# MATHEMATICAL-GEOGRAPHICAL ANALYSIS OF THE ORIENTATION OF ST JOHN'S CHURCH OF THE STUDENICA MONASTERY 

Milutin Tadic ${ }^{1}$ *, Aleksandar Petrovic*<br>* Faculty of Geography, University of Belgrade

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#### Abstract

Considering the fact that ecclesiastical rules do not precisely say that a church must be directed "to the East" or "to sunrise", it should always be checked if there is a connection between the orientation of a church and geometry of the Sun. In this paper, such examination is performed on the example of the church of St. John (the $13^{\text {th }}$ century), one of four churches of the Studenica monastery, in the following way: 1) using gnomon method, the azimuth of the main longitudinal axis of the church is measured; 2) the altitude above the horizon of the point in which the extended axis of the church touches the true horizon is determined by cartometry; 3) the most probable dates when the Sun rises at that point are determined: May $7^{\text {th }}$ according to Gregorian calendar, or April $30^{\text {th }}$ according to Julian calendar, in the $13^{\text {th }}$ century. The applied method is described in details and it can be applied for the analysis of the orientation of any other medieval church. This method can determine the time when the church was founded, as well as the fact if the church is original, or possibly erected on the foundations of some older sacral object.


Key words: mathematical geography, archeoastronomy, church orientation, the Studenica monastery

## Introduction

Contrary to the geometry of the monasteries of Medieval Serbia (Popović, 1994), the orientation of churches, as frameworks of monasteries, has not been systematically studied. It has already been pointed to the need for exact examination of orientation of the main longitudinal axes of medieval churches (Nenadović, 2003), but this examination is only the first step, precondition, for mathematical-geographical analysis in which the position of churches is studied in the context of the geometry of the Sun.

Systematic studies of that kind are not something new. The methodology has been worked out well and, apart from numerous papers which study single

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churches, there are studies in which the churches on the territory of the whole country have been considered, for example on the territory of Austria (Firnies \& Köberl, 1989) or Slovenia (Čaval, 2009). From the point of view of researches which are to be carried out on the territory of the Republic of Serbia, special significance have papers with the methodology of studying the orientation of Orthodox (Byzantine) churches on the territory of Greece (Liritzis \& Vassiliou, 2007; Pantazis, Sinachopoulos, Lambrou, \& Korakitis, 2004).

Studying the orientation of churches in the context of the geometry of Sun, some of these questions can be answered: How much did the proto-master know about the exact methods of orientation? What was the level of exact sciences (Mathematics, Geography and Astronomy) at the time of church construction? When (the day and month) was the church founded? Who was its patron saint? Was the church originally founded at that site or was it on the foundations of some older sacral object?

## The churches of the Studenica monastery and tradition

There are three churches in the complex of the Studenica monastery - the Virgin's Church (erected 1183-1196), the church of Joachimand Ann or the King's Church (erected in 1314), the St Nicholas's Church (erected at the beginning of the $13^{\text {th }}$ century) - and the foundations of the church of St John the Forerunner (probably erected in the $13^{\text {th }}$ century). Obviously, their main longitudinal axes are not parallel (Figure 1).

The church rules do not precisely say that the main longitudinal axis should be set exactly in the direction West-East, nether is eastern point of the horizon (the equinox East) or the equinox line mentioned, but only "to the East" or "to the sunrise" (Mirković, 1996). And, the expression "to the East", if it is not precisely said, refers to, in the narrowest sense, all directions inside the sector of the horizon $45^{\circ}$ of latitude, from NNE to SSE, while the condition "to the sunrise" refers to all directions in the sector of the horizon which has the range of $66.5^{\circ}$ of latitude of the Studenica monastery (the sector between the solstitial points of sunrise, with the eastern direction as a symmetry). Therefore, in accordance with ecclesiastical rules, the main longitudinal axes of churches do not have to be parallel, and, generally speaking, on the territory of Medieval Serbia, they are not (Nenadović, 2003).

