# Reduction of Operational Amplifiers Finite Gain Effects in Switched-Capacitor Biquads 

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#### Abstract

A combined approach for reducing the errors in the pole frequency $f_{p}$, the pole $Q$ - factor $Q_{p}$ and the magnitude at the pole frequency $H_{p}$, of switched capacitor biquads is presented. First, the conventional integrators in the biquads are replaced with gain-and offset-compensated integrators. Next, the errors $\Delta f_{p} / f_{p}, \Delta Q_{p} / Q_{p}$ and $\Delta H_{p} / H_{p}$ are minimized by modifying three capacitances: two feedback capacitances and feed forward capacitance. The effectiveness of this approach is demonstrated by designing a bandpass biquad.


Keywords: Filters, Gain-and offset-compensation, Operational amplifiers, Switched -capacitor integrators.

## 1 Introduction

Over the past twenty years several switched-capacitor (SC) biquads have been reported and used in practical applications. A systematic comparison for the realization of SC filters using most popular SC biquads is given in [1]. The biquads being compared are:

Type-E: Fleischer and Laker's E-type biquad in [2];
Type-F: Fleischer and Laker's F-type biquad in [2];
FGL Type: Modified Fleischer and Laker's biquads in [3];
G-T Type: Gregorian and Temes's biquad in [4];
M-S Type: Martin and Sedra's biquad in [5];
SSGI Type and SSGII Type: Sanchez-Sinencio, Silva-Martinez and Geiger's type-I and type-II biquads in [6]; and
Nagaraj type: Nagaraj's biquad proposed in [7].

[^0]All of these biquads are realized using a feedback loop containing one inverting and one non-inverting conventional integrators.

Two important factors, which limit the performance of the SC integrators, are the finite dc gain $A_{0}$ and the finite bandwidth GB of the operational amplifiers (op amps). However, in SC circuits the distortion, introduced by the finite gain $A_{0}$, is more pronounced than that by the finite bandwidth [8]. On the other hand, a nonzero input-referred op amp offset voltage $V_{O S}$ introduces an output offset voltage that may become a significant limitation to the permissible signal swing. This has led to the development of gain-and offset-compensated (GOC) integrators where the phase error $\theta(\omega)$ is proportional to $1 / A_{0}{ }^{2}$ (in a conventional integrator this is a simple inverse dependence $1 / A_{0}$ ). In most of the GOC integrators, reported in the literature, the reduction in phase error was obtained at the expense of increased gain error $m(\omega)$. According to the authors knowledge, the Betts-91 [9] and the Shafeeu-91 [10] circuits are the two GOC integrators that have the same sample correction property, which results in simultaneous reduction of gain and phase errors. The Betts-91 integrator is however quite complex. The Shafeeu- 91 integrator uses fewer components but requires a four-phase clock. The simple bi-phase GOC Nagaraj-86 [11] and Ki89 [12] integrators can be directly interconnected to form an excellent GOC integrator-pair, without the use of extra clock phases or sampling circuits to satisfy the sampling conditions.

The gain errors of the integrators affect the pole frequency $f_{p}$ of the biquad while the phase errors affect the pole quality factor $Q_{p}$ and the magnitude of the biquad transfer function $H_{p}$ at the pole frequency.

In this paper, an approach for decreasing of the errors in the pole frequency $f_{p}$, in the pole $Q$-factor $Q_{p}$ and in the magnitude at the pole frequency $H_{p}$ of SC biquads is proposed. It is based on the use of simple and fast amplifiers with low but precisely known and stable op amps dc gain [13-15]. The op amp, proposed in [13], has a nominal dc gain of about 40 dB , that varies by $\pm 0.7 \mathrm{~dB}$ for all possible technological spreads and temperature variations. The effectiveness of the approach proposed is demonstrated by designing a bandpass SC biquad.

