Reliability and Its Quantitative Measures

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In this article is made an opening for the software reliability issues, through wide-ranging statistical indicators, which are designed based on information collected from operating or testing (samples). It is developed the reliability issues also for the case of the main reliability laws (exponential, normal, Weibull), which validated for a particular system, allows the calculation of some reliability indicators with a higher degree of accuracy and trustworthiness

Keywords: *Reliability Statistical Indicator, Cdf - Cumulative Distribution Function, Pdf - Probability Density Function, Reliability Function R(T), Weibull Law, Rayleigh Statistical Law, Sampling Plan, Average Lifetime, Testing Time, Reliability Sampling Plans, Statistical Models In Reliability*

1 Preliminaries

More than hundred years ago, the American physicist Willard GIBBS (1839 - 1903) launched a sentence which now is adopted by most of modern engineers and economist as an axiom: "The whole is simpler than the sum of its parts". He was quite a visionary since the huge development of technology and industrial production we are facing nowadays, proved that even a "simple" component of a space rocket (for instance) may be regarded as a complex system itself.

No matter of its simplicity or complexity, a technical entity is created in order to perform failure free at least a given period some specific tasks and at a desirable level of performance. This set of quality features are united under a general concept called reliability. A man-in-the street definition states that reliability of a given product (which may be a desk stapler or a sophisticated limousine) is a probability - that is the probability of the underling item to perform its intended functions at least a given time To without failure and in known conditions of usage.

Formally, if T is the variable representing the time-to-failure of the object, then one may write:

$$T : R(T_0) = \Pr ob\{T \geq T_0\} \qquad (1)$$

Here R(t) - in our case for $t = T_0$ - stands for the reliability function associated to the variable T. Consequently, the complement of R is the so-called non-survival (non-reliability) function: F(t) = 1 - R (t), which represents from statistical viewpoint, the distribution function (df) of T.

Straightforward consequences:

a) the reliability function R(t) is a decreasing one, that is in time, the intrinsic capacities of the item to fulfill its operating duties will diminish;

b) if $t = 0$, then R(0) = 1, that is at the initial moment the entity has to be operational;

c) if $t \to \infty$, then $R(\infty) = 0$, that is from practical point of view after a very large period of time the item will fail for good.

As Blischke and Murthy [3] wrote, there are no means (at least human means!) to stop the process of an "ultimate failure" of any living being or engineered object. Sooner or later - in spite of the best project (design), production process or maintenance activities, a man-made entity will fail.

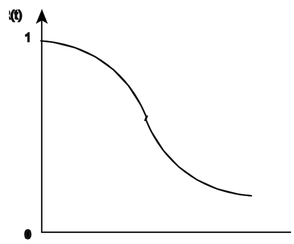


Fig. 1. Process of "ultimate failure"

The theory of reliability deals with all aspects of this process of failure, trying to construct formal/mathematical models for this random phenomenon.
In a broader sense, a failure may be defined as an incident or a situation/condition that causes for a product a process or a service an undesired status when the intended purposes are not performed safely, secure and cost-effectively [21]. In fewer words, the entity is out of order!
The time element is usually considered as the main parameter in describing the reliability behavior of an object, but sometimes the cycles of operation may reflect better this behavior - especially for items/systems which operate intermittently.
Inspired by the domain of demography, reliability theory adopted and adapted the main indicator used there, namely the rate of mortality which is in fact the ratio between the number of deceased persons at the time (t) and the number of survivors at the same moment (t). In a reliability framework, the mortality rate became hazard or failure rate where the "persons" (living beings) are replaced by objects (components, subassemblies a.s.o.). Its analytical expression is:

$$Z(t)=\frac{f(t)}{1-F(t)}=\frac{f(t)}{R(t)} \qquad (2)$$

where f(t) is so-called density function (probability density function) of T and where $F(t)=\Pr ob\{T<t\}$ and $R(t)=1-F(t)$ have been presented above. Since it is known that $f(t)=F'(t)$ (the first derivative of F) and $F'(t)=-R'(t)$ one may write immediately:

$$Z(t)=-\frac{R'(t)}{R(t)}=-[\ln R(t)]' \qquad (3)$$

which provides a general expression of R(t) if z(t) is known:

$$R(t)=\exp\left\{-\int_0^t z(u)du\right\},\quad u\geq 0 \qquad (4)$$

This formula (4) is important from a practical viewpoint since we can observe and record the failure rate of a given kind of objects, expressed usually in **failures per hour**.

