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Superdiffusion revisited in view of collisionless reconnection

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Abstract. The concept of diffusion in collisionless space plasmas like those near the magnetopause and in the geomagnetic tail during reconnection is reexamined making use of the division of particle orbits into waiting orbits and breakouts into ballistic motion lying at the bottom, for instance, of Lévy flights. The rms average displacement in this case increases with time, describing superdiffusion, though faster than classical, is still a weak process, being however strong enough to support fast reconnection. Referring to two kinds of numerical particle-in-cell simulations we determine the anomalous diffusion coefficient, the anomalous collision frequency on which the diffusion process is based, and construct a relation between the diffusion coefficients and the resistive scale. The anomalous collision frequency from electron pseudo-viscosity in reconnection turns out to be of the order of the lower-hybrid frequency with the latter providing a lower limit, thus making similar assumptions physically meaningful. Tentative though not completely justified use of the κ distribution yields $\kappa \approx 6$ in the reconnection diffusion region and, for the anomalous diffusion coefficient, the order of several times Bohm diffusivity.

Keywords. Space plasma physics (magnetic reconnection)

1 Introduction

Anomalous diffusion is the summary heading of all processes where the ensemble averaged mean-square displacement $\langle x^2 \rangle \propto t^\gamma$ deviates from linear time dependence $\gamma = 1$ with the classical (Einstein) diffusion coefficient $D_{\rm cl} = 2T\nu_{\rm c}/m$, with T the temperature, and $\nu_{\rm c}$ the classical binary collision frequency. For $\gamma > 1$ one speaks of superdiffusion, which is of particular importance in the collisionless space plasma where classical diffusion is practically inhibited in all physically interesting processes (the less interesting case in space physics, $\gamma < 1$, refers to subdiffusion, cf., e.g., Sokolov et al., 2002). One of those processes is reconnection, the dominant mechanism for plasma and magnetic field transport across magnetic boundaries represented by thin current sheets/layers.

Reconnection has the enormous advantage over global diffusion of being localised, with the main physics of magnetic merging and plasma mixing taking place in an extraordinarily small spatial region with the linear size shorter than the electron inertial length $< \lambda_{\rm e} = c/\omega_{\rm e}$. On this note, based on available numerical simulations, we demonstrate by estimating the anomalous collision frequency $\nu_{\rm a}$ that magnetic merging during reconnection can well be understood as a localised anomalous diffusion process. This result satisfactorily unifies the two originally different views on plasma transport across an apparently impermeable boundary like the magnetopause.

Anomalous diffusion is also of interest in cosmic ray physics, where it is frequently described as quasilinear diffusion resulting from wave–particle interactions, formulated in the Fokker–Planck phase space-diffusion formalism. Unfortunately, most of the observed diffusive particle spectra (cf., e.g., Christon et al., 1989, 1991, for the most elaborate observations in near-Earth space) barely exhibit the shapes resulting from quasilinear diffusion. They turn out to be power law distributions both in energy and momentum space, most frequently being described best by so-called κ distributions

$$p(\kappa\,|\,\mathbf{x}) = A_\kappa \left(1 + \frac{\mathbf{x}^2}{\kappa \ell^2}\right)^{-(\kappa+1+d/2)} \qquad (1)$$

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with normalisation factor A_κ, d dimensionality, and ℓ correlation length (cf., e.g., Livadiotis and McComas, 2010, 2011, 2013, for an almost complete compilation of the properties of κ distributions) with high-energy/high-momentum slopes to which the parameters κ are related. Estimated κ values from the magnetospheric observations range in the interval $5 < \kappa < 10$ (Christon et al., 1991). Such distributions were introduced by Vasyliunas (1968), following a suggestion by S. Olbert, as best fits.[1] In the time-asymptotic limit, κ distributions were explicitly derived by Hasegawa et al. (1985) and Yoon et al. (2012). Their q equivalent relation to superdiffusion has also been suggested (Tsallis et al., 1995; Prato and Tsallis, 1999; Bologna et al., 2000; Gell-Mann and Tsallis, 2004, and references therein).

For the present purposes we make no direct use of these distributions as they, apparently, play no role in reconnection. Rather, as we demonstrate, anomalous diffusion in reconnection results from processes leading to waiting statistics and causing gyro-viscosity.

