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Preliminary report

RELATIVE-COORDINATE DETERMINATION FOR VISUAL DOUBLE STARS BY APPLYING FOURIER TRANSFORMS

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SUMMARY: We discuss the software developed for the purpose of determining the relative coordinates (position angle θ and separation ρ) for visual double or multiple stars. It is based on application of Fourier transforms in treating CCD frames of these systems. The objective was to determine the relative coordinates automatically to an extent as large as possible. In this way the time needed for the reduction of many CCD frames becomes shorter. The capabilities and limitations of the software are examined. Besides, the possibility of improving is also considered. The software has been tested and checked on a sample consisting of CCD frames of 165 double or multiple stars obtained with the 2m telescope at NAO Rozhen in Bulgaria in October 2011. The results have been compared with the corresponding results obtained by applying different software and the agreement is found to be very good.

Key words. binaries: visual – techniques: image processing – instrumentation: detectors

1. INTRODUCTION

A double star for which the components can be resolved either directly through the telescope or by using additional equipment is said to be a visual double star. Systematic observations of double stars have been carried out for about 200 years. The measuring techniques and methods have been inevitably changed and improved from visual micrometric measurements towards high-angular-resolution techniques. The Washington Double Star Catalog (WDS)[1] contains data for more than 119000 pairs, components of double or multiple stars for which the relative coordinates, position angle and separation have been measured. But only for a small number of pairs, about 2100, the orbital elements have been calculated, i.e. a Keplerian motion has been confirmed. Their orbital elements can be found in the Sixth Catalog of Orbits of Visual Binary Stars[2].

Astronomers from the Belgrade Observatory working in the field of double and multiple stars have used the CCD cameras over the last ten years for the

[1]http://www.usno.navy.mil/USNO/astrometry/optical-IR-prod/wds/WDS

[2]http://www.usno.navy.mil/USNO/astrometry/optical-IR-prod/wds/orb6

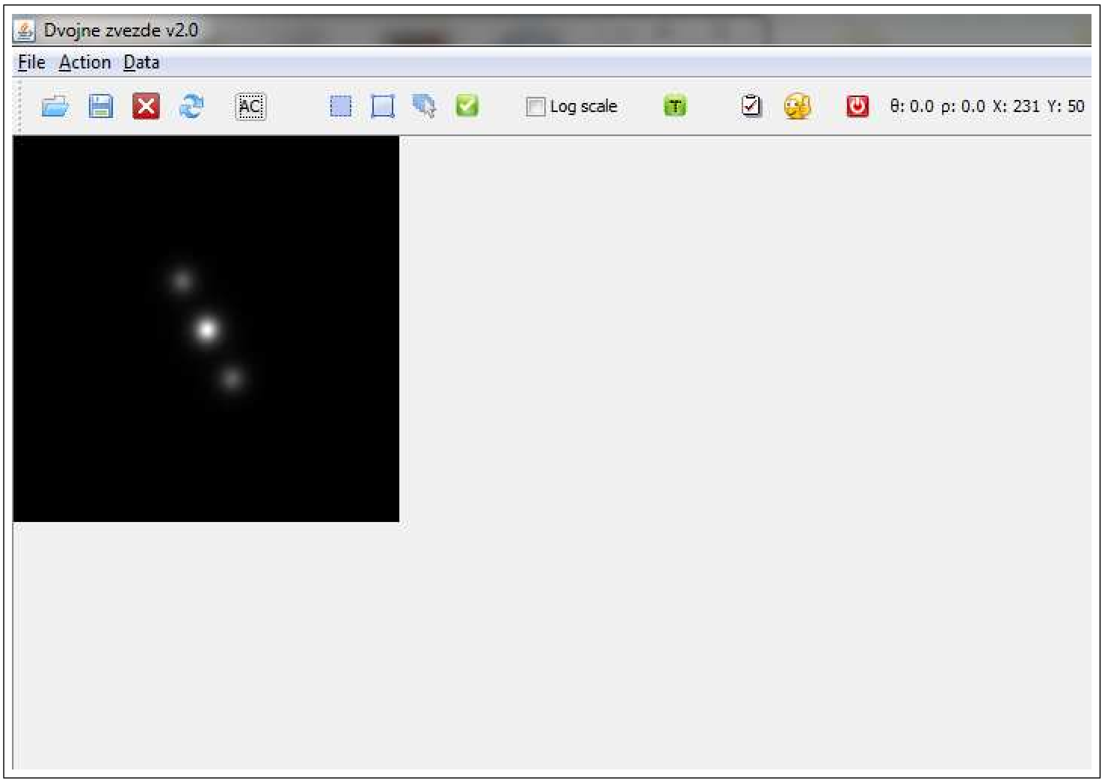

Fig. 1. *The main screen of our application.*

purpose of taking images of these objects. At the beginning the observations were carried out at NAO Rozhen in Bulgaria (Pavlović et al. 2005, Cvetković et al. 2006, 2007, 2010, 2011) but from the middle of 2011 such kind of activity has also been possible at the Astronomical Station of Vidojevica in Serbia. A large number of CCD frames (about 1000) have been collected. Consequently, a lot of time is needed for measuring, or more precisely, determining the relative coordinates for these pairs. Until recently, the frames were measured by using the AIP4WIN (version 2.3.1) software (Berry and Burnell 2002). Now IRAF[3], available on the Internet, is used, but we have also developed a new software. In the following the software will be described and its application will be discussed. The software contains a programme which first reads the corresponding CCD frame in the FITS format. Afterwards, the conversion into the frequency domain is done by applying the Fourier transform (FT) to the frame. Then, applying the autocorrelation function the relative coordinates of interest are obtained from the corresponding result. In Fig. 1 part of the display is presented, where the tools are seen which, if needed, can be used for the purpose of determining the relative coordinates of double stars.

2. THE TWO-DIMENSIONAL FOURIER TRANSFORM

In this software the images are treated in the frequency domain. When speaking of frequency, usually the temporal signal dependence is borne in mind. However, in our case we have a two-dimensional light intensity map registered as an image, i.e. there is a signal dependence in space. Due to this the spatial frequency is not expressed in number of cycles per second, but in number of cycles per pixel or, finally, in number of revolutions per mm. Since a frame has two dimensions, the conversion into the frequency domain requires the two-dimensional FT to be applied (Buil 1991).

