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AN EOQ MODEL FOR TIME-DEPENDENT DETERIORATING ITEMS WITH ALTERNATING DEMAND RATES ALLOWING SHORTAGES BY CONSIDERING TIME VALUE OF MONEY

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Abstract: The present paper deals with an economic order quantity (EOQ) model of an inventory problem with alternating demand rate: (i) For a certain period, the demand rate is a non linear function of the instantaneous inventory level. (ii) For the rest of the cycle, the demand rate is time dependent. The time at which demand rate changes, may be deterministic or uncertain. The deterioration rate of the item is time dependent. The holding cost and shortage cost are taken as a linear function of time. The total cost function per unit time is obtained. Finally, the model is solved using a gradient based non-linear optimization technique (LINGO) and is illustrated by a numerical example.

Keywords: EOQ, Deterioration, Shortages, Two-component Demand.

MSC: 90B05.

1. INTRODUCTION

In recent years, there is a spate of interest in studying the inventory systems with an inventory-level-dependent demand rate. It is observed that large quantities of consumer goods displayed in a supermarket generate higher demands. The impact of shelf-space allocation on retail-product demand has been subject of investigation by many researchers like Levin et al.[11], Silver[18], and Silver and Peterson[19].

An inventory model for stock-dependent consumption rate was discussed by Gupta and Vrat[7]. However, their calculation of the average system cost was wrong. Mandal and Phaujdar [12] suggested corrections to the average system cost in [7]. Another model for deteriorating items with stock-dependent consumption rate was developed by Mandal and Phaujdar[13]. The first rigorous attempt at developing an inventory level with a stock-dependent consumption rate was made by Baker and Urban[1]. Their functional form for the demand rate is realistic and logical from practical as well as economic viewpoints.

Deterioration can not be avoided in business scenarios. Rau et al[16] presented the economic ordering policies of deteriorating items in a supply chain management system. Dye et al[6] developed an EOQ model for deteriorating items allowing shortages and backlogging. An EOQ model for deteriorating items with time varying demand and shortages have been suggested by Chung et al[5]. Skouri et al[21] discussed about EOQ for deteriorating items under delay in payments. Ghare and Schrader[8] categorized the inventory deterioration into three types: direct spoilage, physical depletion and deterioration. Direct spoilage refers to the unstable state of inventory items caused by breakage during transaction or by sudden accidental events. For example quality and effectiveness of some medicines might be reduced in the event of non-functioning of refrigerator caused by sudden load shedding or absence of power supply for hours together. Deterioration on the other hand, refers to the slow but gradual loss of qualitative properties of an item with the passage of time. In fact no inventory item can avoid this kind of deterioration. This is inevitable. Wee[23] considered an inventory problem for deteriorating items with shortages. Reddy et al.[17] considered stock-dependent demand rate in a periodic review inventory system. Subbaiah et al[20] developed an inventory model with stock-dependent demand. Teng et al.[22] discussed an EPQ model for deteriorating items where demand depends on stock and price. Inventory model with stock-dependent demand is developed by Rao et al.[15]. In practice the demand depend not only on stock but also on the types of customers. Pal et al.[14] made an investigation on inventory system of two-component demand rate irrespective of shortages and price breaks. Basu M and Sinha S[2] developed an ordering policy for deteriorating items with two component demand and price breaks allowing shortages. Customers may be classified into two categories:

(i) Those who are motivated by displayed stock level (DSL). They are floating.
(ii) Those who are not motivated by DSL.

So the two-component demand rate is more applicable in real life..

Shortages can not be avoided in practical situations. Jamal et al.[9] presented an EOQ model that focused on deteriorating items with allowable shortages. But they did not consider two-component demand which would make it more applicable in real situation.

Chakrabarti and Sen[3] developed an order inventory model with variable rate of deteriorating and alternating replenishing rates considering shortage. Also Chakrabarti and Sen[4] presented an EOQ model for deteriorating items with quadratic time varying demand and shortages in all cycles. An EOQ model for perishable item with stock and price dependent demand rate was developed by Khanra et al[10].

The objective of the present paper is to determine the optimal order quantity with a deteriorating item by the rate of deterioration as a time function of the on hand inventory. This model will run with time-dependent holding and shortage cost. Here we have taken two-component demand. At the beginning, the demand rate is directly related to the amount of inventory displayed on the board. After a certain time, the demand changes to time-dependent. The objective is to minimize the total average cost function of the inventory system over a long period of time. A numerical example is discussed to illustrate the procedure of solving the model.

2. NOTATIONS AND ASSUMPTIONS

Throughout the paper the following notations and assumptions are used.

2.1 Notations:

- $q(t)$ Inventory level at any time t.
- $S = q(0)$ Stock level at the beginning of each cycle after fulfilling backorders.
- $S_0 = q(0)$ Stock level below which the demand rate is time dependent.
- Q Stock level at the beginning + the amount of shortages.
- T_0 Time epoch at which the demand rate changes.
- T_1 Time until shortage begins.
- T Length of the cycle time.
- $\phi(t) = \theta t; 0 < \theta < 1$, is the time-proportional decay rate of the stock. Since $\theta > 0, (d\phi(t)/dt) = \theta > 0$. Hence, the decay-rate increases with time at a rate θ.
- K Constant ordering cost per order.
- HC Holding cost per cycle where $(h_0 + h_1 t)$ is the holding cost at time t of the on-hand inventory.
- DC Deterioration cost.
- SC Shortage cost where $(c_0 + c_1 t)$ is the shortage cost at time t of the on-hand inventory.

2.2 Assumptions:

- Item cost does not vary with order size.
- Lead time (the time between placing an order for replenishment stock and its receipt) is assumed to be zero. This is a parameter depending on the product as well as the source from which it is available.
- Replenishments are instantaneous.

