Propagation of the initial value perturbation in a cylindrical lined duct carrying a gas flow

Agneta M. BALINT*[,1], Stefan BALINT[2]

*Corresponding author

*[,1]Department of Physics, West University of Timisoara
Blv. V. Pârvan 4, Timisoara, Romania,
balint@physics.uvt.ro

[2]Department of Computer Science, West University of Timisoara,
Blv. V. Pârvan 4, Timisoara, Romania,
balint@balint.uvt.ro

DOI: 10.13111/2066-8201.2013.5.1.4

Abstract: *For the homogeneous Euler equation linearized around a non-slipping mean flow and boundary conditions corresponding to the mass-spring-damper impedance, smooth initial data perturbations with compact support are considered. The propagation of this type of initial data perturbations in a straight cylindrical lined duct is investigated. Such kind of investigations is missing in the existing literature. The mathematical tools are the Fourier transform with respect to the axial spatial variable and the Laplace transform with respect to the time variable. The functional framework and sufficient conditions are researched that the so problem be well-posed in the sense of Hadamard and the Briggs-Bers stability criteria can be applied.*

Key Words: *compressible perfect gas flows, initial data perturbations, Euler equation, axial spatial variable, inviscid non-heat-conducting*

1. INTRODUCTION

Assume, as in [1], that an inviscid non-heat-conducting, compressible perfect gas flows inside an infinitely long, straight and cylindrical lined duct of radius R (Fig. 1.).

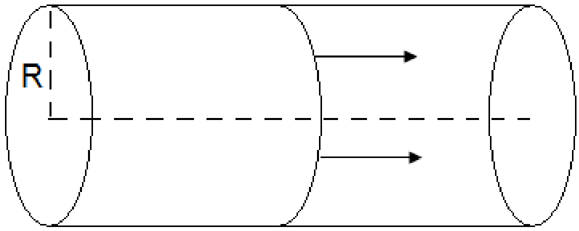

Figure 1: Gas flow in a straight cylindrical lined duct.

Let x, r and θ the axial, radial and circumferential coordinates, u, v and w the projections of the velocity vector on the coordinate axes x, r and θ, and ρ and p, the density and the pressure, respectively. The equations for conservation of mass, and radial, circumferential and axial components of momentum and energy are [1]:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \cdot \frac{\partial (r\rho v)}{\partial r} + \frac{1}{r} \cdot \frac{\partial (\rho w)}{\partial \theta} + \frac{\partial (\rho u)}{\partial x} = 0 \qquad (1)$$

$$\rho \cdot \left(\frac{\partial v}{\partial t} + v \cdot \frac{\partial v}{\partial r} + \frac{w}{r} \cdot \frac{\partial v}{\partial \theta} + u \cdot \frac{\partial v}{\partial x} - \frac{w^2}{r} \right) = -\frac{\partial p}{\partial r} \tag{2}$$

$$\rho \cdot \left(\frac{\partial w}{\partial t} + v \cdot \frac{\partial w}{\partial r} + \frac{w}{r} \cdot \frac{\partial w}{\partial \theta} + u \cdot \frac{\partial w}{\partial x} + \frac{v \cdot w}{r} \right) = -\frac{1}{r} \cdot \frac{\partial p}{\partial \theta} \tag{3}$$

$$\rho \cdot \left(\frac{\partial u}{\partial t} + v \cdot \frac{\partial u}{\partial r} + \frac{w}{r} \cdot \frac{\partial u}{\partial \theta} + u \cdot \frac{\partial u}{\partial x} \right) = -\frac{\partial p}{\partial x} \tag{4}$$

$$\frac{\partial p}{\partial t} + v \cdot \frac{\partial p}{\partial r} + \frac{w}{r} \cdot \frac{\partial p}{\partial \theta} + u \cdot \frac{\partial p}{\partial x} + \gamma \cdot p \cdot \left(\frac{1}{r} \cdot \frac{\partial (rv)}{\partial r} + \frac{1}{r} \cdot \frac{\partial w}{\partial \theta} + \frac{\partial u}{\partial x} \right) = 0 \tag{5}$$

Here: t is time, $\gamma = \frac{c_p}{c_v}$ is the ratio of the specific heat capacities at constant pressure and constant volume, respectively. The pressure, the density and the absolute temperature T satisfy the equation of state of the perfect gas $p = R' \cdot \rho \cdot T$, $R' = c_p - c_v$.

