used for transmission, the directivity index was increased by \( 10 \cdot \log_2 \) since transmitting area was doubled,

\[ DI_2 = DI_1 + 10 \cdot \log 2 = 10dB + 3dB = 13dB \]  

(4.7)

Therefore, when the same electrical voltage as the one input to the single element transmitter was applied on this two element array, the input electrical power was doubled due to two parallel elements in use. The source level should be

\[ SL_2 = 10 \cdot \log_{10} \left( \frac{2I_0}{I_{\text{ref}}} \right) + DI_2 = 169dB + 13dB = 182dB \]  

(4.8)

The source level of system transmission was obtained by practical measurement where both the transmitter and the hydrophone were hung in the middle of the water in the tank, 1m away from each other. The hydrophone whose sensitivity, \( \zeta \), is \(-210dB \) ref \( IV/\mu Pa \) (equivalents to \( 3.1 \times 10^{-11}V/\mu Pa \)) at 40 kHz was set in the direction to which transmitter main lobe points. When a single element was used as the transmitter, the measured output of the hydrophone was 63mV peak-to-peak, thus its \( \text{rms} \) value was

\[ V_{\text{rms}} = \frac{V_{\text{pp}}}{2\sqrt{2}} \approx 22.5mV \]  

(4.9)

Therefore, the source level of single element transmission can be derived from equation (4.2) as

\[ SL_1' = 20 \cdot \log \left( \frac{22.5 \times 10^{-3}}{3.1 \times 10^{-11}} \right) = 177dB \]  

(4.10)

The source level of the two-element transmission was measured with same method as

\[ SL_2' = 183dB \]  

(4.11)
Because all the measurements above were done in the transmitter main lobes, the values of source level include the directivity index effect. Due to the unreliability of the hydrophone calibration and measuring instruments, the measurements resulted in different values from the theory.

4.7.2 The Definition of Target Strength

The target strength $TS$ of a reflecting body is defined by the expression

$$TS = 10 \cdot \log_{10} \left( \frac{I_r}{I_i} \right)$$  \hspace{1cm} (4.12)

where $I_i$ represents the incident intensity and $I_r$ is the reflected intensity at a distance of 1m away from the target acoustic centre. The target strengths of the targets involved in the experiments were calculated from theory [1].

The theoretical $TS$ of a sphere of radius 'a' is given by

$$TS_{sphere} = 10 \cdot \log \frac{a^2}{4}$$  \hspace{1cm} (4.13)

For the table tennis ball, the radius is 1.9cm. It gives a target strength of -39dB.

The $TS$ of the breeze block and the brick were measured in the direction normal to the reflecting surface. Considering them as rectangulars, the theoretical $TS$ definition is

$$TS_{rec} = 10 \cdot \log \left( \frac{a \cdot b}{\lambda} \right)^2 \cdot \left( \frac{\sin \zeta}{\zeta} \right)^2 \cdot \cos^2 \sigma$$  \hspace{1cm} (4.14)

where $a$, $b$ are the sides of the rectangle, $\lambda$ is the wavelength, $\sigma$ is incidence angle to normal in plane containing side $a$, and

$$\zeta = \frac{2\pi}{\lambda} \cdot a \cdot \sin \sigma .$$

For the breeze block and the brick, $a=0.45m$, $b=0.22m$ and $a=0.22$, $b=0.1m$ respectively, $\sigma=0$, thus
\[ TS_{\text{block}} = 20 \cdot \log \left( \frac{0.45 \times 0.22}{0.0375} \right) = 8 \text{db} \]

\[ TS_{\text{brick}} = 20 \cdot \log \left( \frac{0.22 \times 0.1}{0.0375} \right) = -4.6 \text{dB} \]

If \( \sigma = 5^\circ \), the TS for the breeze block is

\[ TS_{\text{block}} = 20 \cdot \log \left( \frac{0.45 \times 0.22}{0.0375} \cdot \sin 6.6 \cdot \cos \frac{5\pi}{180} \right) = -18 \text{db} \]

(4.15)

If \( \sigma = 4.5^\circ \), the TS for the brick is

\[ TS_{\text{brick}} = 20 \cdot \log \left( \frac{0.22 \times 0.10}{0.0375} \cdot \sin 2.9 \cdot \cos \frac{4.5\pi}{180} \right) = -26 \text{db} \]

(4.16)

The oil drum was considered as a finite cylinder which TS is defined as

\[ TS_{\text{cylinder}} = 10 \cdot \log \frac{a \cdot l^2}{2\lambda} \]

(4.17)

where \( a \) is the radius of the cylinder, \( l \) is length of cylinder and direction of incidence is normal to axis of cylinder. For the oil drum, \( a \) is 13.5cm, \( l \) is 47cm, thus

\[ TS_{\text{drum}} = 10 \cdot \log \frac{0.135 \times 0.47^2}{2 \times 0.0375} = -4 \text{dB} \]

which was not too far away from the measured value.

Following the definition of target strength, the target strengths of the targets involved in the experiments were practically measured and listed in table 4.1 against the theoretically calculated values. The practical measurements were made in the tank, both source and target were set in the midwater, 3m away from each other (this distance was not far enough to meet the conditions of some TS definitions[1], however it was limited by the available size of the tank). The source level was first measured at the position of nominal acoustic centre of the target, and then measured at 1m away from the target, in the same
direction as incidence. Because of lack of a precisely controlled mechanism, the incident direction was only nominally set normal to the reflecting surface. Therefore, some measured $TS$ values were more like the ones with a small angle to the normal of the plane, e.g., $TS$ for breeze block and brick (table 4.1, formulas (4.15), (4.16)). The fish pond target has a very complicated shape so that no theoretical $TS$ was given.

<table>
<thead>
<tr>
<th>target</th>
<th>theoretically calculated target strength</th>
<th>measured target strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>table tennis ball</td>
<td>-39dB</td>
<td>-41dB</td>
</tr>
<tr>
<td>breeze lock</td>
<td>-18 ($5^\circ$)</td>
<td>-18dB</td>
</tr>
<tr>
<td>red brick</td>
<td>-26 ($4.5^\circ$)</td>
<td>-28dB</td>
</tr>
<tr>
<td>oil drum</td>
<td>-4dB</td>
<td>-6dB</td>
</tr>
<tr>
<td>20cm diameter buoy</td>
<td>-20dB</td>
<td>-21.5dB</td>
</tr>
<tr>
<td>coated fish pond</td>
<td></td>
<td>-15dB</td>
</tr>
</tbody>
</table>

Table 4.1 Target strength (40 kHz)

4.7.3 The Transmission Loss

Transmission loss indicates the amount of weakening of the signal between a reference point and a point at a distance in the water. If $I_0$ is the intensity of sound at the reference point located $1m$ from the source, and $I_1$ is the intensity at a distant point, then the transmission loss, $TL$, between the source and the distant point is defined as

$$TL = 10 \cdot \log_{10} \left( \frac{I_0}{I_1} \right)$$  \hspace{1cm} (4.18)

From this definition, the experimental $TL$ was measured practically in the following way. Put the same hydrophone 1m and 3m away from the transmitter respectively, the electrical outputs of the hydrophone were 66mv and 25mv peak to peak. The transmission loss was

$$TS_{3m} = 20 \cdot \log_{10} \frac{66}{25} = 8.4dB$$
In theory, the transmission loss is the sum of two quantities, spreading and attenuation. As the targets were set at relatively short distance from the transducer in this experimental system, i.e., a few metres away, the sound propagation was considered as the spherical spreading. The transmission loss due to spreading is defined as

$$TL_{sp} = 10 \cdot \log(r^2) = 20 \cdot \log(r)$$  \hspace{1cm} (4.19)$$

where $r$ is the distance between the measuring point and source, which was $3m$ in this experiment. Therefore,

$$TL_{3m-sp} = 20 \cdot \log 3 = 9.5dB$$

Absorption loss defined as

$$TL_{ab} = \varepsilon r$$  \hspace{1cm} (4.20)$$

in this experiment was

$$TL_{ab} = \varepsilon R = 0.4dB/km \times 3m = 1.2 \times 10^{-3} dB$$

where $\varepsilon$, the coefficient of absorption, was $0.4dB/km$ (fresh water, 40kHz) [1]. Compared to transmission loss due to spreading, the absorption loss is negligible. Thus

$$TL' = TL_{sp} = 9.5dB$$

which was reasonably close to measured value.

4.7.4 Reverberation Level

From theory, the volume reverberation is calculated as

$$RL_v = SL - 40 \cdot \log r + S_v + 10 \cdot \log VM$$  \hspace{1cm} (4.21)$$

$$VM = \frac{cT}{2} \psi r^2$$
where VM is volume and Sv is volume backscattering strength. For a circular element, the equivalent ideal stereo beam angle \( \Psi \) is given by

\[
10 \cdot \log \Psi = 20 \log \left( \frac{\lambda}{2\pi a} \right) + 7.7 \text{ dB ref 1 steradian} \tag{4.22}
\]

where \( a \) is radius of element.

For a single \( \lambda \) diameter circular element, its equivalent ideal stereo beam angle is

\[ \Psi_1 = 0.6 \text{ steradian} \]

For an array composed of two such elements spaced by \( \lambda \), the equivalent ideal stereo beam angle reduces to one half of \( \Psi_1 \) since the array’s beam angle is halved in one dimension.

\[ \Psi_2 = 0.5 \Psi_1 = 0.3 \text{ steradian} \]

For a typical experimental configuration, a \( \lambda \)-diameter circular element was used for transmission producing source level 176 dB, the distance \( r \) was 3m, pulse length \( \tau \) was 120 \( \mu \)s, volume backscattering strength \( S_v \), say, was -70 dB [1]. The corresponding volume reverberation level was

\[
RL_{v1} = 177 - 40 \cdot \log 3 - 70 + 10 \cdot \log \left( \frac{1500 \times 120 \times 10^{-6}}{2} \times 0.6 \times 3^2 \right) = 85 \text{ dB} \tag{4.23}
\]

Where the two-element array used, the source level was 186dB, the corresponding volume reverberation level was

\[
RL_{v2} = 183 - 40 \cdot \log 3 - 70 + 10 \cdot \log \left( \frac{1500 \times 120 \times 10^{-6}}{2} \times 0.3 \times 3^2 \right) = 88 \text{ dB} \tag{4.24}
\]

Meanwhile, the bottom reverberation level was given by

\[
RL_b = SL - 40 \cdot \log r + S_v + 10 \cdot \log A \tag{4.25}
\]
where $A$ is the effective reverberation area and $S_B$ is bottom reverberation strength. For a circular element, its equivalent ideal beam angle is given by

\[ 10 \cdot \log \Phi = 10 \cdot \log \frac{\lambda}{2\pi a} + 6.9 \text{dB} \text{ ref 1 radian.} \quad (4.26) \]