- Inventory system consists of only one item.
- The time horizon of the inventory system is infinite. Only a typical planning schedule of length T is considered and all remaining cycles are identical.
- A time dependent function $\theta t, (0 < \theta < 1)$ of the on-hand inventory deteriorates per unit time and the deteriorated item is lost.
- Shortages are allowed and fully backlogged.
- Two component demand rate is considered here. Demand rate is deterministic and is a known function of the instantaneous inventory level up to a certain interval of time and after that the demand rate is time-dependent. The demand rate D(t) is given by
- $$D(t) = \begin{cases} \alpha\{q(t)\}^{\beta}, 0 \le t \le T_0 \\ (a+bt), T_0 \le t \le T \end{cases}$$
- Where α and β are scale and shape parameters, respectively and a, b are positive constants.

3. MATHEMATICAL MODEL AND ANALYSIS:

The inventory system developed is depicted by the following figure:

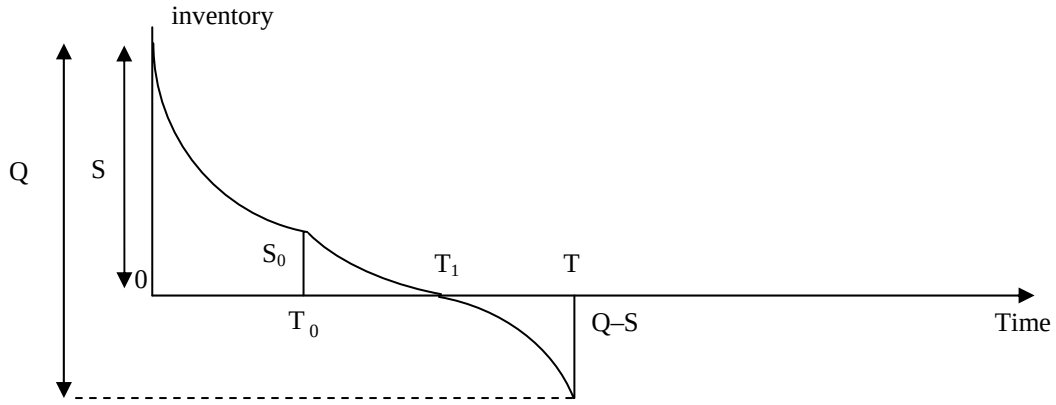

Figure. 1

The inventory level will be depleted at a rate of $\alpha\{q(t)\}^{\beta}$ during the period $[0,T_0]$ where T_0 will be determined by $q(T_0) = S_0$, the corresponding value of S_0 will also be determined. During the period $[T_0,T_1]$ the inventory level will be depleted at a rate of $(a+bt)$. The inventory falls to zero level at time $t = T_1$. Shortages are then

allowed for replenishment up to time $t = T$. Therefore, for a deterioration rate θ_t , the instantaneous inventory level will satisfy the following differential equations

$$\frac{dq_1(t)}{dt} + \theta t q_1(t) = -\alpha\{q_1(t)\}^{\beta}, 0 \le t \le T_0 \tag{1}$$

With the boundary conditions

$$q_1(0) = S, q_1(T_0) = S_0 \tag{1a}$$

On the other hand, in the time interval (T_0, T_1), the system is affected by the combined effect of demand and deterioration. Hence, the change in inventory level is governed by the following differential equation

$$\frac{dq_2(t)}{dt} + \theta t q_2(t) = -(a + bt), T_0 \le t \le T_1 \tag{2}$$

With the boundary conditions

$$q_2(T_0) = S_0, q_2(T_1) = 0 \tag{2a}$$

In the time interval (T_1, T), the system is affected by demand only. Hence, the change in inventory level is governed by the following differential equation

$$\frac{dq_3(t)}{dt} = -(a + bt), T_1 \le t \le T \tag{3}$$

with the boundary conditions

$$q_3(T_1) = 0, q_3(T) = -(Q - S) \tag{3a}$$

The solution of the differential equations (1) with the boundary condition (1a) is

$$q_1(t) = (S^p - \alpha pt)^{\frac{1}{p}}[1 + \frac{\theta}{(S^p - \alpha pt)}(\frac{1}{3}\alpha pt^3 - \frac{1}{2}S^p t^2)] \tag{4}$$

(neglecting θ^2 and higher power)

The boundary condition $q_1(T_0) = S_0$ gives

$$S_0 = (S^p - \alpha pT_0)^{\frac{1}{p}}[1 + \frac{\theta}{(S^p - \alpha pT_0)}(\frac{1}{3}\alpha pT_0^3 - \frac{1}{2}S^p T_0^2)] \tag{4a}$$

The solution of (2) with the help of the condition (2a) gives

$$\begin{aligned} q_2(t) = {} & (S_0 - at + aT_0 - \frac{1}{2}bt^2 + \frac{1}{2}bT_0^2) + \theta(\frac{1}{3}at^3 + \frac{1}{8}bt^4 - \frac{1}{2}aT_0t^2 + \frac{1}{6}aT_0^3 - \frac{1}{4}bt^2T_0^2 \\ & + \frac{1}{8}bT_0^4 - \frac{1}{2}S_0t^2 + \frac{1}{2}S_0T_0^2) \end{aligned} \tag{5}$$

(neglecting θ^2 and higher power)

The condition $q(T_1) = 0$ gives

$$(S_0 - aT_1 + aT_0 - \frac{1}{2}bT_1^2 + \frac{1}{2}bT_0^2) = -\theta(\frac{1}{3}aT_1^3 + \frac{1}{8}bT_1^4 - \frac{1}{2}aT_0T_1^2 + \frac{1}{6}aT_0^3 - \frac{1}{4}bT_1^2T_0^2 + \frac{1}{8}bT_0^4 - \frac{1}{2}S_0T_1^2 + \frac{1}{2}S_0T_0^2) \quad (5a)$$

The solution of (3) with the help of the condition (3a) gives

$$q_3(t) = -a(t - T_1) - \frac{b}{2}(t^2 - T_1^2) \quad (6)$$

The condition $(T) = -(Q - S)$ gives

$$Q - S = a(T - T_1) + \frac{b}{2}(T^2 - T_1^2) \quad (6a)$$

This is the relation between T_1 and T.

The total variable cost comprises of the sum of the ordering cost, holding cost, deterioration cost minus backorder cost.