## 2 Proposed Design Approach

The z-domain biquadratic transfer function has the general form

$$
\begin{equation*}
H(z)=\frac{N_{0}+N_{1} z^{-1}+N_{2} z^{-2}}{D_{0}+D_{1} z^{-1}+D_{2} z^{-2}}=k \frac{1+a_{1} z^{-1}+a_{2} z^{-2}}{1+b_{1} z^{-1}+b_{2} z^{-2}}=\frac{N(z)}{D(z)} \tag{1}
\end{equation*}
$$

where $z=\exp \left(j 2 \pi f / f_{s}\right)$, with $f_{s}$ denoting the sampling frequency.
For a given SC structure, in standard design the op amp gain value $A_{0}$ is assumed to be infinite. Then the coefficients in (1) are functions only of the capacitances. In the depend of the proposed SC biquads the denominator $D(z)$ comprises at least five feedback capacitors: two integrating capacitors around the two op amps, one damping capacitor and two capacitors connected between the two integrators to form a feedback loop. These capacitors determine the pole frequency $f_{p}$ and the pole quality factor $Q_{p}$ in the biquad. In addition to the feedback capacitors the numerator $N(z)$ comprises the feed forward capacitors. These capacitors control the gain $k$ and permit the realization of the different generic biquadratic transfer functions.

For any pair of complex conjugate poles in the $z$-domain, one can write the denominator as

$$
\begin{equation*}
1+b_{1} z^{-1}+b_{2} z^{-2}=1-2 R \cos \theta z^{-1}+R^{2} z^{-2} \tag{2}
\end{equation*}
$$

where $R$ is the radius and $\theta$ is the angle of the pole.
From (2) the following relationships for the pole frequency and the pole $Q$ factor can be derived:

$$
\begin{equation*}
f_{p}=\frac{f_{s}}{2 \pi} \sqrt{\theta^{2}+(\ln R)^{2}} \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
Q_{p}=-\frac{\pi f_{p} / f_{s}}{\ln R} \tag{4}
\end{equation*}
$$

For small ratio $f_{p} / f_{s}$ and high $Q$ - factor the pole frequency $f_{p}$ and the pole factor $Q_{p}$ are approximately given by

$$
\begin{equation*}
f_{p} \approx \frac{f_{s}}{2 \pi} \sqrt{1+b_{1}+b_{2}} \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
Q_{p} \approx \frac{\sqrt{1+b_{1}+b_{2}}}{1-b_{2}} . \tag{6}
\end{equation*}
$$

In the ideal case $\left(A_{0} \rightarrow \infty\right)$, from (5), (6) and (1) the logarithmic sensitivities of $f_{p}, Q_{p}$ and of the magnitude at the pole frequency $H_{p}$ to the capacitances $C_{q}$ can be obtained.

Using the simple classical definition of sensitivity of quantity $y$ on the variable $x$ :

$$
S_{x}^{y}=(\partial y / \partial x)(x / y)
$$

the results are

$$
\begin{gather*}
S_{C_{q}}^{f_{p}}=\frac{C_{q}}{2\left(1+b_{1}+b_{2}\right)}\left(\frac{\partial b_{1}}{\partial C_{q}}+\frac{\partial b_{2}}{\partial C_{q}}\right)  \tag{7}\\
S_{C_{q}}^{Q_{p}}=\frac{C_{q}}{2\left(1+b_{1}+b_{2}\right)}\left[\frac{\partial b_{1}}{\partial C_{q}}+\left(\frac{3+2 b_{1}+b_{2}}{1-b_{2}}\right) \frac{\partial b_{2}}{\partial C_{q}}\right] \tag{8}
\end{gather*}
$$

and

$$
\begin{align*}
& S_{C_{q}}^{H_{p}}=\frac{C_{q}\left[a_{1}+\left(1+a_{2}\right) \cos \left(\omega_{p} T_{s}\right)\right]}{N_{p}} \cdot \frac{\partial a_{1}}{\partial C_{q}}+ \\
& +\frac{C_{q}\left[a_{2}+a_{1} \cos \left(\omega_{p} T_{s}\right)+\cos \left(2 \omega_{p} T_{s}\right)\right]}{N_{p}} \cdot \frac{\partial a_{2}}{\partial C_{q}}-  \tag{9}\\
& -\frac{C_{q}\left[b_{1}+\left(1+b_{2}\right) \cos \left(\omega_{p} T_{s}\right)\right]}{D_{p}} \cdot \frac{\partial b_{1}}{\partial C_{q}} \\
& -\frac{C_{q}\left[b_{2}+b_{1} \cos \left(\omega_{p} T_{s}\right)+\cos \left(2 \omega_{p} T_{s}\right)\right]}{D_{p}} \cdot \frac{\partial b_{2}}{\partial C_{q}}+\frac{\partial k}{\partial C_{q}} \cdot \frac{C_{q}}{k},
\end{align*}
$$

