Modeling, Identification and Control, Vol. 32, No. 2, 2011, pp. 57–77, ISSN 1890–1328

# Semi-decentralized Strategies in Structural Vibration Control

F. Palacios-Quiñonero [1] J.M. Rossell [1] H.R. Karimi [2]

[1] *CoDAlab. Department of Applied Mathematics III, Universitat Politècnica de Catalunya (UPC). Av. Bases de Manresa 61-73, 08242-Manresa, Barcelona, Spain. E-mail:* francisco.palacios@upc.edu, josep.maria.rossell@upc.edu

[2] *Dept. of Engineering, University of Agder, Norway. E-mail:* hamid.r.karimi@uia.no

**Abstract**

In this work, the main ideas involved in the design of *overlapping* and *multi-overlapping controllers* via the *Inclusion Principle* are discussed and illustrated in the context of the *Structural Vibration Control* of tall buildings under seismic excitation. A detailed theoretical background on the Inclusion Principle and the design of overlapping controllers is provided. Overlapping and multi-overlapping LQR controllers are designed for a simplified five-story building model. Numerical simulations are conducted to asses the performance of the proposed semi-decentralized controllers with positive results.

*Keywords:* Semi-decentralized Control; Overlapping Controllers; Multi-overlapping Controllers; Structural Vibration Control

## 1 Introduction

Nowadays, complexity is perhaps one of the most salient features in the field of automatic control. Over the last decades, the focus of interest of control theory and practice has been progressively moving from the initial simple SISO systems to systems of increasing complexity (Åström et al., 2001; Zečević and Šiljak, 2010).

When a complex system can be decomposed into disjoint subsystems, a set of local controllers may be independently obtained to design a decentralized controller. Design and operation of local controllers requires lower-dimension computation, minimizes the information exchange, and increases the global robustness by reducing the effect of perturbations and failures on communications. However, all these potential benefits are severely attenuated by the fact that systems encountered in practical applications rarely admit a perfect disjoint decomposition (Siljak, 1991).

The *overlapping decomposition* can help to overcome this serious drawback by allowing the subsystems to overlap; that is, the requirement of strict disjoint decomposition is relaxed to permit a restricted sharing of states, inputs, and outputs among the subsystems. For systems admitting an overlapping decomposition, the *Inclusion Principle* allows to design *semi-decentralized controllers* which are in accordance with the system structure, and which partially maintain the positive features of decentralized controllers. This approach has proven to be useful in a variety of complex control problems appearing in different fields, such as macroeconomic modeling, electric power generation, automated highway traffic management, civil structural engineering, aerospace structural engineering, and multi-agent robotics (Aybar et al., 1994; Ataslar and İftar, 1999; Bakule and Rodellar, 1995; Bakule et al., 2005; Chen and Stanković, 2005b; Li et al., 1999; Siljak et al., 1999; Stanković et al., 2000; Stipanović et al., 2004).

Structural Vibration Control (SVC) of buildings and civil structures is one of the best examples of large-scale and complex control systems. SVC systems have

doi:10.4173/mic.2011.2.2

proved to be effective in mitigating the dynamic response of large-scale structures to earthquake and wind excitations (Chu et al., 2005; Preumont and Seto, 2008). Last generation SVC systems typically involve a large number of actuation devices and sensors, and a wide and sophisticated communication network. The state-of-the-art actuation devices are semi-active dampers which are capable to produce large actuation forces using only battery power supply. Two good examples of this kind of SVC systems are the 54-story Mori Tower in Tokyo, Japan, with 356 semi-active hydraulic dampers, and the Dongting Lake Bridge in Hunan, China, with 312 semi-active magnetorheological dampers (Housner et al., 1997; Spencer and Nagarajaiah, 2003).

Recently, wireless communications have made a significant impact in SVC. Using wireless communications, instead of the classical coaxial wiring, can critically reduce the installation and maintenance costs; furthermore, it can also add flexibility to the control system, allowing the implementation of new control strategies without costly modifications. However, to improve the communications robustness and to achieve higher sampling frequencies in the real-time control operation, the controllers need to operate using local information provided by neighboring sensors. Consequently, a decentralized control approach is required for a realistic treatment of Wireless Networked Control Systems (WNCS) (Law et al., 2009; Lynch et al., 2008; Swartz and Lynch, 2009; Wang et al., 2006, 2009; Wang, 2011). In this context, the multi-overlapping approach can be specially suitable for large-scale WNCS, reducing the design and operation computational effort and providing semi-decentralized controllers which satisfy the information exchange constraints (Palacios-Quiñonero et al., 2010; Rossell et al., 2010).

The present work has a triple objective: (i) to present the main ideas and theoretical elements involved in the design of overlapping semi-decentralized controllers via the inclusion principle; (ii) to discuss the more general and practically interesting problem of designing multi-overlapping controllers, paying special attention to the longitudinal multi-overlapping case; and (iii) to illustrate the main ideas involved in the design of overlapping controllers in the context of the Structural Vibration Control of tall buildings under seismic excitation. The organization of the paper is as follows: Section 2 gives necessary background results about the inclusion principle and the design of overlapping controllers, with a detailed discussion of the LQR case. Section 3 presents the multi-overlapping problem. In Section 4, state-space models for a particular five-story building with direct and inter-story actuation schemes are derived. For this five-story building, centralized, overlapping, and multi-overlapping LQR controllers are computed in Section 5. Finally, in Section 6, numerical simulations of the free and controlled vibrational response of the five-story building for different seismic disturbances are conducted to assess the performance of the proposed semi-decentralized controllers.

## 2 Overlapping semi-decentralized controllers

In this section we summarize some basic definitions and results related to the *Inclusion Principle* and its application to the design of overlapping controllers. The general theoretical background is complemented with a brief discussion about the design of overlapping LQR controllers. This particular case will later be used in the controller designs and numerical simulations presented in Sections 4 and 5. A rigorous treatment of the design of semi-decentralized controllers via the Inclusion Principle can be found in Bakule et al. (2000a,b); Chen and Stanković (2005a); İftar and Özgüner (1990); Ikeda and Šiljak (1986); Siljak (1991); Stanković and Šiljak (2001).

### 2.1 The Inclusion Principle

Consider a pair of linear systems

$$\mathbf{S}:\begin{cases}\dot{x}(t)=A\,x(t)+B\,u(t),\\ y(t)=C_y\,x(t),\end{cases} \tag{1}$$

$$\tilde{\mathbf{S}}:\begin{cases}\dot{\tilde{x}}(t)=\tilde{A}\,\tilde{x}(t)+\tilde{B}\,\tilde{u}(t),\\ \tilde{y}(t)=\tilde{C}_y\,\tilde{x}(t),\end{cases}$$

where $x(t)\in\mathbb{R}^{n}$, $u(t)\in\mathbb{R}^{m}$, $y(t)\in\mathbb{R}^{l}$ are the state, the input, and the output of $\mathbf{S}$ at time $t\geq 0$; $\tilde{x}(t)\in\mathbb{R}^{\tilde{n}}$, $\tilde{u}(t)\in\mathbb{R}^{\tilde{m}}$, $\tilde{y}(t)\in\mathbb{R}^{\tilde{l}}$ are the state, the input, and the output corresponding to $\tilde{\mathbf{S}}$; $A$, $B$, $C_y$ and $\tilde{A}$, $\tilde{B}$, $\tilde{C}_y$ are $n\times n$, $n\times m$, $l\times n$ and $\tilde{n}\times\tilde{n}$, $\tilde{n}\times\tilde{m}$, $\tilde{l}\times\tilde{n}$ dimensional matrices, respectively. The dimensions of the state, input, and output vectors $x(t)$, $u(t)$, $y(t)$ of $\mathbf{S}$ are supposed to be smaller than those of $\tilde{x}(t)$, $\tilde{u}(t)$, $\tilde{y}(t)$ of $\tilde{\mathbf{S}}$. Let $x(t;x_0,u)$ and $y[x(t)]$ denote the state behavior and the corresponding output of $\mathbf{S}$ for a fixed input $u(t)$ and for an initial state $x(0)=x_0$, respectively; analogous notations $\tilde{x}(t;\tilde{x}_0,\tilde{u})$, $\tilde{y}[\tilde{x}(t)]$ are used for the state and output of the system $\tilde{\mathbf{S}}$.

Let us consider the following linear transformations:

$$\begin{array}{llll} V: & \mathbb{R}^{n} \longrightarrow \mathbb{R}^{\tilde{n}}, & U: & \mathbb{R}^{\tilde{n}} \longrightarrow \mathbb{R}^{n}, \\ R: & \mathbb{R}^{m} \longrightarrow \mathbb{R}^{\tilde{m}}, & Q: & \mathbb{R}^{\tilde{m}} \longrightarrow \mathbb{R}^{m}, \\ T: & \mathbb{R}^{l} \longrightarrow \mathbb{R}^{\tilde{l}}, & S: & \mathbb{R}^{\tilde{l}} \longrightarrow \mathbb{R}^{l}, \end{array} \tag{2}$$

where $V, R, T$ are *expansion matrices* with rank$(V){=}n$, rank$(R){=}m$, rank$(T){=}l$, and $U$, $Q$, $S$ are *contraction matrices* which satisfy $UV{=}I_n$, $QR{=}I_m$, $ST{=}I_l$, with $I_n$, $I_m$, $I_l$ denoting the identity matrices of indicated dimensions. For a given set of expansion matrices $V$, $R$, $T$, a set of contraction matrices may be obtained considering the corresponding pseudoinverses

$$U = (V^TV)^{-1}V^T, Q = (R^TR)^{-1}R^T, S = (T^TT)^{-1}T^T.$$

**Definition 1 (Inclusion Principle)** *A system $\tilde{\mathbf{S}}$ includes the system $\mathbf{S}$ if there exists a quadruplet of matrices $(U, V, R, S)$ such that, for any initial state $x_0$ and any fixed input $u(t)$ of $\mathbf{S}$, the choice of*

$$\tilde{x}_0 = Vx_0,$$
$$\tilde{u}(t) = Ru(t), \quad \textit{for all } t \geq 0$$

*as initial state $\tilde{x}_0$ and input $\tilde{u}(t)$ for the system $\tilde{\mathbf{S}}$, implies*

$$x(t; x_0, u) = U\tilde{x}(t; \tilde{x}_0, \tilde{u}),$$
$$y[x(t)] = S\tilde{y}[\tilde{x}(t)], \quad \textit{for all } t \geq 0.$$

Given a linear system $\mathbf{S}$ and a set of expansion matrices $V$, $R$, $T$, an expanded system $\tilde{\mathbf{S}}$ may be defined by taking the system matrices in the form

$$\tilde{A} = VAU + M, \quad \tilde{B} = VBQ + N, \quad \tilde{C}_y = TC_yU + L, \tag{3}$$

where $U$, $Q$, $S$ are contraction matrices, and $M$, $N$, $L$ are *complementary matrices* of appropriate dimensions. In order to assure that the system $\mathbf{S}$ and the expanded system $\tilde{\mathbf{S}}$ satisfy the Inclusion Principle, the complementary matrices have to fulfil the conditions stated in the following theorem:

**Theorem 1** *A system $\tilde{\mathbf{S}}$ includes the system $\mathbf{S}$ if and only if $UM^iV{=}0$, $UM^{i-1}NR{=}0$, $SLM^{i-1}V{=}0$ and $SLM^{i-1}NR{=}0$ for all $i{=}1, 2, ..., \tilde{n}$.*

A special kind of expansion-contraction scheme, called *restriction*, is particularly simple and suitable for the design of overlapping controllers.

**Definition 2 (Restriction)** *Let $\tilde{\mathbf{S}}$ be an expansion of the system $\mathbf{S}$ defined by the expanded system matrices $\tilde{A}$, $\tilde{B}$, $\tilde{C}_y$, given in (3). The system $\mathbf{S}$ is said to be a restriction of $\tilde{\mathbf{S}}$ if and only if $MV{=}0$, $NR{=}0$ and $LV{=}0$.*

From Theorem 1, it is clear that if the system $\mathbf{S}$ is a restriction of $\tilde{\mathbf{S}}$, then the expanded system $\tilde{\mathbf{S}}$ includes the initial system $\mathbf{S}$.

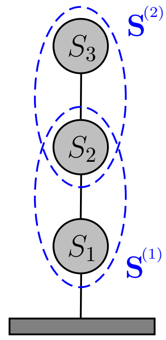



Figure 1: Overlapping decomposition for a three-story building

## 2.2 Decoupled expansions of overlapping decompositions

From an intuitive point of view, a system $\mathbf{S}$ admits an overlapping decomposition if it can be split into three subsystems $S_1$, $S_2$, $S_3$ in such a way that no direct interaction between $S_1$ and $S_3$ may occur; that is, any interaction between $S_1$ and $S_3$ must take place through $S_2$. From the three subsystems $S_i$, two overlapping subsystems $\mathbf{S}^{(1)}{=}[S_1, S_2]$, $\mathbf{S}^{(2)}{=}[S_2, S_3]$ may be defined. The vibrational response of a three story building is a natural and illustrative example in this context. Each story can be seen as a subsystem $S_i$, the vibrational behavior of the first and third stories are clearly influenced by each other, but the interaction may only happen through the second story. An overlapping decomposition for a three-story building system is shown in Fig. 1.

