LEVEL STUDIES OF ^{93}Mo VIA ^{93}Nb(P, nγ)^{93}Mo REACTION AND DENSITY OF DISCRETE LEVELS IN ^{93}Mo

by

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The excited states of ^{93}Mo have been investigated via the ^{93}Nb(P, nγ)^{93}Mo reaction with proton beam energies of 2.5-4.3 MeV. The parameters of the nuclear level density formula were determined from the extensive and complete level scheme of ^{93}Mo. The Bethe formula for the back-shifted Fermi gas model and the constant temperature model are compared with experimental level densities.

Key words: level schemes of ^{93}Mo, ^{93}Nb(P, nγ)^{93}Mo reaction, angular distribution, Fermi gas model

INTRODUCTION

Excited levels of ^{93}Mo have been studied experimentally via β-decay [1, 2] and nuclear reactions with light [3, 4] and heavy ions [5, 6]. On the other hand, in all statistical theories, nuclear level density is the most characteristic quantity and plays a crucial role in the study of nuclear structure.

In this work, we have provided additional experimental information about the existing level structure of ^{93}Mo through the ^{93}Nb(P, nγ)^{93}Mo reaction and then determined nuclear level density parameters of the Bethe formula and the constant temperature model for the ^{93}Mo nucleus.

EXPERIMENTAL PROCEDURE

A self-supporting 0.55 mg/cm thick metal foil of natural, spectroscopically pure ^{93}Nb, was bombarded with a proton beam of 2.5 MeV to 4.3 MeV. The target was placed at an angle of 45° with respect to the beam direction and was thick enough to stop incident protons. Angular distributions were measured at 0°, 30°, 45°, 55°, 75°, and 90°. The γ-rays were detected with a 70 cm coaxial HPGe detector, with a resolution of 1.9 keV for the 1332 keV γ-ray of ^{60}Co. Excitation functions of various γ-rays were measured at 55° with respect to the beam direction, at 2.7 MeV, 3.0 MeV, 3.5 MeV, 4.0 MeV, and 4.3 MeV beam energies, to ascertain that the channel of compound decay is dominant as compared to the Coulomb excitation at the incident proton energy of 4.3 MeV. Other details of the experimental procedure may be found in our previous publications [7-9].

DATA ANALYSIS

Gamma-ray spectra were analyzed using the computer code PEAKFIT [10]. A typical gamma-ray spectrum at 90 degrees for the incident proton energy of 4.3 MeV was given in our previous publication [7-9]. The excitation functions of all observed gamma-rays were carefully analyzed as a function of energy, with those from the (P, nγ) reaction being easily identified by a characteristic rise above their threshold energy. The relative branching ratios used for further analysis are the weighted averages of the respective values at 4.0 MeV and 4.3 MeV bombarding energies.

The extraction of multipole mixing ratios of the observed transitions and the assignment of spin values to the excited levels were made from the χ^2-fitting of angular distribution data at a 4.3 MeV proton beam energy. The optical model parameter sets given by C. M. Perey and F. G. Perey [11], based on the results of F. G. Perey [12] for protons and Wilmore and Hodgson [13] for neutrons, were used to calculate transmission coefficients. Apart from the observed neutron channel, all known (P, P'γ) and (P, nγ) channels were also included as competing channels. The Moldauer with the fluctuation correction [14] was taken into account, as well. Typical experimental angular distributions of

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some of the observed transitions, together with the theoretical curves for different possible spins of these levels and the respective χ - fitting, are given in our previous publication [7-9]. A 0.1% confidence limit was used to exclude unacceptable spins and δ values. Coefficients A_2 and A_4 from the polynomial fits to the experimental distribution, along with multipole mixing ratios (δ), are given in tab. 1.

