

# Intermittent heating of the solar corona by MHD turbulence

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**Abstract.** As the dissipation mechanisms considered for the heating of the solar corona would be sufficiently efficient only in the presence of small scales, turbulence is thought to be a key player in the coronal heating processes: it allows indeed to transfer energy from the large scales to these small scales. While Direct numerical simulations which have been performed to investigate the properties of magnetohydrodynamic turbulence in the corona have provided interesting results, they are limited to small Reynolds numbers. We present here a model of coronal loop turbulence involving shell-models and Alfvén waves propagation, allowing the much faster computation of spectra and turbulence statistics at higher Reynolds numbers. We also present first results of the forward-modelling of spectroscopic observables in the UV.

## 1 Introduction

Although the temperature of the solar corona has been known for more than sixty years to be as much as a million Kelvin (e.g. Edlén, 1943), the precise physical mechanisms allowing its material to reach such high temperatures are still not known precisely. At first view it seems indeed unphysical that the temperature rises when one goes further away from the surface, while the Sun is the primary source of energy in its corona. Several questions arise (Klimchuk, 2006), mainly:

1. What power is necessary to heat the corona to the observed temperatures?
2. Where does the energy come from, how is it transported to the corona, and, if necessary, how is it dissipated there?

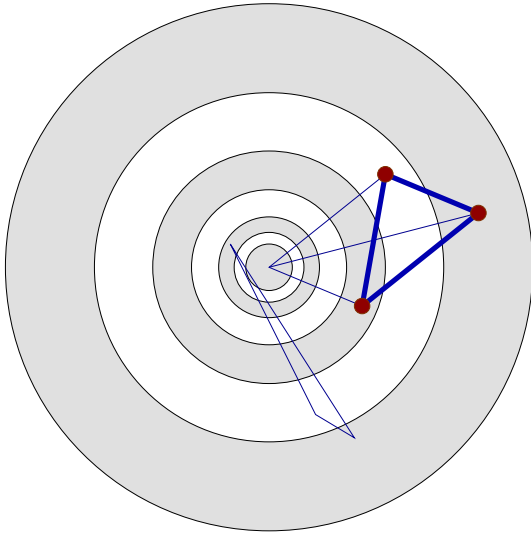
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One can also try to understand whether all structures in the solar atmosphere (loops, active regions, the quiet Sun. . .) are heated by the same mechanisms, and how the coronae of other stars are heated.

According to observations (Withbroe and Noyes, 1977), the power necessary to heat the corona (i.e. the energy losses in the corona) would be between  $3 \times 10^2 \text{ W/m}^2$  in the quiet Sun and  $10^4 \text{ W/m}^2$  in active regions. This answers to the first question, even though observations with new instruments may lead to slightly revise these quantities.

Over the years, different heating mechanisms have been proposed, generally relying on either the dissipation in the corona of waves produced at the photospheric level or the reconnection of current sheets formed by the tangling of the magnetic field lines by the photospheric motions (see e.g. Zirker, 1993; Klimchuk, 2006, for a review). However, all these mechanisms, if considered at the large, observable scales, are not efficient enough to provide the necessary heating power. The solution would then be to consider small scales, at which most dissipation mechanisms are more efficient (for example, the efficiency of classical hydrodynamic viscosity scales as the inverse square of the scale of the structures). The problem is now to produce these small-scale structures, and this could be done thanks to turbulence.

The solar corona seems indeed to be in a very turbulent state, as a rough estimation of the Reynolds number gives  $10^{12}$  to  $10^{14}$ , based on the observed velocities and large structures, and a kinematic viscosity of the order of  $1 \text{ m}^2/\text{s}$ . This is also supported by observations, which have revealed dynamic structures over a wide range of scales, in both space and time. Furthermore turbulence is expected in the corona because it has been detected and measured in media like the photosphere (see Espagnet et al. 1993 for the velocity and Abramenko et al. 2002 for the magnetic field), the transition region (Buchlin et al., 2006) and the solar wind (see Roberts and Goldstein, 1991; Tu and Marsch, 1995, for a review).