For the linear system $\mathbf{S}$ given in (1), the possibility of overlapping decomposition may be stated in terms of the system matrices structure; more precisely, the linear system $\mathbf{S}$ admits an overlapping decomposition if the system matrices $A$, $B$ and $C_y$ present the following block tridiagonal structure:

$$A = \left[\begin{array}{cc:c} A_{11} & A_{12} & 0 \\ \hdashline A_{21} & \multicolumn{1}{:c:}{A_{22}} & A_{23} \\ \hdashline 0 & \multicolumn{1}{:c}{A_{32}} & A_{33} \end{array}\right], \quad B = \left[\begin{array}{cc:c} B_{11} & B_{12} & 0 \\ \hdashline B_{21} & \multicolumn{1}{:c:}{B_{22}} & B_{23} \\ \hdashline 0 & \multicolumn{1}{:c}{B_{32}} & B_{33} \end{array}\right],$$

$$C_y = \left[\begin{array}{cc:c} (C_y)_{11} & (C_y)_{12} & 0 \\ \hdashline (C_y)_{21} & \multicolumn{1}{:c:}{(C_y)_{22}} & (C_y)_{23} \\ \hdashline 0 & \multicolumn{1}{:c}{(C_y)_{32}} & (C_y)_{33} \end{array}\right],$$

where $A_{ii}$, $B_{ij}$, $(C_y)_{ij}$, for $i,j{=}1,2,3$, are $n_i{\times}n_i$, $n_i{\times}m_j$, $l_i{\times}n_j$ dimensional matrices, respectively. The partition of the state $x{=}(x_1^T, x_2^T, x_3^T)^T$ has components of respective dimensions $n_1$, $n_2$, $n_3$, satisfying $n_1{+}n_2{+}n_3{=}n$; the partition of $u{=}(u_1^T, u_2^T, u_3^T)^T$ has components of dimensions $m_1$, $m_2$, $m_3$, such that $m_1+m_2+m_3=m$; and $y=(y_1^T, y_2^T, y_3^T)^T$ has components of respective dimensions $l_1$, $l_2$, $l_3$, satisfying $l_1+l_2+l_3=l$. Note that the explicit dependence on time has been omitted to simplify the new notation; this will also be done in the sequel when convenient.

Given a linear system $\mathbf{S}$ which admits an overlapping decomposition, the design of an overlapping controller starts with a proper definition of the expansion matrices. A usual choice is

$$V=\begin{bmatrix} I_{n_1} & 0 & 0 \\ 0 & I_{n_2} & 0 \\ 0 & I_{n_2} & 0 \\ 0 & 0 & I_{n_3} \end{bmatrix},\ R=\begin{bmatrix} I_{m_1} & 0 & 0 \\ 0 & I_{m_2} & 0 \\ 0 & I_{m_2} & 0 \\ 0 & 0 & I_{m_3} \end{bmatrix},\ T=\begin{bmatrix} I_{l_1} & 0 & 0 \\ 0 & I_{l_2} & 0 \\ 0 & I_{l_2} & 0 \\ 0 & 0 & I_{l_3} \end{bmatrix}.$$

The corresponding pseudoinverse contractions are

$$U=\begin{bmatrix} I_{n_1} & 0 & 0 & 0 \\ 0 & \frac{1}{2}I_{n_2} & \frac{1}{2}I_{n_2} & 0 \\ 0 & 0 & 0 & I_{n_3} \end{bmatrix},\ Q=\begin{bmatrix} I_{m_1} & 0 & 0 & 0 \\ 0 & \frac{1}{2}I_{m_2} & \frac{1}{2}I_{m_2} & 0 \\ 0 & 0 & 0 & I_{m_3} \end{bmatrix},$$

$$S=\begin{bmatrix} I_{l_1} & 0 & 0 & 0 \\ 0 & \frac{1}{2}I_{l_2} & \frac{1}{2}I_{l_2} & 0 \\ 0 & 0 & 0 & I_{l_3} \end{bmatrix}.$$

A first set of expanded matrices are computed in the form

$$\bar{A}=VAU,\ \ \bar{B}=VBQ,\ \ \bar{C}_y=TC_yU,$$

resulting

$$\bar{A}=\left[\begin{array}{cc|cc} A_{11} & \frac{1}{2}A_{12} & \frac{1}{2}A_{12} & 0 \\ A_{21} & \frac{1}{2}A_{22} & \frac{1}{2}A_{22} & A_{23} \\ \hline A_{21} & \frac{1}{2}A_{22} & \frac{1}{2}A_{22} & A_{23} \\ 0 & \frac{1}{2}A_{32} & \frac{1}{2}A_{32} & A_{33} \end{array}\right],$$

$$\bar{B}=\left[\begin{array}{cc|cc} B_{11} & \frac{1}{2}B_{12} & \frac{1}{2}B_{12} & 0 \\ B_{21} & \frac{1}{2}B_{22} & \frac{1}{2}B_{22} & B_{23} \\ \hline B_{21} & \frac{1}{2}B_{22} & \frac{1}{2}B_{22} & B_{23} \\ 0 & \frac{1}{2}B_{32} & \frac{1}{2}B_{32} & B_{33} \end{array}\right],$$

$$\bar{C}_y=\left[\begin{array}{cc|cc} (C_y)_{11} & \frac{1}{2}(C_y)_{12} & \frac{1}{2}(C_y)_{12} & 0 \\ (C_y)_{21} & \frac{1}{2}(C_y)_{22} & \frac{1}{2}(C_y)_{22} & (C_y)_{23} \\ \hline (C_y)_{21} & \frac{1}{2}(C_y)_{22} & \frac{1}{2}(C_y)_{22} & (C_y)_{23} \\ 0 & \frac{1}{2}(C_y)_{32} & \frac{1}{2}(C_y)_{32} & (C_y)_{33} \end{array}\right].$$

Now, we form an expanded system $\tilde{\mathbf{S}}$ as indicated in (3) by adding a set of adequate complementary matrices. If the complementary matrices are chosen in the form

$$M=\begin{bmatrix} 0 & \frac{1}{2}A_{12} & -\frac{1}{2}A_{12} & 0 \\ 0 & \frac{1}{2}A_{22} & -\frac{1}{2}A_{22} & 0 \\ 0 & -\frac{1}{2}A_{22} & \frac{1}{2}A_{22} & 0 \\ 0 & -\frac{1}{2}A_{32} & \frac{1}{2}A_{32} & 0 \end{bmatrix},\ N=\begin{bmatrix} 0 & \frac{1}{2}B_{12} & -\frac{1}{2}B_{12} & 0 \\ 0 & \frac{1}{2}B_{22} & -\frac{1}{2}B_{22} & 0 \\ 0 & -\frac{1}{2}B_{22} & \frac{1}{2}B_{22} & 0 \\ 0 & -\frac{1}{2}B_{32} & \frac{1}{2}B_{32} & 0 \end{bmatrix},$$

$$L=\begin{bmatrix} 0 & \frac{1}{2}(C_y)_{12} & -\frac{1}{2}(C_y)_{12} & 0 \\ 0 & \frac{1}{2}(C_y)_{22} & -\frac{1}{2}(C_y)_{22} & 0 \\ 0 & -\frac{1}{2}(C_y)_{22} & \frac{1}{2}(C_y)_{22} & 0 \\ 0 & -\frac{1}{2}(C_y)_{32} & \frac{1}{2}(C_y)_{32} & 0 \end{bmatrix},$$

then, the system $\mathbf{S}$ is a restriction of $\tilde{\mathbf{S}}$, and the expanded system $\tilde{\mathbf{S}}$ presents an almost-decoupled structure. More specifically, the system matrices of $\tilde{\mathbf{S}}$ have the following block structure:

$$\tilde{A}=\bar{A}+M=\begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix}=\left[\begin{array}{cc|cc} A_{11} & A_{12} & 0 & 0 \\ A_{21} & A_{22} & 0 & A_{23} \\ \hline A_{21} & 0 & A_{22} & A_{23} \\ 0 & 0 & A_{32} & A_{33} \end{array}\right], \tag{4}$$

$$\tilde{B}=\bar{B}+N=\begin{bmatrix} \tilde{B}_{11} & \tilde{B}_{12} \\ \tilde{B}_{21} & \tilde{B}_{22} \end{bmatrix}=\left[\begin{array}{cc|cc} B_{11} & B_{12} & 0 & 0 \\ B_{21} & B_{22} & 0 & B_{23} \\ \hline B_{21} & 0 & B_{22} & B_{23} \\ 0 & 0 & B_{32} & B_{33} \end{array}\right], \tag{5}$$

$$\begin{aligned}\tilde{C}_y=\bar{C}_y+L&=\begin{bmatrix} (\tilde{C}_y)_{11} & (\tilde{C}_y)_{12} \\ (\tilde{C}_y)_{21} & (\tilde{C}_y)_{22} \end{bmatrix}\\ &=\left[\begin{array}{cc|cc} (C_y)_{11} & (C_y)_{12} & 0 & 0 \\ (C_y)_{21} & (C_y)_{22} & 0 & (C_y)_{23} \\ \hline (C_y)_{21} & 0 & (C_y)_{22} & (C_y)_{23} \\ 0 & 0 & (C_y)_{32} & (C_y)_{33} \end{array}\right].\end{aligned} \tag{6}$$

The state, input and output vectors of the expanded system

$$\begin{aligned}\tilde{\mathbf{S}}: \dot{\tilde{x}}(t)&=\tilde{A}\,\tilde{x}(t)+\tilde{B}\,\tilde{u}(t),\\ \tilde{y}(t)&=\tilde{C}_y\,\tilde{x}(t),\end{aligned}$$

can be written in the form $\tilde{x}^T=(x_1^T, x_2^T, x_2^T, x_3^T)$, $\tilde{u}^T=(u_1^T, u_2^T, u_2^T, u_3^T)$ and $\tilde{y}^T=(y_1^T, y_2^T, y_2^T, y_3^T)$. Using the block notation given in (4), (5), (6), we may define two almost-decoupled expanded subsystems

$$\begin{aligned}\tilde{\mathbf{S}}_1: \dot{\tilde{x}}_1(t)&=\tilde{A}_{11}\,\tilde{x}_1(t)+\tilde{B}_{11}\,\tilde{u}_1(t)+\tilde{A}_{12}\,\tilde{x}_2(t)+\tilde{B}_{12}\,\tilde{u}_2(t),\\ \tilde{y}_1(t)&=(\tilde{C}_y)_{11}\,\tilde{x}_1(t)+(\tilde{C}_y)_{12}\,\tilde{x}_2(t),\end{aligned}$$

$$\begin{aligned}\tilde{\mathbf{S}}_2: \dot{\tilde{x}}_2(t)&=\tilde{A}_{22}\,\tilde{x}_2(t)+\tilde{B}_{22}\,\tilde{u}_2(t)+\tilde{A}_{21}\,\tilde{x}_1(t)+\tilde{B}_{21}\,\tilde{u}_1(t),\\ \tilde{y}_2(t)&=(\tilde{C}_y)_{21}\,\tilde{x}_1(t)+(\tilde{C}_y)_{22}\,\tilde{x}_2(t),\end{aligned}$$

where $\tilde{x}_1^T=(x_1^T, x_2^T)$, $\tilde{u}_1^T=(u_1^T, u_2^T)$, $\tilde{y}_1^T=(y_1^T, y_2^T)$, and $\tilde{x}_2^T=(x_2^T, x_3^T)$, $\tilde{u}_2^T=(u_2^T, u_3^T)$, $\tilde{y}_2^T=(y_2^T, y_3^T)$. By removing the interconnection blocks, two decoupled expanded subsystems result

$$\begin{aligned}\tilde{\mathbf{S}}_{\mathrm{D}}^{(1)}: \dot{\tilde{x}}_1(t)&=\tilde{A}_{11}\,\tilde{x}_1(t)+\tilde{B}_{11}\tilde{u}_1(t),\\ \tilde{y}_1(t)&=(\tilde{C}_y)_{11}\,\tilde{x}_1(t),\\ \tilde{\mathbf{S}}_{\mathrm{D}}^{(2)}: \dot{\tilde{x}}_2(t)&=\tilde{A}_{22}\,\tilde{x}_2(t)+\tilde{B}_{22}\,\tilde{u}_2(t),\\ \tilde{y}_2(t)&=(\tilde{C}_y)_{22}\,\tilde{x}_2(t),\end{aligned}$$

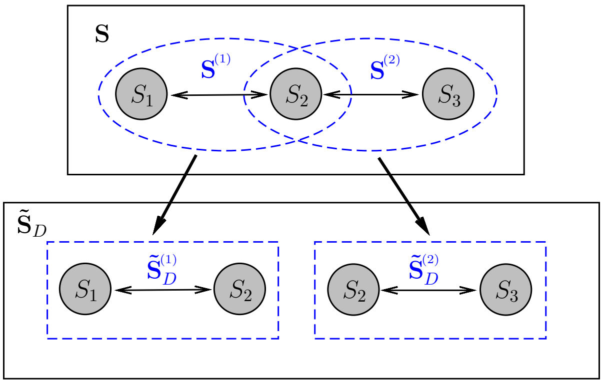



Figure 2: Decoupled expansion of an overlapping decomposition

which define a decoupled expanded system

$$\tilde{\mathbf{S}}_{\mathbf{D}} : \dot{\tilde{x}}(t) = \tilde{A}_D\, \tilde{x}(t) + \tilde{B}_D \tilde{u}(t),$$
$$\tilde{y}(t) = (\tilde{C}_y)_D\, \tilde{x}(t),$$

where $\tilde{A}_D = \mathrm{diag}\{\tilde{A}_{11}, \tilde{A}_{22}\}$, $\tilde{B}_D = \mathrm{diag}\{\tilde{B}_{11}, \tilde{B}_{22}\}$ and $(\tilde{C}_y)_D = \mathrm{diag}\{(\tilde{C}_y)_{11}, (\tilde{C}_y)_{22}\}$. The decoupled expansion process is schematically depicted in Fig. 2.

## 2.3 Design of overlapping controllers

To complete the design of an overlapping controller for $\mathbf{S}$, two additional steps are required: (1) to design a decentralized controller $\tilde{K}_D$ for the expanded decoupled system $\tilde{\mathbf{S}}_{\mathbf{D}}$, and (2) to contract the decentralized expanded controller $\tilde{K}_D$ to a semi-decentralized overlapping controller $K_o$ for $\mathbf{S}$. The design of $\tilde{\mathbf{S}}_{\mathbf{D}}$ can be done by independently computing local controllers for $\tilde{\mathbf{S}}_{\mathbf{D}}^{(1)}$ and $\tilde{\mathbf{S}}_{\mathbf{D}}^{(2)}$; in the next subsection, this step will be considered in detail for the particular case of designing optimal LQR controllers. Regarding to the second step, the concept of *contractibility* is introduced in order to guarantee the correctness of the controller contraction process.