The equations (1)–(5) are considered for: $t \geq 0, x \in (-\infty, \infty), r \in (0, R], \theta \in [0, 2\pi]$. As concerns the mean flow, it is assumed that: $v_0(x, r, \theta, t) \equiv 0$, $w_0(x, r, \theta, t) \equiv 0$ and $u_0(x, r, \theta, t) \equiv U_0(r) \geq 0$, $\rho_0(x, r, \theta, t) \equiv \rho_0 = \text{const} > 0$, $p_0(x, r, \theta, t) \equiv p_0 = \text{const} > 0$, is non-slipping

$$U_0(R) = 0 \tag{6}$$

and $U_0(r)$ is continuously differentiable (once) and decreasing for $r \in [0, R]$.

The linearized Euler equations around the mean flow are:

$$\frac{\partial \rho'}{\partial t} + U_0 \cdot \frac{\partial \rho'}{\partial x} + \frac{1}{r} \cdot \rho_0 \cdot \frac{\partial (rv')}{\partial r} + \rho_0 \left(\frac{1}{r} \cdot \frac{\partial w'}{\partial \theta} + \frac{\partial u'}{\partial x} \right) = 0 \tag{7}$$

$$\rho_0 \cdot \left(\frac{\partial v'}{\partial t} + U_0 \cdot \frac{\partial v'}{\partial x} \right) = -\frac{\partial p'}{\partial r} \tag{8}$$

$$\rho_0 \cdot \left(\frac{\partial w'}{\partial t} + U_0 \cdot \frac{\partial w'}{\partial x} \right) = -\frac{1}{r} \cdot \frac{\partial p'}{\partial \theta} \tag{9}$$

$$\rho_0 \cdot \left(\frac{\partial u'}{\partial t} + U_0 \cdot \frac{\partial u'}{\partial x} + \frac{dU_0}{dr} \cdot v' \right) = -\frac{\partial p'}{\partial x} \tag{10}$$

$$\frac{\partial p'}{\partial t} + U_0 \cdot \frac{\partial p'}{\partial x} + \gamma \cdot p_0 \cdot \left(\frac{1}{r} \cdot \frac{\partial (rv')}{\partial r} + \frac{1}{r} \cdot \frac{\partial w'}{\partial \theta} + \frac{\partial u'}{\partial x} \right) = 0 \tag{11}$$

The known function $U_0(r)$ and the system of five unknown functions $\rho'(x, r, \theta, t)$, $v'(x, r, \theta, t)$, $w'(x, r, \theta, t)$, $u'(x, r, \theta, t)$, $p'(x, r, \theta, t)$ are real valued functions. The last five functions represent the evolution of the density, velocity and pressure, due to an instantaneous perturbation which occurs at a certain instant, say $t = 0$. This means that before the perturbation occurs (i.e. $t < 0$) the unknown functions are equal to zero (so the condition of "strict causality", no output before input [2]. is respected) and the evolution of an instantaneous perturbation occurring at $t = 0$, described by the functions $\rho_0'(x,r,\theta)$, $v_0'(x,r,\theta)$, $w_0'(x,r,\theta)$, $u_0'(x,r,\theta)$, $p_0'(x,r,\theta)$, is given by that solution of (7) – (11) which satisfies the condition:

$$\left(\rho', v', w', u', p'\right)(x,r,\theta,0) = \left(\rho_0', v_0', w_0', u_0', p_0'\right)(x,r,\theta) \tag{12}$$

and the boundary condition:

$$a \cdot \frac{\partial^2 v'}{\partial t^2} + b \cdot \frac{\partial v'}{\partial t} + c \cdot v' = \frac{\partial p'}{\partial t} \text{ for } r = R \tag{13}$$

where a, b, c are positive real constants representing: a - inertance, b - resistance, a - stiffness of the liner [3].