2 Diffusion process

Collisionless dissipation and related diffusion is mediated in a wider sense by collisionless turbulence (cf., e.g., Allegrini et al., 1996). Here binary collision times $\tau_c \gg \tau_a$ by far exceed anomalous interaction times. Any real non-collisional diffusion proceeds at times much shorter than classical (in comparison infinite) diffusion times, with absolute values of anomalous diffusion coefficients being small.

The superdiffusion process can be considered as a sequence of "waiting times" when the particle is in a quasi-stationary trapped state followed by "breakouts" into ballistic motion until the next trapping and waiting period starts (Shlesinger et al., 1987; Klafter et al., 1990). Such particle motions are typical, for instance, for Lévy flights (cf., e.g., Shlesinger et al., 1993).

Working in d dimensions, the probability of a particle occupying a particular volume element during a process, assumed to be caused by some unspecified (nonlinear) interaction between particles and plasma waves, is most conveniently formulated in wave number space $\mathbf{k}$ with probability spectrum

$$p(\mathbf{k}) \propto \exp(-ak^\alpha), \tag{2}$$

where a is some positive constant, and $0 < \alpha \in \mathbb{R}$ a real number. $\alpha \geq 2$ reproduces the classical Gaussian probability spectra (Tsallis et al., 1995). Non-Gaussian spectra have flatter tails implying $\alpha < 2$, indicating superdiffusion. The connection of the above probability spectrum to real-space distributions, in particular to κ distributions, is non-trivial.

The diffusion process can be envisaged as consisting of a sequence of n steps (cf., e.g., Treumann, 1997) bridging the time from $t = 0$ to $t = t_n$ with the particle jumping from first waiting to the nth waiting position; the expectation value of the latter becomes

$$\langle \mathbf{x}^2(n) \rangle = \int \mathbf{x}^2 p(n\,|\,\mathbf{x}) d^d x, \qquad p(n) = \prod_1^n p(i). \tag{3}$$

The nth expectation value is proportional to the random mean square of the displacement $\mathbf{x}^2$ and a power of the elapsed time sequence. Working in Fourier (or momentum) space $\mathbf{k}$, multiplication of the probabilities yields

$$p(n\,|\,\mathbf{k}) = p^n(\mathbf{k}) \propto \exp(-ank^\alpha) \sim \; p(\mathbf{k}') \tag{4}$$

with $p(\mathbf{k}')$ the probability of the nth time step. Hence $\mathbf{k}' = \mathbf{k}n^{1/\alpha}$. Any real-space coordinate therefore scales as $x \to xn^{-1/\alpha}$. For the real-space probability this implies that

$$p(n\,|\,\mathbf{x})\; d^d x \;\longrightarrow\; p\big(\mathbf{x}/n^{1/\alpha}\big)\; d^d x/n^{d/\alpha} \tag{5}$$

yielding from Eq. (3) for the nth displacement expectation value

$$\big\langle \mathbf{x}^2(n) \big\rangle = n^{2/\alpha} \langle \mathbf{x}^2 \rangle \tag{6}$$

with $\alpha < 2$ not precisely known but to be determined below from numerical simulations. The mean-square displacement should be obtained from the second moment of the underlying real-space distribution function, for instance the κ distribution, yielding

$$\langle \mathbf{x}^2 \rangle = \frac{1}{2} d\kappa(\kappa + 1)\ell^2, \tag{7}$$

an expression we will make tentative (not fully justified and for the present purposes marginal) use of only at the very end in application to reconnection.

3 Diffusion coefficient

In using probability steps n, time has been discretised into pieces of free flight, waiting and some kind of interaction. On average the interaction is covered by a fictitious anomalous collision frequency ν_a. Ordinary binary collision frequencies ν_c are very small, suggesting a scaling $\nu_a \gg \nu_c$ with the

[1]Theoretical attempts to justify solar wind κ distributions followed, invoking wave-particle interactions with the inclusion of residual binary collisions (Scudder and Olbert, 1979). Statistical mechanical arguments were based on non-extensive statistical mechanics (Tsallis, 1988; Gell-Mann and Tsallis, 2004). From kinetic theory they were identified as collisionless turbulent quasi-stationary states far from thermal equilibrium resulting from anomalous wave-particle interactions (Treumann, 1999a, b). There the role of the temperature T as a thermodynamic derivative was clarified (see also Livadiotis and McComas, 2010). The relation between the non-extensive q and the κ parameters was first given in Treumann (1997).