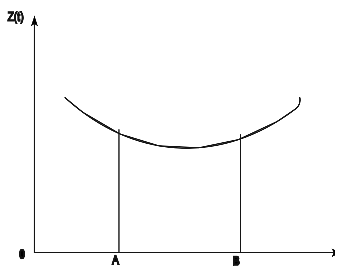


Fig. 2. Graphical image of the hazard rate

The graphical image of the hazard rate (Figure 2) has the same bath-tube form - as in demography, the mortality rate - the difference being of interpretation.
Let us comment now the above curve: the region between O and A point is typical for what is called **early** (or even instant) failures which in demography is known as infant mortality. From A to B, we have the so-called **normal life period** where "accidents" or failures may occur only due to a "bad luck" or due to some unexpected technical or misoperational causes. In this period (A to B) one may observe an almost constant failure rate (the curve is nearly parallel to the horizontal (time) axis. The next (and last) period (B to +∞) is characteristic to the wear-out (or aging) phenomenon, the hazard rate being increasing.

2 Some Elementary Theoretical Results

If T is the random variable describing the time-to-failure of a given technical entity, a time-to-failure model for reliability is the following abstract object:

$$\{T | f(t;\theta) \geq 0, t \geq 0, \theta = (\theta_1, \theta_2, \ldots, \theta_m), \theta_j \in R, \quad 1 \leq j \leq m\} \quad (5)$$

where $f(t;\theta)$ is the p.d.f. of T - that is:

$$\int_0^\infty f(t;\theta)dt = 1 \quad (6)$$

The links between f and F or R are the following:

$$F(t;\theta) = \int_0^t f(x;\theta)dx \quad (7)$$

$$R(t;\theta) = 1 - F(t;\theta) = \int_t^\infty f(x;\theta)dx \quad (8)$$

$$f(t;\theta) = F_t'(t;\theta) = -R_t'(t;\theta) \quad (9)$$

The vector of parameters $\theta = (\theta_1, \theta_2, \ldots, \theta_m)$ individualizes the form of f. Usually, a p.d.f. contains two or three such parameters (θ_j), which are usually location, scale and power/shape ones. To involve more components of θ means to complicate the inference on f.

Figure 3 shows a common form of a p.d.f. which has one modal value (a maximum of f).

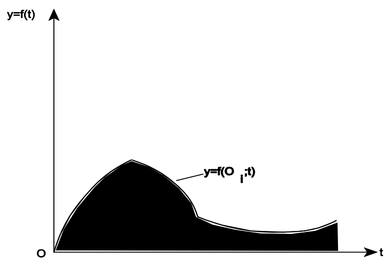


Fig. 3. A common form of a p.d.f.

Figures 4a and 4b show the forms of non-survival and the reliability functions.

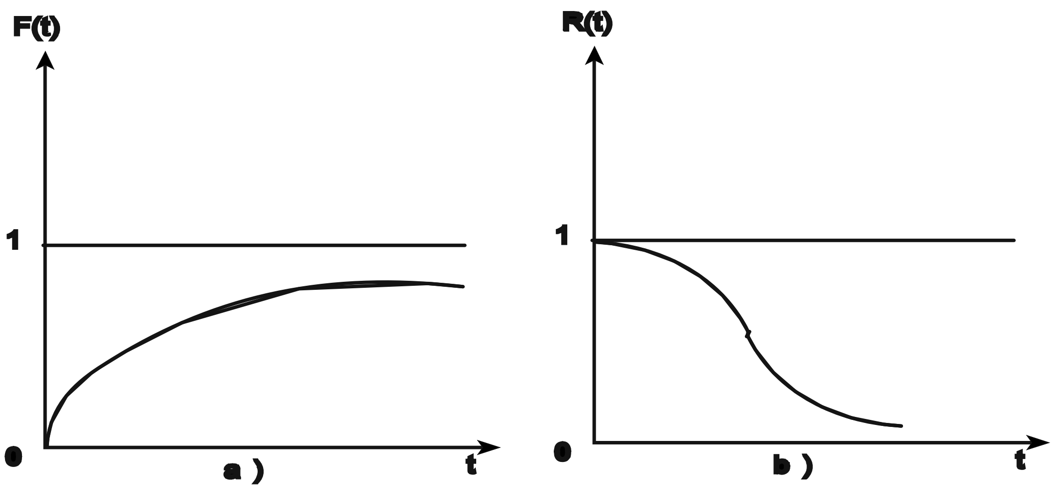


Fig. 4. The forms of non-survival and the reliability functions

Table 1. Some characteristics of the random variable T used in a reliability model