**Definition 3 (Contractibility)** *Suppose that $\tilde{\mathbf{S}}$ is an expansion of the system $\mathbf{S}$. Then, a control law $\tilde{u}(t) = \tilde{K}\, \tilde{x}(t)$ for $\tilde{\mathbf{S}}$ is contractible to the control law $u(t) = Kx(t)$ for $\mathbf{S}$ if there exist transformations as in (2) such that, for any initial state $x_0 \in \mathbb{R}^{\mathrm{n}}$ and any input $u(t) \in \mathbb{R}^{\mathrm{m}}$, if $\tilde{x}_0 = Vx_0$ and $\tilde{u}(t) = Ru(t)$ then $Kx(t; x_0, u) = Q\tilde{K}\tilde{x}(t; Vx_0, Ru)$ for all $t \geq 0$.*

The following proposition expresses the contractibility property in terms of complementary matrices.

**Proposition 1** *Suppose that $\tilde{\mathbf{S}}$ is an expansion of the system $\mathbf{S}$. Then, a control law $\tilde{u}(t) = \tilde{K}\, \tilde{x}(t)$ for $\tilde{\mathbf{S}}$ is contractible to the control law $u(t) = Kx(t)$ for $\mathbf{S}$ if and only if $Q\tilde{K}V = K$, $Q\tilde{K}M^iV = 0$, $Q\tilde{K}M^{i-1}NR = 0$, for $i = 1, \ldots, \tilde{n}$.*

When the original system $\mathbf{S}$ is a restriction of the expanded system $\tilde{\mathbf{S}}$, from Definition 2 and Proposition 1, it is clear that any expanded controller $\tilde{K}$ designed in $\tilde{\mathbf{S}}$ can be contracted to a controller $K = Q\tilde{K}V$ for $\mathbf{S}$.

If $\tilde{K}^{(1)}$ and $\tilde{K}^{(2)}$ are local controllers for the decoupled expanded subsystems $\tilde{\mathbf{S}}_{\mathbf{D}}^{(1)}$ and $\tilde{\mathbf{S}}_{\mathbf{D}}^{(2)}$, then a block diagonal controller can be obtained in the form

$$\tilde{K}_D = \begin{bmatrix} \tilde{K}^{(1)} & 0 \\ 0 & \tilde{K}^{(2)} \end{bmatrix};$$

this expanded controller can be contracted to an overlapping controller

$$K_o = Q\tilde{K}_D V = \left[\begin{array}{c|c|c} K_{11} & K_{12} & 0 \\ \hline K_{21} & K_{22} & K_{23} \\ \hline 0 & K_{32} & K_{33} \end{array}\right], \qquad (7)$$

which has a desired block tridiagonal structure.

## 2.4 Design of LQR overlapping controllers

To design a centralized state-feedback optimal LQR controller for the system (1), we begin by defining the performance index

$$J_c(x(t), u(t)) = \int_0^\infty \left[ x^T(t)\, Q^* x(t) + u^T(t)\, R^* u(t) \right] dt, \qquad (8)$$

where $Q^*$ is a positive-semidefinite real symmetric matrix, and $R^*$ is a positive-definite real symmetric matrix. If the Riccati equation

$$A^T P + PA - PB(R^*)^{-1}B^T P + Q^* = 0$$

has a positive-definite solution $P$, then the control vector

$$u_{opt}(t) = -K_{opt}\, x(t) \qquad (9)$$

with the gain matrix

$$K_{opt} = (R^*)^{-1}B^T P$$

minimizes the index (8) for the trajectories satisfying

$$\dot{x} = Ax + Bu$$

and for all initial state $x_0$. The optimal value of the index corresponding to the initial state $x_0$ is

$$[J_c(x_0)]_{opt} = x_0^T P x_0.$$

If the components of the initial state are considered independent random variables with mean $\mu = 0$ and variance $\sigma^2 = 1$, then the average value of $[J_c(x_0)]_{opt}$ can be computed as

$$[J_c]_{opt} = \text{trace}(P).$$

Any other stable gain matrix $K$ will define a control law $u(t) = -Kx(t)$ with an associated average cost $J_K \geq [J_c]_{opt}$. The value of $J_K$ can be computed as

$$J_K = \text{trace}(P_K),$$

where $P_K$ is the solution of the Lyapunov equation

$$(A - BK)^T P + P(A - BK) + Q^* + K^T R^* K = 0.$$

To design an overlapping LQR controller, we start by computing local optimal LQR controllers for the expanded decoupled subsystems $\tilde{\mathbf{S}}_{\mathbf{D}}^{(1)}$ and $\tilde{\mathbf{S}}_{\mathbf{D}}^{(2)}$. To this end, we consider local quadratic cost functions

$$\tilde{J}_{_D}^{(1)}(\tilde{x}_{_1}(t), \tilde{u}_1(t)) = \int_0^\infty \left[\tilde{x}_1^T(t)\tilde{Q}_1^*\tilde{x}_1(t) + \tilde{u}_1^T(t)\tilde{R}_1^*\tilde{u}_1(t)\right] dt, \quad (10)$$

$$\tilde{J}_{_D}^{(2)}(\tilde{x}_{_2}(t), \tilde{u}_2(t)) = \int_0^\infty \left[\tilde{x}_2^T(t)\tilde{Q}_2^*\tilde{x}_2(t) + \tilde{u}_2^T(t)\tilde{R}_2^*\tilde{u}_2(t)\right] dt, \quad (11)$$

where $\tilde{Q}_1^*$, $\tilde{Q}_2^*$, $\tilde{R}_1^*$ and $\tilde{R}_2^*$ are appropriate expanded weighting matrices. The gain matrices for the control laws

$$\tilde{u}_1(t) = -\tilde{K}_1\, \tilde{x}_1(t), \qquad \tilde{u}_2(t) = -\tilde{K}_2\, \tilde{x}_2(t),$$

that minimize the cost functions (10,11), can be independently computed as

$$\tilde{K}_1 = \left[\tilde{R}_1^*\right]^{-1} \tilde{B}_1^T \tilde{P}_1, \qquad \tilde{K}_2 = \left[\tilde{R}_2^*\right]^{-1} \tilde{B}_2^T \tilde{P}_2,$$

where $\tilde{P}_{_1}$ and $\tilde{P}_{_2}$ are the solutions of the corresponding Riccati equations. In the decoupled expanded system $\tilde{\mathbf{S}}_{\mathbf{D}}$, the gain matrix of the controller $\tilde{u}(t){=}{-}\tilde{K}_{_D}\,\tilde{x}(t)$ which minimizes the cost function

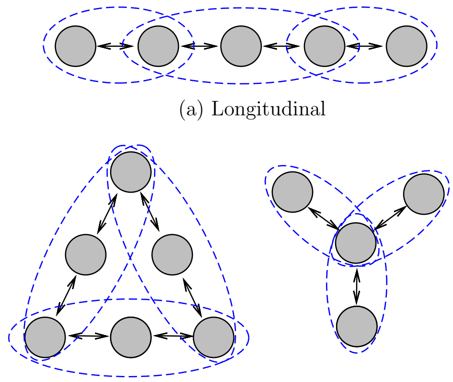
$$\tilde{J}_{_D}(\tilde{x}(t), \tilde{u}(t)) = \int_0^\infty \left[\tilde{x}^T(t)\tilde{Q}_{_D}^*\tilde{x}(t) + \tilde{u}^T(t)\tilde{R}_{_D}^*\tilde{u}(t)\right] dt,$$

with

$$\tilde{Q}_{_D}^* = \text{diag}\{\tilde{Q}_1^*, \tilde{Q}_2^*\}, \quad \tilde{R}_{_D}^* = \text{diag}\{\tilde{R}_1^*, \tilde{R}_2^*\},$$

can be written as a block diagonal gain matrix

$$\tilde{K}_{_D} = \text{diag}\{\tilde{K}_1, \tilde{K}_2\}.$$



(a) Longitudinal

(b) Loop

(c) Radial

Figure 3: Multi-overlapping decompositions

Finally, the controller $\tilde{u}_{_D}(t) = -\tilde{K}_{_D}\,\tilde{x}(t)$ is contracted to an overlapping controller

$$u_o(t) = -K_o\, x(t) \quad (12)$$

that can be implemented into the original system $\mathbf{S}$. The contracted gain matrix is computed as

$$K_o = Q\tilde{K}_{_D} V,$$

and has the desired block tridiagonal structure shown in (7). Obviously, the gain matrix $K_o$ defines a suboptimal controller with an expected cost $J_{K_o} \geq [J_c]_{opt}$. However, the numerical simulations show that the overlapping controller (12) exhibits a remarkably high performance level with respect to the centralized optimal LQR controller (9), despite the restricted information exchange and the lower dimension required in its design.

## 3 Multi-overlapping controllers

Although the vast majority of theoretical results and applications of overlapping decomposition has been formulated for the simple case of two overlapping subsystems, it should be noted that most of the problems appearing in the context of large scale and complex systems lead naturally to the consideration of multi-overlapping structures. The generalization from a simple overlapping to a multi-overlapping approach is by no means straightforward. For three overlapping subsystems, three different overlapping topologies can be considered: *longitudinal*, *loop*, and *radial*. These basic multi-overlapping structures are illustrated in Fig. 3, where the circles in the diagrams may be seen as physical subsystems; the arrows indicate state, input, or output interaction; and the dashed ellipses represent the overlapping subsystems. Obviously, in the general case of $n$ overlapping subsystems, highly complex multi-overlapping structures may appear. An interesting study on multi-overlapping controller design for general multi-overlapping structures can be found in Chen and Stanković (2005a).

The *Expansion-Decoupling-Contraction* (EDC) process involved in the design of multi-overlapping controllers may be carried out following two different approaches: (i) single-step, and (ii) multi-step. The *single-step approach* processes all the subsystems simultaneously, uses a generalized version of the Inclusion Principle, and requires generalized forms of the expansion and complementary matrices whose structure depends on the structure of the particular multi-overlapping decomposition under consideration. The *multi-step approach* breaks the overall EDC process into a set of elemental EDC subprocesses where only two subsystems are involved. The interest of the multi-step approach lies in its balanced combination of theoretical simplicity and computational efficiency: from a theoretical point of view, only the basic theory of two overlapping subsystems is required; from a computational perspective, the same basic procedure is repeatedly used, and part of the computations might be processed in parallel. A detailed study of the design of multi-overlapping controllers for longitudinal multi-overlapping systems following the multi-step approach may be found in Palacios-Quiñonero et al. (2010).

In this section, to illustrate the main ideas involved in the multi-step EDC process, we discuss a particular case of multi-overlapping controller design for a system admitting a longitudinal multi-overlapping decomposition (see Fig. 4); an example of application to the vibrational control of a five-story building will be provided in Section 5. Let us suppose that the system $\mathbf{S}$ given in (1) admits the sequential multi-overlapping decomposition shown in Fig. 4, this means that the system matrices have a tridiagonal block structure. In particular, the state matrix can be written in the form

$$A = \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} & 0 & 0 & 0 \\ \hat{A}_{21} & \hat{A}_{22} & \hat{A}_{23} & 0 & 0 \\ 0 & \hat{A}_{32} & \hat{A}_{33} & \hat{A}_{34} & 0 \\ 0 & 0 & \hat{A}_{43} & \hat{A}_{44} & \hat{A}_{45} \\ 0 & 0 & 0 & \hat{A}_{54} & \hat{A}_{55} \end{bmatrix}; \qquad (13)$$

the input and output matrices $B$, $C_y$, will present an analogous structure with blocks $\hat{B}_{ij}$, $(\hat{C}_y)_{ij}$. The subsystems $\hat{S}_j$ are

$$\hat{S}_1 : \begin{cases} \dot{\hat{x}}_1 = \hat{A}_{11}\hat{x}_1 + \hat{B}_{11}\hat{u}_1 + \hat{A}_{12}\hat{x}_2 + \hat{B}_{12}\hat{u}_2, \\ \hat{y}_1 = (\hat{C}_y)_{11}\hat{x}_1 + (\hat{C}_y)_{12}\hat{x}_2, \end{cases}$$

$$\hat{S}_j : \begin{cases} \dot{\hat{x}}_j = \hat{A}_{jj}\hat{x}_j + \hat{B}_{jj}\hat{u}_j + \hat{A}_{j,j-1}\hat{x}_{j-1} + \hat{A}_{j,j+1}\hat{x}_{j+1} \\ \qquad + \hat{B}_{j,j-1}\hat{u}_{j-1} + \hat{B}_{j,j+1}\hat{u}_{j+1}, \\ \hat{y}_j = (\hat{C}_y)_{jj}\hat{x}_j + (\hat{C}_y)_{j,j-1}\hat{x}_{j-1} + (\hat{C}_y)_{j,j+1}\hat{x}_{j+1}, \end{cases}$$

for $j = 2, \dots 4$, and

$$\hat{S}_5 : \begin{cases} \dot{\hat{x}}_5 = \hat{A}_{55}\hat{x}_5 + \hat{B}_{55}\hat{u}_5 + \hat{A}_{54}\hat{x}_4 + \hat{B}_{54}\hat{u}_4, \\ \hat{y}_5 = (\hat{C}_y)_{55}\hat{x}_5 + (\hat{C}_y)_{54}\hat{x}_4, \end{cases}$$

where $\hat{x}_j$, $\hat{u}_j$, $\hat{y}_j$, are the state, input and output of the subsystem $\hat{S}_j$ with respective dimensions $\hat{n}_j$, $\hat{m}_j$, $\hat{l}_j$, $j = 1, \dots, 5$. The overall state of the system $\mathbf{S}$ is

$$x = (\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4, \hat{x}_5), \text{ with dimension } n = \sum \hat{n}_j;$$

analogously, the overall input and output vectors can be written as

$$u = (\hat{u}_1, \hat{u}_2, \hat{u}_3, \hat{u}_4, \hat{u}_5), \text{ with dimension } m = \sum \hat{m}_j,$$
$$y = (\hat{y}_1, \hat{y}_2, \hat{y}_3, \hat{y}_4, \hat{y}_5), \text{ with dimension } l = \sum \hat{l}_j.$$

To carry out the decoupling extension of the multi-overlapping system, we proceed in two stages.

