MECHANISM TO DRAW MACLAURIN TRISECTRIX

Conf.dr.ing. Mirela **CHERCIU,** University of Craiova, mirela_cherciu2005@uyahoo.com
Prof.dr.ing. Iulian **POPESCU ,** University of Craiova

Abstract: *It is used a geometrical method for generating Maclaurin trisectrix and based on it , the synthesis of a mechanism that can draw it, is made. The structure of the found mechanism is R-R-TRT type, having two driving elements with correlated movements. This mechanism is analysed and the desired curve is obtained just for certain dimensions of the mechanism. The mechanism's movement is studied based on some diagrams and different outputs are obtained for certain initial dimensions of the mechanism's.*

Keywords: Maclaurin trisectrix, 2D curves, mechanisms generating curves

1. INTRODUCTION

Many classic curves obtained by different mathematicians over time, are studied in more detail, including the Internet. Thus, in [2] are given the most popular known 2D curve, indicating their equations in polar and / or Cartesian coordinates, and some aspects regarding their geometric properties. In sec. XIX-XX were designed mechanisms generating such curves as trajectories of some points. Many of these mechanisms are given in [1]. In [3] are also given some mechanisms generating curves with aesthetic qualities. Below it is studied an original mechanism that draws Maclaurin's trisectrix, starting from the geometric considerations given in [2].

2. INITIAL DATA

Maclaurin's trisectrix, he obtained in 1742 (fig.1) [2] has the polar equation:

$$\rho = 2a\frac{\sin 3\theta}{\sin 2\theta} \tag{1}$$


Fig.1 Maclaurin Trisectrix [2]


Fig.2 Geometric data [2]

The equation (1) written in Cartesian coordinates becomes:

$$x = a\frac{3-t^2}{1+t^2} \qquad y = tx \tag{2}$$

The curve is described by the point of intersection of two lines that rotate around a fixed point each, one with angular velocity 3 times higher than the other (fig.2).

3. SINTHESIS OF THE GENERATOR MECHANISM

Based on geometrical construction of fig. 2, we built the mechanism in fig. 3 as follows:
- Lines which are AB and CB rotate around the points A and C;
- The lengths of lines are variable, so sliders 2 and 3 were provided;
- The angle between the lines is variable, therefore between sliders it was provided a rotational joint(in B);
- The straight line AB has the angle φ as generalized coordinate and CB ,respectively, the angle 3φ;
- The correlation of the two angles can be either through a gear or electromagnetic bases (rotations of the driving motors).

4. THE MECHANISM ANALYSIS

Structurally, the mechanism consists (fig. 4) of two driving elements AB and CB and one dyad type TRT ,BBB, ie R-R-TRT.

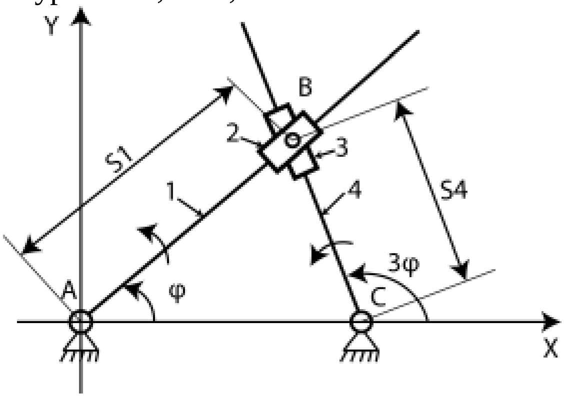


Fig. 3. The achieved mechanism

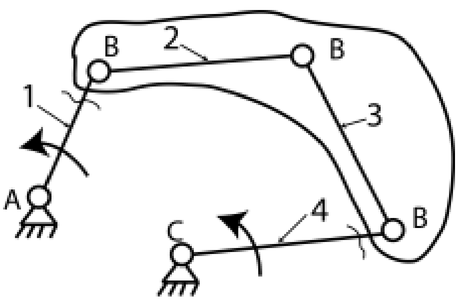


Fig. 4. The structural scheme

The following equations are written:

$$x_B = S_1\cos\varphi = x_C + S_4\cos 3\varphi \tag{3}$$

$$y_B = S_1\sin\varphi = S_4\sin 3\varphi \tag{4}$$

$$\tan\varphi = \frac{S_4\sin 3\varphi}{x_C + S_4\cos 3\varphi} \tag{5}$$

$$S_4 = \frac{-x_C\tan\varphi}{\cos 3\varphi\tan\varphi - \sin 3\varphi} \tag{6}$$