Let $f(x, y)$ be a function, its Fourier transform is:

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-2\pi i(xu+yv)}\mathrm{d}x\mathrm{d}y. \quad (1)$$

The inverse two-dimensional Fourier transform is

[3]http://iraf.noao.edu/

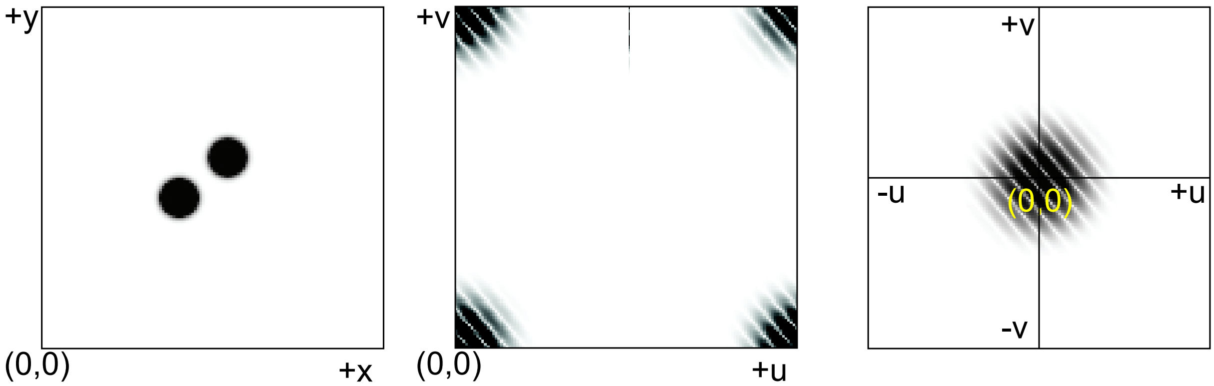

Fig. 2. *The two-dimensional FT for STF 783: a) the negative of the CCD frame, b) FT before translation and c) FT after translation.*

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)e^{+2\pi i(xu+yv)}\mathrm{d}u\mathrm{d}v, \qquad (2)$$

where (x, y) are spatial coordinates, and (u, v) the spatial frequencies.

Since the frames consist of a finite number of pixels (M, N), the conversion into the frequency domain is done by using the discrete FT. In other words the relation (1) becomes:

$$F(u,v) = \sum_{j=0}^{M-1}\sum_{k=0}^{N-1} f(x,y)e^{-2\pi i(\frac{uj}{M}+\frac{vk}{N})}. \qquad (3)$$

The inverse two-dimensional Fourier transform is given by:

$$f(x,y) = \sum_{j=0}^{M-1}\sum_{k=0}^{N-1} F(u,v)e^{+2\pi i(\frac{xj}{M}+\frac{yk}{N})}. \qquad (4)$$

In Fig. 2 we present the results of FT in the case of double star STF 783: a) a negative of its CCD frame, b) its FT before translation and c) its FT after translation. In panel b) the zero frequency is in the lower left corner and in panel c) the zero frequency is moved to the middle of the frame.

3. REDUCTION OF CCD FRAMES OF DOUBLE STARS

The image obtained by using a CCD camera contains the distribution of electromagnetic radiation from a given part of the sky over pixels of a CCD chip which can be described by means of the following relation (Berry and Burnell 2002):

$$I(\rho,\alpha,\delta) = \int_{\Delta S} I_S(\theta)\mathrm{d}\theta \qquad (5)$$

where $I_S(\theta)$ is the intensity in $\theta(\alpha, \delta)$ direction.

The frame quality, in addition to the emission of the observed object, is also affected by other factors such as temperature, atmospheric turbulence, etc. These influences can be very significant in the case of visual double stars, especially when the separation between the components is small. For this reason the image is analysed by applying FT in order to achieve the conversion from the image domain into the frequency one. By using FT it is possible to analyse the same double-star pair on various frames, regardless of their position, because the same result is obtained after the conversion into the frequency domain. This is based on a fundamental property of FT: the invariation of shift in the spatial plane into the frequency plane.

In order to clearly extract the peaks which correspond to the components of a given double star obtained by applying FT, in this software the autocorrelation is applied so that the dominant peaks become even more prominent, i.e. we have one central peak followed by two secondary ones.

According to the Wiener-Khintchine theorem (Saha 2007) the autocorrelation function can be expressed as a product of two FT:

$$AC = \mathrm{IFT}[F(u,v)\cdot\overline{(F(u,v))}] \qquad (6)$$

where $\overline{(F(u,v))}$ is the conjugate-complex $F(u,v)$ and IFT denotes the inverse Fourier transform.

After calculating FT and the autocorrelation it is possible to start determining the separation and position angle for the given double stars. In this procedure one has to extract the secondary peaks and determine their centres because their distance yields the double value of separation. The coordinates of the peak centroids (x, y) are calculated by using the following formulae:

$$x = \frac{\sum_{i=0}^{n-1}\sum_{j=0}^{m-1} i \cdot (AC(i,j) - \overline{s})}{\sum_{i=0}^{n-1}\sum_{j=0}^{m-1} (AC(i,j) - \overline{s})}, \quad (7)$$

$$y = \frac{\sum_{i=0}^{n-1}\sum_{j=0}^{m-1} j \cdot (AC(i,j) - \overline{s})}{\sum_{i=0}^{n-1}\sum_{j=0}^{m-1} (AC(i,j) - \overline{s})} \quad (8)$$

where $AC(i,j)$ is the autocorrelation value on pixel (i,j), n and m are the quantities determining the rectangle containing the peak (for which the centre is determined) and $\overline{s}$ is the arithmetic mean of the intensities of all pixels in the region around the peak:

$$\overline{s} = \frac{\sum_{i=0}^{n-1}\sum_{j=0}^{m-1} AC(i,j)}{n \cdot m}. \quad (9)$$

In the software, after the autocorrelation is carried out, the search for peaks is performed by using 3×3 matrices. The first step is to find the peak top, i.e. the pixel of the maximum intensity because all the neighbouring pixels are of lower intensities. For this reason the whole image is examined by means of the 3×3 matrix for the purpose of looking for such intensities. If a maximum appears at the matrix centre, then a peak has been located and its position is stored. In the final step among stored peaks three with the highest intensity are identified and they correspond to the peaks which have been looked for.