Nr. crt.	**Characteristic**	**Analytic representation**
1	The distribution function of (T)	$F(t) = Prob\{T < t\}, \quad t \geq 0$
2	The probability density function of T (p.d.f.)	$f(t) = F_t'(t) \left(= \frac{dF(t)}{dt} \right)$
3	The reliability function	$R(t) = Prob\{T \geq t\}$
4	The average life be derived easily, using (durability)	$E(T) = \int_0^{\infty} t \cdot f(t)\,dt < +\infty$
5	The variance of T	$Var(T) = E(T^2) - [E(T)]^2 < +\infty$
6	The variation coefficient of T	$CV(T) = \frac{\sqrt{Var(T)}}{E(T)}$

In the Table 1 we shall present the most important characteristics related to the T variable in a time-to-failure frame. As we stated before, if one knows one of functions, the rest may be derived easily, using their mutual relationship.
For instance, f(t) will provide F(t), R(t) = 1 - F(t), the hazard rate z(t) = f(t)/R(t) a.s.o. The knowledge of f means the knowledge of the analytical form of (f), of its parameters $\theta_j, \quad j = \overline{1,m}$ (the value of $t = t_0$ for which we wish to evaluate the reliability, is an additional but an important element of the analysis).

3 Reliability Indicators

The process of reliability evaluation may be done using two major groups of statistical indicators:

a) one, based on the so-called non-parametric indices provided by the descriptive statistics;

b) one, based on parametric indices deduced by the aid of probability models (Table 2).

Table 2. Dependent reliability indicators

	F(t)	**R(t)**	**f(t)**	**e(t)**
F(t)	-	1 - F(t)	$\frac{dF(t)}{dt}$	$\frac{1}{1-F(t)} \bullet \frac{dF(t)}{dt}$
R(t)	1 - R(t)	-	$-\frac{dR(t)}{dt}$	$-\frac{1}{R(t)} \bullet \frac{dR(t)}{dt}$
f(t)	$\int_0^t f(x)\,dx$	$\int_0^{\infty} f(x)\,dx$	-	$\frac{f(t)}{\int_0^{\infty} f(x)\,dx}$

It is important to notice the information on the product reliability are obtained either from an actual operation either from experimental tests or trials. Each procedure has its own advantages and disadvantages. For instance, if we study the behavior of the items in their field exploitation one may observe all events which appear during the operation.

The trial method tries - if it is possible, to simulate the actual exploitation conditions - namely to reproduce some internal stress factors as well as the future environmental conditions in which the item will perform its tasks. Usually, a reliability/durability test consists in the following: a given sample of n elements is subjected to a specified operation and the experimenter waits until all elements of the sample fail. This procedure is known as a “n from n” test. At the end, the time-to-failure data are obtained $\{t_i\}_{1 \leq i \leq n}$: the information is **complete** but sometimes the test is not economical from cost and/or time viewpoint: the objects under the test may be expensive and/or their "natural failure-free life" may by large and therefore it is unpractical to wait until all these items fail. To avoid these situations, there have been imagined two kinds of tests, namely:

a) **censored tests** - which specify previously a number (r), r < n of items allowed to fail; when these (r) objects have failed, then the test is interrupted: hence, at the end, the experimenter obtained $t_1, t_2, \dots t_r$ (r < n) failure times instead of all data $\{t_i\}_{1\le i\le n}$;
b) **truncated tests** - in which the experimenter is allowed to perform the test only during a specified period of time (T_0); when this period is consumed, the experiments is ceased; at the end of kind of test one may obtain a given number of failure times which may be less or even equal to n - the total number of elements subjected to the trial.

It is important to notice that in the case (a), **the duration of the experiment is random** (we do not know in which moment the last $r^{\underline{th}}$ elements will fail); in the case (b), **the number of failed objects is random** (we do not know in advance how many elements will fail during the test period T_0) – see [11].

As it is well known, the majority of consumer goods may be divided into two large categories:

a) durable consumer goods (usually having the possibility to be restored or repaired);
b) items which can be used only once - that is they are not repairable (as for instance electric bulbs).