anomalous timescale $\nu_{\rm a}^{-1} = \tau_{\rm a} \ll \tau_{\rm c} = \nu_{\rm c}^{-1}$ much less than the collision timescale $\tau_{\rm c}$. The diffusion process takes place in a time $t < \tau_{\rm c}$. Replacing the time steps $n \to \nu_{\rm a} t$ the mean square nth displacement becomes

$$\langle \mathbf{x}^2(t) \rangle = \langle \mathbf{x}^2 \rangle (\nu_{\rm a} t)^{2/\alpha}. \tag{8}$$

With $\gamma = 2/\alpha$ it defines the anomalous diffusion coefficient $D_{\rm a}$ when multiplying by $\tau_{\rm a}^{-1} = \nu_{\rm a}$

$$D_{\rm a}(d,t) = \langle \mathbf{x}^2 \rangle (\nu_{\rm a} t)^{2/\alpha} \nu_{\rm a} \equiv D_{\rm ca}(\nu_{\rm a} t)^{2/\alpha} \tag{9}$$

as a function of time $t\nu_{\rm a}$. Since $\nu_{\rm a} \gg \nu_{\rm c}$, it is much less than the classical diffusion coefficient which in this case would correspond to free flight. Under anomalous collisions the free flight is abruptly interrupted and reduced to non-stochastic diffusion by the finite anomalous collision frequency $\nu_{\rm a}$.

4 Evolution

Estimates of diffusion coefficients respectively γ based on observations in space plasma are not only rare but unreliable. They suffer from the practical impossibility of any sufficiently precise determination of particle displacements as a function of time and the subsequent transition to the asymptotic state. In addition they are mostly based on quasilinear theories of particular instabilities (Sagdeev, 1966, 1979; Liewer and Krall, 1973; Huba et al., 1977, 1981; Davidson, 1978; LaBelle and Treumann, 1988; Treumann et al., 1991; Yoon et al., 2002; Matthaeus et al., 2003; Daughton et al., 2004; Ricci et al., 2005; Roytershteyn et al., 2012; Izutsu et al., 2012) which do not properly account for any nonlinear interactions.

We therefore refer to high-resolution particle-in-cell simulations (Scholer et al., 2000) performed in order to determine the cross-magnetic field diffusion of ions near quasi-perpendicular shocks. The results are compiled in Fig. 1.

The right-hand side of the figure shows one macro-particle orbit arbitrarily selected out of the large number of particles used in the simulation to determine their instantaneous displacements from the origins of their trajectories in the simulation as a function of simulation time measured in units of their identical (energy-independent) gyration frequency $\omega_{\rm ci} = eB/m_{\rm i}$ in the total magnetic field, which is the sum of the ambient and the self-consistently generated turbulent wave magnetic field. The particle shifts its position perpendicular to the magnetic field from its start point to the end point in the simulation. It is found in a slowly changing waiting position, performs jumps to new waiting positions, and ends up during a final jump. Such an orbit is neither adiabatic nor stochastic.

The left part of the figure shows the average displacement, ensemble averaged over the entire particle population, as a function of simulation time. After performing an initial oscillation the average displacements settle into an approximately smooth continuously increasing curve of constant slope $\langle (\Delta x)^2 \rangle \propto t^{1.17}$.

The slope of the final evolution of the average displacement is close but by no means identical to classical diffusion, which is shown by the slope of the two straight lines in the figure. Though the deviation in the slope is small, it is nevertheless substantial and statistically significant, indicating a superdiffusive process which deviates from classical diffusion. (We should note that, because of the large number of $\sim 6.3 \times 10^6$ macro-particles used in the simulation of which 525 000 had high energies and contributed most to the mean displacement as well as for the high time resolution, the statistical error of the measurement is smaller than the width of the line in this figure!)

Adopting the probability spectrum-based theory the experimentally determined slope of $2/\alpha \approx 1.17$ of the average displacement in Fig. 1 tells that in these simulations one had

$$\alpha \approx 1.71 \qquad \text{(experimental)} \tag{10}$$

a value substantially far away from the Gaussian limit spectral slope $\alpha = 2$ and being less than it, thus indicating quite strong superdiffusion.