Condition (13) translates the mass-spring-damper type interaction between the liner and the evolving perturbation [3].

The determination of the system of five functions $\rho'(x,r,\theta,t)$, $v'(x,r,\theta,t)$, $w'(x,r,\theta,t)$, $u'(x,r,\theta,t)$, $p'(x,r,\theta,t)$, which satisfies (7)-(13), is an initial – boundary value problem and the system of functions, which is found, represents a solution of the problem. The system of five functions describing the perturbation occurring at $t = 0$ is usually called, initial data. In the case of a given class ID of initial data, if for every initial data the problem (7)-(13) has a unique solution, which depends continuously on the initial data, then (by definition) the problem is well-posed in the sense of Hadamard [4]. The above requirements are reasonable to expect if the problem is to correspond to a well-set physical experiment.

Existence and uniqueness is an affirmation of the scientific determinism and continuous dependence is an expression of the stability [5].

According to [6], the mean flow is stable (in the sense of Lyapunov) if the evolving perturbation $\rho'(x,r,\theta,t)$, $v'(x,r,\theta,t)$, $w'(x,r,\theta,t)$, $u'(x,r,\theta,t)$, $p'(x,r,\theta,t)$ is small for all time, provided it is small initially (i.e. $\rho_0'(x,r,\theta)$, $v_0'(x,r,\theta)$, $w_0'(x,r,\theta)$, $u_0'(x,r,\theta)$, $p_0'(x,r,\theta)$ is small).

The precise meaning of "small" has to be defined by using a topology. In a topology generated by a norm [6], the mean flow is stable if for any ε there exists η (depending on ε) such that if the initial data satisfies:

$$\|\rho'(0)\|,\ \|v'(0)\|,\ \|w'(0)\|,\ \|u'(0)\|,\ \|p'(0)\| < \eta,$$

then the evolving perturbation satisfies

$$\|\rho'(t)\|,\ \|v'(t)\|,\ \|w'(t)\|,\ \|u'(t)\|,\ \|p'(t)\| < \varepsilon$$

for all $t \geq 0$.

The mathematical treatment of the problem (11)-(13) requires fixing a class ID of initial data and a space S for the solutions.

Since the tools, we intend to use, are the Fourier transform with respect to the variable x and the Laplace transform with respect to the variable t ([7], [8]) of the system of five functions representing a solution, we choose for S the space of the set of the systems of five, continuously differentiable functions, defined for $t > 0$, t, x-real, r in $[0, R]$, θ in $[0, 2\pi]$, which are rapidly decreasing with respect to the variable x, for every r and θ fixed, possesses Laplace transform with respect to t and verifies the periodicity condition $(\rho', v', w', u', p')(x, r, 0, t) = (\rho', v', w', u', p')(x, r, 2\pi, t)$. The topology in S is that given by the norm: $\|p'(t)\| = \max_{\substack{x \in R^1, r \in [0,R] \\ \theta \in [0,2\pi]}} |p'(x, r, \theta, t)|$ and so on.

The class ID is the set of the systems of five, infinitely differentiable functions $\rho_0'(x, r, \theta)$, $v_0'(x, r, \theta)$, $w_0'(x, r, \theta)$, $u_0'(x, r, \theta)$, $p_0'(x, r, \theta)$ having compact support in the set $x \in (-\infty, \infty)$, $r \in [0, R)$, $\theta \in [0, 2\pi]$, which verifies the periodicity condition $(\rho'_0, v_0', w_0', u_0', p_0')(x, r, 0) = (\rho'_0, v_0', w_0', u_0', p_0')(x, r, 2\pi)$ and the norm in ID is similar to that in S.