**Fig. 1.** Layout of a shell model, in Fourier space. The triad nonlinear interaction represented by the thin triangle is not taken into account because it is not local.

These ideas have led to the development of models including the effects of magnetohydrodynamic (MHD) turbulence in the coronal heating (Heyvaerts and Priest, 1992; Einaudi et al., 1996). In this paper we present a model of MHD turbulence and nonlinear Alfvén wave propagation in a coronal loop, whose results lead us to think that turbulence is indeed a key player in the coronal heating processes.

## 2 Model for turbulent heating in a coronal loop

### 2.1 Alternatives to Direct Numerical Simulations

Although Direct Numerical Simulations (DNS) of turbulence have been performed in the framework of coronal heating, they have strong limitations due to the complexity of the nonlinear physics involved. The main problem is related to the fact that they are computationally much too expensive: in the space domain, their resolution (at most  $1024^3$  for MHD) is too low compared to what would be needed to simulate a system like the corona with very high Reynolds numbers, and in the time domain, they are too slow to produce long time series suitable for a statistical analysis. An improvement of DNS such as adaptive mesh refinement is not always sufficient because turbulence is not intermittent enough (the most refined grid would be necessary on a large part of the domain). Simulations in 1D (e.g. Suzuki and Inutsuka, 2005) or 2D have their own limitations on the geometry of the structures they can simulate.

For these reasons alternatives have to be considered. A first alternative is to use *Cellular Automata* (CA), i.e., discrete lattice models that evolve according to simple rules involving neighboring cells of the lattice: it is assumed that

MHD in the solar corona has, as many complex systems, a Self-Organized Criticality (SOC) behavior (Bak et al., 1987), and that it can then be modelled by a CA representing a classical SOC system, like a sandpile (Lu and Hamilton, 1991). These models reproduce the slow buildup of the energy in the system under the action of exterior perturbations, until a critical state is reached, and then another perturbation can lead to a sudden energy release. This energy release can be a significant portion of the total energy in the system, depending on the individual perturbations and the critical state that had been reached; the distributions of these releases are then distributed over a very wide range of scales, and their power-law distributions can be identified to those of the energy releases in the solar corona.

In the context of the solar corona, these models have been progressively developed towards a better conformance to the original physical equations (Islaker et al., 2000; Vlahos et al., 1995), including also the propagation of Alfvén waves in a coronal loop (Buchlin et al., 2003). However some of the complexity of the MHD equations is still missed, and other alternatives, like the shell models that we are going to describe now, need to be considered.

### 2.2 Description of the shell model

#### 2.2.1 “Classical” shell models

Shell models have first been developed in order to study the nonlinear behavior of hydrodynamics (Gledzer, 1973; Yamada and Ohkitani, 1987), and they have been extended to incompressible MHD (Gloaguen et al., 1985; Biskamp, 1994; Giuliani and Carbone, 1998). In these models the Fourier space for the fields is divided into concentric shells of radii  $k_n = k_0 \lambda^n$ , with  $\lambda$  the logarithmic spacing (usually  $\lambda=2$ ) between shells (see Fig. 1). In each shell, complex scalars  $u_n$  and  $b_n$  represent the rms amplitude of the velocity and magnetic fields respectively. The Fourier transform of the nonlinear terms of the MHD equations formally gives terms which are convolutions of the fields; introducing the Elsässer-like variables  $Z_n^\pm = u_n \pm b_n$  this gives

$$(d_t Z_n^\pm)_{\text{NL}} = i k_n \left( \sum_{l,m} \alpha_{l,m} Z_l^\pm Z_m^\mp \right)^* \quad (1)$$

where  $\alpha_{l,m}$  are coefficients that still need to be determined.