5 Transition to collisional state

Anomalous diffusion proceeds on a faster than classical timescale with time-dependent diffusion coefficient which justifies the term superdiffusion. In spite of this, the coefficient $D_{\rm ca} = \langle \mathbf{x}^2 \rangle \nu_{\rm a}$ in front of the time factor determining the absolute magnitude of the diffusion is generally small. It does not compensate for the *absolute* smallness of the diffusion coefficient. When, after a long time has elapsed of the order of the classical collision time $\tau_{\rm c}$, classical diffusion takes over scattering some particles to larger, some others back to smaller displacements and putting the collisionless process temporarily out of work. The average displacement of the violently scattered particles whose displacement line has been smeared out suddenly over a large spatial domain may now follow the classical linear temporary increase.

One single elapsed binary collision time may not suffice to stop the nonlinear collisionless interaction process. The widely scattered particle population may still have sufficient freedom to organise again into a softened collisionless diffusion which lasts until the next binary collision time has passed. During this second collisionless period the slope should be flatter than the initial collisionless slope, and after statistically sufficiently many periods of elapsed classical collision times no collisionless mechanisms revive anymore. Diffusion has by then become completely classical. These sequences are schematically shown in Fig. 2.

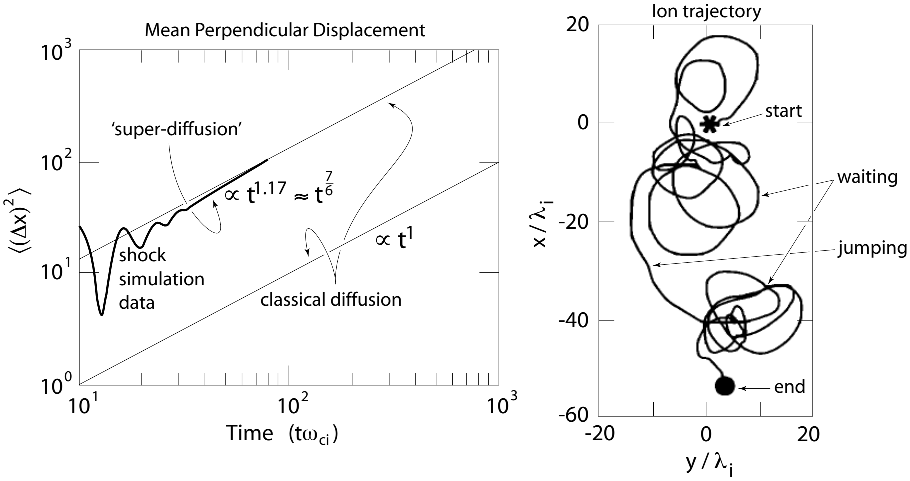

Figure 1. Two-dimensional numerical simulation results of the mean downstream perpendicular displacement of ions near a quasi-perpendicular supercritical shock (shock normal angle $\theta = 87°$), Alfvénic Mach number $M_A = 4$ as a function of simulation time (simulation data taken from Scholer et al., 2000, courtesy American Geophysical Union). Distances are measured in ion inertial lengths $\lambda_i = c/\omega_i$ with ω_i ion plasma frequency. (Left) The particle displacement performs an initial damped oscillation before settling into a continuous diffusive increase at time about $\omega_{ci}t \sim 40$ (in units of the ion gyro frequency ω_{ci}). The further time evolution deviates apparently only slightly from the classical (linear) increase of the mean displacement, following a $\langle(\Delta x)^2\rangle \propto (\omega_{ci}t)^{1.17}$ power law. (Note that simulation-time limitations did not allow monitoring of the long-time evolution of the ensemble-averaged square displacement, thus inhibiting the determination of the final state of the diffusion process.) (Right) Late time trajectory of an arbitrary ion of the sample used. The orbit is projected into the plane perpendicular to the mean magnetic field which consists of a superposition of the ambient and wave magnetic fields. The ion orbit is neither an undisturbed gyro-oscillation nor a smooth stochastic trajectory. It consists of waiting (trapped gyrating) parts and parts when the ion suddenly jumps ahead a long distance cause by some brief but intense interaction between the particle and wave spectrum. This break out of gyration is typical of rare extreme events like those in Lévy flights referred to in the present paper.