For the moment, the individual costs are now evaluated before they are grouped together.

(1) Annual ordering cost= $\frac{K}{T}$

(2) Annual holding cost

$$(\text{HC}) = \frac{1}{T}[\int_0^{T_0}(h_0 + h_1t)q_1(t)dt + \int_{T_0}^{T_1}(h_0 + h_1t)q_2(t)dt]$$

$$= \frac{1}{T}[\frac{h_0}{d_1}(S^{1+p} - Y^{\frac{1+p}{p}}) + \frac{h_0\alpha\theta p}{3}(-\frac{T_0^3}{\alpha}Y^{\frac{1}{p}} - \frac{3T_0^2Y^{\frac{1+p}{p}}}{d_2} - \frac{6T_0Y^{\frac{1+2p}{p}}}{d_3} - \frac{6Y^{\frac{1+3p}{p}}}{d_4} + \frac{6S^{1+3p}}{d_4})$$

$$- \frac{h_0\theta S^p}{2}(-\frac{T_0^2Y^{\frac{1}{p}}}{\alpha} - \frac{2T_0Y^{\frac{1+p}{p}}}{d_2} - \frac{2Y^{\frac{1+2p}{p}}}{d_3} + \frac{2S^{1+2p}}{d_3}) + h_1(-\frac{T_0Y^{\frac{1+p}{p}}}{d_1} - \frac{Y^{\frac{1+2p}{p}}}{d_2} + \frac{S^{1+2p}}{d_2})$$

$$+ \frac{h_1\alpha\theta p}{3}(-\frac{T_0^4Y^{\frac{1}{p}}}{\alpha} - \frac{4T_0^3Y^{\frac{1+p}{p}}}{d_2} - \frac{12T_0^2Y^{\frac{1+2p}{p}}}{d_3} - \frac{24T_0Y^{\frac{1+3p}{p}}}{d_4} - \frac{24Y^{\frac{1+4p}{p}}}{d_5} + \frac{24Y^{1+4p}}{d_5})$$

$$- \frac{h_1\theta S^p}{2}(-\frac{T_0^3Y^{\frac{1}{p}}}{\alpha} - \frac{3T_0^2Y^{\frac{1+p}{p}}}{d_2} - \frac{6T_0Y^{\frac{1+2p}{p}}}{d_3} - \frac{6Y^{\frac{1+3p}{p}}}{d_4} + \frac{6S^{1+3p}}{d_4})$$

$$+h_0\{-\frac{a}{2}(T_1^2-T_0^2)-\frac{b}{6}(T_1^3-T_0^3)+(S_0+aT_0+\frac{b}{2}T_0^2)(T_1-T_0)\}+\theta\{\frac{a}{12}(T_1^4-T_0^4)$$
$$+\frac{b}{40}(T_1^5-T_0^5)-\frac{a}{6}T_0(T_1^3-T_0^3)-\frac{b}{12}T_0^2(T_1^3-T_0^3)+(\frac{a}{6}T_0^3+\frac{b}{8}T_0^4+\frac{S_0}{2}T_0^2)(T_1-T_0)$$
$$-\frac{S_0}{6}(T_1^3-T_0^3)\}+h_1\{(\frac{S_0}{2}+\frac{aT_0}{2}+\frac{bT_0^2}{4})(T_1^2-T_0^2)-\frac{a}{3}(T_1^3-T_0^3)-\frac{b}{8}(T_1^4-T_0^4)\}$$
$$+\theta\{\frac{a}{15}(T_1^5-T_0^5)+\frac{b}{48}(T_1^6-T_0^6)-\frac{a}{8}T_0(T_1^4-T_0^4)+\frac{a}{12}T_0^3(T_1^2-T_0^2)-\frac{b}{16}T_0^2(T_1^4-T_0^4)$$
$$+\frac{b}{16}T_0^4(T_1^2-T_0^2)-\frac{S_0}{8}(T_1^4-T_0^4)+\frac{S_0}{4}T_0^2(T_1^2-T_0^2)\}]$$

where Y= $(S^p-\alpha pT_0)$, $p=1-\beta$,

$$d_1=\alpha(1+p), d_2=\alpha^2(1+p), d_3=\alpha^3(1+p)(1+2p), d_4=\alpha^4(1+p)(1+2p)(1+3p),$$
$$d_5=\alpha^5(1+p)(1+2p)(1+3p)(1+4p)$$

(1) Annual shortage cost (SC) is given by

$$SC=\frac{1}{T}[\int_{T_1}^{T}(c_0+c_1t)q_3(t)dt]$$
$$=\frac{1}{T}[c_0\{-a(\frac{T^2}{2}-T_1T+\frac{T_1^2}{2})-\frac{b}{2}(\frac{T^3}{3}-T_1^2T+\frac{2}{3}T_1^3)\}$$
$$+c_1\{-a(\frac{T^3}{3}-\frac{1}{2}T_1T^2+\frac{1}{6}T_1^3)-\frac{b}{2}(\frac{T^4}{4}-\frac{T_1^2T^2}{2}+\frac{T_1^2}{4})\}]$$

(2) Annual cost of deterioration (DC) is given by

$$DC=\frac{1}{T}\left[q(0)-\{\int_0^{T_0}\alpha(q(t))^\beta dt+\int_{T_0}^{T_1}(a+bt)dt\right]]$$

$$=\frac{1}{T}[S-\{\alpha\{-\frac{Y^{\frac{1+p}{p}}}{d_1}+\frac{S^{1+p}}{d_1}\}+\frac{\alpha^2\theta p}{3}\{\frac{T_0^3Y^{\frac{1}{p}}}{\alpha}-\frac{3T_0^2Y^{\frac{1+p}{p}}}{d_2}-\frac{6T_0Y^{\frac{1+2p}{p}}}{d_3}-\frac{6Y^{\frac{1+3p}{p}}}{d_4}+\frac{6S^{1+3p}}{d_4}\}$$
$$-\frac{\alpha\theta S^p}{2}\{-\frac{T_0^2Y^{\frac{1}{p}}}{\alpha}-\frac{2T_0Y^{\frac{1+p}{p}}}{d_2}-\frac{2Y^{\frac{1+2p}{p}}}{d_3}+\frac{2S^{1+2p}}{d_3}\}-\{a(T_1-T_0)+\frac{b}{2}(T_1^2-T_0^2)\}\}]$$

where Y= $(S^p-\alpha pT_0)$, $p=1-\beta$,

$$d_1=\alpha(1+p), d_2=\alpha^2(1+p), d_3=\alpha^3(1+p)(1+2p), d_4=\alpha^4(1+p)(1+2p)(1+3p),$$
$$d_5=\alpha^5(1+p)(1+2p)(1+3p)(1+4p)$$