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Figure 1. The plan of the Studenica monastery, made by aerial photographs
The main longitudinal axis of St Joachim and Ann's Church is exactly oriented in a direction West-East, while the axis of the Virgin's Church deviates from the eastern direction for about $5^{\circ}$, towards South-East (Tadić \& Babić, 2010). If we take into consideration the gnomonic methods which proto-master could have applied in the Middle Age (Tadić, 2004) and the errors which could not have been avoided then, from the mathematical geographical point of view, it can be said that these two churches are accurately oriented. The main longitudinal axes of other two churches, the St. Nicholas's Church and the St. John the Forerunner's Church, more notably deviate from eastern direction. Therefore, we should not start from the assumption that proto-masters made beginner mistakes in determining eastern direction, but from the assumption that they directed the axes of the churches "to the sunrise" of certain day. From the mathematical-geographical point of view, apart from equinoxes, the key days are summer and winter solstice, and from the religious point of view, these days are the celebration days of the Saint to whom the church is dedicated to.

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Figure 2. Foundations of the church of St John the Forerunner, the $13^{\text {th }}$ century (the St Nicholas's Church is behind)

The subject of further study is the church of St. John the Forerunner ( $43^{\circ} 29^{\prime} \mathrm{N}$, $20^{\circ} 32^{\prime} \mathrm{E}$ ) in which only foundations have been preserved (Figure 2), so contrary to other three churches of the Studenica monastery, the orientation of its longitudinal axis can be directly examined.

## Definition of the task

Working assumption. One morning in the $13^{\text {th }}$ century a proto-master, standing southward from the Virgin's Church directed the main longitudinal axis of the future church to the point of sunrise: above the true horizon of the Studenica monastery.

Task.On that assumption, determine:

- accurate deviation of the main longitudinal axis of the church of St. John the Forerunner from the eastern direction;
- horizon coordinates of the point in which the vertical line of the main longitudinal axis of the church intersects the true horizon;
- dates in a year, according to Gregorian and Julian calendar, in which sun rises at that point, that is, dates in some of the years in the $13^{\text {th }}$ century on which the foundations for the church of St. John the Forerunner could have been made.

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## Measurements of the deviation of the main longitudinal axis of the church of St John the Forerunner from eastern direction

Tending to be closer to a Medieval proto-master, modern technical equipment was set aside, and for examination of the orientation of the church of St. John the Forerunner modernized gnomonic methods were applied. Both methods are based on one measurement of shadow angle of gnomon: the angle of noon shadow in the first method, and the angle of any other shadow in the second method. Considering the fact that the first method is connected to only one moment of a day (apparent noon), the second method was applied.

## Procedure:

- At one moment of the Sun day $(T)$, horizontal angle is measured $\alpha$ which gnomon shadow makes with southern wall of the church, starting from the west in the clockwise direction (Tadić \& Babić, 2010) (Figure 3).
- From ephemerides of the current year the values of the Sun declination $\delta$ and equation of time $e$, for 00:00 h of Universal Time are taken, and then their values are determined by interpolation for the given moment $T$ of Central European Time.
- Apparent sun time is calculated according to the formula $T_{P}{ }^{2}$ :

$$
T p=T+e-\left(15^{\circ}-\lambda^{\circ}\right) \cdot 4 \min
$$

(in which $\lambda$ is longitude of the church), and then hour angle of the Sun $t$ is calculated in degrees, according to the formula,

$$
t=\frac{(T-12 h)}{15}
$$

- Sun altitude $h$ is calculated according to the formula,

$$
\sinh =\sin \varphi \cdot \sin \delta+\cos \varphi \cdot \cos \delta \cdot \cos t
$$

(in which $\varphi$ is latitude of the church), and then astronomical azimuth of the $\operatorname{Sun} A$ is calculated according to the formula,

$$
\sin A=\frac{\sin t \cdot \cos \delta}{\cosh }
$$

taking care of quadrant.

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Figure 3. The relationship between the deviation angle of the main longitudinal axis of the church of St John the Forerunner from eastern direction $(\Delta \mathrm{A})$, astronomic azimuth of the Sun (A) and shadow angle $(\alpha)$

- Finally, the deviation angle $(\Delta \mathrm{A})$ of the main longitudinal axis of the church from eastern direction is calculated according to the formula,

$$
\Delta A=270^{\circ}-A+\alpha
$$

in the morning, or

$$
\Delta A=\alpha-A-90^{\circ}
$$

in the afternoon, considering the fact that the sign "minus" points to the deviation of the main longitudinal axis from the eastern point towards South-East.