where

$$
\begin{aligned}
& N_{p}=1+a_{1}^{2}+a_{2}^{2}+2 a_{1}\left(1+a_{2}\right) \cos \left(\omega_{p} T_{s}\right)+2 a_{2} \cos \left(2 \omega_{p} T_{s}\right) \\
& D_{p}=1+b_{1}^{2}+b_{2}^{2}+2 b_{1}\left(1+b_{2}\right) \cos \left(\omega_{p} T_{s}\right)+2 b_{2} \cos \left(2 \omega_{p} T_{s}\right)
\end{aligned}
$$

The relative deviations in the pole frequency $f_{p}$, in the pole $Q$ - factor $Q_{p}$ and in the magnitude at the pole frequency $H_{p}$ due to the small variation of the capacitances around their nominal values are approximately given by [16],

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$$
\begin{align*}
& \frac{\Delta f_{p}}{f_{p}} \approx \sum_{i=1}^{m_{C}}\left(S_{C_{i}}^{f_{p}} \frac{\Delta C_{i}}{C_{i}}\right)  \tag{10}\\
& \frac{\Delta Q_{p}}{Q_{p}} \approx \sum_{i=1}^{n_{C}}\left(S_{C_{i}}^{Q_{p}} \frac{\Delta C_{i}}{C_{i}}\right) \tag{11}
\end{align*}
$$

and

$$
\begin{equation*}
\frac{\Delta H_{p}}{H_{p}} \approx \sum_{i=1}^{P_{C}}\left(S_{C_{i}}^{H_{p}} \frac{\Delta C_{i}}{C_{i}}\right) \tag{12}
\end{equation*}
$$

where
$m_{C}$ is the number of capacitances in (5);
$n_{C}$ is the number of capacitances in (6); and
$p_{C}$ is the number of capacitances in (1).
The proposed approach for minimization of the errors $\Delta f_{p} / f_{p}, \Delta Q_{p} / Q_{p}$ and $\Delta H_{p} / H_{p}$ in SC biquads consists in the following consecutive steps:

Step 1. First, to reduce the effect of op amp imperfections (dc gain $A_{0}$ and offset voltage $V_{O S}$ ), the conventional integrators in the biquad considered are replaced with Nagaraj-86 [11] and Ki-89 [12] GOC SC integrators. The reduced phase errors of the GOC integrators provide a reduction in the errors $\Delta Q_{p} / Q_{p}$ and $\Delta H_{p} / H_{p}$.

Step 2. For the nominal value $A_{0}$ of the op amps dc gain, the errors $\Delta f_{p} / f_{p}, \Delta Q_{p} / Q_{p}$ and $\Delta H_{p} / H_{p}$ can be further minimized by modifying three capacitances: two feedback capacitances $C_{D_{1}}$ and $C_{D_{2}}$ from the denominator and one feed forward capacitance $C_{N}$ from the numerator of (1). These capacitances are chosen such that the following relation holds:

$$
\begin{equation*}
S_{C_{D_{1}}}^{f_{p}} S_{C_{D_{2}}}^{Q_{p}}-S_{C_{D_{2}}}^{f_{p}} S_{C_{D_{1}}}^{Q_{p}} \neq 0 \tag{13}
\end{equation*}
$$

The sensitivities $S_{C_{D_{i}}}^{f_{p}}, \quad S_{C_{D_{i}}}^{Q_{p}}, \quad S_{C_{D_{i}}}^{H_{p}}$ and $S_{C_{N}}^{H_{p}}$, defined by (7), (8) and (9), are calculated for the standard synthesis values of the capacitances, computed assuming the op amp gain $A_{0}$ to be infinite.