In *Stage 1*, we consider the subsystems $\mathbf{S}^{(1)}$, $\mathbf{S}^{(2)}$, which overlap in subsystem $\hat{S}_3$, and perform a first decoupling expansion as explained in Subsection 2.2 with $n_1 = \hat{n}_1 + \hat{n}_2$, $n_2 = \hat{n}_3$, $n_3 = \hat{n}_4 + \hat{n}_5$; and $m_1 = \hat{m}_1 + \hat{m}_2$, $m_2 = \hat{m}_3$, $m_3 = \hat{m}_4 + \hat{m}_5$. As the output is not relevant in the design of state-feedback controllers, it can be omitted. After adding a suitable set of complementary matrices and properly removing the residual interconnection blocks, we obtain a first pair of expanded decoupled systems $\tilde{\mathbf{S}}_{\mathbf{D}}^{(1)}$, $\tilde{\mathbf{S}}_{\mathbf{D}}^{(2)}$. At this point, expanded controllers $\tilde{K}^{(1)}$, $\tilde{K}^{(2)}$ can be independently computed and contracted to obtain an overlapping controller $K_o$; this approach will be followed in Subsection 5.2 and is depicted in Fig. 7.

In *Stage 2*, we observe that the decoupled expanded systems obtained in Stage 1 admit a new overlapping decomposition: $\tilde{\mathbf{S}}_{\mathbf{D}}^{(1)}$ can be decomposed in the subsystems $\mathbf{S}^{(11)}$, $\mathbf{S}^{(12)}$, which overlap in $\hat{S}_2$; $\tilde{\mathbf{S}}_{\mathbf{D}}^{(2)}$ can be decomposed in the subsystems $\mathbf{S}^{(21)}$, $\mathbf{S}^{(22)}$, which overlap in $\hat{S}_4$. Further decoupling expansions may be performed (in parallel) for $\tilde{\mathbf{S}}_{\mathbf{D}}^{(1)}$ and $\tilde{\mathbf{S}}_{\mathbf{D}}^{(2)}$, resulting the four decoupled expanded systems $\tilde{\mathbf{S}}_{\mathbf{D}}^{(11)}$, $\tilde{\mathbf{S}}_{\mathbf{D}}^{(12)}$, $\tilde{\mathbf{S}}_{\mathbf{D}}^{(21)}$, $\tilde{\mathbf{S}}_{\mathbf{D}}^{(22)}$. Now, four low dimension expanded controllers $\tilde{K}^{(11)}$, $\tilde{K}^{(12)}$, $\tilde{K}^{(21)}$, $\tilde{K}^{(22)}$, can be computed (in parallel). A two-step contraction process follows: firstly, $\tilde{K}^{(11)}$, $\tilde{K}^{(12)}$ are contracted to an overlapping controller $\tilde{K}^{(1)}$ for $\tilde{\mathbf{S}}_{\mathbf{D}}^{(1)}$, and $\tilde{K}^{(21)}$, $\tilde{K}^{(22)}$ are contracted to an overlapping controller $\tilde{K}^{(2)}$ for $\tilde{\mathbf{S}}_{\mathbf{D}}^{(2)}$; secondly, $\tilde{K}^{(1)}$, $\tilde{K}^{(2)}$ are contracted to a multi-overlapping controller

$$K_{mo} = \begin{bmatrix} \hat{K}_{11} & \hat{K}_{12} & 0 & 0 & 0 \\ \hat{K}_{21} & \hat{K}_{22} & \hat{K}_{23} & 0 & 0 \\ 0 & \hat{K}_{32} & \hat{K}_{33} & \hat{K}_{34} & 0 \\ 0 & 0 & \hat{K}_{43} & \hat{K}_{44} & \hat{K}_{45} \\ 0 & 0 & 0 & \hat{K}_{54} & \hat{K}_{55} \end{bmatrix},$$

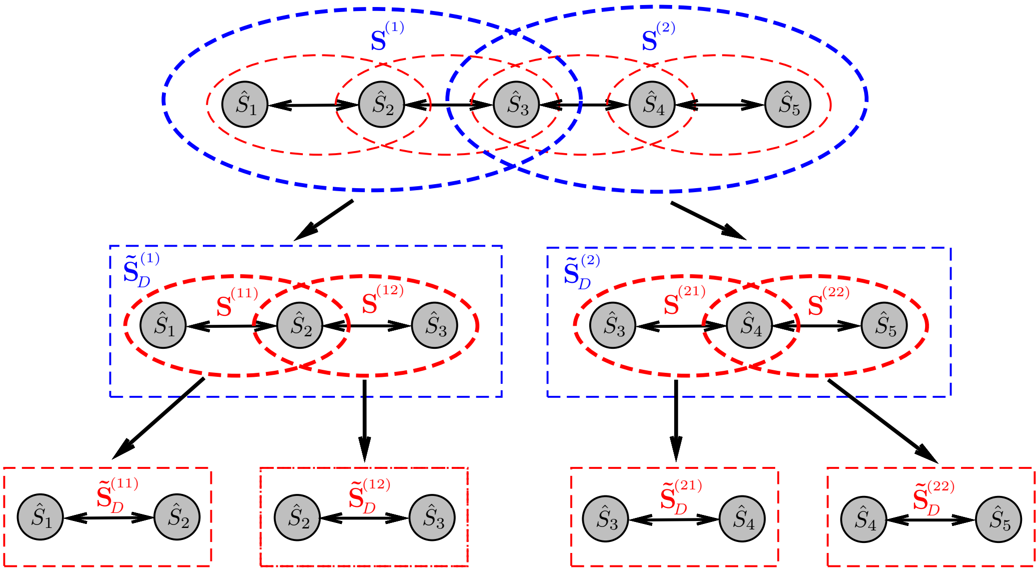



Figure 4: Decoupling expansion for multi-overlapping decomposition

for $\mathbf{S}$ which presents a block structure in accordance with the structure (13) of the system matrices. A multi-overlapping controller $u_{mo} = -K_{mo}\,x$ can be expressed as $u_{mo} = (\hat{u}_1, \hat{u}_2, \hat{u}_3, \hat{u}_4, \hat{u}_5)$, where $\hat{u}_j$ is the control input to the subsystem $\hat{S}_j$. More precisely, we have

$$\begin{cases} \hat{u}_1 = \hat{K}_{11}\hat{x}_1 + \hat{K}_{12}\hat{x}_2, \\ \hat{u}_j = \hat{K}_{jj}\hat{x}_j + \hat{K}_{j,j-1}\hat{x}_{j-1} + \hat{K}_{j,j+1}\hat{x}_{j+1}, j = 2, \dots, 4, \\ \hat{u}_5 = \hat{K}_{55}\hat{x}_5 + \hat{K}_{54}\hat{x}_4, \end{cases}$$

where it can be clearly appreciated that only the states $\hat{x}_j$ corresponding to neighboring subsystems are required to compute the control action for the subsystem $\hat{S}_j$. The presented procedure is schematically depicted in Fig. 8; it will be used in Subsection 5.3 to compute multi-overlapping controllers for a five-story building.

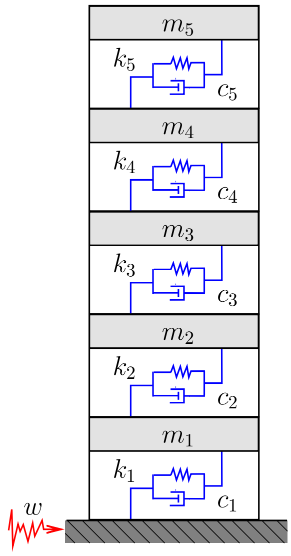



Figure 5: Five-story building model

## 4 Five-story building model

In this section simplified dynamical models for the vibrational response of a five-story building (see Fig. 5) are presented. These models will later be used to compute overlapping and multi-overlapping controllers following the ideas presented in the previous sections; they will also be used in the numerical simulations conducted to assess the vibrational response attenuation achieved by the proposed semi-decentralized controllers. In order to give a clear picture of the structures and dimensions, numerical values of all the relevant matrices are presented. However, to facilitate a one-column display, the numerical format and accuracy in big matrices has had to be conveniently adjusted and the numerical accuracy of different matrices may be inconsistent. In any case, a clear distinction has been made between structural zeros (0) and rounded-to-zero values (0.000).

### 4.1 Second-order model

The building motion can be described by the second-order model

$$M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = T_u u(t) + T_w w(t), \qquad (14)$$

where $M$, $K$, $C$ are, respectively, the mass, stiffness, and damping matrices; the vector of story displacements with respect to the ground is

$$q(t) = [q_1(t),\, q_2(t),\, q_3(t),\, q_4(t),\, q_5(t)]^T,$$

with $q_i(t)$ representing the displacement of the $i$th story; the vector of control forces has a similar structure

$$u(t) = [u_1(t),\, u_2(t),\, u_3(t),\, u_4(t),\, u_5(t)]^T,$$

where $u_i(t)$ denotes the control force exerted by the $i$th actuation device; $T_u$ is the control location matrix; $w(t)$ is the seismic ground acceleration; and $T_w{=}-M\,[1]_{5\times1}$ is the disturbance input matrix, where $[1]_{5\times1}$ denotes a column vector of dimension 5 with all its entries equal to 1.

The mass and stiffness matrices in equation (14) have the following structure:

$$M = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 \\ 0 & 0 & 0 & 0 & m_5 \end{bmatrix},$$

$$K = \begin{bmatrix} k_1{+}k_2 & -k_2 & 0 & 0 & 0 \\ -k_2 & k_2{+}k_3 & -k_3 & 0 & 0 \\ 0 & -k_3 & k_3{+}k_4 & -k_4 & 0 \\ 0 & 0 & -k_4 & k_4{+}k_5 & -k_5 \\ 0 & 0 & 0 & -k_5 & k_5 \end{bmatrix}. \qquad (15)$$

If the damping coefficients $c_j$ were known, a tridiagonal damping matrix $C$ with the same structure as the stiffness matrix (15) might, in principle, be obtained by replacing the stiffness coefficients $k_i$ by the corresponding damping coefficients $c_i$ . However, unlike the case of stiffness properties, the values of the damping coefficients may not be properly estimated from the structural dimensions, structural member sizes, and the damping of the structural materials used. If similar damping mechanisms are distributed throughout the structure (as it is supposed to happen in our multistory building), the damping matrix can be determined from its modal damping ratios. The *Rayleigh damping* approach allows to compute a tridiagonal damping matrix in the form $C = a_0 M + a_1 K$ by setting the value of two damping ratios $\xi_i$, $\xi_j$. If the damping ratios are taken with a common value $\xi$, the coefficients $a_0$, $a_1$, can be computed as

$$a_0 = \xi\frac{2\omega_i\omega_j}{\omega_i+\omega_j}, \qquad a_1 = \xi\frac{2}{\omega_i+\omega_j},$$

with $\omega_j$ denoting the $j$th natural frequency of the structure which can be computed as $\omega_j = \sqrt{\lambda_j}$, where $\lambda_1 \leq \cdots \leq \lambda_n$ are the eigenvalues of the symmetric positive-definite matrix $M^{-1}K$. A commonly accepted value for the damping ration in seismic response simulations (and in earthquake building codes) is $\xi = 5\%$. A detailed treatment of the subject can be found in Chopra (2007).

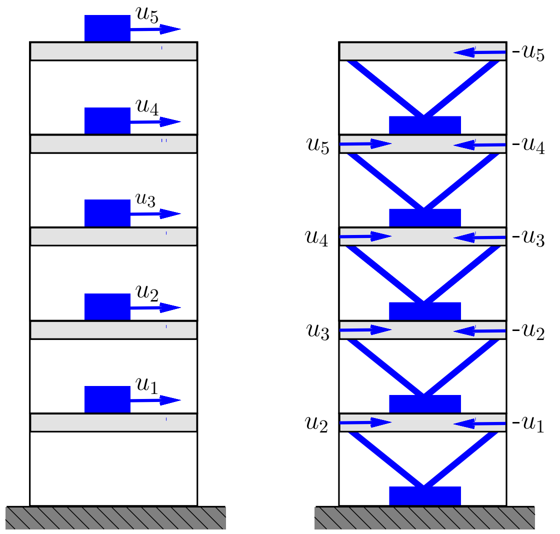



Figure 6: Actuation schemes for a five-story building

In the design of the different controllers and the numerical simulations, we will take the particular values $m_j = 2.156 \times 10^5$ kg, $k_j = 1.5 \times 10^8$ N/m, for $j = 1, \ldots, 5$. The Rayleigh damping matrix corresponding to a damping ratio $\xi = 0.05$ for the first and second natural frequencies (with elements in Ns/m), is

$$C = 10^6 \times \begin{bmatrix} 1.14 & -0.51 & 0 & 0 & 0 \\ -0.51 & 1.14 & -0.51 & 0 & 0 \\ 0 & -0.51 & 1.14 & -0.51 & 0 \\ 0 & 0 & -0.51 & 1.14 & -0.51 \\ 0 & 0 & 0 & -0.51 & 0.63 \end{bmatrix}.$$

Regarding to the control location matrix $T_u$, we will consider two different cases corresponding to the actuation schemes depicted in Fig. 6. In the *direct actuation* scheme, we suppose that an ideal force-actuation device is implemented in each story; the control location matrix is in this case an identity matrix $T_u^{(d)} = I_5$. In the more realistic *inter-story actuation* scheme, ideal force-actuation devices are supposed to be placed between consecutive stories. In this second case, we agree that a positive control actuation $u_j(t)$ will exert a positive force $u_j(t)$ on the $(j-1)$th story, and a negative force $-u_j(t)$ on the $j$th story; this convention is illustrated in Fig. 6, where the positive direction is to the right. The control location matrix corresponding to the inter-story actuation scheme is

$$T_u^{(is)} = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}.$$

### 4.2 First-order state-space model

From the second-order model (14), a first-order state-space model can be derived

$$S_{\scriptscriptstyle I} : \dot{x}_{\scriptscriptstyle I}(t) = A_{\scriptscriptstyle I} x_{\scriptscriptstyle I}(t) + B_{\scriptscriptstyle I} u(t) + E_{\scriptscriptstyle I} w(t), \tag{16}$$

by taking the state vector

$$x_{\scriptscriptstyle I}(t) = \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix}.$$