Since equations (8)-(11) can be decoupled, in the following, instead of (7)-(13) the initial- boundary value problem (8)-(13) will be considered. More precisely, we consider the following initial-boundary value problem:

$$\rho_0 \cdot \left(\frac{\partial v'}{\partial t} + U_0 \cdot \frac{\partial v'}{\partial x} \right) = -\frac{\partial p'}{\partial r} \tag{14}$$

$$\rho_0 \cdot \left(\frac{\partial w'}{\partial t} + U_0 \cdot \frac{\partial w'}{\partial x} \right) = -\frac{1}{r} \cdot \frac{\partial p'}{\partial \theta} \tag{15}$$

$$\rho_0 \cdot \left(\frac{\partial u'}{\partial t} + U_0 \cdot \frac{\partial u'}{\partial x} + \frac{dU_0}{dr} \cdot v' \right) = -\frac{\partial p'}{\partial x} \tag{16}$$

$$\frac{\partial p'}{\partial t} + U_0 \cdot \frac{\partial p'}{\partial x} + \gamma \cdot p_0 \cdot \left(\frac{1}{r} \cdot \frac{\partial (r v')}{\partial r} + \frac{1}{r} \cdot \frac{\partial w'}{\partial \theta} + \frac{\partial u'}{\partial x} \right) = 0 \tag{17}$$

$$(v', w', u', p')(x, r, \theta, 0) = (v_0', w_0', u_0', p_0')(x, r, \theta) \tag{18}$$

$$a \cdot \frac{\partial^2 v'}{\partial t^2} + b \cdot \frac{\partial v'}{\partial t} + c \cdot v' = \frac{\partial p'}{\partial t} \text{ for } r = R \tag{19}$$

where:

- in (14)-(17), $t \geq 0$, $x \in (-\infty, \infty)$, $r \in (0, R]$, $\theta \in [0, 2\pi]$ and $U_0(r)$ is a continuously differentiable real valued function defined for $r \in [0, R]$, it is positive, decreasing and equal to zero for $r = R$, its derivative is zero for $r = 0$

- in (18) $v_0'(x, r, \theta)$, $w_0'(x, r, \theta)$, $u_0'(x, r, \theta)$, $p_0'(x, r, \theta)$ are infinitely differentiable real valued functions defined for $x \in (-\infty, \infty)$, $r \in [0, R]$, $\theta \in [0, 2\pi]$ with compact support in $x \in (-\infty, \infty)$, $r \in [0, R)$, $\theta \in [0, 2\pi]$ and satisfy the periodicity condition $(v_0', w_0', u_0', p_0')(x, r, 0) = (v_0', w_0', u_0', p_0')(x, r, 2\pi)$.

A solution of the problem (14)-(19) is a system of four continuously differentiable real valued functions $v'(x, r, \theta, t)$, $w'(x, r, \theta, t)$, $u'(x, r, \theta, t)$, $p'(x, r, \theta, t)$ defined for $t \geq 0$, $x \in (-\infty, \infty)$, $r \in [0, R]$, $\theta \in [0, 2\pi]$, which satisfies (14)-(17), (18), (19), the periodicity condition $(v', w', u', p')(x, r, 0, t) = (v', w', u', p')(x, r, 2\pi, t)$, are rapidly decreasing with respect to the spatial variable x [7] and possesses Laplace transform with respect to t.

2. CONDITIONS ASSURING THAT THE SOUND ATTENUATION PROBLEM IN CYLINDRICAL LINED DUCT IS WELL-POSED IN SENSE OF HADAMARD AND THE BRIGGS-BERS STABILITY METHOD CAN BE APPLIED

