CONSIDERATIONS ON THE SELECTIVE MAINTENANCE MODEL FOR THE COMPLEX SYSTEMS

Dr. ing CS III Gheorghe **NEGRU**
Military Equipment and Technologies Research Agency

Abstract: *The paper presents the mathematical model of the selective maintenance as well as a numerical application of the method in the case of the remote control systems.*

Keyword: *Selective maintenance, reliability, remote control system*

1. Introduction

The specialists within logistics and resources management are faced with the task of solving the equation "achieving an increased number of requirements with less resource" as budgets allocated to the companies decline, and operational requirements increase. In order to fulfill the missions it is necessary to achieve the full readiness status of the complex systems from the inventory. The changes of the operational scenario generated the changes of the maintenance concept from a repair function to one focuses on the mission. Due to the fact that many of the remote control systems are needed to perform several missions per day the maintenance structures must decide which systems to repair in the time allotted between missions. As well as the problem of solving of an increased number of requirements with reduced resources is applied to the industrial environment in order to ensure the operational efficiency of the technical systems [1], [2]. In this context the present paper proposes a selective maintenance model which can be used to identify the necessary maintenance activities in order to optimize the mission's requirements. However the model can be also applied to the industrial systems which use equipments involved in serial technological operations**.**

Consider a system composed of *m* independent subsystems (subsystem 1, subsystem 2, and subsystem *m)* connected in series. Let each subsystem *i* contain a set of *zi* independent components connected in different ways. Each component in the system is defined by *(i,j)* where *i* define the subsystem number and *j* define the component number. Each component, subsystem, and the system can be in only one of two states: functioning properly or failed. This type of complex system configuration represents a wide variety of equipment utilized in industrial environments [3].

Assume the system is required to perform a sequence of identical missions with breaks of known length between missions. At the beginning of a mission, say mission *k*, the status of a component is defined according to those presented in Table no.1

Table no.1

Component status	Status value	Status significance
Xij(k)	1	Component *i* of the subsystem *j* is working at the beginning of the mission *k*
	0	Other status

The status of a subsystem at the beginning of the mission is defined according to those presented in Table no.2.

Table no.2

Subsystem status	Status value	Status significance
Xi(k)	1	Subsystem *i* is working at the beginning of the mission *k*
	0	Other status

The status of the system at the beginning of the mission is defined according to those presented in Table no. 3.

Table no.3

System status	Status value	Status significance
X(k)	1	System is working at the beginning of the mission *k*
	0	Other status

The status of the system at the beginning of the mission is a resultant of the status of its component subsystems being defined by

$$X(k) = \prod_{i=1}^{m} X_i(k) \qquad (1)$$

Similar can be defined the functioning status at the end of the mission. The status of the component is defined according to those presented in Table no. 4.

Table no.4

Component status	Status value	Status significance
Yij(k)	1	Component *i* of the subsystem *j* is functioning at the end of the mission *k*
	0	Other status

The status of a subsystem at the end of the mission is defined according to those presented in Table no. 5.

Table no.5

Subsystem status	Status value	Status significance
Yi(k)	1	Subsystem *i* is working at the end of the mission *k*
	0	Other status

The system status at the end of the mission is defined according to those presented in Table no. 6.

Table no.6

System status	Status value	Status significance
Y(k)	1	System is working at the end of the mission *k*
	0	Other status

The status of the system at the end of the mission is a resultant of the status of its components subsystems being defined by

$$Y(k) = \prod_{i=1}^{m} Y_i(k) \qquad (2)$$

The performance of a component, subsystem or system can be measured in many ways. For the purposes of maintenance planning, performance is typically measured by reliability. Let *fij* define the probability that component *j* of subsystem *i* does not failed during a particular mission, say mission *k*. Thus,

$$f_{ij} = P\left(Y_{ij}(k)=1 \middle| X_{ij}(k)=1\right) \qquad (3)$$

We emphasize that *fij* is the same for any given mission. This is based on the followings assumptions:

-missions are identical;

-all components have a constant failure rate.