6 Discussion

Waiting statistics offers an approach to anomalous diffusion in various regions of space plasmas where classical (and neo-classical) diffusion processes are inappropriate, violently failing to explain the transport of plasma and magnetic fields. Application to numerical simulations near collisionless shocks determined the value of $\alpha \approx 1.71$, which turns out to be close to but sufficiently far below its classical (Gaussian) limit $\alpha = 2$ for identifying superdiffusion. Superdiffusion coefficients obtained are small but increase with time.

The present theory is based on constant α for the entire diffusion process. This might be unrealistic. Real powers $\alpha\,[W_w(t)]$ will turn out functionals of the time-dependent turbulent wave levels $W_w(t)$ which are generated self-consistently in the underlying turbulent collisionless wave-particle interaction (for a derivation of the phase-space distribution in particular wave-particle interactions cf., e.g., Hasegawa et al., 1985; Yoon et al., 2012, yielding time-asymptotic values of the phase-space power-law index κ depending on wave power W_w).

It may be expected that, with increasing wave level $W_w(t)$, a new collisionless equilibrium will be reached where the diffusion process, in finite time $t \sim \tau_f$, approaches another new and approximately constant diffusivity

$$\lim_{t\to\tau_f} D_a(t) \longrightarrow D_a^{\rm fin}(t \gtrsim \tau_f) < D_c \tag{11}$$

for $\nu_a^{-1}(t=0) \lesssim \tau_f \ll \nu_c^{-1}$, with $W_w(t \gtrsim \tau_f)$, $\alpha\,[W_w(t \gtrsim \tau_f)]$ both either constant or oscillating around their time-averaged mean values $\langle W_w(t \gtrsim \tau_f)\rangle, \langle\alpha(t \gtrsim \tau_f)\rangle$, and the final average diffusion coefficient $\langle D_a^{\rm fin}(\tau_f \lesssim t \lesssim \nu_c^{-1})\rangle$ remaining constant. Under such circumstances the diffusion coefficient in Fig. 2 never approaches the classical limit but settles instead on its much lower anomalous collisionless level $\langle D_a^{\rm fin}\rangle$. The related processes lie outside the present investigation. We may, however, estimate a lower bound on the average final diffusion coefficient $\langle D_a^{\rm fin}\rangle$ assuming $\tau_f \approx \nu_a^{-1}$, which yields

$$D_{ca} \lesssim \left\langle D_a^{\rm fin}\right\rangle. \tag{12}$$

In the following we list a few practical consequences of our theory which focus on one of the most interesting problems in collisionless plasma physics, the mechanism of collisionless reconnection of magnetic fields.

6.1 Resistive scale and relation to reconnection

We may use these arguments to infer briefly about the resistive scale L_ν, a quantity frequently referred to in discussions

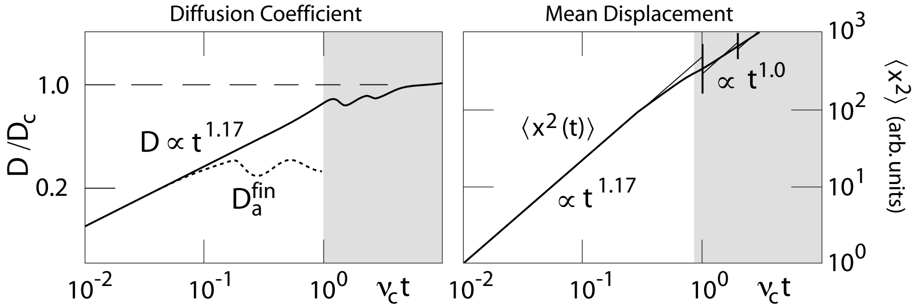

Figure 2. Schematic hypothetical evolution of the diffusion coefficient for the case simulated in Fig. 1 until the collisional classical diffusion state would have been reached. Time is measured here in classical collision times ν_c^{-1}. (Left) The anomalous increase of the diffusion coefficient with time. The growth of the diffusion coefficient gradually comes to rest after the classical collision time has elapsed. Dotted: A time-dependent nonlinear stationary state never approaching classical diffusion. (Right) Time evolution of the average particle displacement increasing as shown in Fig. 1. When approaching the classical collision time, scattering of particles to both larger and smaller displacements widen the displacement range, leading to a reduced increase until the second collision time. Similarly after the second, third, and the following collision times. Finally, the increase in the displacement settles into linear in time, implying classical or stationary diffusion.

