Open forum

Recently published paper in Thermal Science: Awad, M. M., Lage, J. L.: Extending the Bejan Number to a General Form, THERMAL SCIENCE: Year 2013, Vol. 17, No. 2, pp. 631-633, initiated interesting discussion between one of the authors Prof. Mohamed M. AWAD and Prof. Dr.-Ing. habil. Holger Martin, from the Karlsruher Institut fur Technologie (KIT), Karlsruhe, Germany.
Prof. Holger Martin rose an interesting question: "Is Hagen number defined in literature as Hg = =(ΔP/L)/(l^3/ρv^2) (see, for example: H. Martin, Chapter A2-Dimensionless numbers, in: VDI-Gesellschaft Verfahrenstechnik und Chemieingenieurwesen (GVC), Editor, VDI Heat Atlas, Second Edition, Springer-Verlag Berlin Heidelberg, 2010, pp. 11-13), identical with Bejan number defined in paper published in Thermal Science, as Be = (ΔPL^2/ρv^2)"?
Opinion and discussion of the Prof. Mohamad M. Awad is presented in the paper which follows.

HAGEN NUMBER *VERSUS* BEJAN NUMBER

by

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This study presents Hagen number vs. Bejan number. Although their physical meaning is not the same because the former represents the dimensionless pressure gradient while the latter represents the dimensionless pressure drop, it will be shown that Hagen number coincides with Bejan number in cases where the characteristic length (l) is equal to the flow length (L). Also, a new expression of Bejan number in the Hagen-Poiseuille flow will be introduced. At the end, extending the Hagen number to a general form will be presented. For the case of Reynolds analogy (Pr = Sc = 1), all these three definitions of Hagen number will be the same.

Key words   *Hagen number, Bejan number, momentum diffusivity,*
thermal diffusivity, mass diffusivity

The Hagen number (Hg) is named after German physicist and hydraulic engineer Gotthilf Heinrich Ludwig Hagen (March 3, 1797 - February 3, 1884). Hagen's contribution to fluid science is that he published independently of Poiseuille the solution for fully developed laminar flow in a circular pipe. The Hagen-Poiseuille equation is a physical law that gives the pressure drop in a fluid flowing through a long cylindrical pipe. The assumptions of the equation are that the flow is laminar viscous and incompressible and the flow is through a constant circular cross-section that is significantly longer than its diameter. Also, this equation is known as the Hagen-Poiseuille law, Poiseuille law and Poiseuille equation. Poiseuille's law was experimentally derived in 1838 and formulated and published in 1840 [1, 2] and 1846 [3] by Jean Louis Marie Poiseuille (22 April 1797-26 December 1869). Hagen did his experiments in 1839 [4] and

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reported that there might be two regimes of viscous flow. The researcher measured water flow in long three brass pipes of diameters 0.255, 0.401, and 0.591 cm and lengths of 47.4, 109, and 105 cm, respectively. and deduced a pressure-drop law by expressing the pressure drop as the sum of the constant and the entrance effect. He did not realize that this constant was proportional to the fluid viscosity. His formula broke down as Hagen increased volume flow rate (Q) beyond a certain limit, *i. e*., past the critical Reynolds number (Re_{cr}), and he stated in his paper that there must be a second mode of flow characterized by "strong movements of water for which the pressure (P) varies as the second power of the discharge ..." He admitted that he could not clarify the reasons for the change. For more details about Poiseuille law, the reader can see the review article about the history of Poiseuille's law by Sutera and Skalak [5] as well as the article about Poiseuille and his law by Pfitzner [6]. For the first hundred years, the world of science has known this flow regime as "Poiseuille flow", because the applied mathematics literature during the 1800s was in French. Due to boundary layer theory, the literature developed a strong German current in the first half of the 20th century. German authors revised the nomenclature and, quite rightly, named the well-known flow as "Hagen-Poiseuille flow". The mathematical formula of Hagen-Poiseuille flow is:

$$f = \frac{C}{Re} \tag{1}$$

where $C = 16$ for the circular pipe. Other duct cross-sectional shapes have the same eq. (1) but other C values, which do not differ much from 16. For example, the constant (C) is 24 for a rectangular channel with the aspect ratio of 0 while the constant (C) is 14.23 for a rectangular channel with the aspect ratio of 1 (square channel). It should be noted that the pipe diameter (d) is used to calculate the Reynolds number (Re) for circular shapes while the hydraulic diameter (d_h) is used to calculate the Reynolds number (Re) for non-circular shapes. To calculate the constant (C) as a function of the aspect ratio, Shah and London [7] gave the following relation for (fRe) f*or* laminar flow forced convection in rectangular ducts as a function of the aspect ratio (A):

$$f Re = 24(1 - 1.3553A + 1.9467A^2 - 1.7012A^3 + 0.9564A^4 - 0.2537A^5) \tag{2}$$

Roughly 15 years ago, French authors like Jacques Padet proposed to commemorate Poiseuille by naming the constant (C) as the Poiseuille constant (Po), not as the Poiseuille "number".