The next assumption is that the nonlinear interactions are local in Fourier space: only triads<sup>1</sup> of consecutive shells can interact, i.e.,  $\alpha_{l,m} = 0$  if  $\{l, m, n\}$  are not consecutive numbers (in any order). Locality is actually not entirely verified in MHD: DNS have indeed recently allowed to study to what extent these interactions are non-local (Alexakis et al., 2005); however, with  $\lambda=2$  (i.e., direct nonlinear interactions between wavenumbers separated by a factor  $\lambda^2=4$ ), such

<sup>1</sup>Sets of three shells.

non-locality could be sufficiently low for the shell-model approach to remain valid.

Finally, the conservation of invariants of MHD, namely the energy, cross helicity and magnetic helicity in three dimensions (or the anastrophy in two dimensions) gives a unique determination of the coefficients (Giuliani and Carbone 1998; see Buchlin 2004 for a complete derivation):

$$\alpha_{n+1,n+2} = -\lambda\alpha_{n-1,n+1} = (\delta + \delta_M)/2 \quad (2)$$

$$\alpha_{n+2,n+1} = -\lambda^2\alpha_{n-2,n-1} = (2 - \delta - \delta_M)/2 \quad (3)$$

$$\alpha_{n+1,n-1} = -\lambda\alpha_{n-1,n-2} = (\delta_M - \delta)/2\lambda \quad (4)$$

where  $\delta$  and  $\delta_M$  are coefficients depending on  $\lambda$  and on the dimensionality of the model (2-D or 3-D).

Shell models have also been recently extended to Hall-MHD (Hori et al., 2005; Galtier and Buchlin, 2007), but this is out of the scope of this paper.

### 2.2.2 Shell models with Alfvén waves propagation

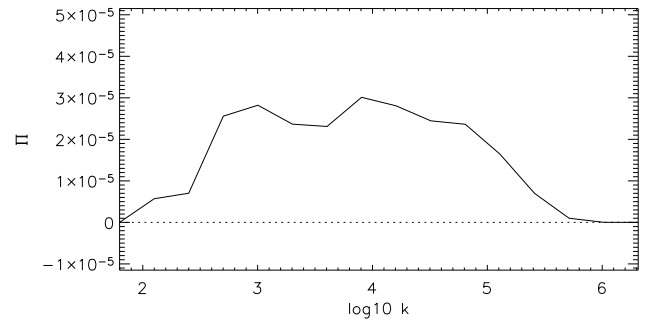
In order to model a coronal loop (as well as other anisotropic systems with a large-scale magnetic field  $\mathbf{B}_\parallel$ ), one needs also to take into account the propagation of Alfvén waves along  $\mathbf{B}_\parallel$ , with an Alfvén speed  $b_\parallel = B_\parallel / \sqrt{\mu_0 \rho}$  that depends on the position  $z$  along the loop of length  $L$  and width  $\ell = 2\pi/k_0$  (the aspect ratio of the loop is  $Lk_0/2\pi$ ). This is done by introducing a dependence of the variables  $Z_n^\pm$  as a function of  $z$ , and by using, for each shell index  $n$ , the equation of linear propagation of Alfvén waves in a stratified atmosphere (Velli, 1993). This includes the reflection along gradients of  $b_\parallel$  and the amplitude variations ensuring the conservation of the energy flux, and, applied to the shell model variables, it gives (Buchlin and Velli, 2007)<sup>2</sup>:

$$(\partial_t Z_n^\pm)_{\text{wave}} = \mp b_\parallel \partial_z Z_n^\pm \mp \frac{1}{4} Z_n^\pm \partial_z (\ln \rho) \mp \frac{1}{2} Z_n^\mp \partial_z b_\parallel \quad (5)$$

Therefore, the final equation for the time derivative of  $Z_n^\pm(z)$  in the model is the sum of the nonlinear terms expressed by Eq. (1) (with coefficients from Eq. (2–4)), of the linear wave propagation expressed by Eq. (5), and of dissipation terms that can take the form of a viscosity  $(\partial_t + \nu k_n^2)u_n = 0$  and of a magnetic diffusivity  $(\partial_t + \eta k_n^2)b_n = 0$ .