The state matrix in (16), has the structure

$$A_{\scriptscriptstyle I} = \begin{bmatrix} [0]_{5\times 5} & I_5 \\ -M^{-1}K & -M^{-1}C \end{bmatrix},$$

while the control and disturbance input matrices are, respectively,

$$B_{\scriptscriptstyle I} = \begin{bmatrix} [0]_{5\times 5} \\ M^{-1}T_u \end{bmatrix}, \quad E_{\scriptscriptstyle I} = \begin{bmatrix} [0]_{5\times 1} \\ -[1]_{5\times 1} \end{bmatrix},$$

where $[0]_{5\times 5}$, $[0]_{5\times 1}$ are zero-matrices of indicated dimensions. Next we define a new state vector

$$x(t) = \mathcal{C} x_{\scriptscriptstyle I}(t)$$

with

$$\mathcal{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

The new state

$$x(t) = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}]$$

groups together the inter-story drifts and inter-story velocities in increasing order

$$\begin{cases} x_{\scriptscriptstyle 1}(t) = q_{\scriptscriptstyle 1}(t),\; x_{\scriptscriptstyle 2}(t) = \dot{q}_{\scriptscriptstyle 1}(t), \\ x_{2j-1}(t) = q_{\scriptscriptstyle j}(t) - q_{\scriptscriptstyle j-1}(t), \text{ for } j = 2, 3, 4, 5, \\ x_{2j}(t) = \dot{q}_{\scriptscriptstyle j}(t) - \dot{q}_{\scriptscriptstyle j-1}(t), \text{ for } j = 2, 3, 4, 5. \end{cases}$$

The new state-space model is

$$S : \dot{x}(t) = Ax(t) + Bu(t) + Ew(t),$$

with

$$A = \mathcal{C} A_{\scriptscriptstyle I} \mathcal{C}^{-1}, \quad B = \mathcal{C} B_{\scriptscriptstyle I}, \quad E = \mathcal{C} E_{\scriptscriptstyle I}.$$

For the particular values of the building parameters given in Subsection 4.1, we obtain the state matrix

$$A = 10^{3} \times \begin{bmatrix} 0 & .001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -.696 & -.003 & .696 & .002 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .001 & 0 & 0 & 0 & 0 & 0 & 0 \\ .696 & .002 & -1.391 & -.005 & .696 & .002 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .001 & 0 & 0 & 0 & 0 \\ 0 & 0 & .696 & .002 & -1.391 & -.005 & .696 & .002 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & .001 & 0 & 0 \\ 0 & 0 & 0 & 0 & .696 & .002 & -1.391 & -.005 & .696 & .002 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .001 \\ 0 & 0 & 0 & 0 & 0 & 0 & .696 & .002 & -1.391 & -.005 \end{bmatrix} \tag{17}$$

the disturbance input matrix

$$E = [0,\, -1,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0]^T,$$

and two different control input matrices

$$B^{(d)} = 10^{-5} \times \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.464 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -0.464 & 0.464 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -0.464 & 0.464 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.464 & 0.464 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.464 & 0.464 \end{bmatrix}, \tag{18}$$

$$B^{(is)} = 10^{-5} \times \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -0.464 & 0.464 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0.464 & -0.928 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.464 & -0.928 & 0.464 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.464 & -0.928 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.464 & -0.923 \end{bmatrix}, \tag{19}$$

corresponding, respectively, to the direct and the inter-story actuation scheme.

## 5 Controllers design

In this section, several controllers with different levels of information exchange are designed for the five-story building model presented in Section 4. More precisely, three kinds of controllers are computed: (i) centralized controllers, (ii) semi-decentralized overlapping controllers, and (iii) semi-decentralized multi-overlapping controllers. In each case, the direct and inter-story actuation schemes are considered. For the sake of clarity, the control design objectives have been restricted to the reduction of maximum absolute inter-story drifts and, in all the cases, the optimal LQR control approach has been followed. To perform a first evaluation of the effectiveness achieved by the semi-decentralized controllers, the corresponding quadratic costs are computed and compared with the optimal costs obtained by the centralized controllers. As in the previous section, the numerical format and accuracy in big matrices has been conveniently adjusted to facilitate a one-column display.

## 5.1 Centralized controllers

To compute the centralized optimal LQR controllers, we consider the quadratic index $J_c$ described in (8) defined by the weighting matrices

$$Q^* = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

and $R^*=10^{-16}\, I_5$. The gain matrix of the optimal LQR controller for the direct actuation scheme computed with the system matrices $A$, $B^{(d)}$ given in (17), (18) is

$$K_c^{(d)} = 10^8 \times$$
$$\begin{bmatrix} .234 & .028 & -.046 & .001 & -.029 & -.002 & -.017 & -.001 & -.008 & -.001\\ .218 & .029 & .235 & .026 & -.063 & .000 & -.037 & -.002 & -.017 & -.001\\ .189 & -.027 & .201 & .028 & .227 & .026 & -.063 & .000 & -.029 & -.002\\ .175 & .026 & .181 & .027 & .201 & .028 & .235 & .026 & -.045 & .001\\ .164 & .026 & .172 & .026 & .189 & .027 & .218 & .029 & .264 & .028 \end{bmatrix},$$

with an optimal cost $\left[J_c^{(d)}\right]_{opt} = 0.2974$. In the case of inter-story actuators, the input matrix $B^{(is)}$ (19) is used, resulting the centralized gain matrix

$$K_c^{(is)} = 10^8 \times$$
$$\begin{bmatrix} -.303 & -.057 & .000 & -.031 & .000 & -.020 & .000 & -.012 & .000 & -.006\\ .000 & -.031 & -.303 & -.045 & .000 & -.024 & .000 & -.013 & .000 & -.006\\ .000 & -.020 & .000 & -.024 & -.303 & -.039 & .000 & -.018 & .000 & -.008\\ .000 & -.012 & .000 & -.013 & .000 & -.018 & -.303 & -.033 & .000 & -.012\\ .000 & -.006 & .000 & -.006 & .000 & -.008 & .000 & -.012 & -.303 & -.025 \end{bmatrix},$$

with an optimal cost $\left[J_c^{(is)}\right]_{opt} = 0.3749$. Note that the full-state is needed to compute the control vector $u_c^{(d)} = -K_c^{(d)} x(t)$. In the inter-story actuation scheme, the gain matrix $K_c^{(is)}$ contains a good number of zero elements; however, to compute the actuation force for the $j$th actuation device, the local inter-story drift and all the inter-story velocities are required. Consequently, a full-range communication system must be used to implement the obtained centralized controllers.

## 5.2 Overlapping controllers

In this subsection, we consider the overlapping decomposition depicted in Fig. 7 and use the ideas presented in Section 2 to design two semi-decentralized overlapping controllers. The considered overlapping decomposition (obviously, other decompositions are also possible) consists of two systems $\mathbf{S}^{(1)} = [1,2,3]$, $\mathbf{S}^{(2)} = [3,4,5]$ which overlap in the third story. The notation $\mathbf{S}^{(1)} = [1,2,3]$ indicates that $\mathbf{S}^{(1)}$ comprises the stories 1–3. Following the notation introduced in Subsection 2.2, the dimensions of the state partition are $n_1 = 4$, $n_2 = 2$, $n_3 = 4$; for the control partition, we have $m_1 = 2$, $m_2 = 1$, $m_3 = 2$. The values of $n_j$, $m_j$ define the expansion matrices

$$V=\begin{bmatrix} I_4 & 0 & 0\\ 0 & I_2 & 0\\ 0 & I_2 & 0\\ 0 & 0 & I_4 \end{bmatrix}, \quad R=\begin{bmatrix} I_2 & 0 & 0\\ 0 & I_1 & 0\\ 0 & I_1 & 0\\ 0 & 0 & I_2 \end{bmatrix}.$$

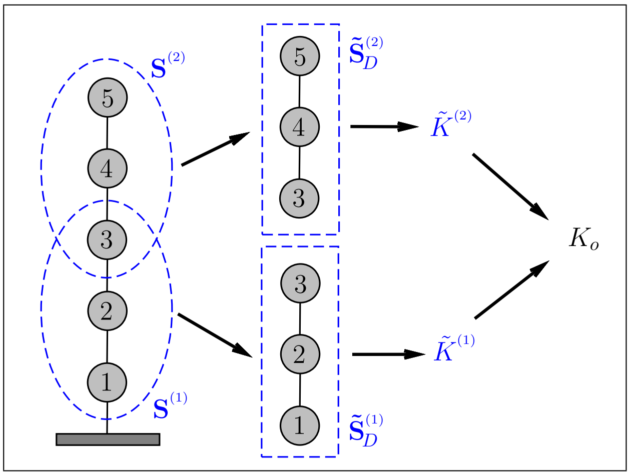



Figure 7: Overlapping controller design

After performing the decoupled decomposition as indicated in Subsection 2.2, we obtain the decoupled expanded systems $\tilde{\mathbf{S}}_D^{(1)}$ and $\tilde{\mathbf{S}}_D^{(2)}$. The decoupled expanded state matrices are

$$\tilde{A}_{11} = 10^3 \times$$
$$\begin{bmatrix} 0 & 0.0010 & 0 & 0 & 0 & 0\\ -0.6957 & -0.0029 & 0.6957 & 0.0024 & 0 & 0\\ 0 & 0 & 0 & 0.0010 & 0 & 0\\ 0.6957 & 0.0024 & -1.3915 & -0.0053 & 0.6957 & 0.0024\\ 0 & 0 & 0 & 0 & 0 & 0.0010\\ 0 & 0 & 0.6957 & 0.0024 & -1.3915 & -0.0053 \end{bmatrix}, \quad (20)$$

$$\tilde{A}_{22} = 10^3 \times$$
$$\begin{bmatrix} 0 & 0.0010 & 0 & 0 & 0 & 0\\ -1.3915 & -0.0053 & 0.6957 & 0.0024 & 0 & 0\\ 0 & 0 & 0 & 0.0010 & 0 & 0\\ 0.6957 & 0.0024 & -1.3915 & -0.0053 & 0.6957 & 0.0024\\ 0 & 0 & 0 & 0 & 0 & 0.0010\\ 0 & 0 & 0.6957 & 0.0024 & -1.3915 & -0.0053 \end{bmatrix}. \quad (21)$$

### 5.2.1 Direct actuation

For the direct actuation scheme, the decoupled expanded input matrices are

$$\tilde{B}_{11}^{(d)} = \tilde{B}_{22}^{(d)} = 10^{-5} \times \begin{bmatrix} 0 & 0 & 0\\ 0.4638 & 0 & 0\\ 0 & 0 & 0\\ -0.4638 & 0.4638 & 0\\ 0 & 0 & 0\\ 0 & -0.4638 & 0.4638 \end{bmatrix}. \quad (22)$$

To compute the LQR expanded controllers, we take the quadratic cost indexes $\tilde{J}_D^{(1)}$, $\tilde{J}_D^{(2)}$ (10,11) with weighting matrices

$$\tilde{Q}_1^* = \tilde{Q}_2^* = \begin{bmatrix} 1&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&0 \end{bmatrix}, \quad \tilde{R}_1^* = \tilde{R}_2^* = 10^{-16.5} \times I_3. \quad (23)$$

For the expanded system $\tilde{\mathbf{S}}_D^{(1)}$, defined by $\tilde{A}_{11}$ and $\tilde{B}_{11}^{(d)}$ (20,22), we obtain the expanded control matrix

$$\tilde{K}_d^{(1)} = 10^8 \times \begin{bmatrix} .7104 & .0491 & -.1649 & -.0019 & -.0673 & -.0025 \\ .5455 & .0472 & .6431 & .0466 & -.1649 & -.0019 \\ .4782 & .0447 & .5455 & .0472 & .7104 & .0492 \end{bmatrix};$$

analogously, for the system $\tilde{\mathbf{S}}_D^{(2)}$ defined by $\tilde{A}_{22}$, $\tilde{B}_{22}^{(d)}$ (21,22), we get

$$\tilde{K}_d^{(2)} = 10^8 \times \begin{bmatrix} .4100 & .0167 & -.3254 & -.0241 & -.1670 & -.0140 \\ .1456 & .0252 & .5413 & .0353 & -.1841 & -.0074 \\ .0252 & .0242 & .4032 & .0382 & .0690 & .0456 \end{bmatrix}.$$

The contraction of the block diagonal expanded control matrix $\tilde{K}_D^{(d)} = \text{diag}\left\{\tilde{K}_d^{(1)}, \tilde{K}_d^{(2)}\right\}$ produces the semi-decentralized overlapping control matrix

$$K_o^{(d)} = Q\tilde{K}_D^{(d)}V = 10^8 \times \begin{bmatrix} .710 & .049 & -.165 & -.002 & -.067 & -.003 & 0 & 0 & 0 & 0 \\ .545 & .047 & .643 & .047 & -.165 & -.002 & 0 & 0 & 0 & 0 \\ .239 & .022 & .273 & .024 & .560 & .033 & -.163 & -.012 & -.083 & -.007 \\ 0 & 0 & 0 & 0 & .147 & .025 & .541 & .035 & -.184 & -.007 \\ 0 & 0 & 0 & 0 & .025 & .024 & .403 & .038 & .690 & .046 \end{bmatrix}. \quad (24)$$

### 5.2.2 Inter-story actuation

For the inter-story actuation scheme, the decoupled expanded input matrices are

$$\tilde{B}_{11}^{(is)} = 10^{-5} \times \begin{bmatrix} 0 & 0 & 0 \\ -0.4638 & 0.4638 & 0 \\ 0 & 0 & 0 \\ 0.4638 & -0.9276 & 0.4638 \\ 0 & 0 & 0 \\ 0 & 0.4638 & -0.9276 \end{bmatrix}, \quad (25)$$

$$\tilde{B}_{22}^{(is)} = 10^{-5} \times \begin{bmatrix} 0 & 0 & 0 \\ -0.9276 & 0.4638 & 0 \\ 0 & 0 & 0 \\ 0.4638 & -0.9276 & 0.4638 \\ 0 & 0 & 0 \\ 0 & 0.4638 & -0.9276 \end{bmatrix}. \quad (26)$$

With the same quadratic indexes $\tilde{J}_D^{(1)}$, $\tilde{J}_D^{(2)}$ given in (10,11,23), the expanded system $\tilde{\mathbf{S}}_D^{(1)}$ defined by the matrices $\tilde{A}_{11}$ and $\tilde{B}_{11}^{(is)}$ (20,25) produces the expanded control matrix

$$\tilde{K}_{is}^{(1)} = 10^8 \times \begin{bmatrix} -.8264 & -.0849 & .0000 & -.0388 & .0000 & -.0168 \\ .0000 & -.0388 & -.8264 & -.0629 & .0000 & -.0219 \\ .0000 & -.0168 & .0000 & -.0219 & -.8264 & -.0461 \end{bmatrix},$$

and for the system $\tilde{\mathbf{S}}_D^{(2)}$, defined by $\tilde{A}_{22}$ and $\tilde{B}_{22}^{(is)}$ (21,26), results

$$\tilde{K}_{is}^{(2)} = 10^8 \times \begin{bmatrix} -.8264 & -.0431 & .0000 & -.0155 & .0000 & -.0062 \\ .0000 & -.0155 & -.8264 & -.0492 & .0000 & -.0155 \\ .0000 & -.0062 & .0000 & -.0155 & -.8264 & -.0431 \end{bmatrix}.$$

After the contraction process, we obtain

$$K_o^{(is)} = 10^8 \times \begin{bmatrix} -.826 & -.085 & .000 & -.039 & .000 & -.017 & 0 & 0 & 0 & 0 \\ .000 & -.039 & -.826 & -.063 & .000 & -.022 & 0 & 0 & 0 & 0 \\ .000 & -.084 & .000 & -.011 & -.826 & -.045 & .000 & -.008 & .000 & -.003 \\ 0 & 0 & 0 & 0 & .000 & -.016 & -.826 & -.049 & .000 & -.016 \\ 0 & 0 & 0 & 0 & .000 & -.006 & .000 & -.016 & -.826 & .043 \end{bmatrix}. \quad (27)$$

Note that, due to the particular structure of the overlapping control matrices (24,27), a local controller with wireless communications system would only need to cover a range of half building to compute the control vectors $u_o^{(d)} = -K_o^{(d)}x(t)$ and $u_o^{(is)} = -K_o^{(is)}x(t)$. The average values of the quadratic index $J_c$ corresponding to the overlapping controllers are $[J_c]_{K_o^{(d)}} = 0.3525$, $[J_c]_{K_o^{(is)}} = 0.4430$.