The relative errors $\Delta f_{p} / f_{p}, \Delta Q_{p} / Q_{p}$ and $\Delta H_{p} / H_{p}$ of the resulting non-ideal GOC biquad (with nominal finite dc gain $A_{0_{1}}=A_{0_{2}}=A_{0}$ ) are substi-

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tuted with opposite sign for the relative deviations into the expression (10), (11) and (12). This results in the following system:

$$
\begin{array}{r}
S_{C_{D_{1}}}^{f_{p}}\left(\frac{C_{D_{1}}^{\prime}}{C_{D_{1}}}-1\right)+S_{C_{D_{2}}}^{f_{p}}\left(\frac{C_{D_{2}}^{\prime}}{C_{D_{2}}}-1\right)=-\left(\frac{\Delta f_{p}}{f_{p}}\right) \\
S_{C_{D_{1}}}^{Q_{p}}\left(\frac{C_{D_{1}}^{\prime}}{C_{D_{1}}}-1\right)+S_{C_{D_{2}}}^{Q_{p}}\left(\frac{C_{D_{2}}^{\prime}}{C_{D_{2}}}-1\right)=-\left(\frac{\Delta Q_{p}}{Q_{p}}\right) \\
S_{C_{D_{1}}}^{H_{p}}\left(\frac{C_{D_{1}}^{\prime}}{C_{D_{1}}}-1\right)+S_{C_{D_{2}}}^{H_{p}}\left(\frac{C_{D_{2}}^{\prime}}{C_{D_{2}}}-1\right)+S_{C_{N}}^{H_{p}}\left(\frac{C_{N}^{\prime}}{C_{N}}-1\right)=-\left(\frac{\Delta H_{p}}{H_{p}}\right) . \tag{14c}
\end{array}
$$

The errors $\Delta f_{p} / f_{p}, \Delta Q_{p} / Q_{p}$ and $\Delta H_{p} / H_{p}$, due to the finite op amp gain $A_{0}$, can be calculated:
a) On account of (1), (2) (3) and (4) if the analytical expression for the nonideal $z$-domain biquadratic transfer function is known;
b) On the basis of the actual frequency response. This curve can be used to determine the actual pole frequency $f_{p a}$ and the actual pole - $Q$ factor $Q_{p a}$ according to the relationship $Q_{p a}=f_{p a} / \Delta f_{b}$, where $\Delta f_{b}$ is the bandwidth of the frequency response.

The set of equations (14) is valid for small variations of the capacitances around their standard synthesis values. That is why the preliminary GOC approach is indispensable for the subsequent compensation by modifying the capacitances.

The errors $\Delta f_{p} / f_{p}, \Delta Q_{p} / Q_{p}$ and $\Delta H_{p} / H_{p}$ can be centered around zero by means of the following iterative procedure:

1) The system (14) with different relative errors on the right hand sides of the equations is solved repeatedly for the modified values $C_{D_{1}}^{\prime}, C_{D_{2}}^{\prime}$ and $C_{N}^{\prime}$ of the three capacitances;
2) The sensitivities $S_{C_{D_{K}}}^{f_{p}}$ and $S_{C_{D_{K}}}^{Q_{p}}$, and the relative errors $\Delta f_{p} / f_{p}$, $\Delta H_{p} / H_{p}$ and $\Delta Q_{p} / Q_{p}$, needed for the $i$-th iteration, are calculated using the capacitance value $C_{D_{1}}^{\prime}, C_{D_{2}}^{\prime}$ and $C_{N}^{\prime}$, obtained in the ( $i-1$ )-th iteration;
3) The sensitivity $S_{C_{N}}^{H_{p}}$ is computed once for the initial values of the capacitances in the biquad considered.

## 3 Application of the Proposed Approach

The approach proposed is illustrated by means of the Sanchez-Sinencio, Silva-Martinez and Geiger's type-I (SSGI) bandpass biquad [6], shown in Fig. 1.