The only input of energy in the model is obtained thanks to the boundary conditions at the footpoints of the loop: a slowly varying  $u_n$  is imposed there at the large scales (small  $n$ ), in a way compatible with the observed motions of the photospheric granulation. This translates into a Poynting flux, which is not imposed a priori as it depends both on the boundary conditions and on the waves arriving at the boundaries.

<sup>2</sup>Nigro et al. (2004) also include the Alfvén wave propagation, but with no dependence of  $b_\parallel$  as a function of  $z$ .



**Fig. 2.** Cross-scale energy flux  $\Pi_n$  in the shell models (positive is towards the small scales) as a function of the perpendicular wavenumber.

## 3 Results on turbulence and coronal heating

### 3.1 Spectra of turbulence

As in classical shell models, spectra of the perpendicular fluctuations develop to power laws over a wide inertial range of perpendicular wavenumbers (more than 4 decades, much more than what can be attained by DNS), and with a slope close to  $-5/3$ . In each cross-section of the coronal loop (one individual shell model), the energy transfer rate towards small scales is governed by the following equation, which is obtained by computing the time-derivative of the energy contained in the shells  $m \geq n$  due to the action of the nonlinear terms (Eq. 1) only:

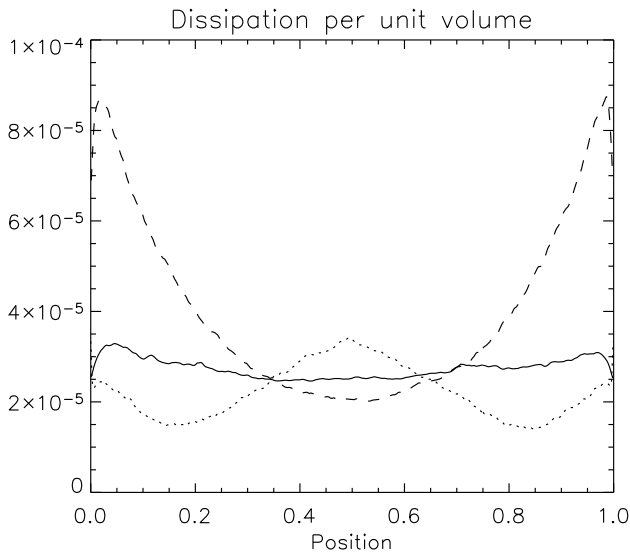
$$\begin{aligned} \Pi_n = -\frac{k_n}{4\lambda^2} \Im \sum_{s=\pm 1} & (\delta_M - \delta) Z_{n-2}^{-s} Z_{n-1}^s Z_n^s \\ & + (2 - \delta - \delta_M) Z_{n-2}^s Z_{n-1}^{-s} Z_n^s \\ & + \lambda ((\delta + \delta_M) Z_{n-1}^s Z_n^s Z_{n+1}^{-s} \\ & + (2 - \delta - \delta_M) Z_{n-1}^s Z_n^{-s} Z_{n+1}^s) \end{aligned} \quad (6)$$

This allows to control that the energy flux in the inertial range is towards the small scales and almost independent on the scale, as can be seen in Fig. 2.

In the parallel direction, no nonlinear cascade is explicitly included in the model equations. However, the propagation of the perpendicular fluctuations produced by the perpendicular cascade also produces a parallel spectrum of the perpendicular fluctuations. We have found that the overall  $(k_\perp, k_\parallel)$  spectrum is anisotropic (Buchlin and Velli, 2007), in a way that could be compatible with the assumption of a “critical balance” between the Alfvén propagation and the perpendicular nonlinear transfer (Goldreich and Sridhar, 1995). The detailed analysis of this anisotropic spectrum will be done in a future work.

