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Guest Editorial

# **Proof in Dynamic Geometry Environments**

Keith Jones, Ángel Gutiérrez and Maria Alessandra Mariotti

Since its creation in 1968, Educational Studies in Mathematics (ESM) has been a leading journal in the field of Mathematics Education. In the pages of ESM the results of new and important research in Mathematics Education have been reported and the most relevant research questions raised and discussed. Somewhat in parallel, the International Group for the Psychology of Mathematics Education (PME) has become, since its inaugural meeting in 1977, one of the main conference forums for researchers in Mathematics Education. Every year at PME, researchers from all over the world discuss the latest research questions and results, and define future research directions. As a result of joint effort by ESM and PME, ESM has initiated a programme of periodically publishing PME special issues aimed at showcasing important and substantial aspects of topics worked out by the PME community.

As editors of this PME special issue, we are very pleased to present the first outcome of the PME Special Issue series. This issue is devoted to analysing the influence of dynamic geometry software (DGS) on students' conceptions of mathematical proof while the students are solving geometry problems involving proofs in an environment mediated by such software. In particular, this Special Issue gathers together, extends and compares a range of recent research, much of it presented at PME conferences or benefiting from discussions in that forum.

The paper by Gila Hanna serves as an introductory paper and deals with some theoretical aspects of questions considered in the four following research papers. Hanna first addresses the question of the role of proof in secondary school mathematics, and the contrast between abstract proofs and heuristics, explorations, and visual proofs. Her remarks on epistemology towards the end of the paper constitute an important element of the background to this Special Issue.

The four research papers explore the central question of whether the opportunities offered by DGS environments to 'see' mathematical properties so easily might reduce or even replace any need for proof or, on the contrary, whether such a facility might open up new ways of meaningful approaches to promoting students' understanding of the need for and the roles of proof. Each of the papers addresses this issue within a distinct theoretical framework.

In the first research paper, and employing a Vygotskian perspective, Maria Alessandra Mariotti presents an analysis of the relevant mediation of several components and commands of the DGS on the interactions in groups of students. She highlights correspondences amongst the DGS menu commands used by students and axioms and theorems they subsequently use in their justifications.

Keith Jones presents an environment where students are faced with the question of the classification of quadrilaterals by means of a set of problems of increasing difficulty. The analysis of students' answers shows that using dynamic geometry software helps students to progress in their understanding of the dependence relationships among components of a figure and amongst families of figures, and so advance towards a progressive abstraction in their justifications.

Ramón Marrades and Ángel Gutiérrez report on a teaching experiment designed to enable students to produce deductive justifications of the correctness of their constructions. An analytic framework, which integrates and expands previous frameworks, is used to analyse student answers and to show that the way in which they use the DGS determines their solutions and justifications, and how the quality of students' justifications improves over time.

Nurit Hadas, Rina Hershkowitz and Baruch Schwarz present an approach to enabling students to produce deductive justifications by presenting them with problems that reveal surprising, contradictory or uncertain results. The authors present different categories of students' answers to this kind of problems when solved in a DGS environment and show how the DGS allows students to move between certainty and uncertainty, between conjecture and checking the conjecture.

In the final paper in this Special Issue, Colette Laborde, by offering a global integrating overview of the four research papers, describes to what extent they complement each other. Using the papers, Laborde illustrates how it is possible to build many different DGS teaching environments, adapted to specific necessities of students, where students can gain a better understanding of the deductive structure of mathematics, and the need for justifications/proofs in mathematics. Furthermore, Laborde highlights the usefulness of DGS in breaking down the traditional separation between action (as manipulation associated to observation and description) and deduction (as intellectual activity detached from specific objects).

While the individual papers in this special issue are the results of their authors' own research activity, the quality of the work has been enhanced by the environment of the PME annual conferences where many valuable discussions have taken place over a number of years. In particular, the editors of this Special Issue would like to record their appreciation of many PME colleagues, particularly Paolo Boero, Celia Hoyles, John Pegg and Michael de Villiers, who contributed to the development of their ideas.

Finally, as guest editors we would especially like to thank Tommy Dreyfus who, as collaborating ESM editor for this first PME Special Issue, worked closely with us, advising us on procedures, and sharing much of the decision-making. His expertise was a great help to us during the time of preparation of this special issue.

This Special Issue provides a range of evidence that working with dynamic geometry software affords students possibilities of access to theoretical mathematics, something that can be particularly elusive with other pedagogical tools. Yet it has to be noted, as Hanna points out in her introductory paper, that the examples of successful access to mathematical theory presented in the four research studies did not happen without carefully designed tasks, professional teacher input, and opportunities for students to conjecture, to make mistakes, to reflect, to interpret relationships among objects, and to offer tentative mathematical explanations. The research presented in this Special Issue needs replication and amplification. In particular, research in the use dynamic geometry software to support the development of students' mathematical thinking could usefully focus on the nature of the tasks students tackle, the form of teacher input and the role of the classroom environment and culture. For teachers in particular, that something works is one thing - further examples of how it can be made to work in the variety of classrooms are crucial.

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