Taking into account these two classes of products, their reliability may be investigated as follows:

1. If the item can be repaired/restored/renewed, its operational status may be expressed via three kinds of indicators:
   - first ones which describes failure-free functionality;
   - second ones describing the restoring situations;
   - third ones providing a measure of availability.
2. If the product is non-restorable, its functionality is described only by failure-free metrics or indices.

The analysis begins usually with some computations regarding the failure structure on operational time intervals, such as:

a) relative frequency of failures:

$$\hat{f}(t_i) = \frac{r_i}{\sum_{i=1}^{m} r_i} \qquad (10)$$

as the ratio between the recorded number of failures occurred in the $i^{\underline{th}}$ interval and the total number of failures. Using these relative frequencies, one may compute:

b) cumulative relative frequency of failures:

$$\hat{F}(t_i) = \frac{1}{N}\sum_{j=1}^{i} r_i \qquad (11)$$

which exhibits the magnitude of failed items at the end of the $i^{\underline{th}}$ interval (of time); this indicator is an increasing function and becomes 1 for the last interval of the series .

c) relative frequency of operating items:

$$\hat{R}(t_i) = 1 - \hat{F}(t_i) = \frac{N_i}{N} \qquad (12)$$

which is in fact the complement of the above indicator and it is known as the so-called experimental reliability function since it shows the weight of still operating elements/items at the end of $i^{\underline{th}}$ time interval and which will fail during the next time intervals. There are necessary also same numerical elements to characterize the central tendency of failures and the spread around this value:

d) average number of failures on a given time interval:

$$\bar{f} = \frac{\sum_{i=1}^{m} r_i}{\sum_{i=1}^{m} t_i r_i} \quad \text{or} \quad \bar{r} = \frac{\sum_{i=1}^{m} t_i r_i}{\sum_{i=1}^{m} t_i} \qquad (13)$$

that is the ratio between the total number of failures $\left(N = \sum_{1}^{i} r_i \right)$ and the total failure-free operational time of all items of the sample.

e) MTBF - Mean Time Between Failures (or average durability, if the item is non-repairable):

$$\bar{t} = \frac{\sum_{i=1}^{m} t_i r_i}{\sum_{i=1}^{m} r_i} = \frac{\sum_{i=1}^{m} t_i r_i}{N} \qquad (14)$$

This MTBF is a straightforward indicator since its magnitude is directly related to the reliability degree of the underlying object: if the reliability is high, it is natural that the average operational span to be large. Lower reliability means a smaller MTBF.

f) Hazard (or failure) rate which is the ratio between the number of failed elements in a given time interval and the number of surviving ones in the same interval

$$Z(t) = \frac{r_i}{\Delta \cdot (n - r_i)} \qquad (15)$$

where $N_i = n - r_i$ is the number of operational item sat the starting point of the $i^{\underline{th}}$ interval.

g) Standard derivation of raw data, given by the expression:

$$\hat{\sigma} = \sqrt{Var(t)} = \sqrt{\frac{\sum (t_i - \bar{t})^2 \cdot r_i}{n}} \qquad (16)$$

This indicator is a measure of spread in the sample, taking as a “milestone” the sample mean, $\bar{t}$.

h) Coefficient of variation - defined as the ratio between standard deviation and the mean. This is an indicator of the relative spread of the raw data:

$$CV = \left(\frac{\hat{\sigma}}{\bar{x}} \right) \text{ or } CV\% = \left(\frac{Var(T)}{MTBF} \right) \times 100 \qquad (17)$$

Observations. It is important to notice that the indices presented above are the **sample** ones or **empirical** ones (as for instance $\bar{x}$, s or $\hat{\sigma}$, CV a.s.o.). They are estimators (from experimental data) of their theoretical correspondents - these later ones being calculated via a **statistical model** personalized by a certain probability density function f.

Some (most usual) statistical models will be presented in the next paragraph.

4 Statistical Models in Reliability

There exists a large variety of reliability - related problems as regards the operational behavior of a given complex item.

To find solutions for these problems one needs the so-called **system approach** which implies the use of mathematical models. The main critical factor in the construction of these models is **modeling the system** (item, product) **failures**.

It is a trivial statement to say that a system is in general a “reunion” of some several parts (components) and the system failure is directly related to their part failures.