### 3.2 Properties of the heating

Another important output of the model is the heating function (the power of energy dissipation per unit volume). First,



**Fig. 3.** Profiles of heating rate per unit volume in a 10 Mm-long model loop obtained by the shell model, in the case of: a loop stratified in density (plain line); a loop stratified in both density and magnetic field (dashed line); a loop with a temperature-dependence of the dissipation coefficients  $\nu$  and  $\eta$  (dotted line). The unit of heating is of the order of  $0.1 \text{ W/m}^3$ , and the unit of length is the length of the loop.

the average heating in the model is sufficient to heat the coronal loop if we compare the average heating per unit area of the cross-section of the loop ( $10^2$  to  $10^3 \text{ W/m}^2$  and higher for smaller loops) to the requirements of Withbroe and Noyes (1977). This is confirmed by further developments of the model, involving the coupling of this heating model (nicknamed SHELLATM) with the HYDRAD hydrodynamic model of Bradshaw and Mason (2003), as seen in Sect. 4, and it is quite remarkable as the power input in the model is not imposed a priori (only the amplitude of photospheric motions is).

Statistical properties of the heating can then be computed. We have shown in Buchlin and Velli (2007) that the time series of the dissipated power in the model (the integral along the loop of the heating function) is intermittent: the distributions of the increments at different time scales of the time series depend on the scale. Furthermore, the energies, durations and waiting-times of “events” of dissipation extracted from the time series of heating are distributed as power laws; however, as we have shown in Buchlin et al. (2005) that such distributions depend on the definition used to extract the events, this result should be taken with care.

Taking a loop length  $L=10 \text{ Mm}$  and width  $1 \text{ Mm}$ , an Alfvén speed of  $10 \text{ Mm/s}$  and a mass density of  $10^{-12} \text{ kg/m}^3$ , the energy dissipated in each event is up to  $10^{17} \text{ J}$ , corresponding to small nanoflares. Larger flares are likely to be produced in loops with different parameters, with different mechanisms, or they can be seen as an unresolved superposition of small nanoflares, as the repetition time of

the small dissipation events in the model is high (of the order of one second) while the conduction and cooling times (Cargill, 1993) are 180 and 4000 s respectively at a temperature of 1 MK for this loop.

### 3.3 Profiles of heating along a loop

The stratification of the atmosphere included in this model can be used to study the dependence of the heating as a function of position along a coronal loop, which is controversial. As seen in Fig. 3, different profiles are obtained, depending on the physical parameters of the loop (Buchlin et al., 2007).

The profile of mass density alone (with a uniform profile of magnetic field  $B_0$ ) has little influence on the heating per unit volume (in Fig. 3 the loop is 30 times denser at the foot-points than at the loop top), which remains quasi-uniform, even though the Alfvén speed profile is then non-uniform. However, when a non-uniform profile of magnetic field is included (the flux tube is expanding and is wider in the corona; in Fig. 3 the apex width is 1.5 times the footpoint width), the heating per unit volume scales roughly as  $B_{\parallel}^2(z)$  (Gudiksen and Nordlund, 2005), but this effect is quite small in practice, especially in small loops, as the magnetic field in coronal loops only weakly depends on the position (Klimchuk et al., 1992).

Furthermore, contrary to DNS, this model can take into account the large non-uniformity in viscosity and magnetic diffusivity than can be expected from the non-uniformity of the temperature profile, despite the dramatic variation of the magnetic Prandtl number between the footpoints and the apex of the loop. This non-uniformity of the diffusion coefficients has an effect on the heating profile, as can be seen in Fig. 3 (dotted line), so it needs to be taken into account by the models even though the absolute values of these coefficients have no effect on the total turbulent dissipation power at the end of the nonlinear cascade (Gudiksen and Nordlund, 2005).

## 4 Towards a better comparison with observations

As inverting a physical quantity like the energy dissipated during an event is difficult (Parnell, 2004) and as the statistical properties of events depend on the way we define events (Buchlin et al., 2005), it would be advisable to perform *forward-modelling* of observed quantities instead of inversions (Patsourakos et al., 2004; Klimchuk, 2006).

