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Robust Approximation to Adaptive Control by Use of Representative Parameter Sets with Particular Reference to Type 1 Diabetes

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Abstract*: This paper describes an approach to adaptive optimal control in the presence of model parameter calculation difficulties. This has wide application in a variety of biological and biomedical research and clinical problems. To illustrate the techniques, the approach is applied to the development and implementation of a practical adaptive insulin infusion algorithm for use with patients with Type 1 diabetes mellitus.*

Keywords*: Adaptive control, Parameter sets, Representative points, Glucose/insulin kinetics, Type 1 diabetes.*

Introduction

Control of many biological processes can involve significant difficulties. Even when a suitable control algorithm has been devised for a model of the system, state and parameter estimation can be difficult because of time-lags in measurement availability due to inherent delays in chemical or other analytical techniques. The problem is further compounded when adaptive control is required due to the likelihood of system parameter changes over time.

An example is the case of control of plasma glucose levels in Type 1 (insulin dependent) diabetic subjects, a nonlinear problem where glucose/insulin model parameter estimation requires clinical tests which take too long to be useful in the dynamic situation. The parameters of the glucose/insulin dynamics model may also vary over time for an individual due to factors such as sleep, exercise and level of health. We describe a straight-forward approach which may be useful in such situations.

Representative parameter sets

Given a control problem where several parameter sets for a model of the system are known for individual cases but the effort involved in calculating them is too high or takes too long to allow for continuous updating, the following robust approximation to adaptive control is suggested:

(a) Using currently available data for model parameter values, determine a number of representative sets that span (in a sense described below) the given data. This can be accomplished before any form of control is attempted and may be reviewed as more relevant parameter data become available.

(b) As measurements of the state of the system are taken, determine which of the representative parameter sets best accounts for previous data. More recent data may be weighted more heavily. The choice of parameter set is then a simple iterative process which must converge (as opposed to most minimization techniques).

(c) Determine an appropriate control strategy on the basis of the chosen parameter set. The parameter set choice may then be reviewed as often as desired in both (a) and (b) above.

To implement (a) we seek a set of representative parameter points that approximates the physical/physiological region of parameter space in the sense that each of the given parameter points is within some maximum radius of one of the representative points.

Given a model with parameters $p_1, p_2, ..., p_k$ let $\mathbf{p}_i = (p_1, p_2, ..., p_k)_i$, $i = 1, \ldots, m$ be the set of currently known parameter points. Also, let the set of representative points be $\mathbf{P}_j$, $j = 1, \ldots, n$, with $n < m$. $\{\mathbf{P}_j\}$ should partition $\{\mathbf{p}_i\}$ so that each $\mathbf{P}_j$ is associated with a disjoint subset of $\{\mathbf{p}_i\}$ for whose elements it is the unique closest point.

$\{\mathbf{P}_j\}$ can be found by minimizing the sum of the squared distances from each existing point to the closest representative point; that is, by minimizing

$$J = \sum_i \min_j \left|\mathbf{p}_i - \mathbf{P}_j\right|^2 .$$

Examples

The following cases illustrate the properties of this cost criterion.

(a) Consider the extreme case of representing two parameter points $\mathbf{p}_1, \mathbf{p}_2$ in a one-dimensional Euclidean parameter space by the one point $\mathbf{P}_1$. It is clear that the result should be $\mathbf{P}_1 = (\mathbf{p}_1 + \mathbf{p}_2)/2$. To minimise J we have

$$\frac{dJ}{d\mathbf{P}_1} = \frac{d}{d\mathbf{P}_1}\sum_{i=1}^{2}(\mathbf{p}_i - \mathbf{P}_1)^2 = -2\sum_{i=1}^{2}(\mathbf{p}_i - \mathbf{P}_1) = 0 \text{ at an extremal point.}$$

Thus, $\mathbf{p}_1 + \mathbf{p}_2 - 2\mathbf{P}_1 = 0$ so $\mathbf{P}_1 = (\mathbf{p}_1 + \mathbf{p}_2)/2$ as required and it is straightforward to check that this is indeed a minimum.

(b) Now consider the one-dimensional case of $\mathbf{p}_1 = 0$, $\mathbf{p}_2 = 2$, $\mathbf{p}_3 = 4$ with two representative points required. Possible candidates for $(\mathbf{P}_1, \mathbf{P}_2)$ are

i) (1; 3) ii) (1; 4) iii) (0; 3) iv) (2; 2) v) (0; 2,5) vi) (0; 3,5).

Choice i) is nicely symmetrical but if $\mathbf{P}_1$ is representing $\mathbf{p}_1, \mathbf{p}_2$ then $\mathbf{P}_2$ does not need to represent both $\mathbf{p}_2$ (again) and $\mathbf{p}_3$. Choices ii) and iii) resolve this and J is in fact lower for these than for i). We also see that J may have multiple minima. Further, it is easy to show that simply summing the squared distances (rather than the minimum squared distances) in J leads to setting all representative points to the centroid of the parameter points, as in iv), which is not satisfactory. Finally, if only the distances (rather than the squared distances) to the nearest point are summed then choices iii), v) and vi) are equivalent. Indeed, any choice of $\mathbf{P}_2$ between $\mathbf{p}_2$ and $\mathbf{p}_3$ will result in the same value of J, which is also unsatisfactory.

Some practical considerations arise:

(a) If numerical methods are used to minimize J then initial estimates of $\{\mathbf{P}_j\}$ will be needed. The possibility of multiple minima needs to be considered.