STATISTICAL FORMULA

Nuclear temperature T can be defined by nuclear level density $\rho(E)$ [15]

$$\frac{1}{T} = \frac{\mathrm{d}}{\mathrm{d}E} \ln \rho(E) \tag{1}$$

The integration yields the constant temperature Fermi gas formula [16]

$$\rho(E) = \frac{1}{T} \exp\left(\frac{E - E_0}{T}\right) \tag{2}$$

Nuclear temperature T and the ground state back-shift E_0 can be determined through experimental data on level density. The Bethe formula of the level density [17] for the back-shifted Fermi gas model [18, 19] can be written as

$$\rho(E) = \frac{\exp[2\sqrt{a(E - E_1)}]}{12\sqrt{2}\sigma \sqrt[4]{a(E - E_1)^5}} \tag{3}$$

In this case, the level density parameter a and the ground state back-shift E_1, are obtained by a fit to the experimental results. The distribution of spins J is determined [16, 17] by the spin cut-off parameter σ^2

$$f(J) = \exp\left(-\frac{J^2}{2\sigma^2}\right) - \exp\left(-\frac{(J+1)^2}{2\sigma^2}\right) \approx \frac{2J+1}{2\sigma^2} \exp\left(-\frac{\left(J+\frac{1}{2}\right)^2}{2\sigma^2}\right) \tag{4}$$

With this spin distribution, the spin-dependent level density is

$$\rho(E,J) = \rho(E) f(J) \tag{5}$$

where, σ^2 is related to an effective moment of inertia I_{eff} and to the nuclear temperature T [15, 18]

$$\sigma^2 = \frac{I_{\mathrm{eff}} T}{\hbar^2} \tag{6}$$

The nuclear moment of inertia for a rigid body is $I_{\mathrm{Rigid}} = (2/5)MR^2$ (where $M = A$, the amu nuclear mass; $R = 1.25A^{1/3}$ fm, (nuclear radius), resulting in [18]

$$\sigma^2 = 0.0150 A^{5/3} T \tag{7}$$

Gilbert and Cameron [16] calculated the spin cut-off parameter for the Bethe formula with the reduced moment of inertia,

$$\sigma^2 = 0.0888 A^{2/3} \sqrt{a(E - E_1)} \tag{8}$$

FIT OF THE LEVEL DENSITY FORMULA

Each of the two level density formulas has two free parameters. They may be obtained by fitting the measured level schemes experimentally. We have applied these formulas to the measured level scheme for ^{93}Mo reported in tab. 1. Our best fit values obtained using the Bethe formula are: the level density parameter a = 11.60 MeV^{-1} and the back-shift E_1 = = 0.941 MeV. Results obtained using the constant temperature formula are T = 0.528 MeV and back-shift E_0 = 0.9199 MeV.

The accumulated levels $N(E)$ as functions of energy are plotted in figs. 1 and 2. The examination of these figures shows that the agreement between theory

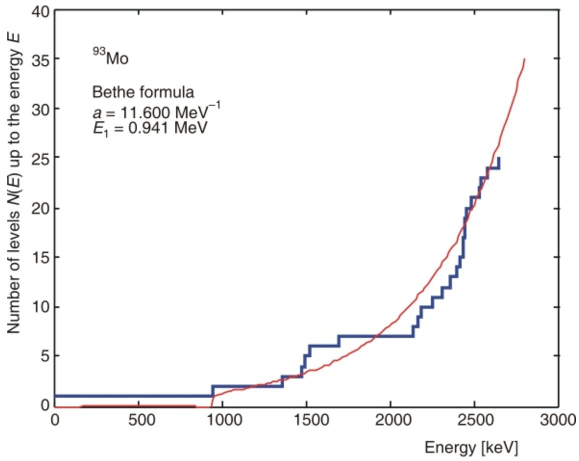

Figure 1. Plot of the number of levels $N(E)$ up to the energy E for ^{93}Mo, along with the fitted curve calculated by the Bethe formula

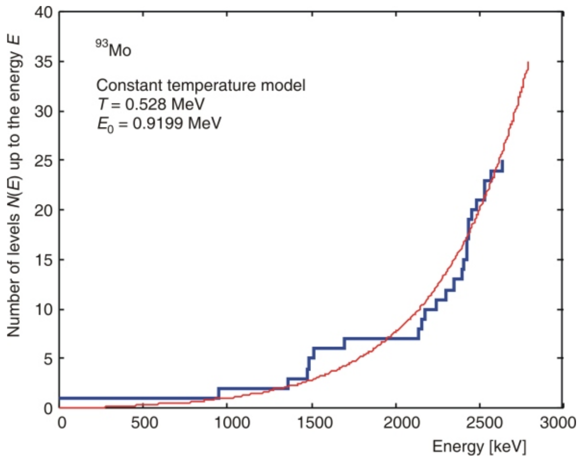

Figure 2. Plot of the number of levels $N(E)$ up to the energy E for ^{93}Mo, along with the fitted curve calculated by the constant temperature model