The coordinates of the secondary-peaks centres $C_1(x_1, y_1)$ i $C_2(x_2, y_2)$ are used to determine the separation and angle with respect to the coordinate system of the image:

$$\rho = \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (10)$$

$$\varphi = \arctan\frac{x_2 - x_1}{y_2 - y_1} \quad (11)$$

In Fig. 3 the peak cross section is given and the distance of the secondary peaks used in the calculation of the separation $\rho = d/2$ is marked.

The separation ρ is the distance in pixels and it is necessary to convert its value into arcseconds taking into account the pixel size and telescope focal length (F). The value obtained for the angle must be reduced to the north direction in order to obtain the position angle θ. The north direction is determined by using the star track.

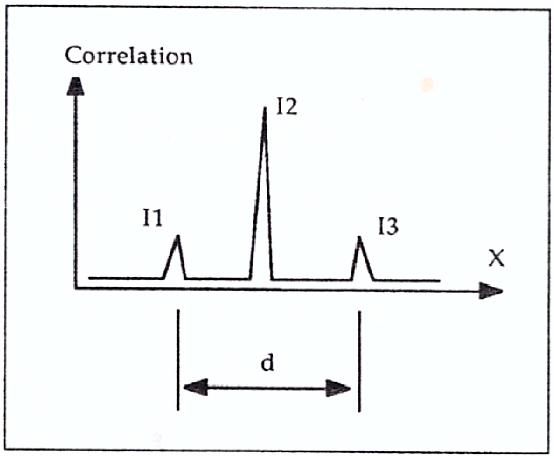

Fig. 3. *The autocorrelation function of a double star and separation determination.*

In the case of close pairs, due to the proximity of the components, the secondary peaks in most cases cannot be clearly distinguished by applying the autocorrelation function, even the relative coordinates cannot be determined. The software was therefore upgraded by adding the option for logarithmic scaling of FT amplitudes so that, in some cases, this method leads to the result. This is possible due to the FT property of being invariant as to scaling. A special attention is payed to the relative-coordinate determination in the case of multiple stars where the autocorrelation function yields several secondary peaks. In such cases the first step is to detach the part of CCD frame which contains a given pair and its data are the software input. It should be said that detaching of pairs in this way, even its measuring, is not always possible. Clearly, these additional activities cannot be automated, instead they are done "manually" resulting in a reduced software efficiency.

Due to this in the case of double stars the determination of relative coordinates is at first done automatically and where the problem of extracting the secondary peaks arises, the procedure is repeated with inclusion of the logarithmic-scaling option for FT amplitudes. In the case of multiple stars the first step is to detach the part of CCD frame containing a given pair and afterwards the procedure is the same as for double stars.

The signal to noise of the object is estimated from the values in the subraster in the following way (Davis 1990):

$$S/N = \frac{N_{\rm object}}{\sqrt{N_{\rm object} + n_{\rm pix} \cdot \sigma_{\rm sky}^2}} \quad (12)$$

where $N_{\rm object}$ is the number of counts in the object above the threshold, $\sigma_{\rm sky}$ is the standard deviation of the pixels in the sky region and $n_{\rm pix}$ is the number of pixels covered by the object. This approximation includes the Poisson noise in the object.

Table 1. Relative coordinates of selected pairs of visual double or multiple stars determined by applying Fourier transforms ρ_{FT} and θ_{FT} and by applying AIP4WIN software ρ_{AIP} and θ_{AIP}; absolute differences $\Delta\rho = \rho_{\text{FT}} - \rho_{\text{AIP}}$ and $\Delta\theta = \theta_{\text{FT}} - \theta_{\text{AIP}}$, and ratio signal to noise S/N.

WDS $\alpha, \delta(2000)$	Discoverer designation	Δm	ρ_{FT} [″]	θ_{FT} [°]	ρ_{AIP} [″]	θ_{AIP} [°]	$\Delta\rho$ [″]	$\Delta\theta$ [°]	S/N
00152+2722	J 868	0.20	5.92	230.36	6.02	230.38	0.10	0.02	139.05
00538+4731	ES 1297	0.10	4.13	213.79	4.36	213.75	0.23	0.04	229.75
05474+2939	BU 560	0.47	1.31	125.24	1.49	127.40	0.18	2.16	119.05
06179+0919	OPI 9	1.32	5.78	241.21	5.64	240.92	0.14	0.29	255.05
19500+0637	J 1336 AB	0.40	5.55	58.89	5.50	58.08	0.05	0.81	217.17
19500+0637	J 1336 AC	0.20	23.81	220.00	23.89	220.00	0.08	0.00	178.76
22013+2751	ES 527	0.10	3.48	212.96	3.42	213.66	0.06	0.70	121.68
23317+1956	WIR 1 AB	1.88	5.33	80.04	5.38	81.33	0.05	1.29	227.74
23317+1956	LMP 24 AC	1.71	31.13	22.42	31.18	22.14	0.05	0.28	220.85
23317+1956	LMP 24 AD	2.54	35.99	344.78	35.92	344.86	0.07	0.08	236.17

4. RESULTS AND DISCUSSION

The results obtained by the application of FT and the software abilities will be demonstrated through the already treated sample of double stars for which the CCD frames were taken in October 2011 at NAO Rozhen in Bulgaria. The observations were performed with the 2m Ritchey-Chretien-Coude (RCC) reflector. The frames were obtained by using the CCD camera VersArray 1300B. The chip dimensions are 1300×1300 pixels, the pixel size is 20×20 micrometers. The angle corresponding to one pixel is 0.261 arcsec.

To assess the quality of the obtained results we compare the relative coordinates ρ and θ obtained by applying FT with the corresponding values obtained by using another software -AIP4WIN. The results for the selected sample of pairs are given in Table 1. In the sample, the selection is carried out to include pairs with various separations, magnitude differences between components and number of components.

The designations used in Table 1 are as follows: WDS - identification in the Washington Double Star Catalog by its coordinates for the epoch 2000.0 in Column 1; Discoverer designation -double-star name after the discoverer with designation for pair components in Column 2; Column 3 gives the magnitude difference between components of the pair; ρ_{FT} and θ_{FT} - separation (in arcseconds) and position angle (in degrees) obtained by using FT in Columns 4-5; ρ_{AIP} and θ_{AIP} - separation and position angle obtained by using the AIP4WIN software in Columns 6-7; Columns 8 and 9 give absolute differences $\Delta\rho = \rho_{\text{FT}} - \rho_{\text{AIP}}$ and $\Delta\theta = \theta_{\text{FT}} - \theta_{\text{AIP}}$ and in the last Column the ratio signal to noise S/N is given.