The total cost per unit time is given by

$$TC(T_0,T)=(OC+HC+DC-SC)$$

$$=\frac{K}{T}+\frac{1}{T}[\frac{h_0}{d_1}(S^{1+p}-Y^{\frac{1+p}{p}})+\frac{h_0\alpha\theta p}{3}(-\frac{T_0^3}{\alpha}Y^{\frac{1}{p}}-\frac{3T_0^2Y^{\frac{1+p}{p}}}{d_2}-\frac{6T_0Y^{\frac{1+2p}{p}}}{d_3}-\frac{6Y^{\frac{1+3p}{p}}}{d_4}+\frac{6S^{1+3p}}{d_4})$$
$$-\frac{h_0\theta S^p}{2}(-\frac{T_0^2Y^{\frac{1}{p}}}{\alpha}-\frac{2T_0Y^{\frac{1+p}{p}}}{d_2}-\frac{2Y^{\frac{1+2p}{p}}}{d_3}+\frac{2S^{1+2p}}{d_3})+h_1(-\frac{T_0Y^{\frac{1+p}{p}}}{d_1}-\frac{Y^{\frac{1+2p}{p}}}{d_2}+\frac{S^{1+2p}}{d_2})$$
$$+\frac{h_1\alpha\theta p}{3}(-\frac{T_0^4Y^{\frac{1}{p}}}{\alpha}-\frac{4T_0^3Y^{\frac{1+p}{p}}}{d_2}-\frac{12T_0^2Y^{\frac{1+2p}{p}}}{d_3}-\frac{24T_0Y^{\frac{1+3p}{p}}}{d_4}-\frac{24Y^{\frac{1+4p}{p}}}{d_5}+\frac{24Y^{1+4p}}{d_5})$$
$$-\frac{h_1\theta S^p}{2}(-\frac{T_0^3Y^{\frac{1}{p}}}{\alpha}-\frac{3T_0^2Y^{\frac{1+p}{p}}}{d_2}-\frac{6T_0Y^{\frac{1+2p}{p}}}{d_3}-\frac{6Y^{\frac{1+3p}{p}}}{d_4}+\frac{6S^{1+3p}}{d_4})$$
$$+h_0\{-\frac{a}{2}(T_1^2-T_0^2)-\frac{b}{6}(T_1^3-T_0^3)+(S_0+aT_0+\frac{b}{2}T_0^2)(T_1-T_0)\}+\theta\{\frac{a}{12}(T_1^4-T_0^4)$$
$$+\frac{b}{40}(T_1^5-T_0^5)-\frac{a}{6}T_0(T_1^3-T_0^3)-\frac{b}{12}T_0^2(T_1^3-T_0^3)+(\frac{a}{6}T_0^3+\frac{b}{8}T_0^4+\frac{S_0}{2}T_0^2)(T_1-T_0)$$
$$-\frac{S_0}{6}(T_1^3-T_0^3)\}+h_1\{(\frac{S_0}{2}+\frac{aT_0}{2}+\frac{bT_0^2}{4})(T_1^2-T_0^2)-\frac{a}{3}(T_1^3-T_0^3)-\frac{b}{8}(T_1^4-T_0^4)\}$$
$$+\theta\{\frac{a}{15}(T_1^5-T_0^5)+\frac{b}{48}(T_1^6-T_0^6)-\frac{a}{8}T_0(T_1^4-T_0^4)+\frac{a}{12}T_0^3(T_1^2-T_0^2)-\frac{b}{16}T_0^2(T_1^4-T_0^4)$$
$$+\frac{b}{16}T_0^4(T_1^2-T_0^2)-\frac{S_0}{8}(T_1^4-T_0^4)+\frac{S_0}{4}T_0^2(T_1^2-T_0^2)\}]$$
$$+\frac{1}{T}[S-\{\alpha\{-\frac{Y^{\frac{1+p}{p}}}{d_1}+\frac{S^{1+p}}{d_1}\}+\frac{\alpha^2\theta p}{3}\{\frac{T_0^3Y^{\frac{1}{p}}}{\alpha}-\frac{3T_0^2Y^{\frac{1+p}{p}}}{d_2}-\frac{6T_0Y^{\frac{1+2p}{p}}}{d_3}-\frac{6Y^{\frac{1+3p}{p}}}{d_4}+\frac{6S^{1+3p}}{d_4}\}$$
$$-\frac{\alpha\theta S^p}{2}\{-\frac{T_0^2Y^{\frac{1}{p}}}{\alpha}-\frac{2T_0Y^{\frac{1+p}{p}}}{d_2}-\frac{2Y^{\frac{1+2p}{p}}}{d_3}+\frac{2S^{1+2p}}{d_3}\}-\{a(T_1-T_0)+\frac{b}{2}(T_1^2-T_0^2)\}\}]$$
$$-\frac{1}{T}[c_0\{-a(\frac{T^2}{2}-T_1T+\frac{T_1^2}{2})-\frac{b}{2}(\frac{T^3}{3}-T_1^2T+\frac{2}{3}T_1^3)\}$$
$$+c_1\{-a(\frac{T^3}{3}-\frac{1}{2}T_1T^2+\frac{1}{6}T_1^3)-\frac{b}{2}(\frac{T^4}{4}-\frac{T_1^2T^2}{2}+\frac{T_1^2}{4})\}] \qquad (7)$$

where Y= $(S^p-\alpha pT_0)$, $p=1-\beta$,

$$d_1 = \alpha(1+p), d_2 = \alpha^2(1+p), d_3 = \alpha^3(1+p)(1+2p), d_4 = \alpha^4(1+p)(1+2p)(1+3p),$$
$$d_5 = \alpha^5(1+p)(1+2p)(1+3p)(1+4p)$$

We now minimize the total cost per unit time $TC(T_0,T)$ under the situation

(1) T_0 is a known point of time.