Table 1. Level energies and the results of the angular distribution measurements in ^{93}Mo

Transitions	Gamma rays [keV]	J_i^π J_f^π	Multipole mixing ratios, σ	A_2	A_4
1363.1 0	1363.1	$\frac{7}{2}$ $\frac{5}{2}$	0.5 $^{0.9}_{0.7}$	–0.03(1)*	–0.01(1)
1477.3 1363.1	114.2	$\frac{9}{2}$ $\frac{7}{2}$	0.05 $^{0.03}_{0.02}$	0.02(2)	0.01(2)
1477.3 0	1477.3	$\frac{9}{2}$ $\frac{5}{2}$	E2	0.25(1)	–0.04(1)
1492.3 0	1492.3	$\frac{3}{2}$ $\frac{5}{2}$	M1	0.06(2)	0.04(2)
1520.3 0	1520.3	$\frac{7}{2}$ $\frac{5}{2}$	1.2 $^{0.5}_{0.3}$	0.34(5)	0.03(5)
1695.0 1363.1	331.9	$\frac{5}{2}$ $\frac{7}{2}$	M1	0.05(2)	0.04(2)
1695.0 0	1695.0	$\frac{5}{2}$ $\frac{5}{2}$	M1	0.08(4)	0.00(5)
2142.0 1363.1	778.9	$\frac{5}{2}$ $\frac{7}{2}$	9.7 0.2 or 0.04 $^{0.01}_{0.02}$	0.07(2)	0.05(3)
2161.9 1477.3	684.6	$\frac{13}{2}$ $\frac{9}{2}$	0.15 $^{0.04}_{0.02}$	–0.32(1)	0.05(1)
2181.3 0	2181.3	$\frac{3}{2}$ $\frac{5}{2}$	M1	0.04(1)	0.05(1)
2247.3 1477.3	770.0	$\frac{11}{2}$ $\frac{9}{2}$	0.1 $^{0.02}_{0.03}$	–0.34(3)	0.08(4)
2304.4 1477.3	827.1	$\frac{11}{2}$ $\frac{9}{2}$	0.2 $^{0.12}_{0.17}$	–0.02(0.3)	–0.00(0.3)
2356.1 1520.3	835.8	$\frac{5}{2}$ $\frac{7}{2}$	0.05 $^{0.02}_{0.03}$	0.12(2)	0.01(2)
2398.1 1492.3	905.8	$\frac{5}{2}$ $\frac{3}{2}$	M1	0.03(0.3)	0.00(0.4)
2398.1 0	2398.1	$\frac{5}{2}$ $\frac{5}{2}$	M1	0.06(3)	0.01(3)
2409.1 2247.3	161.8	$\frac{9}{2}$ $\frac{11}{2}$	M1	0.02(2)	0.01(2)
2409.1 1477.3	931.8	$\frac{9}{2}$ $\frac{9}{2}$	M1	0.08(4)	0.01(4)
2409.1 0	2409.1	$\frac{9}{2}$ $\frac{5}{2}$	E2	0.24(1)	–0.03(1)
2430.0 2161.9	268.1	$\frac{17}{2}$ $\frac{13}{2}$	E2	0.19(1)	–0.00(1)
2431.0 1363.1	1067.9	$\frac{7}{2}$ $\frac{7}{2}$	0.03 0.01 or 1.2 0.01	–0.04(1)	0.01(1)
2431.0 0	2431.0	$\frac{7}{2}$ $\frac{5}{2}$	6.5 $^{0.14}_{0.11}$	–0.06(1)	0.00(2)
2440.6 1363.1	1077.5	$\frac{9}{2}$ $\frac{7}{2}$	9.7 0.12 or 0.05 0.11	–0.02(1)	0.01(1)
2450.2 2247.3	202.9	$\frac{13}{2}$ $\frac{11}{2}$	E1	–0.04(2)	0.00(2)
2479.0 1363.1	1115.9	$\frac{7}{2}$ $\frac{7}{2}$	0.04 0.04 or 0.98 0.11	0.02(2)	0.02(2)
2534.5 1477.3	1057.2	$\frac{9}{2}$ $\frac{9}{2}$	M1	0.04(2)	0.01(3)
2539.3 1492.3	1047.0	$\frac{3}{2}$ $\frac{3}{2}$	1.28 $^{0.15}_{0.14}$	0.24(3)	0.01(4)
2642.0 2430.0	212.0	$\frac{15}{2}$ $\frac{17}{2}$	M1	0.04(1)	0.03(1)
2642.0 2161.9	480.1	$\frac{15}{2}$ $\frac{13}{2}$	0.05 0.07	–021(4)	0.05(5)