of diffusion in the presence of current flow. It plays a role in the diffusive evolution of the magnetic field which from the induction equation is given in its simplest form

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathbf{B} + D_\mathrm{m}\nabla^2\mathbf{B}, \qquad D_\mathrm{m} = \frac{\eta}{\mu_0} = \lambda_\mathrm{e}^2\nu. \qquad (13)$$

The resistive scale is defined as $L_\nu^2 \sim D_\mathrm{m}t = \lambda_\mathrm{e}^2\nu t$ being determined through resistivity $\eta = \nu/\epsilon_0\omega_\mathrm{e}^2$ and electron inertial length $\lambda_\mathrm{e} = c/\omega_\mathrm{e}$, with plasma frequency ω_e. It tells, at what scale resistive diffusion starts affecting the plasma dynamics.

It is interesting to know how the resistive scale evolves with time in a nonlinearly active though collisionless medium. Using the expression for the product $\nu_\mathrm{a}t$ to replace νt gives

$$\frac{L_{\nu_\mathrm{a}}}{\lambda_\mathrm{e}} \sim \left\{\frac{D_\mathrm{a}(t)}{D_\mathrm{ca}}\right\}^{1/2} \sim (\nu_\mathrm{a}t)^{1/\alpha} \qquad (14)$$

for the resistive scale in units of λ_e, expressed through the (time-dependent) diffusion coefficient D_a. This indicates that the resistive scale increases with time from a value $L_{\nu_\mathrm{a}} < \lambda_\mathrm{e}$ until $D_\mathrm{a} \sim \langle D_\mathrm{a}^\mathrm{fin}\rangle$ when $L_{\nu_\mathrm{a}}^\mathrm{fin} \sim \lambda_\mathrm{e}$ approaches the inertial scale.

Small (anomalous) resistive scales imply fast magnetic diffusion as observed in collisionless systems like in reconnection. Since in collisionless plasma there is no resistive diffusion, one concludes that any process causing diffusion will readily reduce the resistive scale to values below the electron inertial scale, causing comparably fast dissipation of magnetic fields and favouring reconnection.

The remaining problem consists in finding an appropriate expression for the *equivalent* anomalous "collision frequency" ν_a under collisionless conditions. Observations (LaBelle and Treumann, 1988; Treumann et al., 1990; Bale et al., 2002) do not indicate any presence of sufficiently high wave amplitudes in collisionless reconnection required (Sagdeev, 1966, 1979) for the quasilinear generation of anomalous resistances. Numerical particle-in-cell simulations (cf. Treumann and Baumjohann, 2013, for a recent review) confirmed instead that in all cases the main driver of fast collisionless reconnection is the electron "pseudo-viscosity" implied by the presence of non-diagonal terms (Hesse and Winske, 1998; Hesse et al., 1999) in the thermally anisotropic electron pressure tensor $\mathbb{P}_\mathrm{e}$ measured in the stationary frame of the reconnecting current layer and accounting for any subtle finite gyro-radius effects in the dynamics of electrons in the inhomogeneous magnetic field of the electron diffusion region where electrons perform bouncing Speiser orbits.

6.2 Gyroviscosity

An expression for the anomalous collision frequency ν_a that is *equivalent* to electron pseudo-viscosity is found referring to the volume viscosity μ_V (or kinematic viscosity $\mu_\mathrm{kin} = \mu_\mathrm{V}/mN$, with N the density) and the molecular collision frequency ν_m (Huang, 1987)

$$\mu_\mathrm{V} = NT/\nu_\mathrm{m} \qquad \text{or} \qquad \mu_\mathrm{kin} = T/m\nu_\mathrm{m}. \qquad (15)$$

Formally, this allows for the determination of ν_a when identifying μ_V with the electron volume "pseudo-viscosity" μ_e (or kinematic pseudo-viscosity $\mu_\mathrm{e,kin} = \mu_\mathrm{e}/Nm_\mathrm{e}$) resulting from the non-diagonal electron pressure tensor elements, a quantity which can be determined either from observation or from numerical particle-in-cell simulations. This yields

$$\nu_\mathrm{a} \approx NT_\mathrm{e}/\mu_\mathrm{e} = T_\mathrm{e}/m_\mathrm{e}\mu_\mathrm{e,kin} \qquad (16)$$

with N the plasma density and T_e the relevant electron temperature for the *pressure tensor-induced equivalent* anomalous collision frequency. Macmahon (1965) derived an MHD form of the full pressure tensor including finite ion-gyroradius contributions in the limit of very strong magnetic fields, barely applicable to the weak magnetic field reconnection site. A simplified version of his expressions neglecting heat fluxes was given by Stasiewicz (1987) based on the implicit assumption that in strong magnetic fields the mean free path is replaced by the ion-gyroradius. In view of reconnection, this form has been used by Hau and Sonnerup (1991) in application to rotational discontinuities (for the role of viscosities in viscous fluids, cf. Landau and Lifshitz, 1987).