5. THE OBTAINED RESULTS

From fig. 3 and the above equations it’s noticed that the only constant is XC dimension. It was chosen the value XC = 50 and it was obtained the seeking curve (fig. 5), therefore the intended mechanism. In fig. 6 are shown the successive positions of the generator mechanism.

In fig. 7 are shown the sliders variations S1 (extreme left, top curve) and S4 (denoted by S2). Jumps to infinity for $\varphi = 90$ and 270 degrees are noticed. The program limited the jumps to 300 mm in order to obtain the diagram. Outer branches of the curve tend to infinity, as shown in fig. 5

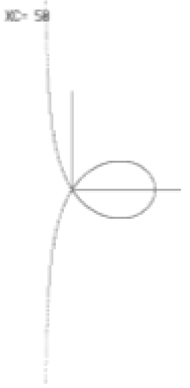


Fig. 5.The desired curve

Fig. 6. Successive positions

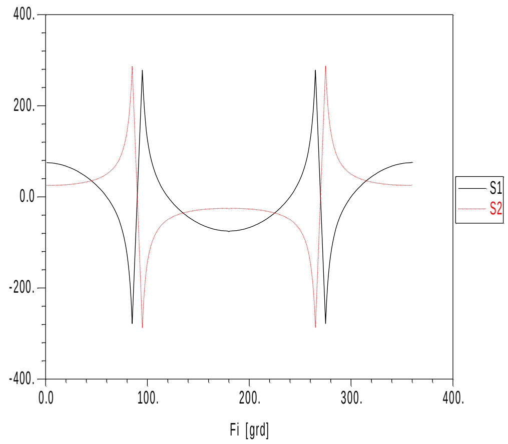


Fig. 7. Curves S1 and S4

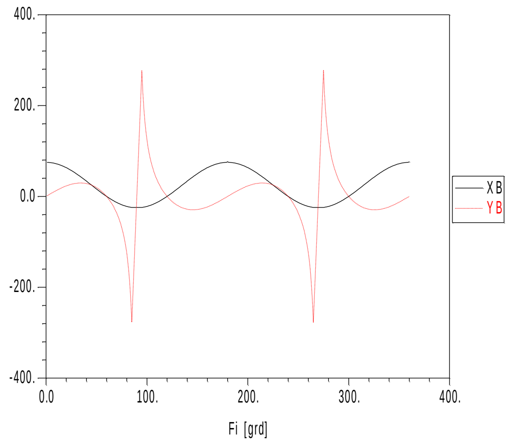


Fig. 8. Curves XB and YB

Variations of the tracer point (B) coordinates are given in fig. 8. It is noted that XB variation is a cosine type and YB should be sine type if the mentioned jumps to infinity shouldn’t exist. If the value of XC loops, the curve generated is of the same type, but with different dimensions. In fig. 9 ... 14 there is increasing loop size and distance from the left branch orderly increase of XC (XC's values are indicated on the figures). For XC = - 50, curve (fig. 14) is symmetrical with that of fig. 5. In this case the variation curve of XB (fig. 15) is also negative and YB curve has the same jumps as for XC = 50.s