For a single $\lambda$ diameter circular element, its equivalent ideal beam angle is

\[ \Phi_1 = 1.6 \text{radian} \]

For an array composed of two such elements spaced by $\lambda$, the equivalent ideal stereo beam angle reduces to one half of $\Phi_1$ as

\[ \Phi_2 = 0.5\Phi_1 = 0.8 \text{radian} \]

For the same experimental configuration used for the volume reverberation calculations, say, bottom backscattering strength $S_B$ was -50 dB at the grazing angle of 30° to the bottom [1]. When a single element was used, the expected bottom reverberation level would be

\[ RL_{B1} = 177 - 40 \cdot \log 3 - 50 + 10 \cdot \log \left( \frac{1500 \times 120 \times 10^{-6}}{2} \times 1.6 \times 3 \right) = 104.4 \text{dB} \quad (4.27) \]

Where the two-element array was used the source level was 186dB, the corresponding bottom reverberation level was

\[ RL_{B2} = 183 - 40 \cdot \log 3 - 50 + 10 \cdot \log \left( \frac{1500 \times 120 \times 10^{-6}}{2} \times 0.8 \times 3 \right) = 107.4 \text{dB} \quad (4.28) \]

Because the beam of transducer aimed down to the bottom with the 30° depression angle, the surface was out of the main lobe of the transducer's beam pattern. Thus, the effect of direct surface reverberation caused by transmission could be neglected. However, some quite strong multipath signals via surface reflection were observed. One
of them was the signal along the path of transmitter-object on floor-surface-receiver (Fig. 4.6, a-b-c). Another was through the twice-reflection path (Fig. 4.6, a-d-e-f). After some geometrical calculations, for the first signal, the values of b, c were 2.86m and 0.43m respectively. The signal incident angle to the surface was 44°, to the transducer main lobe direction is 74°. For the second signal, the values for d, e, f were 2m, 2m and 1.7m. The signal incident angle to the surface was 64°, to the transducer main lobe direction was 33°. The first multi path signal level was given by

\[ ML_1 = SL - TL_a + TS - TL_b - TL_c + S_s + DI \] (4.29)

Say, the surface scattering strength \( S_s \) was -45dB (44°), TS was -6dB (oil drum), directivity index at 74° was -14dB reference to normal. When a single element transducer was in use, this multi path signal level was

\[ ML_1 = 177 - 20 \cdot \log 3 - 6 - 20 \cdot \log 2.86 - 20 \cdot \log 0.43 - 45 - 14 = 100.7dB \] (4.30)

When the vertical two-element array is used, the multi path signal level was

\[ ML_{12} = 183 - 20 \cdot \log 3 - 6 - 20 \cdot \log 2.86 - 20 \cdot \log 0.43 - 45 - 14 = 106.7dB \] (4.31)

where the directivity did not change due to the grating lobe of the two-point array.

The second multi path signal level was given by

\[ ML_2 = SL - TL_a + TS - TL_d - TL_e - TL_f + S_s + S_B + DI \] (4.32)
Say, the surface scattering strength $S_s$ was -40dB ($64^\circ$), the bottom scattering strength $S_b$ was -10dB ($64^\circ$), $TS$ was -6dB (oil drum), directivity index of the single element at $33^\circ$ was -3dB reference to normal, directivity index of the two-element array at $33^\circ$ was -28dB reference to normal. When a single element transducer was in use, this multi path signal level was

$$ML_{21} = 177 - 20 \cdot \log 3 - 6 - 20 \cdot \log 2 - 20 \cdot \log 1.7 - 45 - 10 - 3 = 92dB$$

(4.33)

When the two-element array is used, the multi path signal level was

$$ML_{22} = 183 - 20 \cdot \log 3 - 6 - 20 \cdot \log 2 - 20 \cdot \log 1.7 - 45 - 10 - 3 - 28 = 73dB$$

(4.34)

4.7.5 Noise Level

The spectrum level of thermal noise is given from theory as

$$NL_T = -15 + 20 \cdot \log f - DI - E$$

(4.35)

where $E$ is the efficiency which was -4.4 dB (60%) for the transducers used in the experiment.

For the single element and the array at 40 kHz, they should be

$$NL_{T1} = -15 + 20 \cdot \log 40 - 10 + 4.4 = 11.4dB$$

(4.36)

$$NL_{T2} = -15 + 20 \cdot \log 40 - 13 + 4.4 = 8.4dB$$

(4.37)

which were equivalent to $3 \times 10^{-9} V$ and $2 \times 10^{-9} V$ noise on the transducer ($7.9 \times 10^{-10} V / \mu Pa$) output respectively. Due to the system bandwidth, the actual thermal noise level was expected to be higher.

Since the measuring system self noise was up to $1 \times 10^{-6} V$, there were not any practical measurements available for the thermal noise level.

The experimental system self noise equivalent on transducer output was given by a measurement value as
which obviously dominated the noise level.

### 4.7.6 A Predicted Signal to Noise Ratio at the Input of the Processor

SNR in the typical experimental setting shown in Fig. 4.7 was predicted from theory first. For the case that an oil drum was to be detected against the bottom reverberation and noise background with the single element transducer from the direction normal to its axis, the wanted signal level appearing on receiver \((7.9 \times 10^{-10} \text{ V} / \mu \text{Pa})\) was

\[
WL_{\text{drum}} = SL - 2TL_a + TS = 177 - 2 \times 9.5 - 6 = 110 \text{ dB}
\]  \hspace{1cm} (4.39)

which produces \(28 \mu \text{V}\) on transducer output, and the bottom reverberation produced \(117 \mu \text{V}\) output. Therefore, the signal to noise ratio on the input of the processing was given by

\[
SNR_{\text{drum}} = 20 \cdot \log \frac{28 \times 10^{-3}}{117 \times 10^{-6} + 2.5 \times 10^{-6}} = 47 \text{ dB} .
\]  \hspace{1cm} (4.40)

When the two-element array was used, the source level increased to \(183 \text{ dB}\), and receiver sensitivity was also doubled due to two elements. The final signal to noise ratio was given by

\[
SNR_{\text{drum2}} = 20 \cdot \log \frac{112 \times 10^{-3}}{524 \times 10^{-6} + 2.5 \times 10^{-6}} = 47 \text{ dB}
\]  \hspace{1cm} (4.41)

This showed that using the two-element array did not improve the SNR for detecting objects against the bottom. However, if a target was to be detected by the multi path signals (Section 4.7.4), i.e., to detect bottom against the multi path signal, the usage of a two-element array could improve the SNR by reducing the background level (referring to equations 4.27, 4.28, 4.30, 4.31, 4.33 and 4.34).

### 4.7.7 Typical Experimental Data
Fig. 4.7 shows the practical data collected from the tank under the configuration shown in Fig. 4.6 with the single element transducer, and the detail of the echoes from the area around the target (oil drum) is shown in Fig. 4.8.

The first part of the data about the 1st-50th samples was the reflections of the tank wall behind the transducer. The following part was relatively quiet until about the 450th sample where the echo from the bottom vertically under the transducer arrived at the transducer. This echo reflected between the bottom and surface appeared received signal a few times. Therefore, the bottom reverberation should start coming in after this part was not clearly visible. From about the 950th sample, the echoes due to the target arrived at the receiver followed by the surface reflection through the path b-c (Fig. 4.6) about 50 samples later than main echoes directly from the target. Since the existence of these multipath signals and the effect of multi reflections of side lobe transmission and reception, there was no clear acoustic shadow behind the target observed, and also the measured target to the bottom reverberation ratios did not match the calculated values (Equs. 4.40, 4.41). The measured SNR for one element and two elements setting made from the figures 4.8 and 4.10 (samples 900th-1000th) were about 20dB and 7dB respectively. The echoes starting from about the 1400th sample were the second multipath signal due to path d-e-f (Fig. 4.6) which was significantly depressed by using the narrower beam of two elements array (Fig. 4.9, about 1400th sample).
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Fig. 4.8 The 900th-1600th samples of figure 4.7

Fig. 4.9 The data due to the drum collected by the two-element array

Fig. 4.10 The 900th-1600th samples of figure 4.9
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The reconstructed image by synthetic aperture processing from this experimental data is going to be shown in Chapter Six.

References

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SIMULATIONS

5.1 Introduction

To predict the performance, the processing schemes introduced in Chapter Two and the motion compensation algorithms introduced in Chapter Three were computationally simulated. In these simulations, the system parameters were selected to match the ones in the experimental system (where applicable) in order to compare the two sets of results.

5.2 Simulated Echo Returns

Before any processing can be done, a simulated version of the sonar echo returns must be made available. The method to form the sonar returns is to project the targets in the matrix of the target ground position (x-y) onto a two dimensional memory with track ordinate corresponding to the rows and return slant range to the columns. The procedure of the projection is first to transfer each scattering point \( T_{x,y} \) in the target matrix onto track position-return slant range matrix \((x-R)\), i.e., a single point target in \(x-y\) matrix will correspond to a parabola in the \(x-R\) matrix (Fig. 5.1), and then convolve each column of the \(x-R\) matrix with the transmitted signal waveform. The results of the convolution are saved into two dimensional memory as the sonar returns at the sequential track positions (Fig. 5.3). The mathematical presentation is as follows.

Let \( XY \) denote the target matrix with \(N\) point targets \((x_0,y_0), (x_1,y_1) \ldots (x_N,y_N)\) in it,

\[
XY(x,y) = 0, \text{ where } x \neq x_n \text{ or } y \neq y_n, \quad n = 1, 2 \ldots N.
\]

\[
XY(x,y) = K \neq 0, \text{ where } x = x_n \text{ and } y = y_n, \quad n = 1, 2 \ldots N, \quad (5.1)
\]
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where $K$ is a factor determined by the target strength. For each target point $(x_n, y_n)$, the corresponding parabola in the $x-R$ matrix $XR$ is

$$XR(x, R) = 0,$$
where $R \neq R_n$,

$$XR(x, R) = S \neq 0,$$
where $R = R_n$, \hspace{1cm} (5.2)

where $R_n(x, y_n) = 2 \sqrt{y_n^2 + (x - x_n)^2}$ \hspace{1cm} (5.3)

$x_n - \frac{L}{2} \leq x < x_n + \frac{L}{2}$, $L$ is the length of synthetic aperture.