The reliability of the component *j* of the subsystem *i* during the mission *k* is given by the

$$P\left(Y_{ij}(k)=1\right)= f_{ij} X_{ij}(k) \qquad (4)$$

For a subsystem *i* the reliability during the mission *k* is defined based on the its components reliabilities

$$F_i(k)= P\left(Y_i(k)=1\right) \qquad (5)$$

The reliability of the system for the mission *k* is defined by [4], [5]

$$F(k)= P\left(Y(k)=1\right)= \prod_{i=1}^{m} F_i(k) \qquad (6)$$

At the completion of a particular mission, say mission *k*, each component in the system is either functioning or failed. In the best situation, all failed components (those components having *Yij(k)* = 0) would be repaired prior to the beginning of the next mission. However, it may not be possible to repair all the failed components. Let *tij* define the amount of time required to repair component *j* of subsystem *i*. The total time required to repair all failed components in the system prior to the next mission, mission *k* + 1, is given by

$$T(k)= \sum_{i=1}^{m}\sum_{j=1}^{n_i} t_{ij}\left(X_{ij}(k+1)-Y_{ij}(k)\right) \qquad (7)$$

where $X_{ij}(k+1)$ represent the status of the components at the beginning of the next mission.

Suppose the total amount of time allotted to perform maintenance upon failed components between missions is To time units. If

$$\sum_{i=1}^{m}\sum_{j=1}^{n_i} t_{ij}\left(X_{ij}(k+1)-Y_{ij}(k)\right) \rangle T_0 \qquad (8)$$

then all failed components cannot be repaired prior to beginning the next mission.

In this situation a method is needed to decide which failed components should be repaired prior to the next mission.

2. Mathematical Programming Model

For the case in which the time allotted for maintenance is insufficient to repair all failed components in the system, a mathematical programming model is defined for assisting in the selective maintenance decision. The first step in the formulation of this model is the identification of the decision variables for the model. Given the status of each component at the end of a certain mission (the *Yij(k)* values), the selective maintenance decision consists of identifying the failed components (those components having *Yij(k)*= 0) to be repaired prior to the next mission. This decision can be represented mathematically by specifying the *Xij* values for the next mission. Thus, the $X_{ij}(k+1)$ values serve as the decision variables for the mathematical programming model.

The next step in formulating the mathematical programming model for this selective maintenance problem is the construction of the objective function. Initially, assume that the objective in performing selective maintenance is to maximize the system reliability for the next mission. Therefore, the objective function is given by

$$F(k+1) = \prod_{i=1}^{m} F_i(k+1) \tag{9}$$

where $F(k+1)$ represent the system reliability for the next mission and $F_i(k+1)$ define the reliability function of subsystem i for the next mission.

The final step in formulating this selective maintenance mathematical programming model is the construction of the constraints on the decision variables. Thus we have the followings:

- all maintenance activities be completed within the allotted time.

$$\sum_{i=1}^{m}\sum_{j=1}^{n_i} t_{ij}\left(X_{ij}(k+1) - Y_{ij}(k)\right) \langle T_0 \tag{10}$$

- the decision variables are restricted to binary values

$$X_{ij}(k+1) \;\; binary \quad for \; \forall i, j \tag{11}$$

- a component's status at the beginning of the next mission must be at least as good as its status at the end of the previous mission.

$$X_{ij}(k+1) \geq Y_{ij}(k) \quad for \quad \forall i, j \tag{12}$$

Based on those above presented we can define the selective maintenance problem as follows [6]:

$$Maximize \quad F(k+1) = \prod_{i=1}^{m} F_i(k+1) \tag{13}$$

in the followings conditions

$$\sum_{i=1}^{m}\sum_{j=1}^{n_i} t_{ij}\left(X_{ij}(k+1) - Y_{ij}(k)\right) \langle T_0 \tag{14}$$

$$X_{ij}(k+1) \geq Y_{ij}(k) \quad for \quad \forall i, j \tag{15}$$

$$X_{ij}(k+1) binary \;\; for \; \forall i, j \tag{16}$$

This model is deterministic in that all the model parameters (end-of-mission status values, component reliabilities, component maintenance times, total allotted time for maintenance) are assumed to be known constants.

3. Selective maintenance for the remote control system

In order to present the application of the above mentioned model we consider the case of a remote control system which is designed to equipped the armored vehicles. The components of the remote control system are presented in Table no. 7 .