Results: In the described way, the measurements were performed on $8^{\text {th }}$ June and $6-9^{\text {th }}$ August in 2010 at different time of the day (Table 1).

Analysis: The accuracy of measurements depends on several factors: deficiency of equipment (plummet, spirit level and protractor), measurer's skill, part of the day (angles can be measured less accurately when the Sun is vertical, which makes small angle with vertical line of the main longitudinal axis of the church), length and levelness of wall. Considering all of these, the accuracy in the range $\pm 1^{\circ}$ can be expected. To sum up, the main longitudinal axis of the church of St. John the Forerunner deviates from eastern direction for $12^{\circ}$ towards North-East.

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Table 1. The results received on the basis of measurements performed

| on $8^{\text {th }}$ June and 6- $-9^{\text {th }}$ August in 2010. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Date | T | $\alpha$ | A | $\mathrm{A}_{\mathrm{C}}$ | $\Delta \mathrm{A}$ |
| 08.06 .2010. | $13: 33$ | $135,0^{\circ}$ | $34,0^{\circ}$ | $79,0^{\circ}$ | $11,0^{\circ} \mathrm{NE}$ |
| 06.08 .2010. | $11: 46$ | $71,1^{\circ}$ | $330,4^{\circ}$ | $79,3^{\circ}$ | $10,7^{\circ} \mathrm{NE}$ |
| 07.08 .2010. | $11: 01$ | $55,0^{\circ}$ | $313,3^{\circ}$ | $78,3^{\circ}$ | $11,7^{\circ} \mathrm{NE}$ |
| 09.08 .2010. | $12: 43$ | $102,9^{\circ}$ | $0^{\circ}$ | $77,1^{\circ}$ | $12,9^{\circ} \mathrm{NE}$ |
|  |  |  | Average value | $78,4^{\circ}$ | $11,6^{\circ} \mathrm{NE}$ |

The deviation is significant and, obviously, it is not a consequence of configuration of terrain. Rejecting the assumption that the church of St John the Forerunner was oriented by someone who did not know cardinal points of the world (since it is illogical), the meaning of the orientation should be sought in geometry of the Sun above the horizon of the Studenica monastery.

## Determining horizon coordinates of the point in which vertical of the main longitudinal axis of the church intersects the true horizon

The second part of the task is solved by cartometry, using the adequate sheet of topographic map in a scale of 1:25000 or 1:50000.

## Procedure:

- On the topographic map, from the topographic symbol for the church, under the angle equivalent to azimuth of the main longitudinal axis, a profile line is drawn and above it, natural topographic profile is constructed (Figure 4).


Figure 4. Natural topographic profile along extended axis of the church of St John the Forerunner

- From the starting point of the profile (the church) vertical is drawn down until it touches the profile (the point $C$ ).
- Relative altitude $\Delta H$ of the point $C$ is determined in relation to the starting point and its horizontal distance d from the starting point.

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- At the end, vertical angle $h$ is determined which perceptible line makes with the perceptible horizon (the horizon of the church), using the formula,

$$
\operatorname{tgh}=\frac{\Delta H}{d}
$$

Results: After construction of topographic profile along the extended line which represents extension of the main longitudinal axis of the church of St. John the Forerunner, first the position of the point $C$ (Figure 4) is determined, then its relative altitude in relation to the church ( $\Delta H=564 \mathrm{~m}$ ) and horizontal distance from the church ( $\mathrm{d}=2729 \mathrm{~m}$ ), and finally vertical angle is determined $h=11^{\circ} 41^{\prime}$.

Analysis: If the axis of the church of St John the Forerunner was directed to the sunrise above the true horizon of the Studenica monastery, then it was a moment in which the Sun had horizon coordinates: $h=11^{\circ} 41^{\prime}, \mathrm{A}=270^{\circ}-\Delta \mathrm{A}=258^{\circ}$.