The ideal filter $\left(A_{0} \rightarrow \infty\right)$ has a pole frequency $f_{p}=1634.48 \mathrm{~Hz}$, a quality factor $Q_{p}=15.9993$, a peak gain $H_{p}=3.162208(\sim 10 \mathrm{~dB})$ at $f_{p}$ and sampling frequency $f_{s}=8 \mathrm{kHz}$. The relative capacitances values are $C_{1}=1, C_{2}=11.97$, $C_{3}=2.533, C_{4}=1.195, C_{5}=14.953$ and $C_{6}=1$. It was found that for op amps gains $A_{0_{1}}=A_{0_{2}}=100$ the deviations of $f_{p}, Q_{p}$, and $H_{p}$ from the ideal case are

$$
\Delta f_{p} / f_{p}=-1.831 \%, \quad \Delta Q_{p} / Q_{p}=-25.166 \%, \quad \Delta H_{p} / H_{p}=-25.686 \% .
$$

According to the proposed approach the first integrator in the conventional biquad (Fig. 1) is replaced with the Ki-89 integrator and the second integrator with the Nagaraj-86 integrator. The resulting filter is shown in Fig. 2, where $C_{h_{1}}=C_{1}$ and $C_{h_{2}}=1$.


Fig. 1 - SSGI bandpass biquad with conventional integrators.


Fig. 2 - SSGI bandpass biquad with GOC integrators.

The performance parameters of the GOC biquad for $A_{0_{1}}=A_{0_{2}}=100$ are

$$
\begin{gather*}
\Delta f_{p} / f_{p}=-2.64224 \%, \quad \Delta Q_{p} / Q_{p}=-1.52215 \% \\
\Delta H_{p} / H_{p}=-2.16027 \% \tag{15}
\end{gather*}
$$

The ideal $z$-domain transfer function is

$$
\begin{equation*}
H_{i d}^{22}(z)=-\frac{\frac{C_{3}}{C_{2}}\left(1-z^{-1}\right)}{z^{-2}-\left(2+\frac{C_{6}}{C_{2}}-\frac{C_{4} C_{5}}{C_{1} C_{2}}\right) z^{-1}+1+\frac{C_{6}}{C_{2}}} . \tag{16}
\end{equation*}
$$

From (16) the following approximate expressions for the pole frequency $f_{p}$ and the quality $Q_{p}$ can be derived:

$$
\begin{equation*}
f_{s} \approx \frac{f_{s}}{2 \pi} \sqrt{\frac{C_{4} C_{5}}{C_{1}\left(C_{2}+C_{6}\right)}} \tag{17}
\end{equation*}
$$

and

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$$
\begin{equation*}
Q_{p \approx} \sqrt{\frac{C_{4} C_{5}\left(C_{2}+C_{6}\right)}{C_{1} C_{6}^{2}}} . \tag{18}
\end{equation*}
$$

The corresponding logarithmic sensitivities of $f_{p}$ and $Q_{p}$ to the capacitances are

$$
\begin{gathered}
S_{C_{1}}^{f_{p}}=-0.5, \quad S_{C_{2}}^{f_{p}}=-\frac{C_{2}}{2\left(C_{2}+C_{6}\right)}=-0.4614495, \\
S_{C_{4}}^{f_{p}}=S_{C_{5}}^{f_{p}}=-0.5, \quad S_{C_{6}}^{f_{p}}=-\frac{C_{6}}{2\left(C_{2}+C_{6}\right)}=-0.0385505, \\
S_{C_{1}}^{Q_{p}}=-0.5, S_{C_{2}}^{Q_{p}}=\frac{C_{2}}{2\left(C_{2}+C_{6}\right)}=0.4614495, \\
S_{C_{4}}^{Q_{p}}=S_{C_{5}}^{Q_{p}}=0.5, S_{C_{6}}^{Q_{p}}=-\frac{2 C_{2}+C_{6}}{2\left(C_{2}+C_{6}\right)}=-0.9614495 .
\end{gathered}
$$

The sensitivities of the magnitude at the pole frequency $H_{p}$ to the feed forward capacitance $C_{3}$ and to the feedback capacitances $\mathrm{C}_{1}, C_{2}, C_{4}, C_{5}$ and $C_{6}$ are

$$
\begin{gathered}
S_{C_{1}}^{H_{p}}=-0.181832, S_{C_{2}}^{H_{p}}=-0.174649, S_{C_{3}}^{H_{p}}=1, \\
S_{C_{4}}^{H_{p}}=S_{C_{5}}^{H_{p}}=0.181832, S_{C_{6}}^{H_{p}}=-1.007183 .
\end{gathered}
$$