Table no.7

No.	System	Subsystem
1	Fire control system (S1)	Target tracking system (S11)
		Computing system (S12)
		Main system driving gear system (S13)
2	Main system (S2)	Acting component (S21)
		Feeder (S22)
3	Command and control system (S3)	Acquisition data module from fire control system (S31)
		Acquisition data module from main system (S32)
		Acquisition data module from electrono-optic system (S33)
		Integration/processing/command module at system level (S34)

The reliability diagram of the remote control system defined above is presented in Fig. no. 1

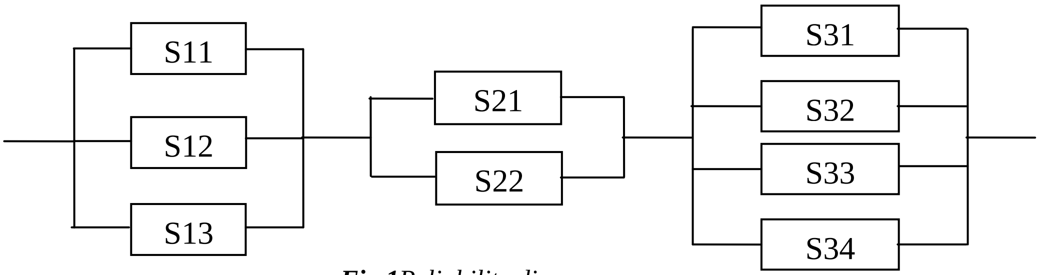

Fig.1*Reliability diagram*

The formulae for the reliability calculus [3] of the component systems of the remote control system are presented in the followings

$$F_1(k+1)=1-(1-f_{11}X_{11}(k+1))(1-f_{12}X_{12}(k+1))(1-f_{13}X_{13}(k+1)) \quad (17)$$

$$F_2(k+1)=1-(1-f_{21}X_{21}(k+1))(1-f_{22}X_{22}(k+1)) \quad (18)$$

$$F_3(k+1)=1-(1-f_{31}X_{31}(k+1))(1-f_{32}X_{32}(k+1))(1-f_{33}X_{33}(k+1))(1-f_{34}X_{34}(k+1)) \quad (19)$$

The values of the specific parameters for the analyzed system are presented in the Table no. 8.

Table no.8

Subsystem(*i*)	Component (*j*)	*fij*	*tij*	*Yij(k)*	*Xij(k+1)*
S1	1	0.8	3	1	1
S1	2	0.8	3	0	1
S1	3	0.8	3	1	1
S2	1	0.7	2	0	1
S2	2	0.7	2	0	1
S3	1	0.9	4	1	1
S3	2	0.9	4	0	0
S3	3	0.9	4	1	1
S3	4	0.9	4	0	0

We consider as given the allotted maintenance time *To*=10 time units. According the above mentioned data the necessary maintenance time for all the failed components before the next mission is 15 time units. In order to solve the selective maintenance problem for the next mission must be repaired the component no. 2 belonging subsystem no. 1, component no. 1 belonging subsystem no. 2 and component no. 2 belonging subsystem no. 2. These repairs spend 7 time units (Table no. 9) and the result for reliability is the value 0.8936 (Table no. 10). The maximum reliability for the next mission is 0.9026 (Table no.11) which is obtained in the situation in which all the components are repaired.

Table no.9

Subsystem(*i*)	Component (*j*)	*tij*	*Yij(k)*	*Xij(k+1)*	*tij[Xij(k+1)-Yij(k)]*
S1	1	3	1	1	0
S1	2	3	0	1	3
S1	3	3	1	1	0
S2	1	2	0	1	2
S2	2	2	0	1	2
S3	1	4	1	1	0
S3	2	4	0	0	0
S3	3	4	1	1	0
S3	4	4	0	0	0
T					7

Table no.10

Subsytem(*i*)	Component (*j*)	*fij*	*Xij(k+1)*	*1-fij*Xij(k+1)*
S1	1	0.8	1	0.2
S1	2	0.8	1	0.2
S1	3	0.8	1	0.2
R1	0.992			
S2	1	0.7	1	0.3
S2	2	0.7	1	0.3
R2	0.91			
S3	1	0.9	1	0.1
S3	2	0.9	0	1
S3	3	0.9	1	0.1
S3	4	0.9	0	1
R3	0.99			
R	0.8936			

Table no.11

Subsytem(*i*)	Component (*j*)	*fij*	*Xij(k+1)*	*1-fij*Xij(k+1)*
S1	1	0.8	1	0.2
S1	2	0.8	1	0.2
S1	3	0.8	1	0.2
R1	0.992			
S2	1	0.7	1	0.3
S2	2	0.7	1	0.3
R2	0.91			