(2) T_0 is a random point of time.

Case I: T_0 is a known point of time.

Hence total cost is given by equation (7).

For minimum total cost, the necessary condition is $\frac{\delta TC(T_0,T)}{\delta T_0} = 0$ and

$$\frac{\delta TC(T_0,T)}{\delta T} = 0 \tag{8}$$

Let T* be the positive real root of the equation (8), then T* is the optimal cycle time. It can also be seen that the sufficient condition for minimum cost

$$\begin{vmatrix} \frac{\partial^2 TC(T_0,T)}{\partial T_0^2} & \frac{\partial^2 TC(T_0,T)}{\partial T_0 \partial T} \\ \frac{\partial^2 TC(T_0,T)}{\partial T \partial T_0} & \frac{\partial^2 TC(T_0,T)}{\partial T^2} \end{vmatrix} > 0 \text{ is satisfied.}$$

Case II: T_0 is a random point of time.

In this case, the cost function $TC(T_0,T)$ is a random variable with respect to T_0. So the expected total cost per unit time is

$$\xi(T) = \frac{K}{T} + \frac{1}{T}[\frac{h_0}{d_1}(S^{1+p} - E(Y)^{\frac{1+p}{p}}) + \frac{h_0\alpha\theta p}{3}(-\frac{1}{\alpha}E(T_0^{\,3}Y^{\frac{1}{p}}) - \frac{3E(T_0^{\,2}Y^{\frac{1+p}{p}})}{d_2} - \frac{6E(T_0Y^{\frac{1+2p}{p}})}{d_3}$$
$$- \frac{6E(Y^{\frac{1+3p}{p}})}{d_4} + \frac{6S^{1+3p}}{d_4}) - \frac{h_0\theta S^p}{2}(-\frac{E(T_0^{\,2}Y^{\frac{1}{p}})}{\alpha} - \frac{2E(T_0Y^{\frac{1+p}{p}})}{d_2} - \frac{2E(Y^{\frac{1+2p}{p}})}{d_3} + \frac{2S^{1+2p}}{d_3})$$
$$+ h_1(-\frac{E(T_0Y^{\frac{1+p}{p}})}{d_1} - \frac{E(Y^{\frac{1+2p}{p}})}{d_2} + \frac{S^{1+2p}}{d_2})$$

$$+\frac{h_1\alpha\theta p}{3}\left(-\frac{E(T_0^{\,4}Y^{\frac{1}{p}})}{\alpha}-\frac{4E(T_0^{\,3}Y^{\frac{1+p}{p}})}{d_2}-\frac{12E(T_0^{\,2}Y^{\frac{1+2p}{p}})}{d_3}-\frac{24E(T_0Y^{\frac{1+3p}{p}})}{d_4}-\frac{24E(Y^{\frac{1+4p}{p}})}{d_5}\right.$$
$$\left.+\frac{24(S^{1+4p})}{d_5}\right)-\frac{h_1\theta S^p}{2}\left(-\frac{E(T_0^{\,3}Y^{\frac{1}{p}})}{\alpha}-\frac{3E(T_0^{\,2}Y^{\frac{1+p}{p}})}{d_2}-\frac{6E(T_0Y^{\frac{1+2p}{p}})}{d_3}-\frac{6E(Y^{\frac{1+3p}{p}})}{d_4}+\frac{6S^{1+3p}}{d_4}\right)$$
$$+h_0[\{-\frac{a}{2}(T_1^{\,2}-E(T_0^{\,2}))-\frac{b}{6}(T_1^{\,3}-E(T_0^{\,3}))+(S_0+aE(T_0)+\frac{b}{2}E(T_0^{\,2}))(T_1-E(T_0))\}$$
$$+\theta\{\frac{a}{12}(T_1^{\,4}-E(T_0^{\,4}))+\frac{b}{40}(T_1^{\,5}-E(T_0^{\,5}))-\frac{a}{6}E(T_0)(T_1^{\,3}-E(T_0^{\,3}))-\frac{b}{12}E(T_0^{\,2})$$
$$(T_1^{\,3}-E(T_0^{\,3}))+(\frac{a}{6}E(T_0^{\,3})+\frac{b}{8}E(T_0^{\,4})+\frac{S_0}{2}E(T_0^{\,2}))(T_1-E(T_0))-\frac{S_0}{6}(T_1^{\,3}-E(T_0^{\,3}))\}]$$
$$+h_1[\{(\frac{S_0}{2}+\frac{E(aT_0)}{2}+\frac{E(bT_0^{\,2})}{4})(T_1^{\,2}-E(T_0^{\,2}))-\frac{a}{3}(T_1^{\,3}-E(T_0^{\,3}))-\frac{b}{8}(T_1^{\,4}-E(T_0^{\,4}))\}$$
$$+\theta\{\frac{a}{15}(T_1^{\,5}-E(T_0^{\,5}))+\frac{b}{48}(T_1^{\,6}-E(T_0^{\,6}))-\frac{a}{8}E(T_0)(T_1^{\,4}-E(T_0^{\,4}))$$
$$+\frac{a}{12}E(T_0^{\,3})(T_1^{\,2}-E(T_0^{\,2}))-\frac{b}{16}E(T_0^{\,2})(T_1^{\,4}-E(T_0^{\,4}))+\frac{b}{16}E(T_0^{\,4})(T_1^{\,2}-E(T_0^{\,2}))$$
$$-\frac{S_0}{8}(T_1^{\,4}-E(T_0^{\,4}))+\frac{S_0}{4}E(T_0^{\,2})(T_1^{\,2}-E(T_0^{\,2}))\}]]$$
$$+\frac{1}{T}[S-\{\alpha\{-\frac{E(Y^{\frac{1+p}{p}})}{d_1}+\frac{S^{1+p}}{d_1}\}+\frac{\alpha^2\theta p}{3}\{\frac{E(T_0^{\,3}Y^{\frac{1}{p}})}{\alpha}-\frac{3E(T_0^{\,2}Y^{\frac{1+p}{p}})}{d_2}-\frac{6E(T_0Y^{\frac{1+2p}{p}})}{d_3}$$
$$-\frac{6E(Y^{\frac{1+3p}{p}})}{d_4}+\frac{6S^{1+3p}}{d_4}\}$$
$$-\frac{\alpha\theta S^p}{2}\{-\frac{E(T_0^{\,2}Y^{\frac{1}{p}})}{\alpha}-\frac{2E(T_0Y^{\frac{1+p}{p}})}{d_2}-\frac{2E(Y^{\frac{1+2p}{p}})}{d_3}+\frac{2S^{1+2p}}{d_3}\}$$
$$-\{a(T_1-E(T_0))+\frac{b}{2}(T_1^{\,2}-E(T_0^{\,2}))\}\}]$$
$$-\frac{1}{T}[c_0\{-a(\frac{T^2}{2}-T_1T+\frac{T_1^{\,2}}{2})-\frac{b}{2}(\frac{T^3}{3}-T_1^{\,2}T+\frac{2}{3}T_1^{\,3})\}$$
$$+c_1\{-a(\frac{T^3}{3}-\frac{1}{2}T_1T^2+\frac{1}{6}T_1^{\,3})-\frac{b}{2}(\frac{T^4}{4}-\frac{T_1^{\,2}T^2}{2}+\frac{T_1^{\,2}}{4})\}] \tag{9}$$