Therefore, the errors $\Delta f_{p} / f_{p}, \Delta Q_{p} / Q_{p}$ and $\Delta H_{p} / H_{p}$ can be further minimized by modifying one of the following groups ( $C_{D_{1}}, C_{D_{2}}, C_{N}$ ) of two feedback capacitances $C_{D_{1}}$ and $C_{D_{2}}$ from the denominator and the feed forward capacitance $C_{N}=C_{3}$ from the numerator of [16]:

$$
\begin{gathered}
\left(C_{1}, C_{2}, C_{3}\right),\left(C_{1}, C_{6}, C_{3}\right),\left(C_{2}, C_{4}, C_{3}\right),\left(C_{2}, C_{5}, C_{3}\right), \\
\left(C_{2}, C_{6}, C_{3}\right)_{5}\left(C_{4}, C_{6}, C_{3}\right) \text { and }\left(C_{5}, C_{6}, C_{3}\right),
\end{gathered}
$$

for which the inequality (13) is valid.
For the first group ( $C_{D_{1}}=C_{1}, C_{D 2}=C_{2}, C_{N}=C_{3}$ ) the new capacitance values $C_{1}{ }^{\prime}, C_{2}{ }^{\prime}$ and $C_{3}{ }^{\prime}$ are the iterative solutions of the system

$$
\begin{equation*}
S_{C_{1}}^{f_{p}}\left(\frac{C_{1}^{\prime}-C_{1}}{C_{1}}\right)+S_{C_{2}}^{f_{p}}\left(\frac{C_{2}^{\prime}-C_{2}}{C_{2}}\right)=-\left(\frac{\Delta f_{p}}{f_{p}}\right) \tag{19a}
\end{equation*}
$$

$$
\begin{gather*}
S_{C_{1}}^{Q_{p}}\left(\frac{C_{1}^{\prime}-C_{1}}{C_{1}}\right)+S_{C_{2}}^{Q_{p}}\left(\frac{C_{2}^{\prime}-C_{2}}{C_{2}}\right)=-\left(\frac{\Delta Q_{p}}{Q_{p}}\right)  \tag{19b}\\
S_{C_{1}}^{H_{p}}\left(\frac{C_{1}^{\prime}-C_{1}}{C_{1}}\right)+S_{C_{2}}^{H_{p}}\left(\frac{C_{2}^{\prime}-C_{2}}{C_{2}}\right)+S_{C_{3}}^{H_{p}}\left(\frac{C_{3}^{\prime}-C_{3}}{C_{3}}\right)=-\left(\frac{\Delta H_{p}}{H_{p}}\right), \tag{19c}
\end{gather*}
$$

where at the beginning of the iterative procedure the terms $\left(\Delta f_{p} / f_{p}\right)$, $\left(\Delta Q_{p} / Q_{p}\right)$ and $\left(\Delta H_{p} / H_{p}\right)$ on the right hand sides of the equations are the relative errors (15) of the GOC biquad for $A_{0_{1}}=A_{0_{2}}=100$.

The modified capacitance values $C_{1}{ }^{\prime}, C_{2}{ }^{\prime}$ and $C_{3}{ }^{\prime}$ for four iterations are given in Table 1.

Table 1
Modified capacitance values $C_{1}{ }^{\prime}, C_{2}{ }^{\prime}$ and $C_{3}{ }^{\prime}$ of the GOC biquad.

| Number of <br> iterations | $C_{1}{ }^{\prime}$ | $C_{2}{ }^{\prime}$ | $C_{3}{ }^{\prime}$ |
| :---: | :---: | :---: | :---: |
| 1 | 0.95835612 | 11.824724 | 2.5631703 |
| 2 | 0.96646629 | 11.826562 | 2.5565461 |
| 3 | 0.96523989 | 11.826449 | 2.5578879 |
| 4 | 0.96542564 | 11.826442 | 2.5576941 |

The corresponding performance parameters of the GOC biquad are summarized in Table 2.