## 5.3 Multi-overlapping controllers

In this subsection, the ideas presented in Section 3 are applied to the design of semi-decentralized multi-overlapping controllers for the longitudinal multi-overlapping decomposition depicted in Fig. 8. Following the terminology of Section 3, *Stage 1* has been completed in Subsection 5.2 and we have now two decoupled expanded decompositions

$$\left\{\tilde{\mathbf{S}}_D\right\}^{(d)} = \left[\left\{\tilde{\mathbf{S}}_D^{(1)}\right\}^{(d)}, \left\{\tilde{\mathbf{S}}_D^{(2)}\right\}^{(d)}\right],$$

$$\left\{\tilde{\mathbf{S}}_D\right\}^{(is)} = \left[\left\{\tilde{\mathbf{S}}_D^{(1)}\right\}^{(is)}, \left\{\tilde{\mathbf{S}}_D^{(2)}\right\}^{(is)}\right].$$

To complete *Stage 2*, we start by introducing a more suitable notation for the system matrices of the subsystems $\tilde{\mathbf{S}}_D^{(j)}$ in the form

$$\tilde{\mathbf{S}}_D^{(j)} = \left[\tilde{A}^{(j)}, \tilde{B}^{(j)}\right];$$

when convenient, notations such as $\left\{\tilde{\mathbf{S}}_D^{(j)}\right\}^{(d)}$ or $\left\{\tilde{B}^{(j)}\right\}^{(is)}$ will be used to indicate the actuation scheme type. With the notations of Subsection 5.2, we have

$$\tilde{A}^{(1)} = \tilde{A}_{11},\ \tilde{A}^{(2)} = \tilde{A}_{22},$$

$$\left\{\tilde{B}^{(1)}\right\}^{(d)} = \tilde{B}_{11}^{(d)},\ \left\{\tilde{B}^{(2)}\right\}^{(d)} = \tilde{B}_{22}^{(d)},$$

$$\left\{\tilde{B}^{(1)}\right\}^{(is)} = \tilde{B}_{11}^{(is)},\ \left\{\tilde{B}^{(2)}\right\}^{(is)} = \tilde{B}_{22}^{(is)}.$$

Now, we observe that each subsystem $\tilde{\mathbf{S}}_D^{(j)}$ admits a new overlapping decomposition and define a second set of expansion matrices

$$V^{(2)} = \begin{bmatrix} I_2 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_2 \end{bmatrix}, \quad R^{(2)} = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_1 & 0 \\ 0 & I_1 & 0 \\ 0 & 0 & I_1 \end{bmatrix},$$

to obtain the decoupled expansions

$$\tilde{\mathbf{S}}_D^{(1)} = \left[\tilde{\mathbf{S}}_D^{(11)}, \tilde{\mathbf{S}}_D^{(12)}\right],\ \tilde{\mathbf{S}}_D^{(2)} = \left[\tilde{\mathbf{S}}_D^{(21)}, \tilde{\mathbf{S}}_D^{(22)}\right],$$

with

$$\tilde{\mathbf{S}}_D^{(ij)} = \left[\tilde{A}^{(ij)}, \tilde{B}^{(ij)}\right],\ i = 1, 2,\ j = 1, 2.$$

Next, we separately discuss the direct actuation and the inter-story actuation cases.

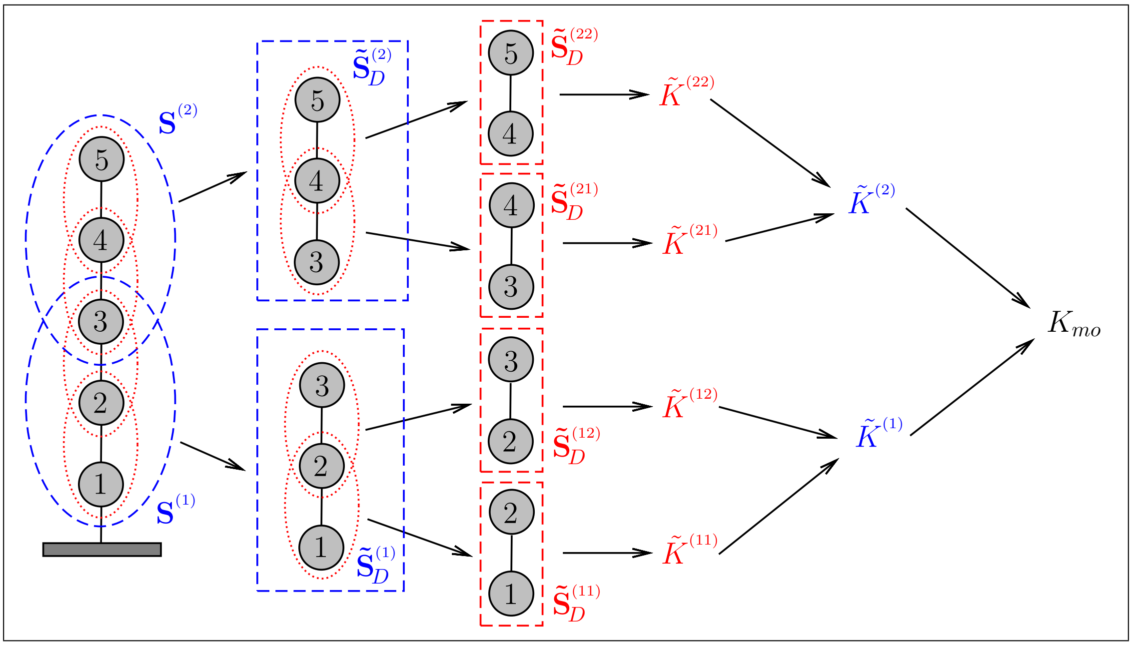



Figure 8: Multi-overlapping expansion-contraction process

### 5.3.1 Direct actuation

After completing the expansion and decoupling process on $\tilde{\mathbf{S}}_D^{(1)}$, $\tilde{\mathbf{S}}_D^{(2)}$, we obtain

$$\tilde{A}^{(11)} = 10^3 \times \begin{bmatrix} 0 & 0.0010 & 0 & 0 \\ -0.6957 & -0.0029 & 0.6957 & 0.0024 \\ 0 & 0 & 0. & 0.0010 \\ 0.6957 & 0.0024 & -1.3915 & -0.0053 \end{bmatrix},$$

$$\tilde{A}^{(12)} = 10^3 \times \begin{bmatrix} 0 & 0.0010 & 0 & 0 \\ -1.3915 & -0.0053 & 0.6957 & 0.0024 \\ 0 & 0 & 0. & 0.0010 \\ 0.6957 & 0.0024 & -1.3915 & -0.0053 \end{bmatrix},$$

$$\tilde{A}^{(21)} = \tilde{A}^{(22)} = \tilde{A}^{(12)},$$

$$\left\{\tilde{B}^{(ij)}\right\}^{(d)} = 10^{-5} \times \begin{bmatrix} 0 & 0 \\ 0.4638 & 0 \\ 0 & 0 \\ -0.4638 & 0.4638 \end{bmatrix}, i = 1, 2, j = 1, 2.$$

To design the expanded LQR controllers for the subsystems $\left\{\tilde{\mathbf{S}}_D^{(ij)}\right\}^{(d)}$, we consider the weighting matrices

$$\tilde{Q}^* = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \tilde{R}^* = 10^{-16.5} \times I_2, \tag{28}$$

obtaining

$$\left\{\tilde{K}^{(11)}\right\}^{(d)} = 10^8 \times \begin{bmatrix} .7424 & .0506 & -.1229 & -.0001 \\ .6195 & .0505 & .7424 & .0506 \end{bmatrix},$$

$$\left\{\tilde{K}^{(12)}\right\}^{(d)} = 10^8 \times \begin{bmatrix} .4074 & .0236 & -.2242 & -.0130 \\ .1740 & .0316 & .6655 & .0448 \end{bmatrix},$$

$$\left\{\tilde{K}^{(21)}\right\}^{(d)} = \left\{\tilde{K}^{(22)}\right\}^{(d)} = \left\{\tilde{K}^{(12)}\right\}^{(d)}.$$

A first contraction step on

$$\left\{\tilde{K}_D^{(1)}\right\}^{(d)} = \mathrm{diag}\left(\left\{\tilde{K}^{(11)}\right\}^{(d)}, \left\{\tilde{K}^{(12)}\right\}^{(d)}\right),$$

$$\left\{\tilde{K}_D^{(2)}\right\}^{(d)} = \mathrm{diag}\left(\left\{\tilde{K}^{(21)}\right\}^{(d)}, \left\{\tilde{K}^{(22)}\right\}^{(d)}\right),$$

produces

$$\left\{\tilde{K}^{(1)}\right\}^{(d)} = 10^8 \times \begin{bmatrix} .7424 & .0562 & -.1230 & -.0001 & 0 & 0 \\ .3098 & .0257 & .5749 & .0371 & -.1121 & -.0065 \\ 0 & 0 & .1740 & .0317 & .6655 & .0448 \end{bmatrix},$$

$$\left\{\tilde{K}^{(2)}\right\}^{(d)} = 10^8 \times \begin{bmatrix} .4074 & .0236 & -.2242 & -.0130 & 0 & 0 \\ .0870 & .0159 & .5364 & .0342 & -.1121 & -.0065 \\ 0 & 0 & .1740 & .0317 & .6655 & .0448 \end{bmatrix},$$

and a second contraction step produces the multi-overlapping controller

$$K_{mo}^{(d)} = 10^8 \times \begin{bmatrix} .742 & .051 & -.123 & .000 & 0 & 0 & 0 & 0 & 0 & 0 \\ .310 & .025 & .579 & .037 & -.112 & -.007 & 0 & 0 & 0 & 0 \\ 0 & 0 & .087 & .016 & .536 & -.034 & -.112 & -.007 & 0 & 0 \\ 0 & 0 & 0 & 0 & .087 & .016 & .536 & .034 & -.112 & -.007 \\ 0 & 0 & 0 & 0 & 0 & 0 & .174 & .032 & .665 & .044 \end{bmatrix}. \tag{29}$$

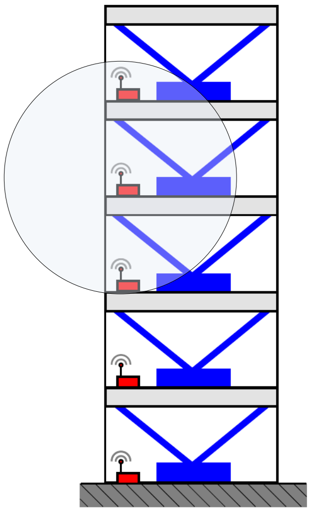


Figure 9: Wireless transmission range required by the multi-overlapping controllers

### 5.3.2 Inter-story actuation

For the inter-story actuation scheme, we obtain the decoupled expanded input matrices

$$\left\{\tilde{B}^{(11)}\right\}^{(is)} = 10^{-5} \times \begin{bmatrix} 0 & 0 \\ -0.4638 & 0.4638 \\ 0 & 0 \\ 0.4638 & -0.9276 \end{bmatrix},$$

$$\left\{\tilde{B}^{(12)}\right\}^{(is)} = 10^{-5} \times \begin{bmatrix} 0 & 0 \\ -0.9276 & 0.4638 \\ 0 & 0 \\ 0.4638 & -0.9276 \end{bmatrix},$$

$$\left\{\tilde{B}^{(21)}\right\}^{(is)} = \left\{\tilde{B}^{(22)}\right\}^{(is)} = \left\{\tilde{B}^{(12)}\right\}^{(is)}.$$

With the weighting matrices given in (28), the expanded LQR controllers for the decoupled subsystems $\left\{\tilde{\mathbf{S}}_D^{(ij)}\right\}^{(is)}$ are

$$\left\{\tilde{K}^{(11)}\right\}^{(is)} = 10^{8} \times \begin{bmatrix} -.8264 & -.0730 & .0000 & -.0255 \\ .0000 & -.0255 & -.8264 & -.0475 \end{bmatrix},$$

$$\left\{\tilde{K}^{(12)}\right\}^{(is)} = 10^{8} \times \begin{bmatrix} -.8264 & -.0416 & .0000 & -.0122 \\ .0000 & -.0122 & -.8264 & -.0416 \end{bmatrix},$$

$$\left\{\tilde{K}^{(21)}\right\}^{(is)} = \left\{\tilde{K}^{(22)}\right\}^{(is)} = \left\{\tilde{K}^{(12)}\right\}^{(is)}.$$

After two contraction steps, the following multi-overlapping controller results

$$K_{mo}^{(is)} = 10^{8} \times \begin{bmatrix} -.826 & -.073 & .000 & -.025 & 0 & 0 & 0 & 0 & 0 & 0 \\ .000 & -.013 & -.826 & -.045 & .000 & -.064 & 0 & 0 & 0 & 0 \\ 0 & 0 & .000 & -.006 & -.826 & -.042 & .000 & -.006 & 0 & 0 \\ 0 & 0 & 0 & 0 & .000 & -.006 & -.826 & -.042 & .000 & -.006 \\ 0 & 0 & 0 & 0 & 0 & 0 & .000 & -.012 & -.826 & -.042 \end{bmatrix}. \quad (30)$$

It is worth to be highlighted that only two four-dimensional LQR problems have been actually solved in the design of the ten-dimensional multi-overlapping controllers. It should also be noted that, due to the block tridiagonal structure of the multi-overlapping control matrices (29,30), to compute the control vectors

$$u_{mo}^{(d)} = -K_{mo}^{(d)} x(t), \quad u_{mo}^{(is)} = -K_{mo}^{(is)} x(t),$$

by means of wireless local controllers, a transmission range of only one story is required (see Fig. 9). The average values of the quadratic index $J_c$ corresponding to the multi-overlapping controllers are $[J_c]_{K_{mo}^{(d)}} = 0.3675$, $[J_c]_{K_{mo}^{(is)}} = 0.4881$.