It follows hence that the first task of the experimenter is the modeling of part failures. We encounter have two situations:

(i) if the part is non-repairable we have to take into account only the first failure: in this instance, the first failure is also the last one and the overused indicator MTBF becomes actual part durability (“Between Failures” - has no sense in this case since we deal only with first and “lethal” failure of the element).

(ii) if the item is repairable, we have to treat separately the first failure since the later ones will depend on the type of maintenance action taken in the field.

The mathematical framework appropriate for modeling the failure event is a distribution function F(t) defined as Prob {T < t} - this quantity leading to density, reliability and other related functions. As has been shown in table 1, it is sufficient to know one of these functions in order to construct completely a reliability model based on the concept of time-to-failure. The relationship (5) from §1 shows that a time-to-failure model can be relatively easily constructed if we know the p.d.f. - of the underlying variable T and its characterizing parameters.

Various data obtained from laboratory or workshop experiments and also field data could suggest - by using classical procedures of descriptive statistics - such as drawing histograms, computing several indicators (coefficient of variation, skewness, kurtosis a.s.o.) - a possible form of the p.d.f. associated to the failure phenomenon.

William Edwards DEMING (1900 - 1993) wrote in one of his books [4] that the object of taking data is to provide **a basis for action**. These data must be analyzed in the frame of a **statistical model** - otherwise we deal only with "a pure raw material" whose generating mechanisms are unknown to us. In a later work [5], he said that such a model has to be a **statistical distribution** personalized by a specific function which describes the behavior of the considered characteristic of interest. Most of these characteristic are measurable ones: "static" product quality characteristics such as hardness, strength, geometric features weights a.s.o. or dynamic ones as for instance durability or time-to-failure of a technical component or system.

Nowadays, the specific literature devoted to statistical distributions, especially to those modeling various measurable variables, is extremely large. We have now the second expanded edition of a four volume collection of the most used distributions (first volume in 1994 - see [14]) and some specialized monographs such as that of Patel and Read [17] about the classical normal (Gauss - Laplace) distribution and the pioneering work of Pollard and Rivoire [18] on the graphical procedures to validate the Weibull reliability model.

For the Romanian speaking readers we mention some useful monographs such as that of Isaic-Maniu [9] on "Weibull method" (which is in fact the first monograph on this distribution we talk about later), and Bârsan-Pipu et al [1] on the failure modeling in general.

From a statistical (theoretical) viewpoint a certain p.d.f. may be obtained in various ways - one of them being to ask for a so-called "system of frequency curves" - the best known being probably that of the British scientist Karl PEARSON (1857 - 1936), derived in 1895 which is a quite simple differential equation:

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = \frac{(x-a)\,f(x)}{b_0 + b_1 x + b_2 x^2} \tag{18}$$

where f(x) is the p.d.f. which will be deduced by some particularization of the parameters a, b_0, b_1 and b_2.

The above relationship may be written:

$$\frac{\mathrm{d}f(x)}{f(x)} = \frac{f'(x)}{f(x)} = \frac{x-a}{b_0 + b_1 x + b_2 x^2}\cdot \mathrm{d}x \tag{19}$$

or $$[\ln f(x)]' = \frac{x-a}{b_0 + b_1 x + b_2 x^2}\cdot \mathrm{d}x \tag{20}$$

which provides immediately (by integration):

$$f(x) = \exp\left\{\int \frac{x-a}{b_0 + b_1 x + b_2 x^2}\cdot \mathrm{d}x\right\} \tag{21}$$

If we take for instance $b_2 = 0$, $b_0 = a$, $b_1 = -1$ we get

$$f(\mathrm{x}) = \exp\left\{\int \frac{\mathrm{x}-a}{\mathrm{a}-\mathrm{x}}\,d\mathrm{x}\right\} = \exp\left\{-\int d\mathrm{x}\right\} = \exp(-\mathrm{x}) = e^{-\mathrm{x}} \quad (22)$$

which is in fact the expression given by (10) for $\theta = 1$, that is a peculiar case of the exponential law. Other details on p.d.f. (s) which could be obtained from Pearson`s equation (20) are given in [22].

In the reliability context we shall use formula (4) from § 1 which by taking the derivative, we obtain

$$\mathrm{f}(\mathrm{t}) = \mathrm{z}(\mathrm{t})\exp\left\{-\int_0^{\mathrm{t}} \mathrm{z}(\mathrm{u})\mathrm{du}\right\} \quad (23)$$

The **EXPONENTIAL statistical law** gives the distribution of time between independent events occurring at a constant rate. Similarly/equivalently, gives the probability distribution of life, assuming or presuming a constant hazard rate.