To achieve this goal, we must first compute the plasma response to the heating, and this can for example be done, as already mentioned in Sect. 3.2, by feeding the HYDRAD hydrodynamic model of Bradshaw and Mason (2003) with the heating produced by the SHELLATM model. As can be seen in Fig. 4 (top), a corona forms, with temperatures of several hundred thousands Kelvin (temperatures  $>1.5 \text{ MK}$  are attained intermittently) and low density. More dissipation,

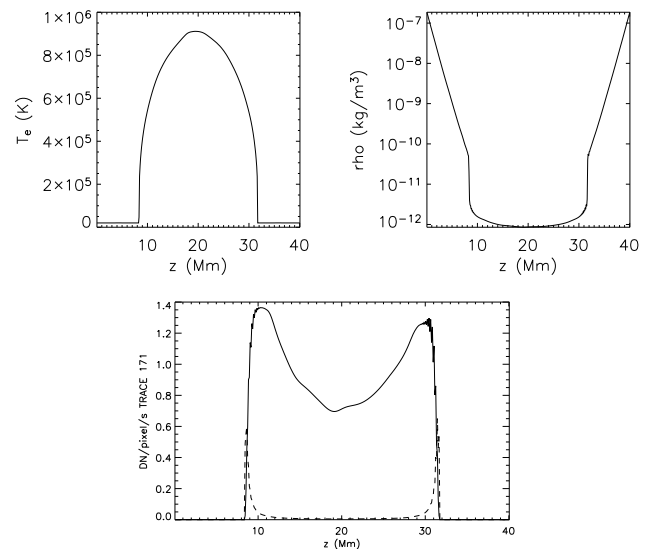
and then higher temperatures may be obtained with a smaller aspect ratio (a thinner loop), a higher density, a higher Alfvén speed or higher footpoint velocities (see Eq. (17) of Buchlin and Velli 2007). There is a sharp transition region between the corona and the chromosphere/photosphere.

Then we compute the line emissions, either using the ability of HYDRAD to compute non-equilibrium ionization states and radiation (with atomic data from CHIANTI, Dere et al., 1997), or by assuming ionization equilibrium and using the CHIANTI atomic model directly. In Fig. 4 (bottom) we show the resulting profile of emission in the TRACE 171Å passband in the case of this relatively cool loop.

These computations allow to perform a direct comparison between observations and the results of models of coronal heating, however such comparisons can yield scientific return on coronal heating problem only if they allow to discriminate between heating mechanisms. For this reason, observable *signatures* of the heating mechanisms have to be determined: the observable variables produced by each of the possible models of coronal heating should be computed, and a good signature would then be a specific feature that would allow to determine non-ambiguously the heating model (hence the physical processes) that was actually used in the computation. If such a signature is found in the analysis of observations, it would allow to determine the heating mechanisms at play in the solar corona.

## 5 Conclusions

We have presented a model for a solar coronal loop based on MHD and including the nonlinear transfer of energy across scales and the propagation of Alfvén waves in a stratified atmosphere. It allows to perform computations of turbulent fields and their nonlinear evolution for much higher Reynolds numbers ( $\approx 10^6$ ) than with direct numerical simulations, and the long time series obtained by this model make it suitable for a statistical analysis in the framework of turbulence. It has furthermore been coupled to a hydrodynamic model in order to compute the plasma response to the heating and the forward-modelled observable variables coming out from the emission of radiation by the plasma. Further work include the determination of suitable signatures of specific heating mechanisms, and this would allow determining the heating mechanisms at play in the real solar corona from the analysis of observational data such as data from the new Hinode and STEREO satellites. The STEREO/SECCHI/EUVI instrument can for example be used to determine UV loops geometry; these loops would be modelled using different types of heating mechanisms (including by the turbulence model presented here); observable quantities would be computed for these different heating mechanisms, and compared to Hinode/EIS spectroscopic observations. This comparison would give an indication on which of the modelled heating mechanisms could be correct.



**Fig. 4.** Top: temperature and density profiles computed by HYDRAD with heating from the SHELLATM model in a 40 Mm-long model loop (the coronal part of the loop is actually only 24 Mm long in this case), at a given time of the simulation. Bottom: the corresponding profiles of the counts per seconds per pixel in this loop as it would be observed by TRACE in the 171Å passband (the plain and dashed lines represent the contributions of the iron and oxygen lines respectively).

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