Table 2
Performance parameters of the GOC biquad with modified capacitances

$$
C_{1}{ }^{\prime}, C_{2}^{\prime} \text { and } C_{3}^{\prime} \text { for } A_{0_{1}}=A_{0_{2}}=100
$$

| Number of <br> iterations | $\Delta f_{p} / f_{p}$ <br> $(\%)$ | $\Delta Q_{p} / Q_{p}$ <br> $(\%)$ | $\Delta H_{p} / H_{p}$ <br> $(\%)$ |
| :---: | :---: | :---: | :---: |
| 1 | 0.4303 | 0.4160 | 0.4150 |
| 2 | -0.06389 | -0.0630 | -0.07573 |
| 3 | $9.594 .10^{-3}$ | $9.646 .10^{-3}$ | $1.1066 .10^{-2}$ |
| 4 | $-1.442 .10^{-3}$ | $-1.364 .10^{-3}$ | $-1.519 .10^{-3}$ |

The capacitance $C_{1}{ }^{\prime}$ can be made equal to the unit capacitance. Then the new value of the capacitance $C_{4}$ after the fourth iteration is $C_{4}{ }^{\prime}=1.237796$.

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By rounding-off the values of the capacitances $C_{2}{ }^{\prime}, C_{3}{ }^{\prime}$ and $C_{4}{ }^{\prime}$ to the third digit after the decimal point finally we obtain

$$
C_{1}^{\prime}=1.0, \quad C_{2}^{\prime}=11.826, \quad C_{3}^{\prime}=2.558, \quad C_{4}^{\prime}=1.238
$$

Table 3 summarizes the performance parameters of the GOC biquad with rounded-off capacitances $C_{2}{ }^{\prime}, C_{3}{ }^{\prime}$ and $C_{4}{ }^{\prime}$, and gain variation $A_{01}=A_{02}=100 \pm 8$.

By proceeding in the same way for the group $\left(C_{5}, C_{6}, C_{3}\right)$ after the fourth iteration the following rounded-off capacitance values are obtained:

$$
C_{5}{ }^{\prime}=15.677, C_{6}{ }^{\prime}=1.013 \text { and } C_{3}{ }^{\prime}=2.590
$$

Table 3

| Performance parameters of the GOC biquad with rounded-off <br> capacitances | $C_{2}{ }^{\prime}, C_{3}{ }^{\prime}$ and $C_{4}{ }^{\prime}$. |
| :---: | :---: | :---: | :---: |

Table 4 summarizes the performance parameters of the GOC biquad with rounded-off capacitances $C_{5}{ }^{\prime}, C_{6}{ }^{\prime}, C_{3}{ }^{\prime}$, and gain variation $A_{01}=A_{02}=100 \pm 8$.

It is seen from the data in Tables 3 and 4 that the modification of the two groups of capacitances results in nearly the same error intervals due to the gain variation

$$
\begin{aligned}
& {\left[\left(\frac{\Delta f_{p}}{f_{p}}\right)_{A_{0}=92},\left(\frac{\Delta f_{p}}{f_{p}}\right)_{A_{0}=108}\right] \approx 0.43 \%,} \\
& {\left[\left(\frac{\Delta Q_{p}}{Q_{p}}\right)_{A_{0}=92},\left(\frac{\Delta Q_{p}}{Q_{p}}\right)_{A_{0}=108}\right] \approx 0.3 \%,} \\
& {\left[\left(\frac{\Delta H_{p}}{H_{p}}\right)_{A_{0}=92},\left(\frac{\Delta H_{p}}{H_{p}}\right)_{A_{0}=108}\right] \approx 0.4 \% .}
\end{aligned}
$$

## Table 4

Performances parameters of the GOC biquad with rounded- off capacitances $C_{5}{ }^{\prime}, C_{6}{ }^{\prime}$ and $C_{3}{ }^{\prime}$.

| $A_{0}$ | $\Delta f_{p} / f_{p}$ <br> $(\%)$ | $\Delta Q_{p} / Q_{p}$ <br> $(\%)$ | $\Delta H_{p} / H_{p}$ <br> $(\%)$ |
| :---: | :---: | :---: | :---: |
|  | -0.2343 | -0.2024 | -0.2601 |
| 100 | $-1.387 .10^{-3}$ | -0.04182 | -0.0388 |
| 108 | 0.1980 | 0.09227 | 0.1482 |