Therefore, it can describe the usage life of some entities - in particular when these are exposed to initial burn-in, and preventive maintenance eliminates some parts/components before their wear-out.

Due to this property of having a constant failure rate, the exponential model reliability is applicable mainly to those products (usually complex one) which have “genetically” a long life span.

It is important to notice that there are a lot of processes and situations when the assumption of a constant hazard rate is not realistic. For instance, in the domain of cutting and grinding tool durability, generally in metalworking, we are facing the irreversible wear-out phenomenon of the items involved: consequently, an increasing failure rate will be adequate.

One of the alternatives to the exponential model was that proposed in 1880 by John William Strutt (1842 - 1919) - better known as Lord RAYLEIGH (Nobel Prize in Physies, 1904) - as the distribution the amplitude resulting from the harmonic oscillations obviously, in those times, no reliability aspects were taken into consideration.

The **RAYLEIGH statistical law** has the p.d.f. and df defined as:

$$f(t;\theta) = \left(t/\theta^2\right)\cdot\exp\left(-t^2/2\theta^2\right); \quad F(t;\theta) = 1-\exp\left(-\mathrm{x}^2/2\theta^2\right) \quad (24)$$

with $t \ge 0,\ \ \theta > 0$ which provide a reliability function and a hazard rates as below:

$$R(t;\theta) = \cdot\exp\left(-t^2/2\theta^2\right); \quad z(t;\theta) = t/\theta^2 \quad (25)$$

The form of $\mathrm{z}(\mathrm{t};\theta)$ shows that it is linearly increasing $(\mathrm{t}_1 \ge \mathrm{t}_2$ implies $\mathrm{z}(\mathrm{t}_1) \le \mathrm{z}(\mathrm{t}_2))$ and hence, the model is useful to describe **the aging** or **wear-out** processes.

The main characteristics of Rayleigh variable (T) are:

Mean $= \mathrm{E}(\mathrm{T}) = \theta(\pi/2)^{1/2} \approx 1.253314\cdot\theta$;

Variance $= \mathrm{Var}(\mathrm{T}) = \theta^2(4-\pi)/2 \approx 0.429204\cdot\theta^2$;

Model value (mode) = θ;

Skewness coefficient ≈ 0.631111;

Kurtosis coefficient ≈ 3.245089.

Remark. A more complex statistical model in reliability has been proposed in 1951 by the Swedish military engineer, Waloddi WEIBULL (see [9]).

The WEIBULL statistical law is defined by the following p.d.f.:

$$\mathrm{f}(\mathrm{t};\gamma,\mathrm{k},\theta) = \left(\mathrm{k}/\theta^{\mathrm{k}}\right)\cdot(\mathrm{t}-\gamma)^{\mathrm{k}-1}\exp\left[-\left(\frac{\mathrm{t}-\gamma}{\theta}\right)^{\mathrm{k}}\right] \quad (26)$$

where $\mathrm{t} \ge \gamma > 0,\ \ \mathrm{k},\theta > 0$.

Here, γ is called a **location parameter**, θ **- a**

scale one and k - shape (or power) parameter. If $\gamma = 0$ we obtain a reduced model:

$$f(t;k,\theta) = \left(k/\theta^k\right)t^{k-1}\exp\left[-\left(\frac{t}{\theta}\right)^k\right] \quad (27)$$

with $t \geq 0, \quad \theta, k > 0$.
The form (26) or (27) provides a more realistic hazard rate function - namely:

$$z(t) = k(t-\gamma)^{k-1}/\theta^k, \quad t \leq \gamma, \theta, k > 0 \quad (28)$$

which depends on time (t) and due to the power parameter (k), it can exhibit an increasing, decreasing or a constant behavior. That is, (28) covers the whole range of the classical bath-tub hazard rate shape.
It is important to notice Weibull`s law includes as peculiar cases the exponential one (for k = 1) and the Rayleigh one (for k = 2).