## Determining the data when the Sun possibly rises at the point in which vertical of the main longitudinal axis of the church intersects the true horizon

When the position of the point $C$ is familiar, in which perceptible line from the church, along its longitudinal axis, touches the true horizon, then only the dates in which the Sun possibly rises at this point, remain to be defined (Figure 5).

## Procedure:

- Declination $\delta$ in which the Sun can have the given horizon coordinates is calculated, according to the formula,

$$
\sin \delta=\sin \varphi \cdot \sinh +\cos \varphi \cdot \cosh \cdot \sin \Delta A
$$

in which latitude of the church is $\varphi$.

- According to the calculated value of the Sun declination ${ }^{3}$, the appropriate dates are found in astronomical ephemerides.

Results: Inserting the values for the church of St John the Forerunner ( $\varphi=43^{\circ}$ $29^{\prime}, \mathrm{h}=11^{\circ} 41^{\prime}, \mathrm{A}=258^{\circ}$ ), the value of the Sun declination is received $\delta=16^{\circ}$ 41'. This value of the Sun declination today, according to Gregorian calendar, corresponds to the dates $7^{\text {th }}$ May and $6^{\text {th }}$ August, and according to Julian calendar, in the $13^{\text {th }}$ century: $30^{\text {th }}$ April and $30^{\text {th }}$ July (Figure 5).

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Analysis: The Sun reaches each value of declination, in the range from $\delta=23^{\circ}$ $27^{\prime}$ to $\delta=23^{\circ} 27^{\prime}$, twice a year, so two dates are determined from appropriate astronomical ephemerides. In this concrete case for the church of St. John the Forerunner of the Studenica monastery, these dates are according to the current calendar: $7^{\text {th }}$ May and $6^{\text {th }}$ August. The first date is more probably the beginning of the construction of the church (the beginning of the summer half of the year and construction season). Taking into consideration the accuracy of a protomaster in the Middle Age, as well as the accuracy in determining the deviation of the longitudinal axis of the church and the constructed topographic profile, it is not correct to say "on $7^{\text {th }}$ May" but "around $7^{\text {th }}$ May" or "around St George's Day".


Figure 5. Celestial hemisphere above the church of St. John in stereographic normal projection with inserted unapparent daily path of the Sun on $7^{\text {th }}$ May ( $6^{\text {th }}$ August)

## Conclusion

The main longitudinal axis of the church of St John the Forerunner was not set exactly in the West-East direction but it deviates from eastern direction for $12^{\circ}$, towards North-East. The terrain is flat and it did not have any influence on such orientation of the church. Obeying the rule that church has to be directed "to the sunrise", the proto-master could have directed the main longitudinal axis to the sunrise of certain day. If he really did that, then he most probably, observed the

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sunrise around $7^{\text {th }}$ May according to the current calendar ( $30^{\text {th }}$ April in the $13^{\text {th }}$ century, according to Julian calendar) that is, around St George's Day. Then, that was the date in which the foundations of the church of St John the Forerunner were made (the construction began).

Additionally, this orientation is logical from the mathematical-geographical point of view since St George's Day ${ }^{4}$ for Serbs, before they were Christianized, had been a feast day dedicated to the Sun when summer part of a year had been celebrated (Janković, 1951).

If the main longitudinal axis of the church was intentionally directed to the point of the sunrise around St George's Day, the day dedicated to St George (Great Martyr St George), then a question arises for archeologists: How certain is the fact that the church with the preserved foundations was originally dedicated to St John the Forerunner?

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[^0]:    ${ }^{1}$ Correspondence to: tadic@gef.bg.ac.rs

[^1]:    ${ }^{2}$ Computation of this and all formulas in this paper can be seen in the book: Tadić, M. (2004). Mathematical Geography. Belgrade: Serbian State Company of Textbooks.

[^2]:    ${ }^{3}$ Theoretically, the value of the Sun declination above the range $23^{\circ} 27^{\prime} \geq \delta \geq-23^{\circ} 27^{\prime}$, excludes the possibility that the church axis was set towards the sunrise.

[^3]:    ${ }^{4} \ldots$ andrein [horse on which the Sun goes over the Sky] holds St George, who is probably the heir of a Sun divinity (Janković, 1951)