The Hagen number (Hg) represents the dimensionless pressure gradient and can be written as follows:

$$Hg = \frac{\Delta P}{L} \frac{l^3}{\rho v^2} \tag{3}$$

Regarding using the Hagen number (Hg) in the literature, Martin [8] recast pressure drop correlations for flow normal to inline and staggered plain tube bundles in terms of the Hagen number per tube row that were developed by Gaddis and Gnielinski [9]. Later, the equations of Martin [8] appeared in the section about tube bundles in the book of Shah and Sekulic [10] about fundamentals of heat exchanger design in 2003. Also, Shah and Sekulic [10] used the Hagen number in Chapter 7 in their table about "Important Dimensionless Groups for Internal Flow Forced Convection Heat Transfer and Flow Friction, Useful in Heat Exchanger Design". Shah and Sekulic [10] mentioned that the Hagen number does not have any velocity explicitly in its definition, so it avoids the ambiguity of velocity definitions. This may be advantageous in flow normal to the tube bank and other external flow geometries having some ambiguity in defining the maximum velocity, as needed in the definition of Fanning friction factor (f). In addi-

tion, Shah and Sekulic [10] related the Hagen number to the Euler number (Eu) and the Reynolds number (Re) for flow normal to a tube bundle. Any reference velocity may be used as long it is the same in both the Euler and Reynolds number definitions. Moreover, Shah and Sekulic [10] mentioned the relations of Fanning friction factor, Darcy-Weisbach friction factor, and Hagen number definitions with Reynolds number for Hagen-Poiseuille flow (fully developed laminar flow in a circular pipe).

For the case of natural convection flows, $\Delta P/L$ is the static pressure gradient $g\Delta\rho$ or $g\rho\beta\Delta T$ in a gravity field, and the Hagen number becomes an Archimedes number (Ar) or a Grashof number (Gr) as follows [11]:

$$\text{Hg} = g\Delta\rho \frac{l^3}{\rho v^2} = \frac{gl^3\Delta\rho}{\rho v^2} = \text{Ar} \tag{4}$$

or

$$\text{Hg} = g\rho\beta\Delta T \frac{l^3}{\rho v^2} = \frac{g\beta\Delta T l^3}{v^2} = \text{Gr} \tag{5}$$

For the case of forced convection flows, the linear Hagen-Poiseuille law of fully developed forced laminar tube flow can be written as:

$$\frac{\Delta P}{L} = \frac{4f}{d} \frac{1}{2} \rho U^2 \tag{6}$$

$$f = \frac{16}{\text{Re}} \tag{7}$$

$$\text{Re} = \frac{Ul}{v} \tag{8}$$

Substituting eqs. (6-8) into eq. (3) with using the internal pipe diameter (d) as the characteristic length (l), we obtain:

$$\text{Hg} = 2f\text{Re}^2 = 32\text{Re} \tag{9}$$

At a critical Reynolds number of $\text{Re}_{cr} = 2300$, the critical Hagen number (Hg_{cr}) for the transition of laminar to turbulent flow is equal to:

$$\text{Hg}_{cr} = 32 \times 2300 = 73600 \tag{10}$$

On the other hand, the Bejan numbers (Be) is named after Duke University Professor Adrian Bejan. It represents the dimensionless pressure drop along a channel of length L. Historically, Bhattacharjee and Grosshandler [12] performed the scale analysis of a wall jet in 1988. The researchers discovered the new dimensionless group:

$$\text{Be} = \frac{\Delta P L^2}{\mu v} \tag{11}$$

They recognized the general importance of this group throughout forced convection (note the ΔP), and they named it "Bejan number" because of the method of scale analysis that they employed based on Bejan's 1984 book [13].

While unaware of Bhattacharjee and Grosshandler's discovery of the Be dimensionless group, Bejan and Sciubba [14] discovered in 1992 the same dimensionless group (more generally, for any Pr) in the scale analysis and intersection of asymptotes of parallel plates channels with optimal spacings and forced convection. They recognized the general role of this group, and named it pressure drop number (Π). The coincidence between refs. [12] and [14], and

the fact that "optimal spacings" became a fast growing field is why the Bejan number term gained wide acceptance. Today, the Bejan number has spread because it is general, like the scale analysis, which gave birth to it.