Statement 1. (necessary condition for the existence of solution). If for every initial data from ID the problem (14)-(19) has a solution $v'(x,r,\theta,t)$, $w'(x,r,\theta,t)$, $u'(x,r,\theta,t)$, $p'(x,r,\theta,t)$ in S, for which the Laplace transform L_t commutes with its Fourier transform F_x,, i.e. $\bar{u}(\alpha,r,\theta,\omega) = \mathrm{F}_x \circ \mathrm{L}_t(u'(x,r,\theta,t)) = \mathrm{L}_t \circ \mathrm{F}_x(u'(x,r,\theta,t))$ and so on, then for every initial data there exists a real number μ (depending upon the initial data) such that for every complex number ω with $\mathrm{Im}\,\omega > \mu$ and every real number α the Fourier coefficients of the expansions with respect to $\exp(m \cdot i \cdot \theta)$ (m integer) [9], satisfy:

$$\frac{d^2\bar{p}_m}{dr^2} + \left(\frac{1}{r} + \frac{2\alpha}{\omega - \alpha U_0} \cdot \frac{dU_0}{dr}\right) \cdot \frac{d\bar{p}_m}{dr} + \left[\frac{(\omega - \alpha U_0)^2}{c_0^{\,2}} - \alpha^2 - \frac{m^2}{r^2}\right] \cdot \bar{p}_m = f_m(\alpha, r.\omega) \tag{20}$$

$$i \cdot \rho_0 \cdot \omega \cdot \bar{p}_m(\alpha, R, \omega) - Z(\omega) \cdot \frac{dp}{dr}(R) = 0 \tag{21}$$

$$\bar{u}_m = \frac{\alpha}{\rho_0(\omega - \alpha U_0)} \cdot \bar{p}_m - \frac{1}{\rho_0(\omega - \alpha U_0)^2} \cdot \frac{dU_0}{dr} \cdot \frac{d\bar{p}_m}{dr} + \frac{i}{\omega - \alpha U_0} \cdot \bar{u}_m^0 + \frac{1}{(\omega - \alpha U_0)^2} \cdot \frac{dU_0}{dr} \cdot \bar{v}_m^0 \tag{22}$$

$$\bar{v}_m = -\frac{i}{\rho_0(\omega - \alpha U_0)} \cdot \frac{d\bar{p}_m}{dr} + \frac{i}{\omega - \alpha U_0} \cdot \bar{v}_m^0 \tag{23}$$

$$\bar{w}_m = \frac{m}{r} \cdot \frac{1}{\rho_0(\omega - \alpha U_0)} \cdot \bar{p}_m + \frac{i}{\omega - \alpha U_0} \cdot \bar{w}_m^0 \tag{24}$$

where f_m is given by

$$f_m = i \cdot \rho_0 \cdot \alpha \cdot \bar{u}_m^0 + \rho_0\left[\frac{2\alpha}{\omega - \alpha U_0} + \frac{1}{r}\right] \cdot \bar{v}_m^0 + \rho_0 \cdot \frac{d\bar{v}_0^m}{dr} + \frac{i(\omega - \alpha U_0)}{c_0^{\,2}} \cdot \bar{p}_m^0 + \frac{i \cdot \rho_0}{r} \cdot \bar{w}_m^0 \tag{25}$$

α- is the F_x transform variable; $\omega = iz$ and z-is the L_t transform variable; $Z(\omega) = -ai\omega + b - \dfrac{c}{i\omega}$ is the liner impedance.

In the half plane $\mathrm{Im}\,\omega > \mu$ the complex valued function f_m, given by (25), is an analytic function with respect to ω and it is rapidly decreasing with respect to α [7]. Moreover, for $r = R$ the function f_m is equal to zero.

Comments:

1. *Statement 1*. can be proved by calculus.

2. Taking $\mu_0 = \max\{0, \operatorname{Im}\omega_1, \operatorname{Im}\omega_2\}$, where ω_1, ω_2 are the roots of the equation $Z(\omega) = 0$, for every real number α and every m –integer, the right hand side of the equation (20) is an analytic function in the half plane $\operatorname{Im}\omega > \mu_0$ and for an arbitrary solution of the problem (20), (21) there exists a complex number K such that the solution is equal to the solution of (20), corresponding to the initial data $p_m(R) = K$, $\frac{dp_m}{dr}(R) = \frac{i \cdot \rho_0 \cdot \omega}{Z(\omega)} K$.