The double star WDS 00152+2722 = J 868 is chosen as an example of a wide pair, for which the values of relative coordinates are obtained automatically, i. e. by applying FT its components are clearly separated, regardless of the low S/N ratio (Table 1). The next double star WDS 00538+4731 = ES 1297 has a lower separation (about 4″) than the previous one. In its case the results are also obtained through the programme, automatically. This is, among others, due to a very small difference in the brightness of its components ($\Delta m = 0.2$ and $\Delta m = 0.1$ for the first and second pairs, respectively). Fig. 4 gives the CCD frame, form of FT, 2D and 3D autocorrelations for double star WDS 00538+4731 = ES 1297.

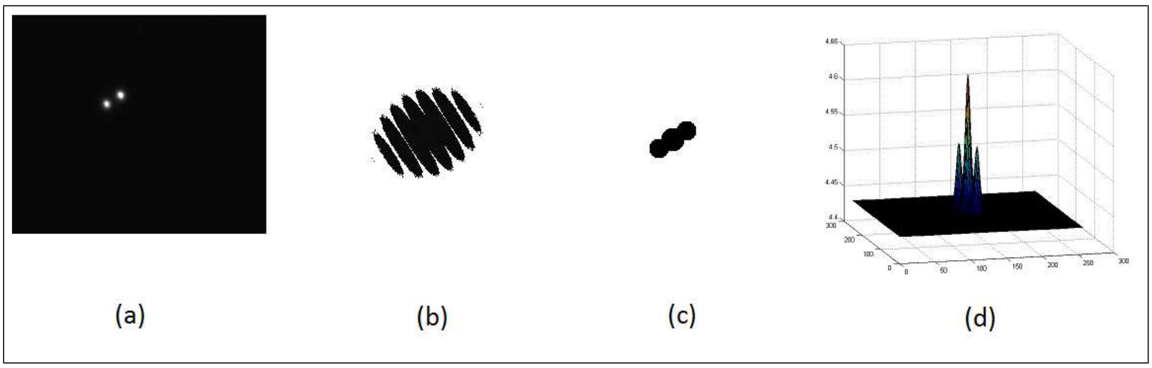

Fig. 4. *Double star WDS 00538+4731: a) CCD frame; b) result of its FT; c) autocorrelaton in 2D and d) autocorrelation in 3D.*

The pair WDS 05474+2939 = BU 560 is chosen because it has a low separation (less than $1''.5$). Due to the proximity of the components it is impossible to clearly indicate the secondary peaks by means of autocorrelation, even to determine the relative coordinates. The results are obtained by applying the logarithmic scaling. Besides, the S/N value is small, which led to rather high coordinate differences, especially $\Delta\theta$.

In the case of the double star WDS 06179+0919 = OPI 9 it is also necessary to carry out the logarithmic scaling in order to calculate the relative coordinates. The reason is not the separation, but a rather high difference in the brightness of the components ($\Delta m = 1.32$).

WDS 19500+0637 = J 1336 is composed of three components. The pair AB has a separation less than $6''$, whereas the pair AC is much wider. All three components have approximately the same brightness. The coordinates of both pairs are determined automatically after detaching parts of the image.

The pair WDS 22013+2751 = ES 527 has a small magnitude difference between the components ($\Delta m = 0.1$). However, by using the programme it is not possible to determine the relative coordinates automatically. Although the components have almost the same brightness, the separation is relatively small, also a low ratio $S/N = 121.68$ has contributed to the insufficient separation of the peaks. The result is obtained by applying the logarithmic scaling.

The multiple system WDS 23317+1956 consists of 4 components. In Fig. 5 the negative of the CCD frame for this system is presented. The AB pair was discovered by Wirtanen in 1941 and its designation is WIR 1AB. Two more pairs, AC and AD, were discovered in 1953 (Lampen and Strigachev 2001), their designations are LMP 24AC and LMP 24AD, respectively. Since this is a multiple star, the measurements concerning individual pairs were obtained after detaching the corresponding part of the image. In this way the influence of the other components on the result is removed. All magnitude differences between components exceed 1.5 but there was no need for the logarithmic scaling because these pairs are wide enough with separations above $5''$. However, the magnitude difference $\Delta m = 1.88$ for the pair AB affected the position angle determination. The difference $\Delta\theta = \theta_{\rm FT} - \theta_{\rm AIP}$ is equal to $1^\circ.29$. The CCD frame possesses a high quality which is confirmed by the S/N ratio, it exceeds 200 for all three pairs.

The results of the CCD frame measurements for 154 double or multiple stars are given in Table 2 in which the columns are designated in the same way as in Table 1. The first part of the table contains the results of automatic determination, whereas the second one contains those obtained by applying the logarithmic scaling. These results have an asterisk by the WDS identification.

The coordinate values obtained by using two softwares (FT and AIP4WIN) differ by small amounts which can be seen in Figs. 6 and 7. From these figures one can notice a better agreement in the case of separations than in the case of position angles. Larger differences concern either the pairs with small separation or the pairs with higher magnitude difference.

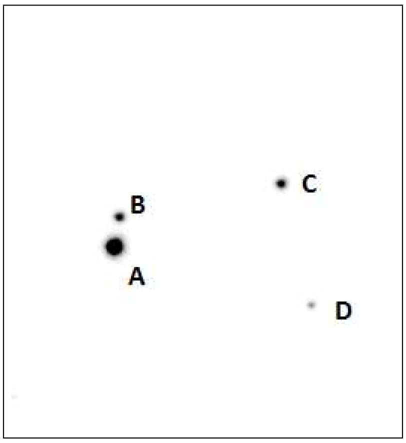

Fig. 5. *Negative of CCD frame of double star WDS 23317+1956 with component designations.*

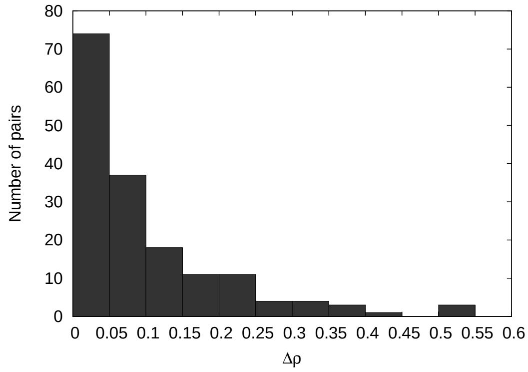

Fig. 6. *Distribution of separation differences* $\Delta\rho = \rho_{FT} - \rho_{AIP}$.