Assuming the transducer has a rectangular beam pattern, the transmitted signal waveform $TW$ is raised-cosine weighted pulse (Fig. 5.2) which is

$$TW(R) = \sin \left( \frac{2\pi \cdot rR}{\lambda \cdot f_{sig} / f_{sample}} \right) \cdot \frac{1}{2} \cdot \cos \left( 1 - \frac{2\pi \cdot R}{M} \right) \cdot [u(0) - u(R - M)] \hspace{1cm} (5.4)$$

68
where $u$ is the step function, $M$ is the pulse length in sampling points, and $f_{\text{sig}}, f_{\text{sample}}$ are the signal frequency and sampling frequency respectively. Thus, the sonar returns $E(x,R)$ are

$$E(x_n, R) = TW \otimes XR(x_n, R)$$

(5.5)

where $x_n$ is a track position, $E(x_n, R)$ and $XR(x_n, R)$ is the nth column of $E(x, R)$ and $XR(x, R)$ respectively, $\otimes$ denotes convolution.

![Fig. 5.3 Sonar returns due to a point target](image)

A sonar return data array $E_0(x, R)$ was formed for the further simulation study, where the system parameters were:

- signal frequency: $f_{\text{sig}} = 40kHz$
- sampling frequency: $f_{\text{sample}} = 240kHz$
- pulse length: $\tau = 120\mu s$
- target configuration: two point targets, their the slant ranges at closest approach point are both 3m.
- track sampling spacing: 2cm
- reflection factor: $K=1$
- receiver: single $\lambda$ diameter circular element
- transmitter: single $\lambda$ diameter circular element
5.3 Coherent Addition

As discussed in early chapters, this algorithm is a simple, efficient implementation of synthetic aperture processing. Each pixel of the image is taken as the amplitude of the summation over the data collected from the data array $E_o(x,R)$ along a certain parabola (Section 2.4.1). The images shown in this section are all 100×20 pixels where the pixel resolution is 1cm×1cm. Fig. 5.4 and Fig. 5.6 show the reconstructed images synthesised with a 2m aperture and a 3m aperture respectively. Fig. 5.5 shows the range shell of Fig. 5.4 in which the two point targets are. It can be observed from the graph in Fig. 5.5 that the 3 dB width of the main lobe is about three pixels, i.e., 3cm, which approximately agrees with the theoretical value (refer Equ. 2.3)

$$\delta x = 2.8 \text{cm}.$$  \hfill (5.6)
Similarly, the resolution of the 3m synthetic aperture can be obtained from the graph shown in Fig. 5.7. The 3 dB width of the main lobe is about two pixels, i.e., 2cm, which approximately agrees with the theoretical value

\[ \delta x \approx 1.9 \text{cm} \quad (5.7) \]

The beam pattern of the synthetic aperture is in the shape of \( \frac{\sin x}{x} \). If appropriate weighting had been used, the side lobe could have been reduced.

![Fig. 5.6 The image reconstructed with 3m aperture](image1)

![Fig. 5.7 The range shell containing the targets](image2)

Because the focusing function, in other words the shape of the parabola, is range dependent whereas the echo due to a point target spreading \( ct \) in return path range axis
(τ is the transmitting pulse length, c is sound speed in water), each wavefront in the entire pulse length cannot be properly focused simultaneously. It is important to focus on the point of the pulse where the maximum absolute amplitude of the pulse occurs, i.e. the 15th sample in Fig. 5.2. or the thick line in Fig. 5.8 (a). This problem is explained in Fig. 5.8 where picture (a) shows the data band due to a point target, and picture (b) shows a group of parabolas corresponding to the range dependent focusing function. In picture (a), the curves denote the wavefronts due to the point target where solid lines stand for the peak amplitude wavefront in the local cycle whereas dotted lines stand for the zero amplitude wavefront. The pulse centre wave front where the amplitude reaches the maximum is indicated by the thick line in (a). The curvatures of each wavefront in (a) are all the same, which are decided by the range from the target to the aperture, $R_0$.

Each curve in (b) represents a focusing function for that particular range shell where the thick line stands for focusing function of range $R_0$. Because the focusing function is range dependent, the curvatures of the parabolas are different from one another. Since the thick lines in (a) and (b) are both due to range $R_0$, they have same curvature.

The array focusing here is partially to fit that group of parabolas (b) onto the two dimensional data array (a). When the group of focusing parabola is fitted onto the echo data array in such a way that two thick lines overlay, the pulse centre wavefront will be focused properly so that the coherent addition has the best output. If the thick line in (b) overlay on the other curve in (a), the thick line in (a) will be overlaid on by the other parabola in (b) which has a different curvature. This range mismatching will lead the array output to be non optimum by focusing on to a non-maximum amplitude wavefront.

![graph](image_url)
The mismatching can be caused by not using a pulse centre, e.g. the front edge of the pulse, as the reference point of the time of arrival, or an imprecise time of starting edge of range sampling window.

Fig. 5.9 shows the effects of the range mismatching in coherent addition SAS imaging. All images were formed from $E_0(x,R)$ with the 2m aperture. The mismatches in (a), (b), (c) and (d) were uniform and $\lambda/2$, $\lambda$, a half pulse length (9.5cm) and a pulse length (19 cm). It can be seen that the main lobe was getting broader with increasing mismatch, and obvious twin peaks appeared when the mismatch was up to the pulse length.
(b) Range mismatching of $\lambda$

(c) Range mismatching of half pulse length

(d) Range mismatching of pulse length

Fig. 5.9 Effects of range mismatching on coherent addition
Fig. 5.10 shows the effects of range sampling rate upon the image reconstruction of coherent addition algorithm. Image (a) was formed from the data sampled on triple carrier frequency, i.e. 120 kHz, image (b) was formed with sampling frequency of 160 kHz and image (c) with 480 kHz. Together with the image in Fig. 5.4 which was formed from the data sampled six pointed per cycle (240kHz), they suggested that the sampling rate of four point per cycle was necessary for obtaining a reasonably useful image, six samples per cycle sampling gave a quite good results, and twelve points per cycle sampling seemed unnecessary since no dramatic improvement was observed except the speckle back ground was smoothed by more accurate phase information.
5.4 I-Q Processing

The sonar return data set used in this section is the $D_0(x,R)$ which is the $E_0(x,R)$ demodulated version where carrier frequency is removed. The images shown in this section are all in the size of 100×20 pixels where pixel resolution is 1cm×1cm. Fig. 5.11(a) and Fig. 5.12(a) show the reconstructed images synthesised with 2m aperture and 3m aperture respectively. Fig. 5.11(b) shows the range shell of Fig. 5.11(a) in which the two point targets are. It can be observed from the graph in Fig. 5.11(b) that the 3 dB width of the main lobe is about three pixels, i.e., 3cm, which agrees with the theoretical value (Equ. 5.6).
Similarly, the resolution of the 3m synthetic aperture can be obtained from the graph shown in Fig. 5.12(b). The 3 dB width of the main lobe is about two pixels, 2cm, which approximately agrees with the theoretical value (Equ. 5.7).
(a) Effect of $\lambda/2$ range mismatching

(b) Effect of $\lambda$ range mismatching

(c) Effect of half pulse length range mismatching
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Fig. 5.13 Effect of range mismatching on I-Q processing

Sharing the same focusing principle with the coherent addition, the I-Q processing suffers from the mismatching of range shell focusing as well. However, since the carrier component has been removed by the demodulation, the reconstructed image is much less sensitive to the range mismatch, and the mismatch broadened the main lobe rather than splitting it. From this point of view, the correlation processing can tolerate more error in the platform trajectory than coherent addition. Especially for the contrast based autofocus, avoiding peak splitting can prevent high contrast from indicating a false optimum focus of the image. The mismatching in Figures 5.13(a)-(d) was \( \lambda/2, \lambda \), a half pulse length and a pulse length respectively.

5.5 Broad-band Low-Q Transmission

This SAS scheme was suggested to avoid the ambiguities caused by under sampling along the track direction. The following simulations were to reconstruct the range shell containing two point targets 3m away. The signal frequency was 40kHz, and the aperture was synthesised on 2m (effective transducer dimension was D=6cm). To save computational time, only 360 pixels (bearing -10° to 26°) were calculated where the 50th pixel corresponded to the 0° bearing.
Fig. 5.14 Effect of under sampling the aperture
The first set of results shown in Fig. 5.14 were formed under the condition that the transmitted pulse was infinitely long, and the along-track sampling spacing was 2cm (a), 3cm (b), 4cm (c), 6cm (d) and 12cm (e) respectively. The second set of images shown in Fig. 5.15 resulted from the constant along-track spacing of 12cm and the transmitted pulse with 10kHz, 17kHz, 38kHz and 75 kHz bandwidth respectively.

![Graphs showing effect of pulse bandwidth](image.png)

**Fig. 5.15** Effect of pulse bandwidth when the aperture undersampled
It was observed that the broadband width pulse can reduce ambiguities significantly.

5.6 Frequency Domain Processing Using CTFM

The advantages of the CTFM SAS are shown by figures 5.16 (a), 5.16 (b), 5.17 (a) and 5.17 (b) containing two point targets 3m away from the aperture. The sizes of the imaging windows in Fig. 5.16 (a) and Fig. 5.16 (b) are both 102cm (along-track) by 100cm (across-track). The pixel along-track resolution was artificially set to 2cm whereas its across-track resolution was decided by the FFT resolution. To form the image in Fig. 5.16 (a), the target strength versus range information at each track position was obtained from a FFT of 400 along range samples which gave the range shell resolution of 5cm, and each pixel was synthesised from a three-metre long aperture. Fig. 5.16 (b) shows the range shell in Fig. 5.16 (a) containing the two point targets.

![Diagram](image_url)

Fig. 5.16 The image formed with 400 point FFT
The image in Fig. 5.17 (a) was reconstructed by the same method except the data set used only had 200 range samples at each track position. Thus, the range shell resolution was 10 cm, as twice coarse as it was in Fig. 5.16 (a). Similarly, Fig. 5.17 (b) displays the range shell containing the targets of Fig. 5.17 (a). It was observed that the azimuth resolution remains reasonably constant (1 pixel, 2 cm) with reducing range samples. As a consequence of reducing the number of range samples, the time taken by the sonar platform to traverse through the aperture of a given length is shortened by sacrifice of the range resolution. However, since the focusing function (or say the matched filter) is range dependent, the less precise range information due to the coarser range shell will smear the focus point. Therefore, the azimuth resolution can not remain constant with unlimited reduction of range samples. Fig. 5.18 which has the same size imaging window as the previous figures shows this effect resulting from data set with 100 range samples per track position where azimuth resolution was two pixels, i.e. 4 cm.
5.7 Envelope Processing

The simulation for envelope processing shared the same processing program with coherent addition. The sonar return array used for the processing input was the envelope of signal of \( E_0(x,R) \). To show the along-track resolution of envelope processing being relevant to the range resolution, two types of transmitted signal, 125\( \mu \)s raised-cosine weighted pulse (8kHz frequency bandwidth) and 50\( \mu \)s raised-cosine weighted pulse (20kHz frequency bandwidth) were used in the simulations. The effective pulse length of the raised-cosine weighted pulse is 0.4 times the length of the pulse envelope.
Table 5.2 shows the theoretical and simulated values of along-track resolution due to different aperture and pulse length.