In Tables 1 and 2, the average quadratic cost for all the designed controllers are collected and compared. As could be expected, the obtained semi-decentralized controllers are suboptimal: the overlapping controllers produce a moderate increase, with respect to centralized optimal controller, of about 18%; additional increases of 5.1% and 12.1% for the direct and inter-story actuation schemes, respectively, are introduced by the multi-overlapping controllers. However, the relatively higher values associated to the multi-overlapping controllers might be seen in the light of its remarkable features: reduced information exchange, short-range transmission requirements, and computational efficiency.

## 6 Numerical simulations

In this section, a set of numerical simulations are conducted in order to gain a clearer insight on the behavior of the semi-decentralized controllers designed in Section 5; in all the cases, the behavior of the corresponding centralized controllers have been taken as reference. For clarity and simplicity, the study has been restricted to a couple of relevant aspects: *maximum absolute inter-story drifts*, and *maximum absolute control efforts.*

Ground acceleration records of four real earthquakes have been used as seismic disturbances: (i) *Hachinohe,* North-South (NS) component recorded at Hachinohe City during the Tokachi-oki earthquake of May 16, 1968 (Magnitude 8.5); (ii) *El Centro,* NS component recorded at the Imperial Valley Irrigation District substation in El Centro, California, during the Imperial Valley, California earthquake of May 18, 1940 (Magnitude 5.5); (iii) *Northridge,* NS component recorded at Sylmar County Hospital in Sylmar, California, during the Northridge, California earthquake of January 17, 1994 (Magnitude 6.5); and (iv) *Kobe,* NS component recorded at the Kobe Japanese Meterological Agency

Table 1: Quadratic cost: direct actuation

| | $[J_c^{(d)}]_{opt}$ | $[J_c]_{K_o^{(d)}}$ | $[J_c]_{K_{mo}^{(d)}}$ |
|---|---|---|---|
| Value | 0.2974 | 0.3525 | 0.3675 |
| % increase | — | 18.5 | 23.6 |

Table 2: Quadratic cost: inter-story actuation

| | $[J_c^{(is)}]_{opt}$ | $[J_c]_{K_o^{(is)}}$ | $[J_c]_{K_{mo}^{(is)}}$ |
|---|---|---|---|
| Value | 0.3794 | 0.4430 | 0.4881 |
| % increase | — | 18.1 | 30.2 |

station during the Hyogoken-Nanbu earthquake of January 17, 1995 (Magnitude 6.7). (Ohtori et al., 2004; USGS, 2011).

Hachinohe and El Centro are far-field records measured at far-from-epicentre seismic stations, the respective absolute acceleration peaks are 2.25 and 3.42 m/s$^2$. Northridge and Kobe are near-field records corresponding to close-to-epicentre stations. These near-field records present large acceleration peaks of 8.27 and 8.18 m/s$^2$, respectively, associated to large displacement pulses that are extremely destructive to tall building structures (Huang and Chen, 2000).

Figures 10–13 display the seismic disturbance records and the corresponding simulation outputs. The graphics of simulated outputs present the maximum absolute inter-story drifts together with the associated maximum absolute control efforts obtained for the controlled building with: (i) centralized controller (blue circles), (ii) overlapping controller (green triangles) (iii) multi-overlapping controller (black asterisks); the maximum absolute inter-story drifts of the uncontrolled building response (red squares) are also included as reference. Results corresponding to the direct and inter-story actuation scheme, are presented in separate frames.

The simulation results show that the overlapping and multi-overlapping controllers achieve levels of performance similar to those obtained by their centralized counterparts. Attending to the seismic acceleration peak magnitude, the behavior of the semi-decentralized controllers is excellent for the low-peak far-field disturbances, and still quite good for the large-peak near-field ones. Specially remarkable is the behavior of the semi-decentralized controllers under the inter-story actuation scheme, for which particularly well shaped output patters are obtained.

Regarding to the control efforts, the controllers with direct actuation scheme demand lower levels of actuation forces; however, the interest of this case is merely theoretical, since no technical means are currently available to practically implement this type of control systems. The situation is completely different for the inter-story actuation scheme, in this case a variety of large-scale semi-active devices are currently available and under development. Moreover, measures of the inter-story drifts and velocities can be obtained by stroke sensors (Wang, 2011), active actuation systems driven by LQR controllers may be properly implemented by semi-active devices (Ou and Li, 2010), and large actuation forces may be generated with minimal or even null power supply. For instance, 1MN ($10^6$N) damping forces may be obtained with the semi-active hydraulic dampers installed in the Kajima Sizuoka Buildign (Kurata et al., 1999; Spencer and Nagarajaiah, 2003), and 2MN damping forces can be produced by the passive hydraulic damper with semi-active characteristics reported in Kurino et al. (2004); the 1MN semi-active damper only needs an electric power of 70W, and no power is required to operate the second. From this perspective, the control effort demanded by the inter-story control systems in the Hachinohe and El Centro simulations might be considered as practically implementable while, in a realistic simulation for the Northridge and Kobe near-field seismic excitations, the saturation of the actuator devices should be considered. Nevertheless, it has to be noted that in these cases the centralized controller would also be affected by the same saturation problem.

## Conclusions

In this work, the main ideas involved in the design of *overlapping* and *multi-overlapping controllers* via the *Inclusion Principle* have been presented. In wireless implementations of large-scale Structural Vibration Control systems, the use of semi-decentralized control strategies is highly convenient. In the particular case of vibrational control of tall buildings under seismic excitations, the multi-overlapping approach has shown to be a specially suitable choice, being able to reduce the design and operation computational effort and providing semi-decentralized controllers which satisfy the information exchange constraints imposed by wireless implementations. To illustrate the application of the theoretical ideas presented in the paper, a simplified dynamical model of a five-story building has been introduced. For this model, overlapping and multi-overlapping LQR controllers have been designed and numerical simulations, using four different earthquake records as input disturbances, have been conducted to asses the performance of the proposed semi-decentralized controllers with positive results.

## Acknowledgments

This work was supported in part by the Spanish Ministry of Science and Innovation through the grant DPI2008-06699-C02.

## References

Åström, K., Albertos, P., Isidori, A., Shaufelberger, W., and Sanz, R. *Control of Complex Systems*. Springer Verlag, 2001. doi:10.1007/978-1-4471-0349-3.

Ataslar, B. and İftar, A. Decentralized routing controller design using overlapping decompositions. *International Journal of Control*, 1999. 72(13):1175–1192.

Aybar, A., İftar, A., and Uzman, M. Decentralized routing control using overlapping decompositions. In *Proceedings of the 7th Mediterranean Electrotechnical Conference*, volume 2. pages 671–674, 1994.

Bakule, L., Paulet-Crainiceanu, F., Rodellar, J., and Rossell, J. M. Overlapping reliable control for a cable-stayed bridge benchmark. *IEEE Transactions on Control Systems Technology*, 2005. 13(4):663–669. doi:10.1109/TCST.2004.841678.

Bakule, L. and Rodellar, J. Decentralized control and overlapping decomposition of mechanical systems. Part 1: System decomposition. Part 2: Decentralized stabilization. *International Journal of Control*, 1995. 61(3):559–587. doi:10.1080/00207179508921918.

Bakule, L., Rodellar, J., and Rossell, J. M. Generalized selection of complementary matrices in the inclusion principle. *IEEE Transactions on Automatic Control*, 2000a. 45(6):1237–1243. doi:10.1109/9.863615.

Bakule, L., Rodellar, J., and Rossell, J. M. Structure of expansion-contraction matrices in the inclusion principle for dynamic systems. *SIAM Journal on Matrix Analysis and Applications*, 2000b. 21(4):1136–1155. doi:10.1137/S0895479898340548.

Chen, X.-B. and Stanković, S. Decomposition and decentralized control of systems with multi-overlapping structure. *Automatica*, 2005a. 41(10):1765–1772. doi:10.1016/j.automatica.2005.01.020.

Chen, X.-B. and Stanković, S. Structural control for overlapping interconnected power systems. In *Preprints of the International Conference on Control and Automation (ICCA2005)*, volume 2. Budapest, Hungary, pages 391–396, 2005b.

Chopra, A. K. *Dynamics of Structures. Theory and Applications to Earthquake Engineering. 3rd Edition.* Prentice Hall, 2007.

Chu, S., Soong, T., and Reinhorn, A. *Active, Hybrid and Semi-Active Structural Control*. Wiley, 2005.

Housner, G., Bergman, L., Caughey, T., and Chassiakos, A. Structural control: Past, present, and future. *Journal of Engineering Mechanics*, 1997. 123(9):897–971. doi:10.1061/(ASCE)0733-9399(1997)123:9(897).

Huang, C.-T. and Chen, S.-S. Near-field characteristics and engineering implications of the 1999 Chi-Chi earthquake. *Earthquake Engineering and Engineering Sysmology*, 2000. 2(1):23–41.

İftar, A. and Özgüner, U. Contractible controller design and optimal control with state and input inclusion. *Automatica*, 1990. 26(3):593–597. doi:10.1016/0005-1098(90)90031-C.

Ikeda, M. and Šiljak, D. Overlapping decentralized control with input, state, and output inclusion. *Control-Theory and Advanced Technology*, 1986. 2(2):155–172.

Kurata, N., Kobobi, T., Takahashi, M., Niwa, N., and Midorikawa, H. Actual seismic response controlled building with semi-active damper system. *Earthquake Engineering and Estructural Dynamics*, 1999. 28(11):1427–1447.

Kurino, H., Matsunaga, Y., Yamada, T., and Tagami, J. High performance passive hydraulic damper with semi-active characteristics. Paper No. 33. In *13th World Conference on Earthquake Engineering, Vancouver, B. C., Canada*. 2004.

Law, K., Lynch, J., and Wang, Y. Decentralized control strategies with wireless sensing and actuation. In *Proceedings of 2009 NFS CMMI Engineering Research and Innovation Conference, Honolulu, Hawaii*. 2009.

Li, K., Kosmatopoulos, E., Ioannou, E., and Boussalis, H. Centralized, decentralized, and overlapping control design for a segment telescope. In *Proceedings of the 1999 IEEE International Symposium on Intelligent Control/Intelligent Systems and Semiotics*. Cambridge, MA, pages 173–178, 1999. doi:10.1109/ISIC.1999.796650.

Lynch, J., Wang, Y., Swartz, R., Lu, K., and Loh, C. Implementation of a closed-loop structural control system using wireless sensor networks. *Structural Control and Health Monitoring*, 2008. 15(4):518–539. doi:10.1002/stc.214.

Ohtori, Y., Christenson, R., Spencer, B., and Dyke, S. Benchmark control problems for seismically excited nonlinear buildings. *Journal of Engineering Mechanics*, 2004. 130(4):366–385. doi:10.1061/(ASCE)0733-9399(2004)130:4(366).

Ou, J. and Li, H. Analysis of capability for semi-active or passive damping systems to achieve the performance of active control systems. *Structural Control and Health Monitoring*, 2010. 17(7):778–794. doi:10.1002/stc.408.

Palacios-Quiñonero, F., Rodellar, J., and Rossell, J. M. Sequential design of multi-overlapping controllers for longitudinal multi-overlapping systems. *Journal of Applied Mathematics and Computation*, 2010. 217(3):1170–1183. doi:10.1016/j.amc.2010.01.130.

Preumont, A. and Seto,K. *Active Control of Structures*. Wiley, UK, 2008. doi:10.1002/9780470715703.

Rossell, J. M., Palacios-Quiñonero, F., and Rodellar, J. Semi-decentralized output feedback $H_\infty$ control strategy for large building structures. In *Proceedings of the 5th. World Conference on Structural Control and Monitoring*. Shinjuku,Tokio, 2010.

Siljak, D. *Decentralized Control of Complex Systems*. Academic Press, New York, USA, 1991.

Siljak, D., Mladenović, S., and Stanković, S. Overlapping decentralized observation and control of a platoon of vehicles. In *Proceedings of the American Control Conference*. San Diego, California, USA, pages 4522–4526, 1999.

Spencer, B. and Nagarajaiah, S. State of the art of structural control. *Journal of Structural Engineering*, 2003. 129(7):845–856. doi:10.1061/(ASCE)0733-9445(2003)129:7(845).

Stanković, S., Stanojević, M., and Šiljak, D. Decentralized overlapping control of a platoon of vehicles. *IEEE Transactions on Control Systems Technology*, 2000. 8(5):816–832.

Stanković, S. and Šiljak, D. Contractibility of overlapping decentralized control. *Systems & Control Letters*, 2001. 44(3):189–199.

Stipanović, D., Inalhan, G., Teo, R., and Tomlin, C. Decentralized overlapping control of a formation of unmanned aerial vehicles. *Automatica*, 2004. 40(8): 1285–1296. doi:10.1016/j.automatica.2004.02.017.

Swartz, R. and Lynch, J. Strategic network utilization in a wireless structural control system for seismically excited structures. *Journal of Structural Engineering*, 2009. 135(5):597–608. doi:10.1061/(ASCE)ST.1943-541X.0000002.