It follows that for the Fourier coefficient p_m, which satisfies (20), (21), there exists a complex number K_0 (depending upon α, m, ω) such that the Fourier coefficient p_m coincides with the solution of (20), corresponding to the initial data $p_m(R) = K_0$, $\frac{dp_m}{dr}(R) = \frac{i \cdot \rho_0 \cdot \omega}{Z(\omega)} K_0$.

The complex number K_0 is unique, because to different initial data corresponds a different solution of the equation (20). If $K = K_0$, then the solution p_m of (20), which corresponds to the initial data (and coincides with the Fourier coefficient), has finite limit at zero and the functions u_m, v_m, w_m, given by (22), (23), (24), are well defined on $[0, R]$.

Beside this, the convergence of the Fourier series, which coefficients are p_m, u_m, v_m, w_m, as well the existence of the inverse Fourier and Laplace transforms of the sums of these series are ensured.

3. The ideea is, to build up the solution $v'(x, r, \theta, t)$, $w'(x, r, \theta, t)$, $u'(x, r, \theta, t)$, $p'(x, r, \theta, t)$ of (14)-(19), solving the boundary value problem (20), (21). From the above comment it follows that it would be necessary to ensure the existence of a real number $\mu > \mu_0$ having the following properties:

i). for every real number α, every m –integer and every complex number ω with $\operatorname{Im}\omega > \mu$ there exists a unique complex value for K (depending on α. m, ω), for which the solution of the equation (20), corresponding to the initial data $p_m(R) = K$, $\frac{dp_m}{dr}(R) = \frac{i \cdot \rho_0 \cdot \omega}{Z(\omega)} K$ has a finite limit at zero.

ii). the functions u_m, v_m, w_m, given by (22), (23), (24), are well defined on $[0, R]$

iii). the Fourier series, which coefficients are p_m, u_m, v_m, w_m converge and the inverse Fourier and Laplace transforms of the sums of these series exist.

4. If there exists a real number $\mu > \mu_0$ having the property i), then the unique solution of the homogeneous problem:

$$\frac{d^2 q_m}{dr^2} + \left(\frac{1}{r} + \frac{2\alpha}{\omega - \alpha U_0} \cdot \frac{dU_0}{dr} \right) \cdot \frac{dq_m}{dr} + \left[\frac{(\omega - \alpha U_0)^2}{c_0^2} - \alpha^2 - \frac{m^2}{r^2} \right] \cdot q_m = 0 \tag{26}$$

$$i \cdot \rho_0 \cdot \omega \cdot q_m(\alpha, R, \omega) - Z(\omega) \cdot \frac{dq_m}{dr}(\alpha, R, \omega) = 0 \tag{27}$$

is equal to zero.

5. In [9] the existence of non- zero solutions of a problem similar to (26), (27) and the behavior of these solutions for r tending to zero are analyzed.

It is found numerically, in case of a mean flow, constant in the central part of the duct, that for every μ there exist α, m, ω with $\operatorname{Im}\omega > \mu$ such that for (26), (27) non-zero solution, having finite limit for r tending to zero, exists.

So, the fact that zero is the unique solution of (26), (27) having finite limit at zero has to be ensured by hypothesis.

Statement 2. If there exists $\mu > \max\{0, \operatorname{Im}\omega_1, \operatorname{Im}\omega_2\}$ having the following properties:

i). for every real number α, every m –integer and every complex number ω with $\operatorname{Im}\omega > \mu$ there exists a unique complex value for K (depending on α, m, ω) for which the solution of the equation (20), corresponding to the initial data $p_m(R) = K$, $\frac{dp_m}{dr}(R) = \frac{i \cdot \rho_0 \cdot \omega}{Z(\omega)} K$ has a finite limit at zero

ii). the functions u_m, v_m, w_m, given by (22), (23), (24), are well defined on $[0, R]$

iii). the Fourier series, which coefficients are p_m, u_m, v_m, w_m, converges and the inverse Fourier and Laplace transforms of the sums of these series exist, then for every initial data from ID the problem (14)-(19) has a unique solution in S, the problem is well- posed in the sense of Hadamard and the Briggs-Bers method can be applied.