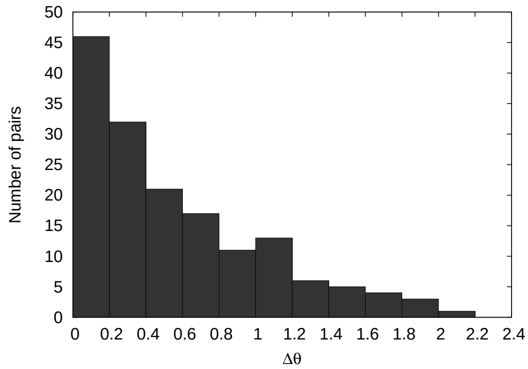

Fig. 7. *Distribution of position-angle differences* $\Delta\theta = \theta_{\rm FT} - \theta_{\rm AIP}$.

Table 2. Relative coordinates of selected pairs of visual double or multiple stars determined: by applying Fourier transforms ρ_{FT} and θ_{FT}; by applying AIP4WIN software ρ_{AIP} and θ_{AIP}; absolute differences $\Delta\rho = \rho_{\mathrm{FT}} - \rho_{\mathrm{AIP}}$ and $\Delta\theta = \theta_{\mathrm{FT}} - \theta_{\mathrm{AIP}}$ and ratio signal/noise S/N.

WDS $\alpha, \delta(2000)$	Discoverer designation	Δm	ρ_{FT} [$''$]	θ_{FT} [°]	ρ_{AIP} [$''$]	θ_{AIP} [°]	$\Delta\rho$ [$''$]	$\Delta\theta$ [°]	S/N
00017+6309	MLB 241 AB	2.20	7.10	277.55	7.30	277.01	0.20	0.54	234.66
00057+4549	STT 547 AB	0.17	6.04	186.66	6.04	186.56	0.00	0.10	238.03
00059+1805	STF3060 AB	0.33	3.32	136.12	3.39	134.45	0.07	1.67	250.43
00152+2722	J 868	0.20	5.92	230.36	6.02	230.38	0.10	0.02	139.05
00159+5233	ES 865 AB	0.23	4.00	102.55	4.17	102.26	0.17	0.29	234.03
00159+5233	ES 865 BC	0.53	8.94	53.37	8.85	53.59	0.09	0.22	234.03
00169+4427	ES 1481	1.62	7.18	58.15	7.18	58.87	0.00	0.72	228.49
00187+2545	HJ 1015 AB	1.17	5.52	290.51	5.43	289.55	0.09	0.96	242.45
00251+1824	HJ 621	1.20	5.33	5.35	5.33	5.46	0.00	0.11	230.63
00321+6715	VYS 2 AB	1.58	3.92	178.01	3.96	179.38	0.04	1.37	235.92
00336+4509	A 912 AB	1.29	15.35	204.31	15.41	203.73	0.06	0.58	233.45
00458+5459	WAL 9 AD	3.61	41.87	93.38	41.80	93.27	0.07	0.11	235.94
00492+2150	J 1439	0.31	6.83	180.08	6.85	180.12	0.02	0.04	224.82
00535+2536	BRT 119	0.77	5.26	297.80	5.71	297.50	0.45	0.30	241.46
00538+4731	ES 1297	0.10	4.13	213.79	4.36	213.75	0.23	0.04	229.75
00569+6152	HJ 1056 AB	1.82	8.13	177.55	8.24	177.13	0.11	0.42	240.56
00570+5729	ES 44	2.02	9.21	264.72	9.33	265.46	0.12	0.74	222.86
01054+3204	SEI 12	0.46	3.51	117.78	3.58	116.52	0.07	1.26	235.89
01088+6145	ES 1945	0.27	3.81	165.28	3.76	164.25	0.05	1.03	246.84
01098+3704	J 515	0.02	2.89	6.44	3.11	7.12	0.22	0.68	223.57
01184+1529	J 225 AB	0.40	2.69	210.29	2.97	210.07	0.28	0.22	236.82
01269+0332	STF 122	2.86	5.86	328.86	5.91	328.28	0.05	0.58	250.83
01393+5257	STF 139 AB	0.38	9.31	39.40	9.29	39.41	0.02	0.01	229.22
01411+1817	A 2321	2.70	7.09	235.03	7.21	234.90	0.12	0.13	241.75
01485+2811	BRT 6	0.40	5.24	73.82	5.29	73.94	0.05	0.12	225.06
01535+1918	STF 180 AB	0.06	7.33	1.18	7.42	0.85	0.09	0.33	255.73
02118+6529	HJ 1108	0.74	8.75	239.07	8.75	239.12	0.00	0.05	234.38
02180+3116	FOX 123 AC	4.26	52.73	309.19	52.68	309.26	0.05	0.07	286.56
02246+5959	STF 255	1.15	6.80	1.23	6.79	1.21	0.01	0.02	197.28
02291+6724	STF 262 AC	4.42	7.12	115.53	7.17	116.09	0.05	0.56	250.01
02332+3554	AG 41	0.20	3.96	83.64	4.48	83.13	0.52	0.51	247.13
02338+4252	ES 554	0.21	6.02	0.77	6.10	0.77	0.08	0.00	223.57
02410+6539	STF 282 AB	0.03	6.79	293.91	6.68	294.97	0.11	1.06	229.43
02489+3823	ALD 10	0.30	3.23	257.21	3.26	257.18	0.03	0.03	223.00
03069+3952	MLB 14	0.30	5.29	10.27	5.34	10.34	0.05	0.07	246.00
03088+2758	BRT 131	0.00	4.93	239.78	4.83	240.22	0.10	0.44	230.85
03088+3528	STF 352	1.85	3.62	361.00	3.87	359.93	0.25	1.07	219.49
03122+3713	STF 360	0.27	2.71	125.01	2.84	125.81	0.13	0.80	214.45
03162+5810	MLB 115 AB	0.39	4.98	4.25	5.02	3.55	0.04	0.70	220.48
03207+1736	HJ 3246 AD	1.64	9.25	223.95	9.34	224.48	0.09	0.53	212.53
03267+4110	J 889	0.00	1.99	68.05	2.34	69.79	0.35	1.74	108.52
03384+6033	ES 1816	0.00	3.28	120.45	3.35	122.56	0.07	2.11	232.58
03435+2935	A 989 AB	0.54	3.14	356.48	3.21	356.05	0.07	0.43	246.35
03492+3651	J 2721	0.61	2.89	356.03	2.88	354.15	0.01	1.88	232.18
04384+1900	BRT2317	1.30	3.66	181.24	3.79	180.97	0.13	0.27	214.32
04385+2656	STF 572 AB	0.15	4.26	188.80	4.33	189.26	0.07	0.46	234.39
04404+6830	HJ 1148	0.00	8.98	322.08	8.96	322.11	0.02	0.03	227.55
04448+0517	STF 589	0.14	4.21	278.33	4.57	277.35	0.36	0.98	226.29

Table 2. Continued.