* The numbers in the brackets in the last two columns indicate the uncertainties in the deduced values of the coefficients

and experiment is very good and that both formulas fit the measured level scheme equally well.

Furthermore, the spin cut-off parameter σ^2 has been obtained by fitting the known spin distribution, $N(J)$ of tab. 1, with the theoretical expression (4) shown in fig. 3. Our best fit value for this parameter is σ^2 = 13.17. This deduced value is different from its corresponding rigid body value of σ^2 = 10.02. This finding is opposite to the claim made by some authors that the spin cut-off parameter reduces to its rigid body value at lower energies.

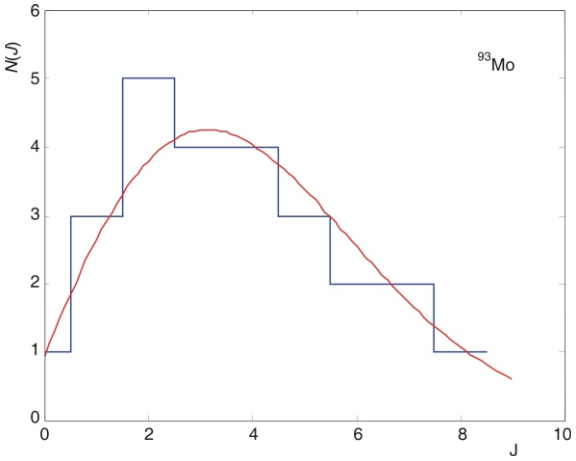

Figure 3. Spin distribution of low-lying states. The histogram is a representation of experimental data (data from tab. 1). The curve corresponds to the statistical distribution where σ^2 = 13.17

CONCLUSIONS

The purpose of this study was to provide additional experimental information on the existing level structure of ^{93}Mo through the (P, nγ) reaction. We have measured γ-ray energies, branching ratios, and multipole mixing ratios of various transitions in ^{93}Mo.

This complete and extensive nuclear level scheme of ^{93}Mo provides a valid basis for statistical interpretations of low energy nuclear level schemes based on various tests of statistical theories. The level density near the ground state is well reproduced by the Bethe formula, as well as by the constant temperature formula, if the two parameters are fitted.

The spin cut-off parameter of ^{93}Mo has been determined from the analysis of the experimental data on spins of low-lying states given in tab. 1. This has not been confirmed by its corresponding rigid-body value.

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ПРОУЧАВАЊЕ НИВОА ^{93}Mo ПОМОЋУ РЕАКЦИЈЕ ^{93}Nb(P, nγ)^{93}Mo И ГУСТИНЕ ДИСКРЕТНИХ НИВОА У ^{93}Mo

Побуђена стања ^{93}Mo истраживана су посредством реакције ^{93}Nb(P, nγ)^{93}Mo, са протонским снопом енергија 2.5-4.3 MeV. Параметри формуле за густину нуклеарних нивоа били су одређени на основу екстензивне и потпуне шеме нивоа ^{93}Mo. Бетеова формула за модел уназад сифтованог Фермијевог гаса и модел константне температуре упоређени су са експерименталним вредностима густине нивоа.

Кључне речи: шема нивоа ^{93}Mo, ^{93}Nb(P, nγ)^{93}Mo реакција, угаона расподела, модел Фермијевог гаса