As Gertsbahh remarks (see [6]) a logarithmic transformation of the Weibull random variable produces a random variable which belongs to the so-called location-scale family which has several very good features for statistical analysis.
Indeed, if T is Weibull variable, denoted $W(\lambda, \beta)$ where p.d.f. of T is:

$$f(t;\lambda,\beta) = \lambda^\beta \beta\, t^{s-1}\exp\left[-(\lambda t)^\beta\right] \quad (29)$$

then x = log T has the below p.d.f:

$$\Pr ob\{x \leq t\} = F(t) = 1 - \exp\left[\frac{t-a}{b}\right], \quad (30)$$

where $a = -\log\lambda$ and $b = 1/\beta$.
For the form (29), the failure rate is:

$$z(t) = \lambda^\beta \beta t^{\beta-1}, \quad t \geq 0, \quad \lambda, \beta > 0 \quad (31)$$

and has the following characteristics:
(i) for $\beta > 1$, $z(t)$ is increasing in t (that is we have an IFR type);
(ii)for $\beta = 1$, $z(t)$ is constant $(z(t) = \lambda)$ which means that we deal with the exponential law;
(iii)for $\beta < 1$, $z(t)$ is decreasing in t (that is we have a DFR type).
The mean - value and the variance of a Weibull variable are

$$E(T) = \lambda^{-1}\Gamma(1+1/\beta) \text{ and}$$
$$Var(T) = \lambda^{-2}\left[\Gamma(1+2/\beta) - \Gamma^2(1+1/s)\right] \quad (32)$$

where $\Gamma(\cdot)$ is the well-known Gamma function proposed by the famous Swiss mathematician Leonhard EULER (1707 - 1783): the so-called CLT (Central Limit Theorem) which states that the average value of n
The **NORMAL** or **GAUSS-LAPLACE law** represents - as it is known - a basic distribution in statistical theory and practice.

Many of its applications arise from an important theorem - that is observations/measurements approaches normal distribution - irrespective of the form of initial/original distribution of those measurements, under quite general conditions. The p.d.f. of a normal variable T is:

$$T: f(t;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(t-\mu)}{2\sigma^2}\right], \quad (33)$$
$$t \in R, \mu \in R, \sigma > 0$$

where μ and σ^2 have special significances, namely $\mu = E(T)$ - the theoretical mean-value and $\sigma^2 = Var(T)$ - the theoretical variance.

As Ireson states in [13], in reliability theory, the normal model in used to approximate the wear-out failure, since its hazard rate increases with time. One of the advantages of T as a normal variable lies in the fact that μ

and σ^2 can be immediately estimated by their sample correspondents, namely

$$\bar{t} = \frac{1}{n}\sum_{1}^{n} t_i \text{ and } s^2 = \frac{1}{n}\sum_{1}^{n}\left(t_i - \bar{t}\right)^2 \qquad (36)$$

If $\mu = 0$ and $\sigma = 1$ we are dealing with the nor med or standardized Gauss-Laplace variable for which p.d.f. and c.d.f. are:

$$f_0(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2} \quad \text{and}$$
$$F_0(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} \exp\left(-u^2/2\right) du \qquad (37)$$

All useful indicators presented in Table 3.

Table 3. Key indicators of reliability for the models shown

	The model		
	Exponential	**Weibull**	**Normal**
1	2	3	4
F(t)	1 - exp(-λt)	$1 - \exp\left[-\left(\frac{t-\gamma}{\theta}\right)^k\right]$	$\phi\left(\frac{t - m_0}{\sigma_0}\right)$
R(t)	exp(-λt)	$\exp\left[-\left(\frac{t-\gamma}{\theta}\right)^k\right]$	$\phi\left(\frac{m_0 - t}{\sigma_0}\right)$
z(t)	λ	$\frac{k(t-\gamma)^{k-1}}{\theta^k}$	$\frac{\frac{1}{\sigma_0\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{t-m_0}{\sigma_0}\right)^2\right]}{\phi\left(\frac{t-m_0}{\sigma_0}\right)}$
MTBF	$\frac{1}{\lambda}$	$\gamma + \theta \bullet V_k$	m_0
$\sqrt{Var(t)}$	$\frac{1}{\lambda}$	$\theta \bullet g_k$	σ_0

5 Final Comments

All elements on reliability presented in the above paragraphs concern the **hardware reliability** - that is time to failure, modeling at the component and system levels, evaluation/calculation of reliability indicators a.s.o.

As a consequence of its historical development, reliability theory and practice has its roots in this "hardware framework": basic definitions, terms, methods a.s.o. arose in this hardware context,

Software reliability has its own characteristics but many of terms and procedures may be translated (with an appropriate interpretation) from hardware reliability theory to the software one.