Now assume that the distribution function of T_0 to be rectangular distribution. Then its probability density function $f(x)$ is given by

$$f(x)=\frac{1}{l_2-l_1}, l_1 \le x \le l_2$$

=0, elsewhere

Then equation (9) reduces to

$$\xi(T)=\frac{K}{T}+\frac{1}{T}[\frac{h_0}{d_1}\{S^{1+p}+\frac{(x_1^{\frac{1+2p}{p}}-x_2^{\frac{1+2p}{p}})}{\alpha l(1+2p)}\}+\frac{h_0\alpha\theta p}{3}\{-\frac{1}{\alpha l}(\frac{T_0^{3}(x_1^{\frac{1+p}{p}}-x_2^{\frac{1+p}{p}})}{\alpha(1+p)}$$

$$+\frac{3T_0^{2}(x_1^{\frac{1+2p}{p}}-x_2^{\frac{1+2p}{p}})}{\alpha^2(1+p)(1+2p)}+\frac{6(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha^3(1+p)(1+2p)(1+3p)}+\frac{6(x_1^{\frac{1+4p}{p}}-x_2^{\frac{1+4p}{p}})}{\alpha^4(1+p)(1+2p)(1+3p)(1+4p)})$$

$$-\frac{3}{dl_2}(\frac{T_0^{2}(x_1^{\frac{1+2p}{p}}-x_2^{\frac{1+2p}{p}})}{\alpha(1+2p)}+\frac{2T_0(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha^2(1+2p)(1+3p)}+\frac{2(x_1^{\frac{1+4p}{p}}-x_2^{\frac{1+4p}{p}})}{\alpha^3(1+2p)(1+3p)(1+4p)})$$

$$-\frac{6}{d_3l}(\frac{T_0(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha(1+3p)}+\frac{(x_1^{\frac{1+4p}{p}}-x_2^{\frac{1+4p}{p}})}{\alpha^2(1+3p)(1+4p)}\}-\frac{6}{d_4l}\{\frac{(x_1^{\frac{1+4p}{p}}-x_2^{\frac{1+4p}{p}})}{\alpha(1+4p)})+\frac{6S^{1+3p}}{d_4}\}$$

$$-\frac{h_0\theta S^{p}}{2}\{-\frac{1}{\alpha l}(\frac{T_0^{2}(x_1^{\frac{1+p}{p}}-x_2^{\frac{1+p}{p}})}{\alpha(1+p)}+\frac{2T_0(x_1^{\frac{1+2p}{p}}-x_2^{\frac{1+2p}{p}})}{\alpha^2(1+p)(1+2p)}+\frac{2(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha^3(1+p)(1+2p)(1+3p)})$$

$$-\frac{2}{d_2l}(\frac{T_0(x_1^{\frac{1+2p}{p}}-x_2^{\frac{1+2p}{p}})}{\alpha(1+2p)}+\frac{(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha^2(1+2p)(1+3p)})-\frac{2}{d_3}\frac{(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha l(1+3p)}+\frac{2S^{1+2p}}{d_3}\}$$

$$+h_1\{-\frac{1}{d_1l}(\frac{T_0(x_1^{\frac{1+2p}{p}}-x_2^{\frac{1+2p}{p}})}{\alpha(1+2p)}+\frac{(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha^2(1+2p)(1+3p)})-\frac{1}{d_2}(\frac{(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha l(1+3p)})+\frac{S^{1+2p}}{d_2}\}$$

$$+\frac{h_1\alpha\theta p}{3}\{-\frac{1}{\alpha l}(\frac{T_0^{4}(x_1^{\frac{1+p}{p}}-x_2^{\frac{1+p}{p}})}{\alpha(1+p)}+\frac{4T_0^{3}(x_1^{\frac{1+2p}{p}}-x_2^{\frac{1+2p}{p}})}{\alpha(1+p)(1+2p)}+\frac{12T_0^{2}(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha^3(1+p)(1+2p)(1+3p)}$$