WDS $\alpha, \delta(2000)$	Discoverer designation	Δm	$\rho_{\rm FT}$ [″]	$\theta_{\rm FT}$ [°]	$\rho_{\rm AIP}$ [″]	$\theta_{\rm AIP}$ [°]	$\Delta\rho$ [″]	$\Delta\theta$ [°]	S/N
04495+3914	STF 594	1.70	9.25	332.24	9.21	331.82	0.04	0.42	212.13
04562+0304	BAL1654 AB	0.55	5.98	248.04	6.23	248.23	0.25	0.19	226.13
05013+5015	STF 619	0.37	4.22	159.39	4.21	158.38	0.01	1.01	214.16
05233+3445	SEI 229	0.12	6.98	178.66	6.96	178.50	0.02	0.16	205.42
05345+3726	SEI 332	1.85	7.97	140.23	8.02	140.18	0.05	0.05	216.41
05364+2200	STF 742	0.38	4.16	274.71	4.08	274.30	0.08	0.41	240.64
05446+2901	STF 783	1.65	8.36	0.95	8.33	0.93	0.03	0.02	234.34
05456+2141	J 1905	0.35	5.78	266.04	5.87	264.93	0.09	1.11	231.85
05458+2130	J 2731 AB	0.90	5.19	42.15	5.11	41.51	0.08	0.64	240.13
05458+2130	J 2731 AC	0.02	6.32	246.80	6.29	247.13	0.03	0.33	241.45
05563+4353	ES 1531 AB	0.40	4.00	302.85	4.29	303.35	0.29	0.50	138.65
05565+5256	STF 810	0.39	2.41	245.75	2.73	246.85	0.32	1.10	206.03
06092+6424	MLB 259	0.30	5.62	26.52	5.64	27.58	0.02	1.06	226.43
06206+2327	POU1237	1.35	6.81	3.46	6.90	3.48	0.09	0.02	238.19
06220+1440	HO 232	1.50	7.32	2.28	7.44	3.21	0.12	0.93	237.99
06241+3733	AG 110	0.17	9.10	289.68	9.20	290.00	0.10	0.32	224.12
06277+2249	J 1092 AB	0.90	6.66	226.24	6.63	226.73	0.03	0.49	200.07
06284+0834	AG 114	0.77	4.30	1.26	4.85	1.26	0.55	0.00	238.40
06325+0820	J 2395	0.48	5.96	309.12	6.01	309.93	0.05	0.81	238.40
06571+5438	HJ 2350	2.03	6.29	198.17	6.41	197.84	0.12	0.33	230.04
07106+1543	J 703	1.97	10.20	293.86	10.26	293.67	0.06	0.19	225.34
18080+2406	POU3350	0.20	9.24	66.13	9.35	66.49	0.11	0.36	231.12
18278+2415	POU3411 AB	1.84	9.79	35.37	9.90	35.06	0.11	0.31	165.25
18278+2415	POU3412 AC	0.84	13.10	245.19	13.26	244.83	0.16	0.36	221.59
18438+2309	POU3509	0.99	6.95	35.53	6.96	36.39	0.01	0.86	190.33
19028+3123	STF2441 AB	1.93	5.80	263.51	6.14	264.17	0.34	0.66	240.92
19030+3729	J 766 AC	1.30	16.66	176.72	16.30	176.39	0.36	0.33	228.93
19054+3803	AG 227	0.52	6.91	30.71	7.04	29.97	0.13	0.74	228.28
19060+4549	STF2463 AB	1.73	9.67	359.69	9.69	359.85	0.02	0.16	222.85
19060+4549	STF2463 AC	3.83	24.21	279.86	24.39	279.84	0.18	0.02	212.98
19069+2210	STF2455 AB	2.02	9.35	27.79	9.36	27.06	0.01	0.73	233.11
19079+3043	HLM 16 AB	1.89	8.00	309.61	8.00	309.40	0.00	0.21	199.88
19079+3656	HJ 1369	0.19	9.74	154.18	9.73	154.14	0.01	0.04	224.04
19197+4422	STF2507 AC	0.88	29.45	147.83	29.36	147.58	0.09	0.25	226.56
19197+4422	STF2507 BC	1.28	6.47	105.28	6.42	105.58	0.05	0.30	205.89
19246+2131	STF2515 AB	1.81	5.71	138.66	5.82	138.77	0.11	0.11	237.67
19266+2530	STF2525 AB	0.38	5.29	82.71	5.39	82.70	0.10	0.01	211.18
19323+3417	HU 946	1.72	8.48	214.93	8.52	214.76	0.04	0.17	227.66
19383+2542	ES 492	0.36	4.93	213.90	4.98	214.05	0.05	0.15	227.95
19500+0637	J 1336 AB	0.40	5.55	58.89	5.50	58.08	0.05	0.81	217.17
19500+0637	J 1336 AC	0.20	23.81	220.00	23.89	220.00	0.08	0.00	178.76
20066+1147	J 503	1.10	5.00	265.23	5.53	265.62	0.53	0.39	153.97
20087+1223	J 1338	0.55	9.68	72.32	9.68	72.43	0.00	0.11	172.94
20210+1028	J 838	0.48	6.43	117.81	6.52	118.07	0.09	0.26	212.91
20346+2914	J 565 AC	3.50	15.70	89.34	15.92	88.97	0.22	0.37	216.47
20462+1554	STF2725 AB	0.66	5.85	11.50	6.06	11.10	0.21	0.40	241.02
20520+4346	STT 416	0.30	9.36	118.18	9.49	118.17	0.13	0.01	233.83
21066+3436	POP 22 AC	1.24	30.00	15.88	29.94	15.79	0.06	0.09	231.25
21070+4125	BU 988 AC	0.18	9.45	6.02	9.48	6.94	0.03	0.92	228.57
21121+4543	BU 160 BC	0.20	6.27	24.59	6.38	24.44	0.11	0.15	242.45

Table 2. Continued.