In spite of the fact that there is no unanimity amongst the authors dealing with this subject matter, software reliability theory is nowadays an important chapter of the general reliability theory (see [19]).

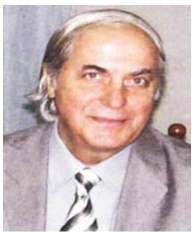
References

[1] N. Bârsan-Pipu, Al. Isaic-Maniu, V. Gh. Vodă, "Grubbs type statistics for detecting outliers in exponential population", *Ec. Comp. Econ. Cyb. Stud. Res.*, nr. 1-4, 1995.

[2] N. Bârsan-Pipu, Al. Isaic-Maniu, V.Gh. Vodă, *Defectarea. Modele statistice cu aplicații (The Failure. Statistical Models with Applications)*, Editura Economică, București, 1999.

[3] W.R. Blischke and D.N.P. Murty, *Reliability. Modeling, Prediction and*

Optimization, John Wiley and Sons Inc., New York, 2000, pg. 3.
[4] W.E. Deming, *Statistical Adjustment of Data*, Dover Publications Inc., New York, 1964.
[5] W.E. Deming, "On a probability as a basis for action", *The American Statistician*, vol. 29, no. 4, pp. 146 – 152, 1975.
[6] I. Gertsbakh, *Reliability Theory With Applications to Preventive Maintenance*, SPRINGER - Verlag, Berlin, 2000, pg. 33.
[7] G.J. Hahn and S.S. Shapiro, *Statistical Models in Engineering*, John Wiley and Sons, New York, 1967
[8] K. Yang and K.C. Kapur, *Customer driven reliability: integration of QFD and robust design.* Proceeding of the Annual Reliability and Maintainability Symposium, page 339 – 345, 1997
[9] Al. Isaic-Maniu, *Metoda Weibull. Aplicații (The Weibull Method. Applications)*, Editura ACADEMIEI Republicii Socialiste România, 1983.
[10] Al. Isaic-Maniu, V.Gh. Vodă, *Fiabilitatea – şansă şi risc*, *(Reliability - Chance and Risk)*, Editura Tehnică, Bucureşti, 1986.
[11] Al. Isaic-Maniu, V.Gh. Vodă, *Proiectarea statistică a experimentelor. Fundamente şi studii de caz (Statistical Design of Experiments and Case studies)*, Editura Economică, Bucureşti, 2006.
[12] Al. Isaic-Maniu, V.Gh. Vodă, *Manualul calității*, Editura Economică, Bucureşti, 1997.
[13] W.G. Ireson, *Reliability Handbook*, Mcgraw-Hill book Co., New York, 1997, pg. 4 – 30.
[14] N.L. Johnson, S. Kotz, and N. Balakrishan, *Distributions in Statistics, Continuous Univariate Distributions*, vol. 1 (Second Edition, John Wiley and Sons Inc., New York, 1997
[15] K. Krishnamoorthy, *Handbook of Statistical Distributions with Applications*, Chapman & Hall/CRC-Taylor & Francis Group, 2006
[16] M. Modarres, Ma. Kaminskiy, V. Krivtsov, *Reliability Engineering and Risk Analysis: A Practical Guide*, Second Edition, CRC Press, 2009.
[17] J.K. Patel, and C.B. Reed, *Handbook of the Normal Distribution* (Second Edition, Revised and Expanded), Marcel DEKKEE Inc., New York, 1996.
[18] A. Pollard, and C. Rivoire, C. *Fiabilité et statistique previsionelle. Méthode de Weibull*, EYROLLES, Paris, 1971.
[19] N. Singpurwalla, and S.P. Wilson, "Models for assessing reliability of computer software", *Naval Research Reviews*, vol. 44/45, pg. 21 – 29, 1992.
[20] U. Wiener, Al. Isaic-Maniu, V. Vodă, *Aplicații ale rețelelor probabilistice în tehnică*, *(Engineering Applications of Probability Papers)*, Editura Tehnică, Bucureşti, 1983.
[21] C.E. Witherell, *Mechanical Failure Avoidance*, Mc. Graw-Hill Book Co., New York, 1994, pg. 11.
[22] Al. Isaic-Maniu, *Dicționar de Statistică Generală*, Editura Economică, Bucureşti, 2003, pp. 55 – 65.



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