<table>
<thead>
<tr>
<th></th>
<th>125μs pulse length, δR=0.038cm</th>
<th>50μs pulse length, δR=0.015cm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>theoretical</td>
<td>simulated</td>
</tr>
<tr>
<td>2m aperture</td>
<td>23cm</td>
<td>20.5cm</td>
</tr>
<tr>
<td>3m aperture</td>
<td>15cm</td>
<td>14cm</td>
</tr>
</tbody>
</table>

Table 5.1 Along-track resolutions due to different configurations

Fig. 5.19 and Fig. 5.20 are images formed with the 125μs pulsed transmission, 2m and 3m aperture respectively. The images in Fig. 5.21 were formed with the 50μs transmission and 2m aperture.
Fig. 5.20 Reconstructed image with 3m aperture-125µs pulse

(a) 3D plotting

(b) The range shell containing the targets

Fig. 5.20 Reconstructed image with 3m aperture-125µs pulse
5.8 Contrast Optimisation Autofocusing

Since the I-Q processing provided smoother image than coherent addition (refer Sections 5.3 and 5.4), the contrast of a image formed by I-Q processing would indicate the focus of the image more appropriately. Therefore, the I-Q processing method was used in this section. The sonar return data set used in this section is the $D(x,R)$ which is the $E(x,R)$ demodulated version in which the carrier frequency has been removed. Its properly focused image is shown in Fig. 5.12(a) with 100 pixels×20 range-shells where the two point targets are in the 10th range shell.

5.8.1 Depth of Velocity

If there is an error on the along-track velocity of sonar platform, and the data is still
sampled on a constant time basis, the actual along-track sampling spacing will be
different from the nominal value. Equation (3.7) can be arranged as

$$\frac{L_{su}}{S} = \frac{\delta x}{\delta s}$$  \hspace{1cm} (5.8)

where $S$ is sampling spacing and $\delta x$ is called the depth of spacing. In the simulation
where the contrast of each range shell was shown, the length of the synthetic aperture is
3m, the azimuth resolution is 2cm, the nominal sampling spacing is 2cm. Thus, the depth
of sampling is

$$\delta s = 0.013cm$$  \hspace{1cm} (5.9)

Fig. 5.22 shows the contrast of the range shell containing the point target against the
actual along-track sampling spacing varying around the nominal value of 2cm $\pm 5\delta s$
(where $\delta s$ is the depth of spacing). Fig. 5.23 shows the target range shell of the image
with the spacing error of five times the depth of spacing.

![Graph](image1.png)

Fig. 5.22 The contrast target range shell versus the spacing error

![Graph](image2.png)

Fig. 5.23 The target range shell reconstructed with $5\delta s$ spacing error

5.8.2 Contrast Optimisation

The first step of contrast optimising is selecting a number of range shells whose contrast
will be used to indicate the quality of the whole image’s focus. To avoid such a range
shell where the high contrast accidentally given by the defocusing effects is not a reliable
indicator of structural content, a shorter aperture should be used to process the whole image with the initial estimate.

To show the effect of defocusing on the structural range shell selection, two images were reconstructed from $D_0(x,R)$ with a spacing error of five times the depth of spacing corresponding to the 3m aperture. The first one synthesised with the 3m aperture, and the second one with the 0.6m aperture. The contrasts of each range shell are plotted in Fig. 5.24 and Fig. 5.25 respectively. Since the lower azimuth resolution processing can tolerate more focusing error (the five times depth of spacing for the 3m aperture equals to a depth of spacing for a 0.6m aperture; Equ. 3.6), the contrast of coarser resolution range shell indicated the right high structural range shell in which the point target is, i.e., the 10th range shell.

Fig. 5.24 The contrast of range shells in the image formed with 3m aperture

Fig. 5.25 The contrast of range shells in the image formed with 0.6m aperture

Fig. 5.26 The image processed with a spacing error
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When the indicating range shells (only one range shell was selected in this example) have been selected, the contrast optimisation at the chosen resolution, i.e., 3cm with 2m aperture, can commence. The optimisation will be terminated by the criterion which is that difference between the present best estimate and the previous one is less than a quarter of the depth of spacing plus the spacing between the present three estimates less than a quarter of the depth of spacing. Fig. 5.26 shows the initial image processed with the uniform spacing error in terms of five times depth of spacing, and Fig. 5.27 is the image autofocused by the contrast optimisation. It is clearly shown that the focus of the image was dramatically improved by autofocusing. Fig. 5.28 shows the history of the three spacing values in each estimate, in which the final spacing compensation for the spacing error of 0.07475cm was -0.0748cm.

---

Fig. 5.27 The autofocused image

Fig. 5.28 The history of autofocus parameter
Since the spacing error was uniform in the last simulation, the whole width of the image (100 pixels) was autofocused in one piece.

![Fig. 5.29 The simulation geometry](image)

In most practical situations, the motion errors are more likely to be non-uniform. A simple example is that the along-track sampling spacing changes linearly relative to the along-track position. Before starting any further simulations, here is an introduction to the observation geometry of the simulations (Fig. 5.29). The scene (S-S') to be observed was 100cm long by 20cm wide with 5 point targets at the same range, 3m away from the sonar track. The sonar platform traversed through a 5m nominal track (dashed line) from D to D'. The actual sonar trajectory was nominally indicated by a solid freeform line. A 2m long synthetic aperture (thick solid line) slid from P to P' to form each pixel in the scene S-S'. The image formed with error free aperture is shown in Fig. 5.30.

![Fig. 5.30 The error free image](image)
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Table 5.2 List of some spacing errors

<table>
<thead>
<tr>
<th>Description of spacing error</th>
<th>error image</th>
<th>autofocused image</th>
</tr>
</thead>
<tbody>
<tr>
<td>linearly changing form -8 times depth of spacing to +8 times depth of spacing during sonar moving from D to D'</td>
<td>Fig. 5.31</td>
<td>Fig. 5.32 (a)</td>
</tr>
<tr>
<td>changing in sin form, one cycle within the track D-D', with the amplitude 16 times depth of spacing</td>
<td>Fig. 5.33</td>
<td>Fig. 5.34 (a)</td>
</tr>
<tr>
<td>changing in sin form, two cycles within the track D-D', with the amplitude 16 times depth of spacing</td>
<td>Fig. 5.35</td>
<td>Fig. 5.36 (a)</td>
</tr>
<tr>
<td>changing in sin form, four cycles within the track D-D', with the amplitude 16 times depth of spacing</td>
<td>Fig. 5.37</td>
<td>Fig. 5.38 (a)</td>
</tr>
</tbody>
</table>

The fairly simple examples for the sonar trajectory with motion errors is that the platform traversed along the straight line with some non-uniform sampling spacing errors. Table 5.2 lists some non-uniform sampling spacing errors, and Fig. 5.31 to Fig. 5.34 show their effects on the image respectively.

In these simulations, the two-cycle in track sin form spacing error caused most severe defocus, the four-cycle in track sin form spacing error defocused the image least, linear error and one-cycle sin form error gave some medium effects. This indicated that the most harmful spacing error was the periodic like errors with the wavelength similar to the synthetic aperture length, and the high frequency like error seemed to cancel each other.

Fig. 5.31 Image formed from data with spacing error
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Fig. 5.32(a) Autofocused image

Fig. 5.32(b) History of autofocusing

Fig. 5.33 Image formed from data with spacing error
Fig. 5.34(a) Autofocused image

Fig. 5.34 (b) History of autofocusing

Fig. 5.35 Image formed from data with spacing error
Fig. 5.36(a) Autofocused image

Fig. 5.36(b) History of autofocusing

Fig. 5.37 Image formed from data with spacing error
Part (a) of Fig. 5.35 to Fig. 5.38 shows the autofocused images corresponding to Fig. 5.31 to Fig. 5.34. In each autofocusing, the image of whole scene was split into five azimuth strips to be focused in order to compensate the non-uniform error. Part (b) of these four figures shows the history of the autofocus parameter of each image strip, and each time the space between triple estimates jumped to a bigger value it indicated the beginning of a new image strip. The best estimate of the previous strip was used as the initial estimate for the present azimuth strip, but a triple estimate space of several depth of spacing was used for initial processing for each strip to avoid locking onto the side lobe of the contrast curve. The significant improvements on image focus were shown in these simulation results.
For most practical situations, the actual trajectory of the towfish would not be straight but a kind of periodic like curved line (Fig. 5.29), e.g. the error motion caused by a surface wave. Some such kinds of track with lateral displacement from the nominal track are listed in Table 5.3, and the simulated results are shown in Fig. 5.39 to Fig. 5.43. To insure that no data will be lost at any track position of any aperture, the lateral errors were limited by $2\lambda$, i.e., well within a pulse length (pulse length is $4.5\lambda$).

<table>
<thead>
<tr>
<th>description of track error</th>
<th>Relative figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin form, one cycle within the track D-D', with the amplitude of $2\lambda$</td>
<td>5.39</td>
</tr>
<tr>
<td>sin form, two cycles within the track D-D', with the amplitude of $2\lambda$</td>
<td>5.40</td>
</tr>
<tr>
<td>sin form, five cycles within the track D-D', with the amplitude of $2\lambda$</td>
<td>5.41</td>
</tr>
<tr>
<td>sin form, ten cycles within the track D-D', with the amplitude of $2\lambda$</td>
<td>5.42</td>
</tr>
<tr>
<td>periodic like random, displayed in Fig.5.43(c), nominal track on x-axis</td>
<td>5.43</td>
</tr>
</tbody>
</table>

Table 5.3 List of some track errors

Since non uniform across-track displacement from the nominal track was caused by non zero across-track acceleration, these track errors can be corrected by compensating for the across-track acceleration. According to Section 3.3.3, the across-track acceleration can be compensated for by its equivalent along-track velocity error, further by the equivalent along-track sampling spacing error. Therefore, the along-track spacing was still used in the autofocus processing to correct the track error.