In this form, rewritten for the relevant electron dynamics, one has $\mu_e \simeq T_e/m_e\omega_{ce}$, which identifies $\nu_a = \nu_{gv} \sim \omega_{ce}$ as an electron gyro-viscous MHD collision frequency of the order of the electron cyclotron frequency $\omega_{ce} = eB/m_e$ – indeed much larger than any Coulomb collision frequency. It suggests that gyro-viscous superdiffusion means Bohm diffusion.

6.3 Estimates of transport quantities

Instead, use can be made of available numerical simulations (Pritchett, 2005) which quantitatively determined the contribution of the electron-pressure tensor-induced pseudo-viscosity to the dissipative generation of the parallel electric field in guide-field reconnection (cf. Treumann and Baumjohann, 2013, for a critical discussion). Pritchett (2005) obtained for the maximum non-diagonal pressure-generated field $E_{\parallel,P}$ in the inner part of the reconnection site (or electron exhaust region)

$$E_{\parallel,P} \;=\; (eN)^{-1}|\nabla\cdot\mathbb{P}_e| \;\lesssim\; 0.4\,V_A B_0, \tag{17}$$

where N, B_0, and V_A are the respective density, magnetic field outside the current layer, and Alfvén velocity based on B_0. The width of the current layer was $L_s \sim 2\lambda_i = 2\sqrt{M_s}\lambda_{es}$, with simulation mass ratio $M_s = m_i/m_{es} = 64$. On using index s for simulation quantities, real electron masses become $m_e = r\,m_{es}$, with $r = 64/1840$. With current $\mathbf{J}$, we may put

$$E_{\parallel,P} \;=\; \eta_{as}|\mathbf{J}| \;\sim\; \frac{\eta_{as}}{\mu_0}\frac{B_0}{L_s} \;=\; \frac{\lambda_{es}\nu_{as}B_0}{2\sqrt{M_s}} \tag{18}$$

Thus, the anomalous collision frequency corresponding to the pressure-induced pseudo-viscosity in the simulation of the reconnection process was of the order of

$$\nu_{as} \;\lesssim\; 0.8\;\sqrt{M_s}\;(V_A/c)\;\omega_{es} \;=\; 0.8\,\omega_{ce,s} \tag{19}$$

with the second form of the right-hand side resulting when accounting for the identity $(V_A/c)\sqrt{M} = \omega_{ce}/\omega_e$. In terms of real electron masses the last expression becomes

$$\nu_a \;= \nu_{as} r \;\lesssim\; 0.03\,\omega_{ce}. \tag{20}$$

This value is more than one order of magnitude smaller than the one of ν_{gv} obtained above from gyro-viscous MHD theory, rewritten for electrons. Still, its value is uncertain for the unknown dependence on mass ratio of the reconnection electric field $E_{\parallel,P}$ in the simulations. Assuming that this dependence is moderate, the agreement is surprisingly reasonable. For the wanted pseudo-viscosity this gives

$$\mu_{e,kin} \approx T_e/m_{es}\nu_{as} = T_e/m_e\nu_a \gtrsim 1.25\;T_e/m_e\omega_{ce} \tag{21}$$

with the factor r in the denominator cancelling, a form similar to gyro-viscosity for both simulation and real plasma applications.

Adopting the above numerical estimate of ν_a, the anomalous diffusion coefficient becomes

$$D_a(t) = 1.65\times10^{-2} D_{ca}\;(\omega_{ce}t)^{1.17}. \tag{22}$$

It increases slowly with time measured in electron cyclotron periods.

6.4 Digression on κ

With the last formula we have, in principle, achieved our goal.