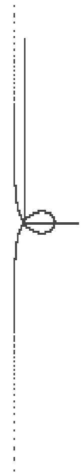


Fig. 9. XC=10

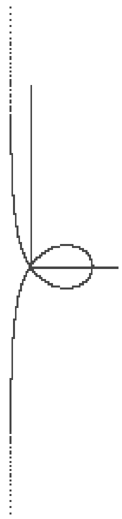


Fig. 10. XC=20

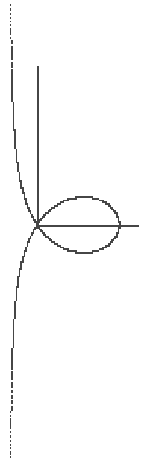


Fig. 11. XC=30

Fig. 12. XC=80

Fig. 13. XC=120

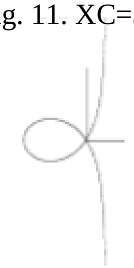


Fig. 14. XC= - 50

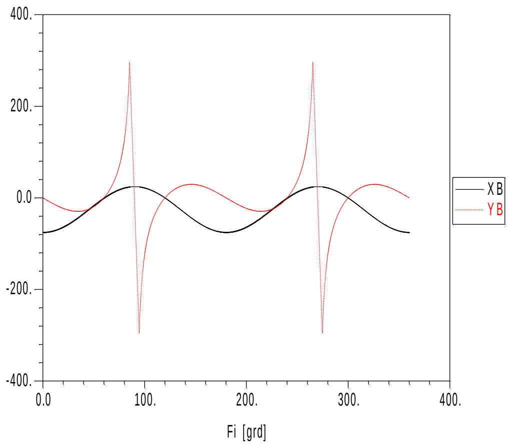


Fig. 15. Curves XB and YB

6. OTHER POSSIBILITIES OF THE MECHANISM

If point C of the mechanism in fig. 3, is not on the abscissa axis, so $YC \neq 0$, then the mechanism has other possibilities. Thus, in Fig. 16 ... 19 are shown curves plotted at different values of YC. The following are found:

- There are two branches of the curve: one loop and the other similar to a branch of a hyperbole;
- By increasing YC, the loop size decreases;
- For YC <0 is changing positions of the two branches, that remain symmetrical with the positive values of YC.

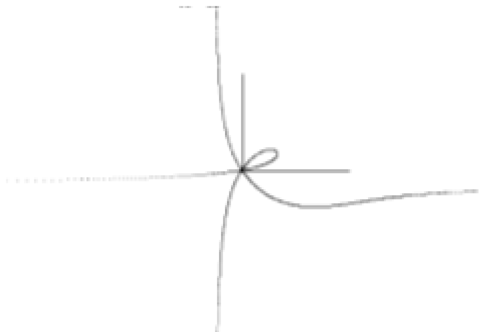


Fig. 16. YC=10

Fig. 17. YC=50

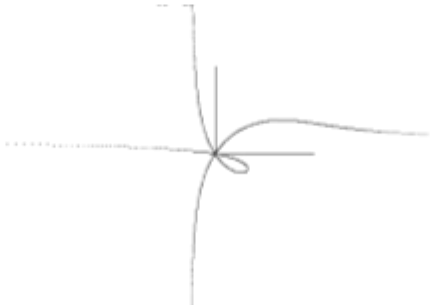


Fig. 18. YC=-10

Fig. 19. YC=-80

In fig. 20 are shown the coordinates variations of tracer point for the YC = 10. This time both XC and YC are areas that tend to infinity. Thus, these trends occur for XC = 0, 180, 360, and for the YC = 90, 270 degrees**.** In fig. 21 are represented the mechanism positions for YC = -80. Areas increases to infinity are noticed.