$$+\frac{24T_0(x_1^{\frac{1+4p}{p}}-x_2^{\frac{1+4p}{p}})}{\alpha^4(1+p)(1+2p)(1+3p)(1+4p)}+\frac{24(x_1^{\frac{1+5p}{p}}-x_2^{\frac{1+5p}{p}})}{\alpha^5(1+p)(1+2p)(1+3p)(1+4p)(1+5p)})$$

$$-\frac{4}{d_2l}(\frac{T_0^{3}(x_1^{\frac{1+2p}{p}}-x_2^{\frac{1+2p}{p}})}{\alpha(1+2p)}+\frac{3T_0^{2}(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha^2(1+2p)(1+3p)}+\frac{6T_0(x_1^{\frac{1+4p}{p}}-x_2^{\frac{1+4p}{p}})}{\alpha^3(1+2p)(1+3p)(1+4p)}$$

$$+\frac{6(x_1^{\frac{1+5p}{p}}-x_2^{\frac{1+5p}{p}})}{\alpha^4(1+2p)(1+3p)(1+4p)(1+5p)})-\frac{12}{d_3 l}(\frac{T_0^{\,2}(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha(1+3p)}$$

$$+\frac{2T_0(x_1^{\frac{1+4p}{p}}-x_2^{\frac{1+4p}{p}})}{\alpha^2(1+3p)(1+4p)}+\frac{2(x_1^{\frac{1+5p}{p}}-x_2^{\frac{1+5p}{p}})}{\alpha^3(1+3p)(1+4p)(1+5p)})$$

$$-\frac{24}{d_4 l}(\frac{T_0(x_1^{\frac{1+4p}{p}}-x_2^{\frac{1+4p}{p}})}{\alpha(1+4p)}+\frac{(x_1^{\frac{1+5p}{p}}-x_2^{\frac{1+5p}{p}})}{\alpha^2(1+4p)(1+5p)})$$

$$-\frac{24}{d_5 l}(\frac{(x_1^{\frac{1+5p}{p}}-x_2^{\frac{1+5p}{p}})}{\alpha(1+5p)})+\frac{24(S^{1+4p})}{d_5}\}$$

$$-\frac{h_1\theta S^p}{2}\{-\frac{1}{\alpha l}(\frac{T_0^{\,3}(x_1^{\frac{1+p}{p}}-x_2^{\frac{1+p}{p}})}{\alpha(1+p)}+\frac{3T_0^{\,2}(x_1^{\frac{1+2p}{p}}-x_2^{\frac{1+2p}{p}})}{\alpha^2(1+p)(1+2p)}+\frac{6(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha^3(1+p)(1+2p)(1+3p)}$$

$$+\frac{6(x_1^{\frac{1+4p}{p}}-x_2^{\frac{1+4p}{p}})}{\alpha^4(1+p)(1+2p)(1+3p)(1+4p)})-\frac{3}{d_2 l}(\frac{T_0^{\,1}(x_1^{\frac{1+2p}{p}}-x_2^{\frac{1+2p}{p}})}{\alpha(1+2p)}+\frac{2T_0(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha^2(1+2p)(1+3p)}$$

$$+\frac{2(x_1^{\frac{1+4p}{p}}-x_2^{\frac{1+4p}{p}})}{\alpha^3(1+2p)(1+3p)(1+4p)})-\frac{6}{d_3 l}(\frac{T_0(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha(1+3p)}+\frac{(x_1^{\frac{1+4p}{p}}-x_2^{\frac{1+4p}{p}})}{\alpha^2(1+3p)(1+4p)})$$

$$-\frac{6}{d_4 l}(\frac{(x_1^{\frac{1+4p}{p}}-x_2^{\frac{1+4p}{p}})}{\alpha(1+4p)})+\frac{6S^{1+3p}}{d_4}\}$$