USGS. Historic world earthquakes. 2011. URL `http://earthquake.usgs.gov/earthquakes/world/`.

Wang, Y. Time-delayed dynamic output feedback $H_\infty$ controller design for civil structures: A decentralized approach through homotopic transformation. *Structural Control and Health Monitoring*, 2011. 18(2): 121–139. doi:10.1002/stc.344.

Wang, Y., Lynch, J., and Law, K. Decentralized $H_\infty$ controller design for large-scale civil structures. *Earthquake Engineering and Structural Dynamics*, 2009. 38(3):377–401. doi:10.1002/eqe.862.

Wang, Y., Swartz, A., Lynch, J., Law, K., Lu, K.-C., and Loh, C.-H. Wireless feedback structural control with embedded computing. In *SPIE 13th Annual International Symposium on Smart Structures and Materials, San Diego, CA*. 2006.

Zečević, A. and Šiljak, D. *Control of Complex Systems. Structural Constraints and Uncertainty*. Springer, 2010.

Figure 10: Hachinohe 1968 seismic record. Maximum inter-story drifts and control efforts

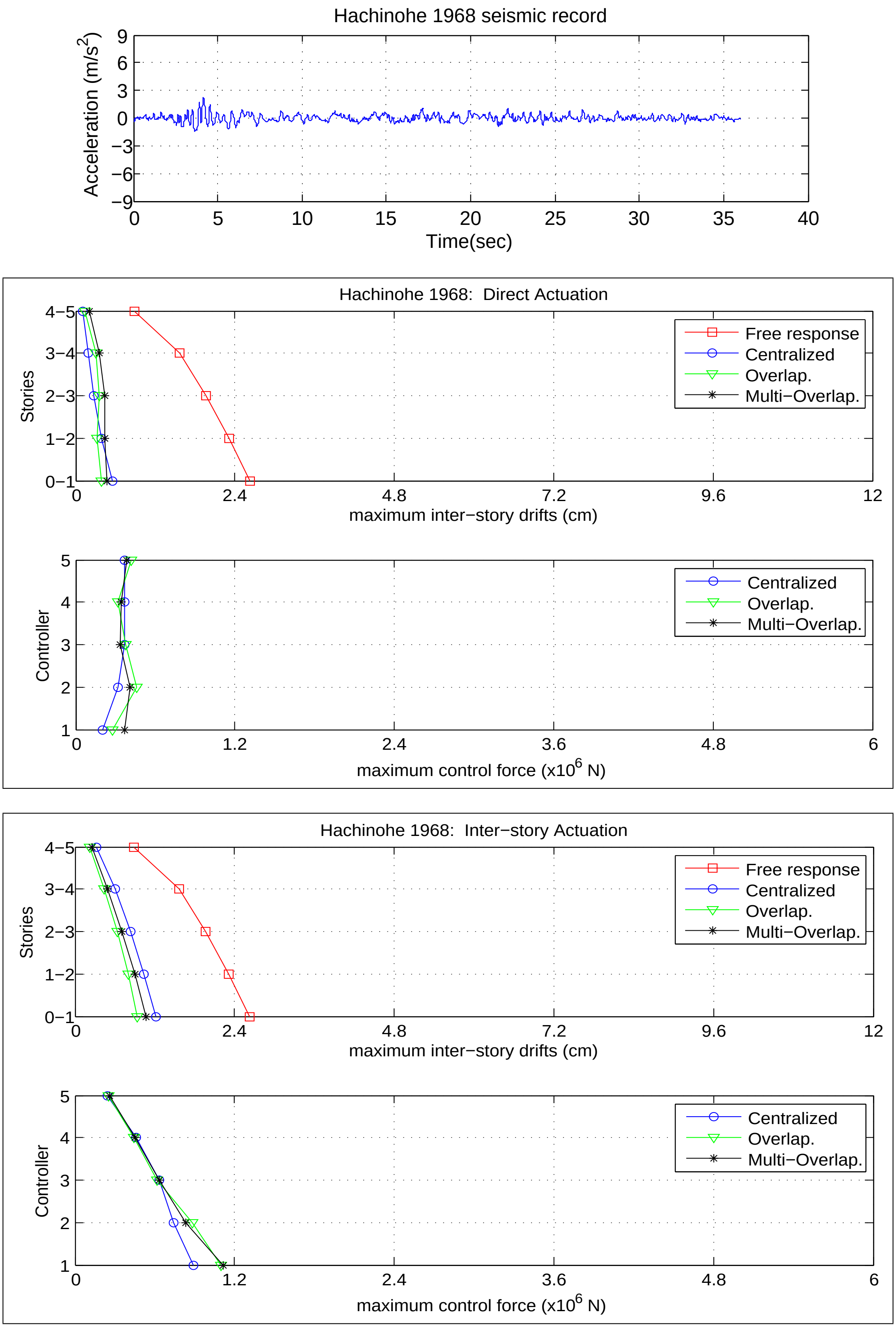
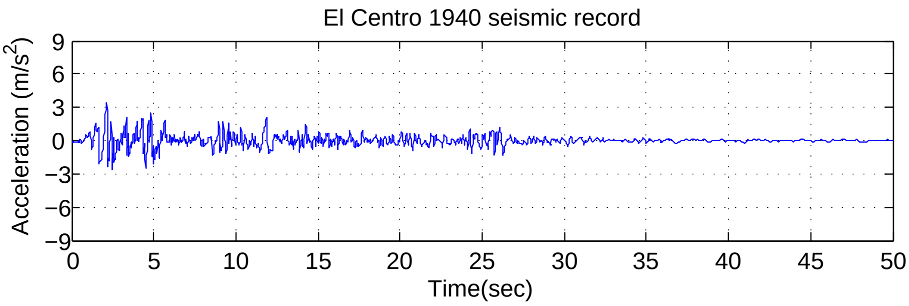
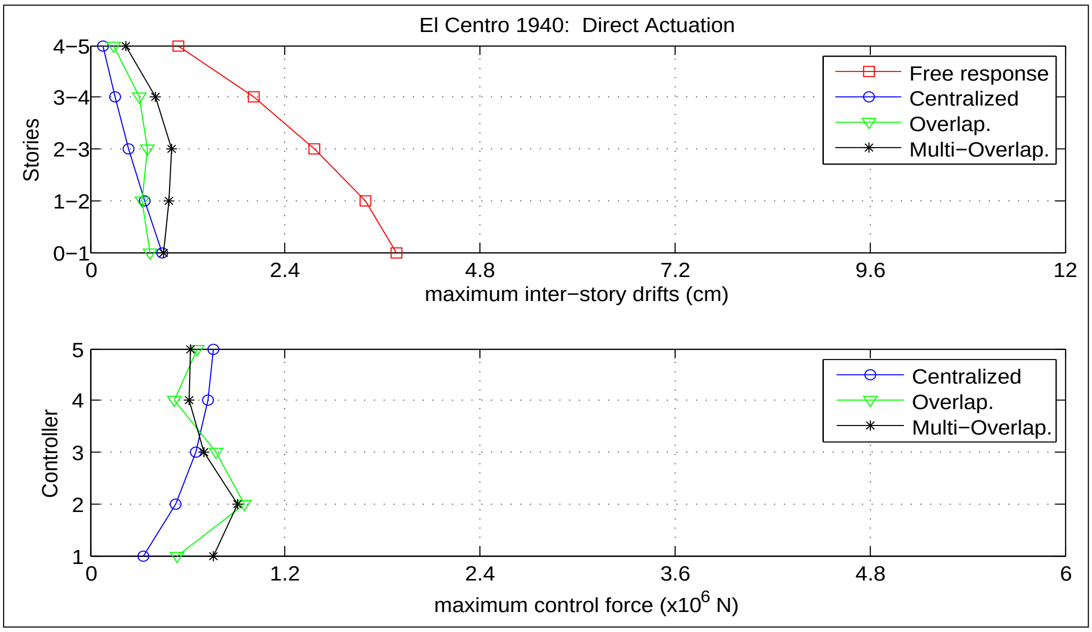


Figure 11: El Centro 1940 seismic record. Maximum inter-story drifts and control efforts



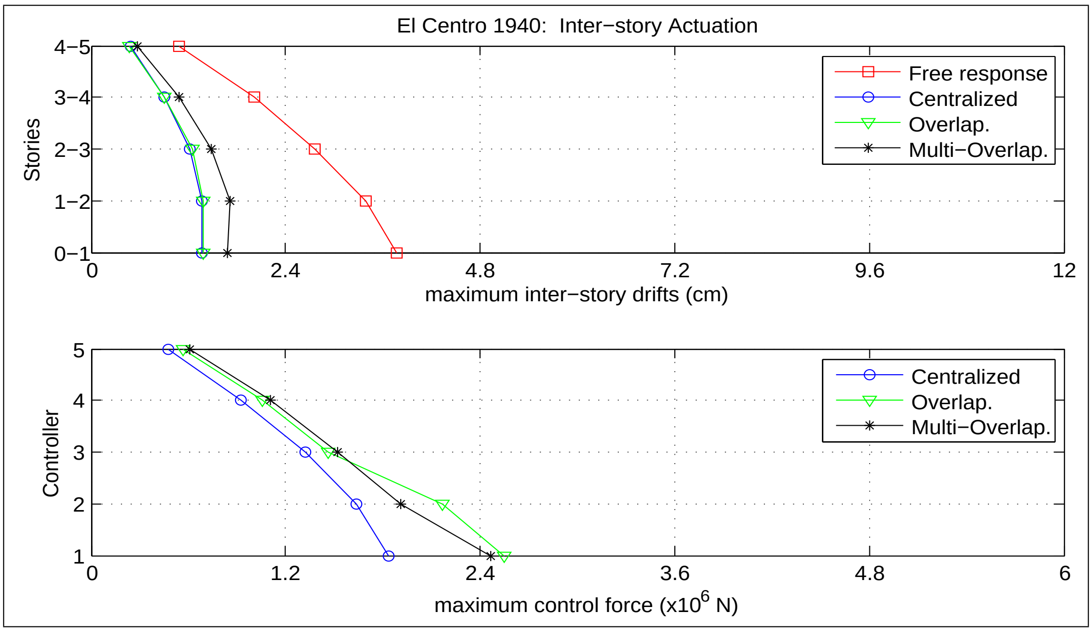



Figure 12: Northridge 1994 seismic record. Maximum inter-story drifts and control efforts

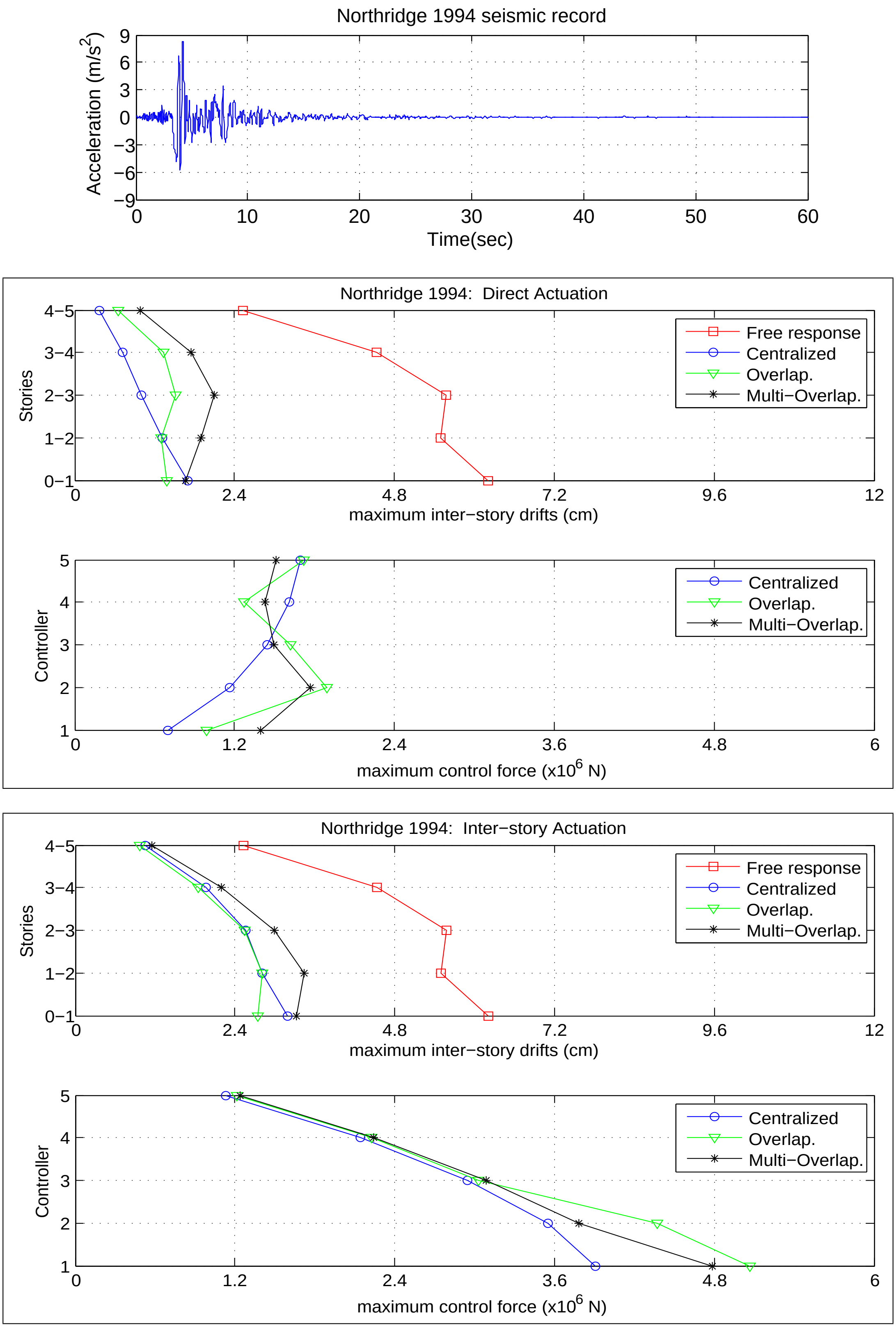


Figure 13: Kobe 1995 seismic record. Maximum inter-story drifts and control efforts