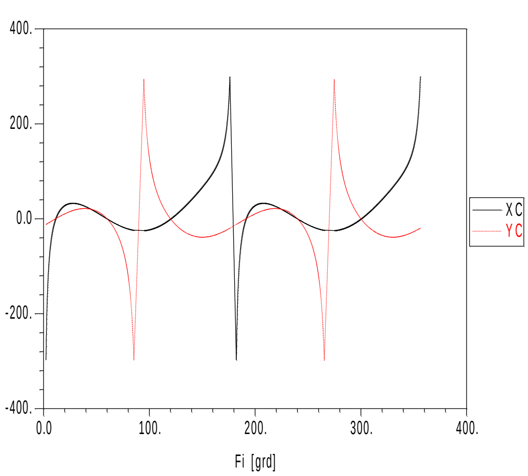


Fig. 20. Curves for XC and YC

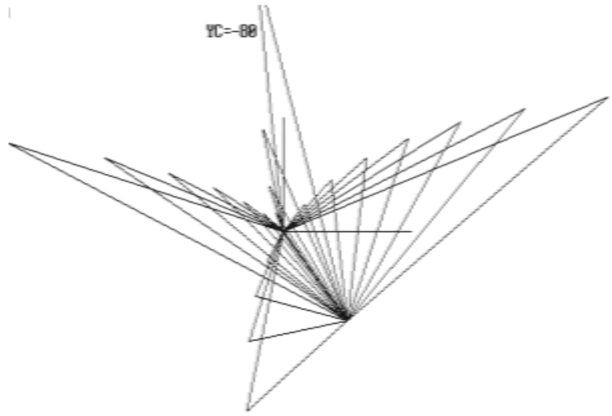


Fig. 21. The positions of the new mechanism

In fig. 22, 23 are shown curves obtained at different values of YC, as follows: fig. 22 (YC = 50), fig. 23 (YC = - 50).

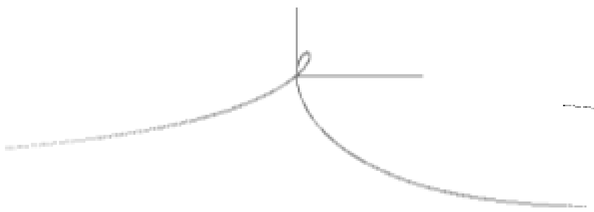


Fig. 22 YC=50

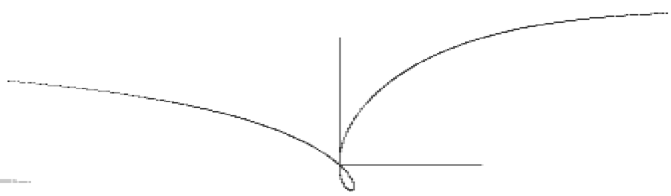
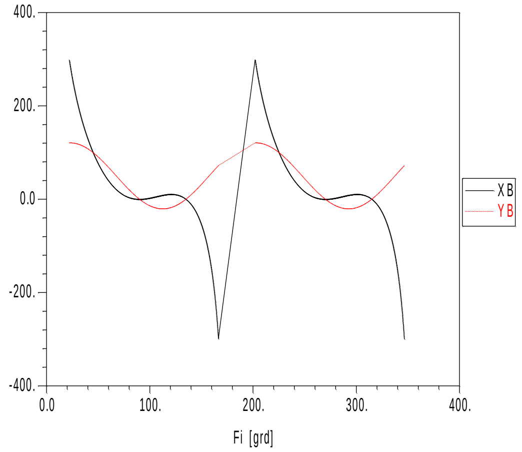


Fig. 23 YC= - 50

In fig. 24 are shown the curves of variation of tracer point coordinates YC = - 50, observing that appear increases to infinity just for XB curve, which has a break around φ = 180 degrees, and jumps to infinity at φ = 0 and 360 degrees. These findings can be seen in successive positions of the mechanism of fig. 25, where it is noted the area where the mechanism does not work.



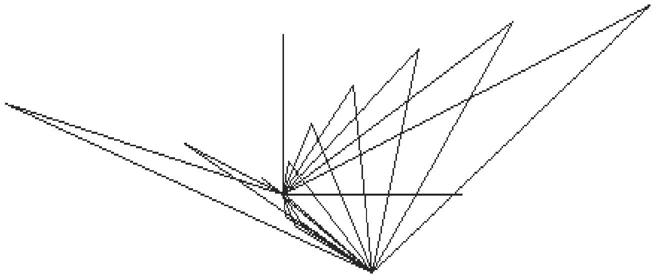


Fig. 24 Curves for XB and YB

Fig. 25 Successive positions

7. CONCLUSIONS

- It started from the curve described in [2] and was done the synthesis of mechanism that draw it;
- The found mechanism is R-R-TRT type, so it has two driving elements with correlated movements;
- The mechanism draws the desired curve;
- The analysis of successive positions and diagrams found that the mechanism is working properly;
- Making some changes in the mechanism, resulted other curves, this time with 4 branches;

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