$$+h_0\{-\frac{a}{2}(T_1^2-\frac{1}{3l}(l_2^3-l_1^3))-\frac{b}{6}(T_1^3-\frac{1}{4l}(l_2^4-l_1^4))+S_0T_1+aT_1\frac{1}{2l}(l_2^2-l_1^2)+\frac{b}{6}T_1\frac{1}{l}(l_2^3-l_1^3)$$
$$-S_0\frac{1}{2l}(l_2^2-l_1^2)-a\frac{1}{3l}(l_2^3-l_1^3)-b\frac{1}{8l}(l_2^4-l_1^4)+\theta(\frac{a}{12}(T_1^4-\frac{1}{5l}(l_2^5-l_1^5))$$
$$+\frac{b}{40}(T_1^5-\frac{1}{6l}(l_2^6-l_1^6))-\frac{a}{12}T_1^3\frac{1}{l}(l_2^2-l_1^2)-bT_1^3\frac{1}{6l}(l_2^3-l_1^3)+b\frac{1}{12l}(l_2^6-l_1^6)$$
$$+aT_1\frac{1}{24l}(l_2^4-l_1^4)+bT_1\frac{1}{40l}(l_2^5-l_1^5)+S_0T_1\frac{1}{6l}(l_2^2-l_1^2)-b\frac{1}{48l}(l_2^6-l_1^6)-S_0\frac{1}{8l}(l_2^4-l_1^4)$$
$$+S_0\frac{1}{24l}(l_2^4-l_1^4))\}+h_1\{\frac{S_0}{2}T_1^2+aT_1^2\frac{1}{4l}(l_2^2-l_1^2)+bT_1^2\frac{1}{12l}(l_2^3-l_1^3)-\frac{S_0}{6l}(l_2^3-l_1^3)$$
$$-a\frac{1}{8l}(l_2^4-l_1^4)-b\frac{1}{20l}(l_2^5-l_1^5)-\frac{a}{3}T_1^3+a\frac{1}{12l}(l_2^4-l_1^4)-\frac{b}{8}T_1^4+b\frac{1}{40l}(l_2^5-l_1^5)$$
$$+\theta(\frac{a}{15}(T_1^5-\frac{1}{6l}(l_2^6-l_1^6))+\frac{b}{48}(T_1^6-\frac{1}{7l}(l_2^7-l_1^7))-\frac{a}{8}(T_1^4-\frac{1}{5l}(l_2^5-l_1^5))+\frac{a}{12}T_1^2\frac{1}{4l}(l_2^4-l_1^4)$$
$$-\frac{a}{12}.\frac{1}{6l}(l_2^6-l_1^6)-\frac{b}{16}T_1^4\frac{1}{3l}(l_2^3-l_1^3)+\frac{b}{16}T_1^2\frac{1}{5l}(l_2^5-l_1^5)-\frac{S_0}{8}T_1^4+\frac{S_0}{8}.\frac{1}{5l}(l_2^5-l_1^5)$$
$$+\frac{S_0}{4}T_1^2\frac{1}{3l}(l_2^3-l_1^3)-\frac{S_0}{4}.\frac{1}{5l}(l_2^5-l_1^5))\}$$
$$+\{S+\frac{\alpha}{d_1}(\frac{x_1^{\frac{1+2p}{p}}-x_2^{\frac{1+2p}{p}}}{\alpha l(1+2p)})-\frac{\alpha S^{1+p}}{d_1}+\frac{\alpha\theta p}{3l}(\frac{T_0^3(x_1^{\frac{1+p}{p}}-x_2^{\frac{1+p}{p}})}{\alpha(1+p)}+\frac{3T_0^2(x_1^{\frac{1+2p}{p}}-x_2^{\frac{1+2p}{p}})}{\alpha^2(1+p)(1+2p)}$$
$$+\frac{6(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha^3(1+p)(1+2p)(1+3p)}+\frac{6(x_1^{\frac{1+4p}{p}}-x_2^{\frac{1+4p}{p}})}{\alpha^4(1+p)(1+2p)(1+3p)(1+4p)})+\frac{\alpha^2\theta p}{d_2l}(\frac{T_0^2(x_1^{\frac{1+2p}{p}}-x_2^{\frac{1+2p}{p}})}{\alpha(1+2p)}$$
$$+\frac{2T_0(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha^2(1+2p)(1+3p)}+\frac{2(x_1^{\frac{1+4p}{p}}-x_2^{\frac{1+4p}{p}})}{\alpha^3(1+2p)(1+3p)(1+4p)})$$
$$+\frac{2\alpha^2\theta p}{d_3l}(\frac{T_0(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha(1+3p)}+\frac{(x_1^{\frac{1+4p}{p}}-x_2^{\frac{1+4p}{p}})}{\alpha^2(1+3p)(1+4p)})$$
$$+\frac{2\alpha^2\theta p}{d_4l}(\frac{(x_1^{\frac{1+4p}{p}}-x_2^{\frac{1+4p}{p}})}{\alpha(1+4p)}-\frac{2\alpha^2\theta pS^{1+3p}}{d_4}-\frac{\theta S^p}{2l}(\frac{T_0^2(x_1^{\frac{1+p}{p}}-x_2^{\frac{1+p}{p}})}{\alpha(1+p)}$$
$$+\frac{2T_0(x_1^{\frac{1+2p}{p}}-x_2^{\frac{1+2p}{p}})}{\alpha^2(1+p)(1+2p)}+\frac{2(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha^3(1+p)(1+2p)(1+3p)})$$
$$-\frac{\alpha\theta S^p}{d_2l}(\frac{T_0(x_1^{\frac{1+2p}{p}}-x_2^{\frac{1+2p}{p}})}{\alpha(1+2p)}+\frac{(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha^2(1+2p)(1+3p)})$$
$$-\frac{\alpha\theta S^p}{d_3}(\frac{(x_1^{\frac{1+3p}{p}}-x_2^{\frac{1+3p}{p}})}{\alpha l(1+3p)})+\frac{\alpha\theta S^{1+3p}}{d_3}-aT_1+\frac{a(l_2^2-l_1^2)}{2l}-\frac{b}{2}T_1^2+\frac{b}{6l}(l_2^3-l_1^3)\}$$

$$+c_0a(\frac{T^2}{2}-T_1T+\frac{T_1^2}{2})+\frac{c_0b}{2}(\frac{T^3}{3}-T_1^2T+\frac{2}{3}T_1^3)+c_1a(\frac{T^3}{3}-\frac{1}{2}T_1T^2+\frac{1}{6}T_1^3)$$
$$+\frac{c_1b}{2}(\frac{T^4}{4}-\frac{T_1^2T^2}{2}+\frac{T_1^2}{4})] \qquad (10)$$

where $x_1=(S^p-\alpha pl_1)$, $x_2=(S^p-\alpha pl_2)$.

The necessary condition for $\xi(T)$ to be minimum is that $\frac{\partial\xi(T)}{\partial T}=0$, and the sufficient condition for $\xi(T)$ to be minimum is that $\frac{\partial^2\xi(T)}{\partial T}>0$ is satisfied.

Hence, using a suitable computer program, we can solve numerically the problem of Case I and Case II.

4. NUMERICAL EXAMPLE

A numerical example is considered to illustrate the effect of the developed model.

Case I:

For this model let,

$\alpha=11,\beta=0.3,a=2,b=7$,

$h_0=1,h_1=2,c_0=1,c_1=2,k=100,Q=530,S=479,S_0=80,\theta=0.003$

The model is now solved for the above parameter values using a gradient based non-linear optimization technique (LINGO), which yields the following optimal solution: $TC=718.6391,T_0=3.89849,T=11.06686$.

It is numerically verified that this solution satisfies the convexity condition.

Case II:

An equation(10) is now solved for the above parameter values(in Case I) using a gradient based non-linear optimization technique (LINGO), which yields the Global optimal solution:

Optimal cost(TC)= $\xi(T)$ =621.541, Optimal Time(T*)= 9.348.

It is numerically verified that this solution satisfies the convexity condition for $\xi(T)$.

5. CONCLUDING REMARKS

In this paper, a perishable inventory model with two components demand (stock dependent and time dependent), and time dependent holding and shortage cost is developed for an infinite planning horizon. This is justified for the products such as electronic components, radioactive substances, volatile liquids etc. which are not only costly but also require more sophisticated arrangements for their security and safety. The effect of deterioration is also considered here.

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