However, someone might want to know the explicit form of the diffusion coefficient. For this one needs to determine the coefficient D_{ca}, which requires knowledge of $\langle\mathbf{x}^2\rangle$ in the electron exhaust. Since, from the simulations, no information is available on displacements, one has to refer to model assumptions for the distribution function $p(\mathbf{x})$.

Among the limited number of such functions available one may adopt the κ distribution Eq. (1), even though it is rather improbable that in the tiny reconnection region and for the restricted reconnection time any stationary κ distributions will have sufficient time to evolve.

Nevertheless, in the absence of any better choice, one may tentatively evoke the relation $\alpha/2 = \kappa(\kappa + d/2)^{-1}$ between α and κ, as proposed from non-extensive statistical mechanics (Tsallis et al., 1995; Prato and Tsallis, 1999; Bologna et al., 2000; Livadiotis and McComas, 2013) to hold in the superdiffusion range $\alpha < 2$, and apply it as well to our particular reconnection problem.

Then, on using the measured value of α, we have $\kappa \approx 5.9$ for $d = 2$. This gives the two-dimensional κ superdiffusion coefficient from Eqs. (22), (7), and (9), with squared correlation length $\ell^2 = 2T_e/m_e\nu_a$, as

$$D_{a\kappa}(t) \approx 11\;D_B\,(\omega_{ce}t)^{1.17}, \tag{23}$$

where $D_B \approx T_e/m_e\omega_{ce}$ is of the order of the Bohm diffusion coefficient. This value of ten times (!) the Bohm diffusion is excessively large, implying the presence of extraordinarily strong anomalous diffusion at the reconnection site, though not in unacceptable disagreement with exceptionally fast spontaneous reconnection. For a Gaussian probability distribution one had $\langle x^2\rangle = \ell^2 d/2$ and thus $D_a(t) \approx D_B\,(\omega_{ce}t)^{1.17}$.

It should, however, be kept in mind that the derivation of the κ diffusion coefficient Eq. (23) is based on the arbitrary assumption that the unknown distribution of displacements in the narrow electron exhaust would indeed be of the family of κ distributions. While the determination of the anomalous collision frequency from the simulations used is very well justified, there is however no observational or any theoretical justification for this ad hoc assertion.

6.5 Lower limit on ν_a in reconnection

The above numerical simulation-based estimates can be directly applied to observations of reconnection in the magnetotail current sheet in order to infer about the anomalous collision frequency generated in reconnection. From an applicational geophysical point of view this is most interesting. Observed magnetic fields across the tail plasma sheet vary between $1\,\mathrm{nT} < B_0 < 10\,\mathrm{nT}$. With these values one obtains the following range for the anomalous collision frequencies during reconnection in the plasma sheet:

$$4.9\,\mathrm{Hz} < \nu_a < 50\,\mathrm{Hz}, \qquad \omega_{lh} \approx 4.1\,\mathrm{Hz}. \tag{24}$$

These reasonably high values follow directly from analysis of the simulations, compared to the lower-hybrid frequency ω_{lh} given on the right for the lower value $B = 1\,\mathrm{nT}$ only. This estimated anomalous collision frequency at the magnetotail reconnection site is the result of non-stochastic processes in the electron exhaust diffusion region which generate the out-of-diagonal pseudo-viscous terms in the electron pressure tensor. It is responsible for the necessary superdiffusion at the reconnection site which is required in the collisionless reconnection process.

The closeness of the lower-hybrid frequency ω_{lh} to the range of anomalous collision frequencies indicates the collisionless electric coupling between electrons and ions in any reconnection process.

In addition, it provides an important lower limit

$$\omega_{lh} \lesssim \min_{\mathrm{rec}} (\nu_a) \tag{25}$$

on ν_a in collisionless reconnection, thereby a posteriori justifying the frequently found surprising closeness (e.g. Huba et al., 1977; LaBelle and Treumann, 1988; Treumann et al., 1991; Yoon et al., 2002, and others) to the lower-hybrid frequency of the rough estimates of anomalous collision frequencies from the analysis of spacecraft observations of reconnection, which are necessary for explaining the timescale of the observed dissipation of energy.

Considered in this spirit, collisionless reconnection is understood as an *equivalent anomalous local super*-diffusion process in collisionless plasma. From a general physical point of view, this interpretation ultimately re-unifies the initially considered mutually excluding collisionless reconnection and diffusion theories in satisfactory concordance with fundamental electrodynamics.

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