![Fig. 5.39(a) Image formed with track error](image-url)
Fig. 5.39 (b) Autofocused image

Fig. 5.39(c) History of autofocusing

Fig. 5.40 (a) Image formed with track error
Fig. 5.40 (b) Autofocused image

Fig. 5.40 (c) Autofocusing history

Fig. 5.41 (a) Image formed with track error
Fig. 5.41(b) Autofocused image

Fig. 5.41(c) Autofocusing history

Fig. 5.42(a) Image formed with the track errors
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Fig. 5.42 (b) Autofocused image

Fig. 5.42(c) Autofocusing history

Fig. 5.43(a) The periodic like random track
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Fig. 5.43 (b) Image with track error

Fig. 5.43 (c) Autofocused image

Fig. 5.43 (d) Autofocusing history
In figures 5.39-5.43, part (a) shows the ill focused image, part (b) is the autofocused image and part (c) displays the autofocusing history. The image of the whole scene was also split into five strips in order to let autofocusing cope with the non uniform track errors. The autofocusing processing on each strip used the best autofocus parameter of the previous strip, but always started with a wide triple estimate spacing, i.e. several depths of spacing.

In these simulations, the track error with one or two cycles in track caused the least defocus, therefore, the improvement on the focus of their autofocused image was not significant. The image corresponding to the track error of five cycles in track was severely defocused, and the autofocusing improved its focus dramatically. For the error with ten cycles in track or the periodic like random error (Fig. 5.43 (a)), the performances of the autofocus processing were not good. It seemed to agree with the suggestion made by I. P. Finley and J. W. Wood [1] which was that the autofocusing cannot cope with the motion error with a structure much finer than the synthetic aperture.

It was noticed that there were the geometrical distortions in the final images, since the autofocus only attempts to resolve the defocus problem. However, the geometrical distortion can be removed by other processing which was not the interest of this particular research but available in some other publications.

5.9 Phase Differential SAS

To simulate the performance of the phase differential SAS, some images were reconstructed using this algorithm to form the aperture with lateral position errors. The simulation geometry was still the one shown in Fig. 5.29, and some track error descriptions are listed in Table 5.4.

Since four and half cycles (40kHz) long pulsed transmission was used in the simulations, the track errors were limited to within two wavelengths to avoid totally losing the information about the targets at any track position. Fig. 5.44 showed the image formed with phase differential synthetic aperture without track error which gave no advantage over the conventional synthetic aperture without the presence of the track errors.
(comparing with Fig. 5.4). Since the phase integration introduced more errors into the phase history, the image formed by the phase differential synthetic aperture was actually worse than conventional one under this circumstance.

<table>
<thead>
<tr>
<th>description of track error</th>
<th>Relative figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin form, 1 cycle within the track D-D', symmetrical about the nominal track with the amplitude of $2\lambda$.</td>
<td>5.45</td>
</tr>
<tr>
<td>sin form, 1 cycles within the track D-D', asymmetrical about the nominal track with the amplitude of $2\lambda$.</td>
<td>5.46</td>
</tr>
<tr>
<td>sin form, 10 cycles within the track D-D', symmetrical about the nominal track with the amplitude of $2\lambda$.</td>
<td>5.47</td>
</tr>
<tr>
<td>sin form, 10 cycles within the track D-D', asymmetrical about the nominal track with the amplitude of $2\lambda$.</td>
<td>5.48</td>
</tr>
<tr>
<td>periodic like random, displayed in Fig. 5.49 (c), nominal track is on x-axis</td>
<td>5.49</td>
</tr>
<tr>
<td>normally distributed random, displayed in Fig. 5.50 (c), x-axis is nominal track</td>
<td>5.50</td>
</tr>
</tbody>
</table>

Table 5.4 Descriptions of track errors

![Fig. 5.44 Track error free image formed by phase differential method](image)

In figures 5.45-5.50, part (a) shows the image formed with the normal I-Q processing method and part(b) is the image reconstructed using the phase differential synthetic aperture processing.
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Fig. 5.45 (a) The image formed by the conventional synthetic aperture

Fig. 5.45 (b) The image formed by phase differential synthetic aperture

Fig. 5.46 (a) The image formed by the conventional synthetic aperture
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Fig. 5.46 (b) The image formed by phase differential synthetic aperture

Fig. 5.47(a) The image formed by the conventional synthetic aperture

Fig. 5.47(b) The image formed by phase differential synthetic aperture
Fig. 5.48 (a) The image formed by the conventional synthetic aperture

Fig. 5.48 (b) The image formed by phase differential synthetic aperture

Fig. 5.49 (a) The image formed by the conventional synthetic aperture
Fig. 5.49 (b) The image formed by phase differential synthetic aperture

Fig. 5.49 (c) Track errors

Fig. 5.50 (a) The image formed by the conventional synthetic aperture
It has been shown that the advantage of the phase differential synthetic aperture was significant over the conventional synthetic aperture where the track error displacements were greater than $\lambda$ and asymmetrical about the nominal track, especially when the error were periodic like random or normally distributed random. Under these circumstances, the two point targets in the image formed by conventional synthetic aperture were severely smeared, the ones formed by phase differential synthetic aperture were still reasonably resolvable. However, where the track error were symmetrical about the nominal track, the phase differential synthetic aperture did not show great advantages over conventional one. This was mainly due to the conventional synthetic aperture did not suffer much from the error, since they cancelled each other.
5.10 Performance of Envelope Processing on Motion Errors

Since the envelope processing uses no phase information but only the envelope of the echoes to synthesise the image, its performance is expected to be not sensitive to the period of error motion. It should only be sensitive to the error motion with amplitude of the same order of transmitted pulse length.

<table>
<thead>
<tr>
<th>description of track error</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin form, one cycle within the track D-D', with the amplitude of $\lambda$</td>
<td>5.51</td>
</tr>
<tr>
<td>sin form, one cycle within the track D-D', with the amplitude of $2\lambda$</td>
<td>5.52</td>
</tr>
<tr>
<td>sin form, one cycle within the track D-D', with the amplitude of $3\lambda$</td>
<td>5.53</td>
</tr>
<tr>
<td>sin form, ten cycles within the track D-D', with the amplitude of $\lambda$</td>
<td>5.54</td>
</tr>
<tr>
<td>sin form, ten cycles within the track D-D', with the amplitude of $2\lambda$</td>
<td>5.55</td>
</tr>
<tr>
<td>sin form, ten cycles within the track D-D', with the amplitude of $3\lambda$</td>
<td>5.56</td>
</tr>
<tr>
<td>random with small amplitude, shown in Fig. 5.50(c), nominal track on x-axis</td>
<td>5.57</td>
</tr>
<tr>
<td>random with large amplitude, shown in Fig. 5.58(b), nominal track on x-axis</td>
<td>5.58</td>
</tr>
</tbody>
</table>

Table 5.5 List of the track errors

To examine these predictions, some track errors were selected as listed in Table 5.5. The simulation results are shown in figures 5.51-5.58 (referring Fig. 5.19 (a) for an error free image). The same geometry shown in Fig. 5.29 was used in the simulation of this section.

When the errors changed rather slow along the track, e.g. one cycle within a 2.5m track, the effects on images of the error smaller than the pulse length were invisible, and the error greater than the pulse length shifted the position of the images rather than broaden them (Fig. 5.51, 5.52, 5.53). However, when the errors changed much faster, e.g. ten cycles along the 2.5m track, the effects on the image were obvious, especially the error greater than the pulse length severely smeared the image (Fig. 5.54, 5.55, 5.56). When the errors were in the form of a normally distributed random wave (Fig. 5.58 (b)), the effects on the image were dependent of their standard deviation. If the measured amplitude was sufficiently small, e.g. less than $\lambda$, the damage on the image was invisible (Fig. 5.57). But, if the amplitude was greater than the pulse length, the image would be totally smeared (Fig. 5.58 (a)).
Fig. 5.51 Image formed with track error

Fig. 5.52 Image formed with track error
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Fig. 5.53 Image formed with track error

Fig. 5.54 Image formed with track error

Fig. 5.55 Image formed with track error

112
Fig. 5.56 Image formed with track error

Fig. 5.57 Image formed with track error

Fig. 5.58 (a) Image formed with track error
References

CHAPTER SIX

EXPERIMENTAL RESULTS

6.1 Introduction

Some experiments were done in the department’s tank with the experimental synthetic aperture system which was described in Chapter Four. All algorithms discussed in Chapter Two and Chapter Three were tested on this system except the CTFM system due to the limit of tank size. The experimental results are presented in this chapter, and the same system geometry (Fig. 5.29) used as was used in the simulation.

6.2 System test

A 2cm diameter solid ball was set in midwater 3m away from the aperture for system testing. Fig. 6.1 is the target range shell of the reconstructed image with 3m aperture where the 3dB width of the main lobe was measured as 2cm which agreed with the theoretical value (Equ. 5.7) and the simulated results (Fig. 5.7).

Fig. 6.1 Experimental result of the single ball target
Furthermore, a pair of such balls were set 4cm away from each other 3m away from the aperture. Again, the aperture used to form the target range of the image was 3m long. Fig. 6.2 is the result which shows that two balls are clearly resolved.

6.3 The Raw Data

One of the targets used in the experiments was composed of some table tennis balls glued on a aluminum sheet in the form of 'LUT' (Fig. 6.3), which presented a relatively low signal/background ratio. The raw data collected over this 'LUT' target along a four metres long track with 2cm sampling spacing is shown in Fig. 6.4. The target corresponds to the position around 75-125 (along track), 600-800 (range samples) in the raw data matrix. It is hard to detect the presence of a target from these raw data.
Another target used in the experiments to present a relatively high signal/background ratio was composed of three buoys which had diameters of 16cm, 17cm and 30cm respectively. The three buoys were set up in a form of an isosceles triangle with the bottom closest to the sonar track. Due to the higher signal/background ratio, the parabolic echo history of the target clearly showed up in Fig. 6.5.

Fig. 6.4 The raw data of the 'LUT' target

Fig. 6.5 The raw data due to three balls target
Both sets of data were collected when the transducer carriage was moving along a nominal straight line. However, due to the distortion of the nylon rail and instability of the carriage mechanism, a lateral error motion of up to $\lambda/2$ and an along track sampling spacing error up to 5% did exist.

### 6.4 Coherent Addition

The raw data of 'LUT' shown in Fig. 6.4 was first processed by the coherent addition algorithm with the nominal along track spacing of 2cm. The result is shown in Fig. 6.6. Due to the existence of the error motion, the image was defocused although it is still recognizable.

![Image](image.png)

**Fig. 6.6** The image of the 'LUT' target formed by coherent addition with the nominal spacing

### 6.5 I-Q Processing

The same raw data set of 'LUT' target was processed by the I-Q processing also with nominal 2cm sampling spacing. Although it is defocused as well by the error motion, it gave a better image than coherent addition. In the I-Q processing, the focusing parabola sweeping through the bandwidth dependent envelope of the pulse varying slowly
relative to carrier frequency in a I-Q demodulated data matrix. And, in the coherent addition processing, the data collected along the parabola over the carrier, the amplitude of the echo changed rapidly on the frequency of carrier. This made coherent addition processing more sensitive to the error motion, e.g. the range mismatching broadened the main lobe in I-Q processing but split it in the coherent processing (referring to Sections 5.3 and 5.4).