WDS $\alpha, \delta(2000)$	Discoverer designation	Δm	$\rho_{\rm FT}$ [″]	$\theta_{\rm FT}$ [°]	$\rho_{\rm AIP}$ [″]	$\theta_{\rm AIP}$ [°]	$\Delta\rho$ [″]	$\Delta\theta$ [°]	S/N
21174+3203	HJ 931	1.14	9.96	356.60	9.98	356.60	0.02	0.00	226.56
21199+3957	SEI1506	0.50	7.48	61.85	7.47	61.58	0.01	0.27	228.91
21401+2928	BRT 57	0.10	4.81	316.24	5.00	316.66	0.19	0.42	236.73
21462+2817	MLB 540 AC	0.08	8.48	143.13	8.71	142.44	0.23	0.69	240.19
21555+2942	HO 609 AB	0.30	2.89	176.05	3.26	176.17	0.37	0.12	170.01
21558+3716	HO 174 AB	0.37	7.60	334.68	7.61	334.77	0.01	0.09	230.03
21559+3141	ES 2360	0.10	4.07	226.79	4.10	227.76	0.03	0.97	235.81
21564+3156	HJ 1707	1.67	9.52	351.63	9.46	351.03	0.06	0.60	193.02
21565+2900	HJ 1706	0.99	9.85	260.81	9.95	261.33	0.10	0.52	225.61
22042+3806	SEI1555	0.03	7.33	179.19	7.32	178.73	0.01	0.46	232.40
22092+4734	HJ 1737	0.06	6.79	165.59	6.72	164.77	0.07	0.82	230.99
22112+5347	A 1456 AB	1.11	6.36	171.80	6.31	172.11	0.05	0.31	205.95
22138+4520	BRT1159	0.30	4.19	232.58	4.41	233.66	0.22	1.08	228.06
22166+5831	HJ 1748	0.39	5.02	309.90	5.22	309.65	0.20	0.25	171.01
22265+3837	HO 185 AB	2.50	4.07	221.09	4.19	221.60	0.12	0.51	237.04
22280+5742	KR 60 AH	3.97	53.87	265.67	53.78	265.61	0.09	0.06	113.67
22284+3533	HJ 1770	0.02	6.97	286.31	6.92	285.41	0.05	0.90	114.56
22423+1116	J 181	1.60	8.26	263.25	8.29	263.16	0.03	0.09	251.51
22455+1112	BU 711 AB	1.18	2.46	350.99	2.50	349.77	0.04	1.22	220.81
22513+2914	HJ 1819 AB	2.46	14.72	71.52	14.59	71.75	0.13	0.23	213.47
22547+1812	J 621 AC	1.61	15.67	105.74	15.79	105.55	0.12	0.19	114.80
23212+3526	STF3006 AB	0.79	7.26	151.03	7.24	151.70	0.02	0.67	224.75
23317+1021	HJ 3198	0.80	9.77	75.71	9.78	76.04	0.01	0.33	220.05
23317+1956	WIR 1 AB	1.88	5.33	80.04	5.38	81.33	0.05	1.29	227.74
23317+1956	LMP 24 AC	1.71	31.13	22.42	31.18	22.14	0.05	0.28	220.85
23317+1956	LMP 24 AD	2.54	35.99	344.78	35.92	344.86	0.07	0.08	236.17
23356+2816	BRT 230	0.15	3.15	6.00	3.17	5.81	0.02	0.19	220.87
23372+5633	HJ 1895	0.29	8.11	292.01	8.11	291.26	0.00	0.75	148.45
23375+4832	ES 859 BC	0.10	3.17	81.78	3.34	83.74	0.17	1.96	231.34
23390+5640	MLB 103	0.43	4.68	334.67	4.91	334.25	0.23	0.42	231.22
23479+1703	STF3041 AB	0.70	60.24	357.75	60.18	357.67	0.06	0.08	231.30
23479+1703	STF3041 BC	0.13	3.39	356.78	3.45	356.42	0.06	0.36	260.49
23556+2137	COU 345	1.22	7.49	184.06	7.54	184.08	0.05	0.02	230.23
23581+2840	HJ 995	2.50	8.06	126.98	8.10	126.39	0.04	0.59	236.08
00424+0410*	STT 18 AB	1.85	1.49	209.50	1.89	208.75	0.40	0.75	119.85
01017+4635*	A 927	0.81	2.36	8.10	2.36	8.12	0.00	0.02	121.45
01246+2450*	J 639	0.40	4.82	126.15	4.78	125.02	0.04	1.13	166.35
01467+3310*	STF 158 AB	0.44	2.15	270.38	2.20	270.04	0.05	0.34	122.19
01579+3310*	A 1920	0.58	1.49	235.07	1.66	236.67	0.17	1.60	228.28
02446+2928*	STF 300	0.19	3.08	313.99	3.00	315.77	0.08	1.78	127.34
02475+1922*	STF 305 AB	0.73	3.52	307.02	3.49	307.46	0.03	0.44	126.83
03086+6028*	STI 428	0.42	2.13	75.44	2.07	76.17	0.06	0.73	108.72
03247+2033*	J 931	0.20	4.74	80.35	4.65	79.83	0.09	0.52	181.65
05378+2322*	J 147 AB	0.18	3.61	359.94	3.39	358.91	0.22	1.03	118.81
05474+2939*	BU 560	0.47	1.31	125.24	1.49	127.40	0.18	2.16	119.05
06179+0919*	OPI 9	1.32	5.78	241.21	5.64	240.92	0.14	0.29	255.05
06324+0329*	J 982	0.50	3.13	214.34	3.01	215.90	0.12	1.56	122.11
06583+1341*	J 1058	0.15	2.45	349.38	2.42	349.69	0.03	0.31	117.65
07018+6637*	MLB 401	1.07	3.46	143.91	3.31	143.96	0.15	0.05	120.04

Table 2. Continued.