(b) An appropriate number of representative points will need to be determined. Too few and some system behaviour will be modelled poorly, too many and it may take too long to determine the appropriate representative parameter set.

(c) In some instances the only available parameter data may have been derived from a variety of (perhaps not entirely compatible) sources and exhibit a much larger overall variation in the parameters than occurs in the individual case to be controlled.

As with other adaptive control approaches some time may be required to learn about the system and a certain amount of caution in control required in these early stages. As more relevant data become available, more appropriate representative sets can be determined.

The approach described above was used to reduce 306 existing parameter sets, each calculated by fitting a four parameter model of glucose/insulin kinetics to intravenous glucose tolerance test data, to 31 representative parameter sets. These were then used in conjunction with a control algorithm previously developed for plasma glucose control in Type 1 diabetic subjects [1]. Fig. 1 illustrates the results of an insulin infusion regimen as prescribed by the resulting adaptive optimal control algorithm with 10min glucose sampling and in the presence of a predetermined glucose infusion challenge designed to simulate a meal. The goal was to keep plasma glucose levels at 5mmol/l. The adaptive property is illustrated by the changing representative parameter set.

Conclusion

A robust approach to adaptive optimal control in the presence of model parameter calculation difficulties has been described and applied to the development and implementation of a practical adaptive insulin infusion algorithm for some actual patients with Type 1 diabetes. Other biological control situations can benefit from the techniques presented here.

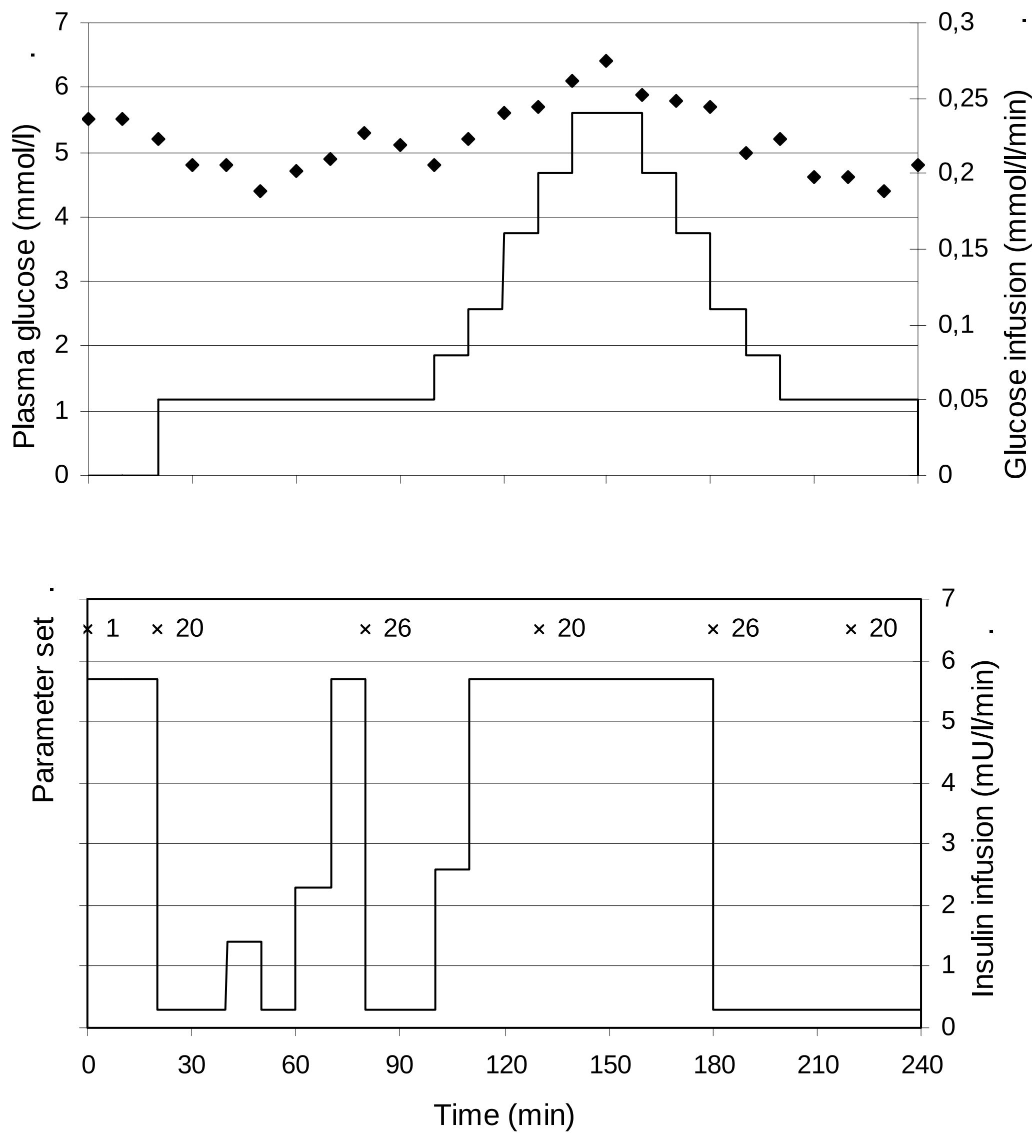

Fig. 1 Four-hour glucose profile of a Type 1 diabetic subject.
Insulin infusion was controlled adaptively by computer program incorporating representative parameter sets. Parameter set 1 was the centroid of all existing parameter sets.

References

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