![Image](image.png)

Fig. 6.7 I-Q processed 'LUT' image with nominal sampling spacing

### 6.6 Contrast Optimization Autofocusing

Autofocus was first applied on the coherent addition processing, the focus of the image was significantly improved. However, some of the table tennis ball echoes were still very weak against the background (Fig. 6.8).

When the same raw data went through the I-Q processing based autofocus, the result was excellent. Each table tennis ball clearly showed up in Fig. 6.9. Comparing with the raw data shown in Fig. 6.4, the effect of the synthetic aperture I-Q processing and autofocusing were very successful.

The effective along track sampling spacing error in these two experiments were considered as uniform. The final best estimate for effective spacing was 1.045 times of
nominal spacing. This showed that a quite small error (4.5% error in spacing) can defocus the image significantly.

To study further the performance of the autofocusing, some artificial error were added in sonar track. Table 6.1 lists a series of sonar tracks with error and corresponding experimental results. In these experiments, the I-Q processing was used for the image reconstruction.

---

Fig. 6.8 Autofocused 'LUT' image based on the coherent addition

Fig. 6.9 Autofocused 'LUT' image based on the I-Q processing
description of track error | Relative figure
---|---
sin form, one cycle within the track, with the amplitude of $2\lambda$ | 6.10
sin form, two cycles within the track, with the amplitude of $2\lambda$ | 6.11
sin form, five cycles within the track, with the amplitude of $2\lambda$ | 6.12
sin form, ten cycles within the track, with the amplitude of $2\lambda$ | 6.13
periodic like random, displayed in Fig.5.43(a), nominal track on x-axis | 6.14

Table 6.1 List of some track errors

Part (a) of Fig. 6.10 to Fig. 6.14 shows the image ill focused by the nominal spacing, part (b) is the autofocused image. The image of the whole scene (110 cm long) was split into five strips to let autofocusing cope with the non uniform track errors. In one case the processing was done treating the whole image as a image piece (Fig. 6.10 (c)). In this case, the focus of the image was still improved, but not as good as the one processed as five image strips (Fig. 6.10 (b)). In each autofocus processing, sixteen range shells with highest contrast were used to trial each estimate and obtain the finally best one.

Fig. 6.10 (a) Image formed with track error
The performance of the autofocusing was reasonable when the error was uniform within the track, i.e. Fig. 6.9. When the non uniform error changed slowly, i.e. one cycle within the track, the result was also reasonably good (Fig. 6.10). However, the
performance of the autofocusing dropped with increasing frequency of the error changing.

Fig. 6.11 (a) Image formed with track error

Fig. 6.11 (b) Autofocused image

Most of the behavior of the autofocusing in the experiments approximately corresponded with the computer simulated results. Only under the circumstance where
the error was periodic like random, the experimental result seemed better than the simulated one. Two reasons were suggested for this improvement. One was that the uncontrollable mechanical random errors and artificial random errors happened to cancel each other. Another was that the 'LUT' target got more structural range shells than the simulated target (five single points in one range shell) so that the autofocusing for the experimental target had a better contrast indicator during the processing.

Fig. 6.12 (a) Image formed with track error

Fig. 6.12 (b) Autofocused image
The experimental results showed that the autofocus processing worked well when the image was blurred but still recognizable. If the image was smeared so badly that the target structure was totally lost, a correctly focused image could not be produced by the autofocus. This meant that the reasonable application for autofocus was to
remove residual defocus in the image having been compensated by using the navigation information. It was too hard for autofocusing to compensate for severe motion error directly. Although the shorter synthetic aperture could be used to tolerate larger motion errors, the poor resolution of the image due to the short aperture would lose the information about the target structure.

Fig. 6.14 (a) Image formed with track error

Fig. 6.14 (b) Autofocused Image
6.7 Under sampling Aperture Effects

The set of results shown in Fig. 6.15 were for the 'LUT' target reconstructed with I-Q processing where the transmitted pulse had a 12.6kHz bandwidth, and the along-track sampling spacing was 2cm, 4cm, 6cm, 8cm, 12cm and 16cm respectively.

(a) \lambda/2 spacing

(b) \lambda (or D) spacing
It was observed that for this imaging window, the result was reasonably useable when sampling spacing was up to $2\lambda$. Even when the spacing was $3\lambda$, the target was still recognizable with some prior knowledge. The D spacing gave a quite good result, which could double the mapping rate in this experimental system.
Fig. 615. Effect of under sampling the aperture with the transmitting pulse of 12.6 kHz bandwidth
The other set of images (Fig. 6.16) resulted from the constant along-track spacing of $\lambda$ (4cm) and the transmitted pulse with 24 kHz, 12.6 kHz, 6.3 kHz and 3.1 kHz bandwidth respectively.

(a) 24 kHz

(b) 12.6 kHz
The results showed that the broad band signal did improve the quality of the image. Under the same along-track sampling rate, the image reconstructed from the narrower band transmission was totally unrecognizable (Fig. 6.16 (d)) while the one formed from the wider band signal was satisfactory (Fig. 6.16 (a)).
6.8 Envelope Processing

The first target used for the envelope processing experiments was the 'LUT' target. However, the envelope processing was found not to work under a low signal/background ratio situation. Fig. 6.17 shows the image processed by envelope processing from the same 'LUT' raw data used in the previous sections. The 'LUT' could not be recognized from this image.

Fig. 6.17 The envelope processed 'LUT'

Fig. 6.18 The envelope processed three buoys target without the artificial track error
To provide a higher signal/background ratio situation, a three-buoy target configuration (referring to Section 6.3) was used for the envelope processing experiments. Fig. 6.18 shows the image formed by this algorithm without the artificial track error. Some artificial errors were added in the track for further experiments, which were listed in Table 6.2 the same as these in the simulations (Table 5.5). The results are shown in Fig 6.19 to Fig 6.26.

<table>
<thead>
<tr>
<th>description of track error</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin form, one cycle within the track, with the amplitude of $\lambda$</td>
<td>6.19</td>
</tr>
<tr>
<td>sin form, one cycle within the track, with the amplitude of $2\lambda$</td>
<td>6.20</td>
</tr>
<tr>
<td>sin form, one cycle within the track, with the amplitude of $3\lambda$</td>
<td>6.21</td>
</tr>
<tr>
<td>sin form, ten cycles within the track, with the amplitude of $\lambda$</td>
<td>6.22</td>
</tr>
<tr>
<td>sin form, ten cycles within the track, with the amplitude of $2\lambda$</td>
<td>6.23</td>
</tr>
<tr>
<td>sin form, ten cycles within the track, with the amplitude of $3\lambda$</td>
<td>6.24</td>
</tr>
<tr>
<td>random with small amplitude, shown in Fig. 5.50(c), nominal track on x-axis</td>
<td>6.25</td>
</tr>
<tr>
<td>random with large amplitude, shown in Fig. 5.58(b), nominal track on x-axis</td>
<td>6.26</td>
</tr>
</tbody>
</table>

Table 6.2 List of the track errors

![Image](image.png)

Fig. 6.19 Image formed with track error
It was shown that the performance of the envelope processing was not very promising. The target was recognizable only when track error was small and changing slowly. This algorithm was very dependent on signal/background ratio. When the signal is below the background, no gain could be obtained from this processing. From this point of
view, the coherent processing, e.g. I-Q processing is definitely better than the envelope processing. Therefore, the envelope processing seemed to be a processing which should be used only with the absence of the phase information.

Fig. 6.22 Image formed with track error

Fig. 6.23 Image formed with track error
Fig. 6.24 Image formed with track error

Fig. 6.25 Image formed with track error
6.9 Phase Differential Synthetic Aperture

The phase differential synthetic aperture was found to be very dependent on signal/background ratio during the experiments. Fig. 6.27 shows the image processed by this algorithm from the 'LUT' raw data. The target was invisible in the image.
To provide a suitable data for the processing, the three-buoy target used for the envelope processing was also used for the phase differential processing experiments. Fig. 6.28 (b) shows the reconstructed target without artificial track error comparing
with the image formed by conventional I-Q processing under the same circumstance (Fig. 6.28 (a)).

<table>
<thead>
<tr>
<th>description of track error</th>
<th>Relative figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin form, 1 cycle within the track, symmetrical about the nominal track with the amplitude of 2λ</td>
<td>6.29</td>
</tr>
<tr>
<td>sin form, 1 cycles within the track, asymmetrical about the nominal track with the amplitude of 2λ</td>
<td>6.30</td>
</tr>
<tr>
<td>sin form, 10 cycles within the track, symmetrical about the nominal track with the amplitude of 2λ</td>
<td>6.31</td>
</tr>
<tr>
<td>sin form, 10 cycles within the track, asymmetrical about the nominal track with the amplitude of 2λ</td>
<td>6.32</td>
</tr>
<tr>
<td>periodic like random, displayed in Fig. 5.49 (c), nominal track is on x-axis</td>
<td>6.33</td>
</tr>
<tr>
<td>normally distributed random, displayed in Fig. 5.50 (c), x-axis is nominal track</td>
<td>6.34</td>
</tr>
</tbody>
</table>

Table 6.3 Descriptions of track errors

In the further experiments, some artificial errors described in Table 6.3 were added into the track, which were the same as the ones used in simulations (Table 5.4).

In figures 6.29-6.34, part (a) shows the image formed with the normal I-Q processing method and part (b) is the image reconstructed using the phase differential synthetic aperture processing.

From these results, it is hard to say that the phase differential synthetic aperture processing gives any advantages over the I-Q processing except when the errors were fast changing cycled movement (Fig. 6.31, 6.32). Under these circumstances, the I-Q processed images were severely smeared while the phase differential synthetic aperture gave more or less similar results for all six situations, but the quality of all these images produced by the phase differential synthetic aperture were poor. This algorithm seemed not to be a good solution for those kinds of error motions appeared in these experiments.
(a) Image formed by I-Q processing

(b) Image formed by phase differential algorithm

Fig. 6.29 Image formed with track error
(a) Image formed by I-Q processing

(b) Image formed by phase differential algorithm

Fig. 6.30 Image formed with track error
(a) Image formed by I-Q processing

(b) Image formed by phase differential algorithm

Fig. 6.31 Image formed with track error
(a) Image formed by I-Q processing

(b) Image formed by phase differential algorithm

Fig. 6.32 Image formed with track error
Fig. 6.33 Image formed with track error
Fig. 6.34 Image formed with track error

(a) Image formed by I-Q processing

(b) Image formed by phase differential algorithm
6.10 Miscellaneous Results

The image presented in this section were all formed by the I-Q processing. A nine-rung aluminum ladder was used as the target (Fig. 6.35). First it was laid on tank floor with its rungs parallel to the aperture. The result is shown in Fig. 6.36 where each rung showed up. Secondly, the ladder was laid on the tank floor at an angle to the aperture. Due to the specular nature of the rungs as a target, the reflection from the ladder did not illuminate the synthetic aperture well. Therefore, the rungs and side bars were both weak in the image (Fig. 6.37).