## 4 Conclusion

A combined approach for reducing the effects of op amps finite gain in switched-capacitor biquads has been presented. First, the conventional integrators in the biquads have been replaced with gain-and offset- compensated integrators. Next, the errors in the pole frequency $f_{p}$, in the pole $Q$-factor $Q_{p}$ and in the magnitude $H_{p}$ at the pole frequency have been minimized by modifying three capacitances: two feedback capacitances which control the frequency $f_{p}$ and the pole quality factor $Q_{p}$ of the ideal biquad, and a feedforward capacitance which controls the gain of the biquad. The approach proposed has been illustrated for two groups of three capacitances in a bandpass biquad. The filter with modified capacitances has approximately an order of magnitude smaller relative errors. The modification of the two groups of capacitances results in nearly the same error intervals due to the gain variation.

## 5 References

[1] L. Yue, J. I. Sewell: A comparison study of SC biquads in the realisation of SC filters, in Proc. 1994 IEEE ISCAS, pp. 711-714, 1994.
[2] P.E. Fleischer, K.R. Laker: A family of active switched-capacitor biquads building blocks, Bell Syst. Tech. J., Vol. 58, pp. 2235-2269, 1979.
[3] K.R. Laker, A. Ganesan, P. E. Fleischer: Design and implementation of cascaded switched capacitor delay equalisers, IEEE Transactions on Circuit and Systems, Vol. CAS-32, pp. 700-711, 1985.
[4] R. Gregorian, G.C. Temes: Analog MOS integrated circuits for signal processing, John Wiley \& Sons, 1986.

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[5] K. Martin, A.S. Sedra: Exact design of switched-capacitor bandpass filters using coupledbiquad structures, IEEE Trans. on Circuits and Syst., Vol. CAS-27, No. 6, pp. 469-475, 1980.
[6] E. Sanchez-Sinencio, J. Silva-Martinez, R. Geiger: Biquadratic SC filters with small GB effects, IEEE Trans. on Circuits and Syst., Vol.CAS-31, No. 10, pp. 876-883, 1984.
[7] K. Nagaraj: A parasitic -insensitive area- efficient approach to realizing very large time constants in switched-capacitor circuits, IEEE Trans. on Circuits and Syst., Vol. CAS-36, No. 9, pp. 1210-1216, 1989.
[8] K. Martin, A.S. Sedra: Effects of finite gain and bandwidth on the performance of switchedcapacitor filters, IEEE Transaction on Circuits and Systems, Vol. CAS-28, pp. 822-829, 1981.
[9] A.K. Betts, H. Shafeeu, J. T. Taylor: Amplifier gain insensitive SC integrators with "samesample correction" of both gain and phase errors for singlepath and multipath circuits, Electron. Lett., Vol. 27, No. 16, pp. 1424-1425, 1991.
[10] H. Shafeeu, A.K. Betts, J. T. Taylor: Novel amplifier gain insensitive switched capacitor integrator with same sample correction properties, Electron. Letrt., Vol. 27, No. 24, pp. 2277-2279, 1991.
[11] K. Nagaraj, J. Vlach, T.R. Viswanathan, K. Singhal: Switched-capacitor integrator with reduced sensitivity to amplifier gain, Electron. Lett., Vol. 22, No. 21, pp. 1103-1105, 1986.
[12] W. -H. Ki, G. C. Temes: Low-phase-error offset-compensated switched-capacitor integrator, Electron. Lett., Vol. 26, No. 13, pp. 957-959, 1990.
[13] A. Baschirotto, R. Alini, R. Castello: BiCMOS operational amplifier with precise and stable dc gain for high-frequency switched capacitor circuits, Electron. Lett., Vol. 27, No. 15, pp. 1338-1340, 1991.
[14] A. Baschirotto, R. Castello, F. Montecchi: Exact design of high-frequency SC circuits with low-gain op amps, in Proc. 1993 IEEE ISCAS, Vol. 2, pp. 1014-1017, 1993.
[15] A. Baschirotto: Considerations for the design of switched-capacitor circuits using precisegain operational amplifiers, IEEE Trans. on Circuits and Syst., Vol. CAS-43, pp. 827-832, 1996.
[16] S. Sedra, P.O. Brackett: Filter Theory and Design: Active and Passive, Matrix Publishers, Portland, Oregon, p. 482, 1982.


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