Comments:

1. The proof of the above sentence is not difficult. For the Brigss-Bers method, in a similar case, see for example [10].

2. The above statement contains requirements which are not present in the papers concerning the subject [11]. In general, in the published papers the requirement is the existence of $\mu > \max\{0, \operatorname{Im}\omega_1, \operatorname{Im}\omega_2\}$ having the property that for every real number α, every m–integer and every complex number ω with $\operatorname{Im}\omega > \mu$, the unique solution of (26), (27) is equal to zero. This condition is a necessary one, but it is not sufficient.

3. The comments appearing after the first statement show that the requirements appearing in the second statement (sufficient conditions) are also justified.

3. CONCLUSION

If the set of the initial data contains systems of four infinitely differentiable functions, which are periodic in θ, with compact support in $x \in (-\infty, \infty)$, $r \in [0, R)$, $\theta \in [0, 2\pi]$ and the set of solutions is the set of the systems of four, continuously differentiable functions, defined for $t > 0$, t, x-real, r in $[0, R]$, θ in $[0, 2\pi]$, which are rapidly decreasing with respect to the variable x, possesses Laplace transform with respect to t and verifies the periodicity condition with respect to θ, then the sound attenuation problem is well- posed in the sense of Hadamard and the Brigss-Bers method can be applied.

4. ACKNOWLEDGEMENT

This work was supported by a grant of the Romanian National Authority for Scientific Research, CNCS-UEFISCDI, project number PN-II-ID-PCE-2011-3-0171.

5. REFERENCES

[1] S. W. Rienstra, G. G. Vilenski, Spatial Instability of Boundary Layer Along Impedance Wall, *14th AIAA/CEAS Aeroacoustics Conference, 5-7 May 2008, The Westin Bayshore Vancouver, Vancouver, Canada., AIAA-2008-2932*.

[2] J. S. Toll, Causality and Dispersion Relation: Logic Foundations, *Physical Review*, Vol. **104**, Number 6, pp.1760-1770, 1956.

[3] S. W. Rienstra, Impedance models in time domain including the extended Helmholtz Resonator model, *AIAA paper 2006-2686 Vibrations*, Vol. **9**, No. 1, pp. 28-48, DOI: http://dx.doi.org/10.1016/0022-460X(69)90260-0, 1969.

[4] G. E. Ladas, V. Lakshmikantham, *Differential Equations in Abstract Spaces*, Academic Press, New York and London, pp.21-22, 1972.

[5] E. Hille, R. S. Philips, Functional analysis and semi-groups, Colloq.Amer.Math.Soc. **31**, 1957.

[6] P. G. Drazin, W. N. Reid, *Hydrodynamic stability*, Cambridge University Press, Cambridge, pp. 8-21, 1995.

[7] K. Yosida, *Functional Analysis*, Springer, Berlin, Gottingen, Heidelberg, 1965.

[8] D. Ditkine, A. Prondnikov, *Calcul operationnel*, Ed. Mir, Moscow, 1979.

[9] A. M. Balint, St. Balint, L. Tanasie, *Existence of strange modes in cylindrical lined duct*, in preparation.

[10] A. M. Balint, St. Balint, M. Darau, Partial Fourier and Laplace transform in linear stability analysis of a non slipping mean flow in a 2D straight lined duct with respect to perturbations of the initial value by disturbances spatially distributed in a bounded neighborhood of a point in the duct, *Mathematics in Engineering, Science and Aerospace Journal*, Vol. **3**, No.3, pp.239-258, 2012.

[11] E. J. Brambley, Well-posed boundary condition for acoustic liners in straight lined ducts, *AIAA Journal*, Vol. **49**, No.6, pp.1272-1282, June 2011.