Fig. 6.35 The nine-rung ladder used for the target

Fig. 6.36 The experiment on the ladder laid on tank floor
It was noted that the side bar did not show up in the Fig. 6.36 where the rungs were quite clear. It was considered as that the side bars were flat frame rather than round pipe, therefore the most reflected energy did not go to the aperture direction unless the bars was perpendicular to the transducer beam. To prove that, the ladder was laid on the
floor with its side bar parallel to the aperture. One bar was clearly observed in the resulted image (Fig. 6.38), another was weak. It showed the illuminating angle, not only the azimuth but also the depression angle, was critical for the side bars.

Fig. 6.39 The image of the bicycle

Fig. 6.40 The image of the oil drum
Fig. 6.39 is the image of a bicycle. Due to the complexity (for 4cm wavelength) of this target, the bicycle is not recognizable in the image.

The target in Fig. 6.40 was an oil drum, sized 47cm long × 27cm diameter. The closest and strongest target in the image was due to the direct reflection from the oil drum. The one following was due to the angle constructed by the oil drum and the tank floor underneath. The targets which appeared further away and having a similar shape were from the multi path signal caused by the reflection between the surface and the floor. Because of the existence of multi path signals and the wide transmitter beam, there was no significant acoustic shadow observed behind the oil drum.

The experimental results of bathymetric synthetic aperture sidescan sonar will be presented in the next chapter, Chapter Seven.
CHAPTER SEVEN

SYNTHETIC APERTURE BATHYMETRIC SIDE SCAN SONAR

7.1 Introduction

Measurement of depth is probably the most important aspect of a seabed survey. Until recently the main means of determining the depth have been the echo sounder which, since it only measures the depth directly beneath the ship, leaves a large undetected area between survey lines. However the introduction of swath bathymetry has changed this situation. Using sidescan sonar [1, 2], P. N. Denbigh (1983) reported a BAthymetric Sidescan Sonar (BASS) which makes simultaneous measurements of the depth of the sea bed throughout the sidescan area. This is achieved by the use of two (or more) transducers to determine the angle of the returning wavefront at each instant in time and combining the result with the range measurement. However in the conventional sidescan sonar it is necessary to use a narrow beam requiring either a physically large array or the use of a high operating frequency.

An alternative approach to obtain a narrow beam is the use of the synthetic aperture technique which has been applied very successfully in Radar. Several studies have shown that it is possible to apply synthetic aperture processing in Sonar [5, 6, 7, 8]. Synthetic Aperture Sonar (SAS) can provide a very good horizontal resolution independent of range and frequency without requiring a physically large array. This would be an important advantage for a bathymetric system. Hence, the combination of synthetic aperture processing and a bathymetric sidescan sonar, a Synthetic Aperture BAthymetric Sidescan Sonar (SABASS), could be an attractive alternative. There have been a number of studies of the operation and analysis of swath bathymetry [1, 2, 3, 4], as well as synthetic aperture systems [5, 6, 7, 8].
7.2 The Basic Concept of BASS

By measuring the slant range and declination angle of each small area of the sea floor the depth relative to the sonar can be deduced. Assuming an echo is arriving at any instant from one direction only, a simple method of determining the angle of arrival is to measure the phase difference between two vertically spaced receivers. The declination angle can be determined from this phase measurement. The slant range is available from the two way propagation time delay and the depth is given by $R \times \sin \xi$, where $\xi$ is the declination angle and $R$ is slant range.

Fig. 7.1 gives a close view of the wavefront from a point on the seabed arriving at the pair of receivers. The phase difference, $\Delta \phi$, will be proportional to small difference in the path distances to the point from which the echo comes, $\Delta R$.

\[
\Delta R = \frac{\Delta \phi \cdot \lambda}{2\pi}
\]  

(7.1)

![Fig. 7.1 A close view to a wave front arriving at the receivers](image)

From the triangle law, the angle $\alpha$ can be determined as

\[
\alpha = \cos^{-1}\left(\frac{d^2 + (R + \Delta R)^2 - R^2}{2d \cdot (R + \Delta R)}\right),
\]  

(7.2)
where \( d \) is the distance between the two transducers. Assuming far field conditions, where \( R \gg d \), formula (7.1) and (7.2) together can be simplified into

\[
\Delta \phi = \frac{2\pi d \cdot \cos \alpha}{\lambda}.
\]  

(7.3)

And the declination angle \( \xi \) is

\[
\xi = \pi - \beta - \alpha.
\]  

(7.4)

7.3 Synthetic Aperture BASS – SABASS

Earlier the assumption was made that the echo was arriving at any instant from one direction only. Thus the main lobe of the sonar system has to be narrow and the range resolution should be good. The use of a short pulse, or a long FM pulse with compression, can meet the later requirement provided there is sufficient bandwidth. To form a narrow beam in a conventional sidescan sonar system requires either a large array or a high operating frequency. However, increasing the length of the array causes operating problems and is costly while increasing the operating frequency may cause an unacceptable loss of maximum range due to propagation loss. In addition the cross range resolution of a conventional sidescan system gets poorer with increasing range as is seen in equation (7.5).

\[
\delta x = R \frac{\lambda}{D}
\]  

(7.5)

where \( \delta x \) is the horizontal resolution, \( \lambda \) is the wavelength, \( R \) is the slant range and \( D \) is the dimension of the transducer.

Synthetic aperture processing offers the possibility of obtaining a good cross range resolution which in theory is independent of the range and wavelength and which does not require an expensive long array or the use of an impractical high frequency. In a SABASS system, only a single element or a short array is used in each of the pair of receivers. A point on the sea floor will be illuminated and detected while the wide swath
sweeps across it. By processing range dependent apertures, the along-track resolution will remain as $D/2$ [8, 9].

The signals from each of the receivers are processed independently to form two synthetic aperture images. The phase difference between the resulting focused signals from any particular point can then be extracted from the two apertures.

### 7.4 Transducer Pair Spacing

Equation (7.3) shows that the sensitivity of the measurement is improved with a large value of $d$. Fig. 7.2 shows wavefronts returning from a single point on the seabed. In the absence of interference these wave fronts would be spherical centred about that point, each with a small curvature that effectively makes it planar over the dimensions of the receiver arrays. Because of the interference however these wavefronts become crinkled, and it is this which leads to errors in determining their true direction of arrival. It is seen that closely spaced receivers such as at A and B are much more prone to giving a substantial error in the apparent direction of arrival than the widely spaced receivers at A and C are. Widely spaced receivers are effective in averaging out irregularities in the wavefronts that are caused by interference. However an unfortunate consequence of their use is that directional ambiguities can arise, and these must then be eliminated.

![Wavefronts](image.png)

**Fig. 7.2 Wavefront apparent to receivers**
7.5 Simulations

The simulations of the SABASS have been carried out using Matlab on a 486 PC. In order to compare results to those obtained in the experiment discussed later the dimensions used in the simulation are those of the experimental system. The height of the sonar transducers above the tank bottom was 1.5m and the transducers were set with a tilt angle of 30° with the target about 3.5m away in slant range. The two receivers were spaced by 8cm which was equivalent to two wavelengths at the operating frequency of 40 kHz. This wider spacing is used to average out irregularities in the wavefront caused by interference. The transducers used were circular of diameter approximately 1 wavelength giving a beamwidth of some 60 degrees. It is assumed that there are no reflections from the bottom or the surface, that there was no reverberation and that there were no secondary reflections from the targets. For simplicity it was assumed that the elements of the receiver had an ideal rectangular beam pattern although as will be seen later weighting of the beam pattern improved the results. While the transducers swept the illuminated area, the echoes appearing at the two receivers were stored in two data arrays simultaneously. After data acquisition, echoes were demodulated into complex form. Matched filtering was employed to obtain the back scattering strength. A threshold was necessary to reject the signals which were too weak for the interferometer. For each point whose back scattering amplitude was over the threshold, its echoes were coherently summed along a parabola due to this point in the two data arrays respectively. Subsequently, the phase difference, was taken to deduce the depth with slant range. When the synthetic aperture beam is formed by focusing on any particular point a contribution will be received from other targets at the same range due to the side lobes of the synthesized beam pattern. Weighting on the summing window in the phase measuring stage was found to reduce this interference between targets. This effect would appear naturally in practice due to the shape of the element beam pattern. Fig. 7.3 shows the simulation model.

The first simulation was mainly set to examine the algorithm itself. A number of point targets spaced by 10 cm where arranged in the form of the letters of "LUT" in which the "L", "U", and "T" were 3 cm, 5 cm and 10 cm high respectively. Samples were taken every 2 cm along the track and the aperture was 2.56m long. Prior knowledge of phase information was used for phase unwrapping. For example, in Fig. 7.4, the measuring window is 1.5m long (return path) and 3m (single path) away from the aperture, that is
shown by shadow part. Suppose point A at seabed level is on the boresight axis of the transducer pair, angle $\alpha$ (see Fig. 7.1) is $90^\circ$, phase difference $\delta \varphi$ is zero (equation 7.3). When the receiver pair spacing is 8 cm, the first ambiguity will happen where $\delta \varphi = +/- 2\pi$, $\alpha = \cos^{-1}(+/1/2) = 60^\circ$ or $120^\circ$. From the geometry of Fig. 7.4, it can easily be observed that any objectives lower than 1.5m located at right side of the line A-A’ will not be ambiguous with the target at the same side if A-A’, and no phase unwrapping is required for the target lower than 1.5m located at a ground range further than 2.6m.

Fig. 7.3 Simulation model

Fig. 7.4 Model of phase unwrapping with prior knowledge
Fig. 7.5 shows the simulation result which are reconstructed "LUT".

7.6 Experimental Results

An experimental SABASS system has been set up in the department's tank. The system parameters are configured as below:
Operational Frequency=40kHz
Pulse Length=100μs
Aperture=2m
Cross-range Sampling Space=2cm
Range Sampling Frequency=240kHz
Each transducer is one wavelength diameter spaced by two wavelengths
A very basic experimental result is shown in Fig. 7.6. The parameters were as discussed in the first simulation in the last section and Fig. 7.4. There is no phase unwrapping involved in the reconstruction. The targets were two buoys set at a height of 20 cm and 25 cm respectively.