Expressing the dynamic viscosity (μ) in the denominator as a product of the fluid density (ρ) and the momentum diffusivity of the fluid (ν), Awad and Lage [15] wrote the original Bejan number as:

$$\mathrm{Be} = \frac{\Delta P L^2}{\rho \nu^2} \tag{12}$$

This new form is more akin the physics it represents and has the advantage of having one single viscosity coefficient in it.

From eqs. (3) and (12), it can be seen that Hagen number coincides with Bejan number in cases where the characteristic length (l) is equal to the flow length (L) although their physical meaning is not the same because the former represents the dimensionless pressure gradient while the latter represents the dimensionless pressure drop.

Substituting eqs. (6-8) into eq. (13) with using the internal pipe diameter (d) as the characteristic length (l), we obtain:

$$\mathrm{Be} = 2 f \, \mathrm{Re}^2 \left(\frac{L}{d} \right)^3 = 32 \, \mathrm{Re} \left(\frac{L}{d} \right)^3 \tag{13}$$

Equation (13) shows that the Bejan number in the Hagen-Poiseuille flow is indeed a dimensionless group, not recognized previously.

Similar to the Petrescu definition [16] of the Bejan number for heat transfer processes, Awad and Lage [15] extended the Bejan number to heat transfer processes by replacing the momentum diffusivity of the fluid (ν) with the thermal diffusivity (α) as:

$$\mathrm{Be} = \frac{\Delta P L^2}{\rho \alpha^2} \tag{14}$$

In a similar way, the Hagen number can be extended to heat transfer processes by replacing the momentum diffusivity of the fluid with the thermal diffusivity in eq. (3) as

$$\mathrm{Hg} = \frac{\Delta P}{L} \frac{l^3}{\rho \alpha^2} \tag{15}$$

It should be noted that the ratio of the Hagen number, eq. (3), to the definition, eq. (15), is equal to:

$$\frac{\alpha^2}{\nu^2} = \left(\frac{1}{\mathrm{Pr}} \right)^2 \tag{16}$$

Similar to the Awad definition [17] of the Bejan number for mass transfer processes, Awad and Lage [15] extended the Bejan number to mass transfer processes by replacing the thermal diffusivity (α) with the mass diffusivity (D) as:

$$\mathrm{Be} = \frac{\Delta P L^2}{\rho D^2} \tag{17}$$

In a similar way, the Hagen number (Hg) can be extended to mass transfer processes by replacing the momentum diffusivity of the fluid (ν) with the mass diffusivity (D) in eq. (3) as:

$$\mathrm{Hg} = \frac{\Delta P}{L} \frac{l^3}{\rho D^2} \tag{18}$$

It should be noted that the ratio of the Hagen number, eq. (3), to the definition, eq. (18), is equal to:

$$\frac{D^2}{v^2} = \left(\frac{1}{\text{Sc}}\right)^2 \tag{19}$$

For the case of Reynolds analogy (Pr = Sc = 1), it is clear that all these three definitions of Hagen number, eqs. (3), (15), and (18), are the same.

Finally, it can be seen that what a powerful tool analogy between the quantities that appear in the formulation and solution of fluid convection, heat convection, and mass convection that can be in the study of natural phenomena.

Nomenclature

A – aspect ratio
Ar – Archimedes number
Be – Bejan number
C – constant
D – mass diffusivity, [m^2s^{-1}]
d – pipe diameter, [m]
Eu – Euler number
f – fanning friction factor
g – acceleration due to gravity, [ms^{-2}]
Gr – Grashof number
Hg – Hagen number
L – flow length, [m]
l – characteristic length, [m]
P – pressure, [Pa]
ΔP – pressure drop, [Pa]
Po – Poiseuille constant
Pr – Prandtl number
Q – volume flow rate, [m^3s^{-1}]

Re – Reynolds number
Sc – Schmidt number
ΔT – temperature difference, [K]
U – fluid velocity, [ms^{-1}]

Greek symbols

α – thermal diffusivity, [m^2s^{-1}]
β – thermal expansion coefficient, [K^{-1}]
μ – dynamic viscosity, [$kgm^{-1}s^{-1}$]
v – momentum diffusivity, [m^2s^{-1}]
Π – pressure drop number
ρ – density, [kgm^{-3}]
$\Delta\rho$ – change in density, [kgm^{-3}]

Subscripts

cr – critical
h – hydraulic

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