WDS $\alpha, \delta(2000)$	Discoverer designation	Δm	$\rho_{\rm FT}$ [″]	$\theta_{\rm FT}$ [°]	$\rho_{\rm AIP}$ [″]	$\theta_{\rm AIP}$ [°]	$\Delta\rho$ [″]	$\Delta\theta$ [°]	S/N
07023+1030*	J 21	0.61	3.22	275.64	3.19	274.85	0.03	0.79	146.93
07142+0533*	J 2039	0.00	2.30	201.35	2.25	201.28	0.05	0.07	110.97
07154+1221*	ROE 25	0.90	3.09	8.46	3.05	8.03	0.04	0.43	232.86
18443+3940*	STF2383 CD	0.95	2.26	78.71	2.16	80.23	0.10	1.52	241.65
19012+1253*	J 1279	0.00	3.00	350.19	2.72	348.83	0.28	1.36	111.03
19030+3729*	J 766 AB	0.60	2.95	0.50	2.89	1.65	0.06	1.15	112.83
19111+3847*	STF2481A,BC	0.08	4.56	19.91	4.58	20.08	0.02	0.17	207.28
20306+2158*	BRT2478	0.10	3.60	296.24	3.35	297.21	0.25	0.97	233.23
21068+3408*	STF2760 AB	0.84	4.85	33.19	4.68	32.61	0.17	0.58	210.12
21111+4530*	BRT1146	0.10	2.66	347.45	2.31	345.98	0.35	1.47	114.21
21208+3227*	STT 437 AB	0.27	2.42	21.55	2.14	20.42	0.28	1.13	125.08
21330+2043*	STF2804 AB	0.34	3.32	357.01	3.22	356.81	0.10	0.20	123.82
21363+2917*	BRT 56	0.10	3.06	306.87	2.99	306.64	0.07	0.23	156.68
22013+2751*	ES 527	0.10	3.48	212.96	3.42	213.66	0.06	0.70	121.68
22190+4107*	ES 1587	0.10	4.17	302.22	4.16	300.94	0.01	1.28	120.00
22449+5035*	ES 848	0.51	2.41	48.87	2.38	50.25	0.03	1.38	115.52
23148+2447*	J 624	0.32	2.74	196.87	2.75	195.83	0.01	1.04	121.70

5. ANALYSIS OF THE RESULTS

In the previous section systems with different characteristics are dealt with (different separations, different magnitudes and number of components). They served for the purpose of demonstrating the capabilities and limitations of the software for determining the relative coordinates, as well as the properties of the method including the FT application.

Here, we want to consider the influence of separation and magnitudes on the software application to the determination of relative coordinates. In the case of pairs with high separations it is possible to calculate the coordinates ρ and θ automatically, with no need for scaling or any other intervention (for instance, WDS 00152+2722 and WDS 00538+4731).

In the case of lower separations an important role is played by the magnitude difference Δm. If the magnitude difference is rather high, then the resolving of components may fail, which means that the coordinates cannot be determined. However, when the magnitude difference is small enough, ρ and θ can be calculated by applying the logarithmic scaling (for instance, WDS 06179+0919).

In the case of small separations ($\rho < 3''$) to apply the logarithmic scaling is most often needed because it is not easy to resolve very close components.

Out of the total of 165 double and multiple stars there were 11 where no measurements were possible. These are double stars with separations mostly below $1.''5$.

In addition to separation and magnitudes the application of FT is also affected by the quality of the frame itself. In the case of the pairs with low S/N ratio in the coordinate calculation additional software options, such as logarithmic scaling can be useful (for instance, WDS 19500+0637).

The problems just described can also be explained by the characteristics of FT. In the case of large brightness difference between system components the FT application will result in losing the fainter star because the peak of the brighter star is significantly more prominent. In the case of the FT application when the signal is close to the surrounding noise, it may become indistinguishable from the noise. When very small separations are in question, the problem is due to "coalescence" of peaks and their detaching by using FT is not possible.

For multiple stars (WDS 23317+1956) the same remarks, as given in the preceding paragraphs, are valid but the determination of relative coordinates becomes difficult due to the necessity of detaching a part of the image in order to avoid the influence of other components on the result. However, the spatial distribution of components does not always allow to detach any pair.

6. CONCLUSION

The software developed here makes it possible to calculate the coordinates by applying the Fourier transform. The Fourier transform due to its characteristics offers the possibility of resolving the double-star components and, in this way, the separation and position angle can be calculated.

The main limitations of this programme concern a high magnitude difference between the components, small separation and low quality of the frame. Some of these problems can be avoided by applying the logarithmic scaling which offers a possibility of better resolving the secondary peaks. The objective of this paper is automatisation in measuring the relative coordinates of double stars. In this way, much less time is needed for reduction of CCD frames and this has been achieved to a satisfactory extent.

Future improvement of the programme should make it possible to calculate the relative coordinates for multiple stars more easily. This could be done by using the already implemented procedure with 3×3 matrices because, by applying the Fourier transform to a multiple star, several secondary peaks corresponding to different pairs within the multiple star are obtained. At the moment the programme offers the possibility to determine automatically the most prominent secondary peaks only, but the later versions will offer the possibility of determining other peaks as well. It is also necessary to study and conceive the ways of how to solve the problems mentioned in Section 5 (calculation in the case of smaller separations, etc).

For the future programme improvement, better algorithms for peak resolving, also for a better determining their centroids, are needed. Better results with new algorithms for peak resolving are also expected in the case of multiple stars. The user interface should also be improved in order to contain more options for automatisation of measurements, which would simplify the use of the application significantly.

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ОДРЕЂИВАЊЕ РЕЛАТИВНИХ КООРДИНАТА ВИЗУЕЛНО ДВОЈНИХ ЗВЕЗДА ПРИМЕНОМ ФУРИЈЕОВИХ ТРАНСФОРМАЦИЈА

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Претходно саопштење

У раду је приказан софтвер развијен за одређивање релативних координата визуелно двојних или вишеструких звезда (позициони угао θ и угловно растојање ρ) који је заснован на примени Фуријеових трансформација при обради CCD снимака таквих система. Циљ развоја овог софтвера је био да се постигне што већи степен аутоматског одређивања релативних координата како би се смањило време потребно за обраду великог броја добијених CCD снимака. Испитане су могућности и ограничења развијеног софтвера. Осим тога, даје се предлог за нека побољшања. Истраживања су рађена на узорку CCD снимака за 165 двојних и вишеструких звезда снимљених 2-m телескопом на НАО Рожен у Бугарској у октобру 2011. године. Резултати су упоређени са одговарајућим резултатима добијеним применом других софтвера и слагање је веома добро.