The second experimental object comprised breeze blocks and bricks as shown in Fig. 7.7.

![Fig. 7.7 Sketch of the experimental target](image)

![Fig. 7.8 Image of the target back scattering strength of the brick set](image)
All the system parameters retain the same values as in the previous experiment. Fig. 7.8 shows the image of back scattering strength reconstructed with SAS. Because the surfaces of the breeze blocks and bricks are smooth corresponding to system wavelength, i.e. 4cm, the visible reflections only came from the edges of blocks or bricks. The breeze block at the bottom had the lower density with more air bubbles inside, which gave a stronger reflection than the other one. The reflection from the bricks was even weaker, which could hardly be seen in Fig. 7.8. However, the height measurement obtained from SABASS clearly showed the three step stairs in Fig. 7.9. The image pixel resolutions are 1cm by 1cm in both Fig. 7.8 and 7.9, and height is presented in cm. The SABASS image obviously gave better information about the structure of the target.

An oil drum, 47cm longx27cm diameter, was used as a target in the next experiment. The oil drum was laid on the tank floor with the axis of the cylinder parallel to the sonar track. Fig. 7.10 shows the target back scattering strength of the oil drum, and Fig. 7.11 shows the height measurement where the pixel size is 2cm by 2cm, height is in cm. The height below 50cm was due to oil drum. It did not match the physical height of the target since the precise transducer tiled angle was barely accurately due to mechanical
difficulties. The height above 1.5m was caused by the surface reflection, i.e. the signal through the path a-b-c in Fig. 4.6. Since the surface reflection came from the direction above the receiver, the height derived from the angle of arrival was much higher than the real target.

To give a target with more continuous variation of height, a set of bottles and bricks were mounted on a mesh set in the tank as shown in Fig. 7.12. Fig. 7.13 shows the backscattering strength of the bottle-brick set, and Fig. 7.14 is the measured height where the pixel size is 2cm by 2cm, the height is shown in cm. Obviously, the height
measurement showed more information about the structure of the target than the target strength reconstruction.

Fig. 7.12 The setting of the bottles and bricks set

Fig. 7.13 The back scattering strength of the bottles and bricks set

Fig. 7.14 height measurement of the bottles and bricks set
Fig. 7.15 is the backscattering strength of the ladder shown in Fig. 6.35, and Fig. 7.16 is the measured height where the pixel size is 1cm by 1cm, the height is shown in cm. Since the ladder was tilted, only four rungs appeared within the vertical beamwidth of the transducer. The second rung from the near side gave the strongest reflection because it was positioned closest to the centre of the transducer beam. A wrong impression is sometimes taken from the 3D plotting of the target back scattering strength (Fig. 7.15(b)), this rung was the highest in height, but it was actually not. Obviously, the height measurement of the ladder in Fig. 7.16 gave the more correct impression about the structure of the target.
Fig. 7.16 The height measurement of the ladder

References


CHAPTER EIGHT

CONCLUSION AND FURTHER CONSIDERATION

8.1 Introduction

The main objective of this research was to make a study of synthetic aperture sonar with particular interest in methods of increasing the towing speed, reducing the effect of track errors and on the use of bathymetric sonar. The task has been accomplished by use of the computer simulations and experiments carried out in the department's tank.

The simulation work in this thesis has been mainly done with Matlab based on a SUN station, and some early computation was carried out with Matlab on a PC486.

A experimental synthetic aperture sonar system has been set up in the department's tank sized 9m×5m×2m. The system is wholly computer based where the processing could be done on-line or off-line giving a flexibility to test the different processing algorithm and the system configurations.

This chapter is devoted to the conclusion of the research and the consideration of further topics.

8.2 Conclusions Arising from the Study

To accomplish the first objective of this research, broad band transmission was studied by using the computer simulation and experiments. There are two opinions about this subject. One is that the broad band signal can reduce the ambiguities when the aperture is under sampled since the grating lobes are frequency dependent so that they will smear
each other [1, 2, 3, 4, 5, 6]. The other one considers the broad band signal does improve the image reconstructed form under sampled data since they think that the quality of the synthetic aperture sonar should not be measured by the conventional criteria, i.e. normally azimuth resolution and ambiguity. According to their analysis, the broad band signal does not ease the under sampling situation [7].

To investigate the effect of broad band signals on reducing the ambiguities caused by an under sampled synthetic aperture which was proposed to increase the towing speed, the transmissions with various bandwidth was studied based on the I-Q processing. It was found in the computer simulations that the broad band transmission could reduce the ambiguities when the aperture was under sampled. The extreme example of broad band signal, Continuous Transmission Frequency Modulation, was investigated by the computer simulation. It was shown that in this processing, the range resolution could be traded off to increase the towing speed. However, the sacrifice of the range resolution was not unlimited since the azimuth resolution was getting poorer with the coarser range resolution due to the range dependence of the focusing function for the synthetic aperture array.

Transmitting signals with different bandwidth were tested when the aperture was under sampled, i.e. along track sampling spacing was horizontal dimension of the transducer, D (4cm). The broad band signal was found to improve the quality of the image within the imaging window of the experiments. Although it cannot be said that the broad band signal is the total solution for towing speed increasing, the broad band signal is definitely helpful in reducing the effects of under sampling the aperture. Unfortunately, the CTFM was not able to be tested in practice due to the limited size of the tank.

Secondly, the various synthetic aperture algorithms which are the coherent addition, the I-Q processing, the phase differential synthetic aperture and the envelope processing were studied by the computer simulations.

The two algorithms which share the same basic principle, the coherent addition and the I-Q processing, were compared in their performance. They were all based on delay-add procedure, but the coherent addition worked on the carrier frequency whereas the I-Q processing did it on the base band. The simulations showed the two algorithm producing the same results under the situation where there was no motion error, noise and any
other turbulence. When the track errors presented, e.g. the range mismatching, both processes suffered from the phase error introduced by the extra propagation path since they used the same delay procedure. However, since the coherent addition did the addition over the carrier frequency, the amplitude of the sum was sensitive to the track error in the order of $\lambda/4$. Meanwhile, the I-Q processing used the I-Q demodulated signal over the base band so that the output amplitude was only sensitive to the error up to the half length of the pulse envelope. This concluded that the I-Q processing was the better processing than the coherent addition in practice.

Through the experiments, the I-Q processing showed the better performance than the coherent addition, which agreed with the simulations. Especially for the autofocus processing, the I-Q processing was the better image reconstruction algorithm to be based on than the coherent addition.

The phase differential synthetic aperture was computationally simulated without unwanted background to compare with the conventional I-Q processing. Under some certain circumstances, i.e. where track errors were periodical and asymmetrical about the nominal track, or random like, the phase differential synthetic aperture produced some slightly better images.

However, the performance of phase differential synthetic aperture processing in the experiments was more disappointing. First it did not work at a low signal/background ratio, e.g. where the table tennis balls had to be detected against the tank floor. Even in a higher signal/background ratio situation, i.e. some buoys to be detected against the tank floor, it still did not show any advantages over the conventional I-Q processing in the experiments where some unwanted back ground signal was present. Considering that the interferometer on which the phase differential synthetic aperture is based on can only work properly under the assumption that there is only one strong target in each range shell, it is not difficult to understand that this algorithm did not perform well in practice.

The envelope only processing was investigated in the term of track error tolerance. In the simulations where no unwanted back ground was present, it could tolerate track error with small amplitude (up to $\lambda$), long and short period, or large amplitude up to pulse length and long period, i.e. one cycle within the track. Due the nature of the processing, the azimuth resolution was range dependent.
Chapter Eight

The envelope processing was proved not a practical method of processing for the bottom imaging in these experiments. Due to the nature of the processing which is that the image reconstruction only depends on the history of the echo envelope, it cannot resolve the target from the unwanted background when the signal/background ratio is low, especially when the target is a distributed one. Even when the experimental target was higher than the background, the image was still disturbed by the background. It is concluded that this processing is not suitable for the seabed imaging.

As a motion compensating algorithm, the contrast optimisation autofocus was tested by computer simulations. For the uniform track error and slowly (relative to the synthetic aperture) changing non-uniform track error, it worked very well. Some improvements were still observed when the fast changing track errors presented. However, it could not cope with the errors with the structure much finer than the synthetic aperture.

The experimental results of the autofocus processing basically agreed with the simulated ones. The contrast optimisation autofocus worked very well on the uniform track error and the slowly changing non-uniform track errors (relative to the length of the synthetic aperture). Within those processing schemes studied in this thesis to reduce the effect of track errors, the autofocus gave the best performance. Although it did not work well for the rapidly changing errors, it still could be used to remove the residual motion error after the motion compensation based on the navigation information has been applied.

Finally, the Synthetic Aperture BAthymetric Sidescan Sonar (SABASS) was proposed in this thesis to provide a good constant horizontal resolution and depth measurement without the requirement for a large array or high operating frequency. The results presented in this thesis were very encouraging although there is much further work to be done.

8.3 Consideration of Items for Further Study

In the experiments, some bottom back scattering due to the tank floor were observed as large as in order of a 20cm diameter buoy (-21.5dB). It was not entirely understood why the back scattering was so large since the floor was very smooth relative to the 4cm wavelength. One suggestion was that the strong scattering was caused by the structure
under the concrete surface. It will be very useful to carry out further investigation to understand this acoustic phenomena in the tank and explain the experimental results better. To help in this study, it should be possible to build a model of the acoustic tank in software.

The targets used in the research were more or less isolated targets. This was mainly because that it is difficult to build a target rough enough for the 4cm wavelength to get the reasonable back scattering, big enough to be a distributing target and also small enough to be fitted in the experimental scale in the tank. From this point of view, a higher operating frequency or a large scale experiment in a more open space is demanded.

Providing improved mechanism of the experimental sonar platform, more research could be done on effects of the quantified track errors, which should be able to give a idea about of which kind of track error effect could be tolerated or removed by each algorithm. Also, some other autofocus methods should be investigated.

An experimental system with a reasonable internal navigation unit in a more open space should be the next-step system. Together with autofocus processing, it could produce some more sensible results from the more practical environment, e.g. a reservoir or a dock yard. It is worth seeing the performance of these algorithms in a practical sonar environment, especially for the autofocusing and the broad band signal under sampling.

Providing a target which is continuously visible to the sonar, the research on phase unwrapping could be carried out in the experimental environment, which is crucial for the SABASS application.

After all, it seems worth doing experiments in sea. The more proof for the performance of various algorithm and indication for further research can be found in the real data.
Reference