S3	1	0.9	1	0.1
S3	2	0.9	1	0.1
S3	3	0.9	1	0.1
S3	4	0.9	1	0.1
R3	0.9999			
R	0.9026			

4. Generalization of the Selective Maintenance Model

According those above presented a finite amount of time, *To,* was allotted for making repairs to the failed components. However, in many cases, both time and cost constrain the maintenance activities performed between missions. Let *cij* define the cost to repair component *j* of subsystem *i*. *S*uppose the total cost of repairs between two missions may not exceed *Co*. The total cost of repairing selected failed components in the system prior to the next mission, say mission *k* + 1, is given by

$$C(k)=\sum_{i=1}^{m}\sum_{j=1}^{n_i}c_{ij}\left(X_{ij}(k+1)-Y_{ij}(k)\right) \quad (20)$$

The cost constraints which can be added to the selective maintenance model suppose that all maintenance activities are required to be completed within the allotted cost. Thus

$$\sum_{i=1}^{m}\sum_{j=1}^{n_i}c_{ij}\left(X_{ij}(k+1)-Y_{ij}(k)\right)\langle C_0 \quad (21)$$

Adding this constraint to the model result a new selective maintenance optimization problem as follows [6]:

$$Maximize \qquad F(k+1)=\prod_{i=1}^{m}F_i(k+1) \quad (22)$$

in the following conditions

$$\sum_{i=1}^{m}\sum_{j=1}^{n_i}t_{ij}\left(X_{ij}(k+1)-Y_{ij}(k)\right)\langle T_0 \quad (23)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n_i}c_{ij}\left(X_{ij}(k+1)-Y_{ij}(k)\right)\langle C_0 \quad (24)$$

$$X_{ij}(k+1)\geq Y_{ij}(k) \; for \;\; \forall i,j \quad (25)$$

$$x_{ij}(k+1) \;\; binary \qquad for \; \forall i,j \quad (26)$$

In the above presented problem the objective is to maximize the system reliability in terms of time and cost constraints.

A similar variant of this problem could be defined by minimizing total repair time subject to cost and reliability constraints. Thus the new problem for selective maintenance optimization will be [6]:

$$Minimize \qquad C(k)=\sum_{i=1}^{m}\sum_{j=1}^{n_i}c_{ij}\left(X_{ij}(k+1)-Y_{ij}(k)\right) \quad (27)$$

in the following conditions

$$\sum_{i=1}^{m}\sum_{j=1}^{n_i}t_{ij}\left(X_{ij}(k+1)-Y_{ij}(k)\right)\langle T_0 \quad (28)$$

$$\prod_{i=1}^{m}F_i(k+1)\geq F_0 \quad (29)$$

$$X_{ij}(k+1) \geq Y_{ij}(k) \quad for \quad \forall i, j \tag{30}$$

$$X_{ij}(k+1) \quad binary \quad for \quad \forall i, j \tag{31}$$

Another variant of the selective maintenance will be obtained by minimizing the total repairing time in terms of time and cost constraints.

$$Minimize \quad T(k) = \sum_{i=1}^{m} \sum_{j=1}^{n_i} t_{ij} \left(X_{ij}(k+1) - Y_{ij}(k) \right) \tag{32}$$

in the following conditions

$$\sum_{i=1}^{m} \sum_{j=1}^{n_i} c_{ij} \left(X_{ij}(k+1) - Y_{ij}(k) \right) \langle C_0 \tag{33}$$

$$\prod_{i=1}^{m} F_i(k+1) \geq F_0 \tag{34}$$

$$X_{ij}(k+1) \geq Y_{ij}(k) \quad for \quad \forall i, j \tag{35}$$

$$X_{ij}(k+1) \quad binary \quad for \, \forall i, j \tag{36}$$

5. Conclusions

The paper introduces and applies the selective maintenance concept for the complex systems and proposes a series of models in order to be used at the optimization of the decisions in that field. These can be applied to the remote control systems with fixed mission lengths and limited time between missions for maintenance. The calculus example emphasized the way by which the components could be selected in order to be repaired.

The selective maintenance concept could contribute to the enhancement of the remote control system performances as well as to the cost reducing during the life cycle system belonging to the current inventories. As well as the presented models underline the intricacy and validity of the selective maintenance problem.

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