AN OFF-LINE DUAL MAXIMUM RESOURCE BIN PACKING MODEL FOR SOLVING THE MAINTENANCE PROBLEM IN THE AVIATION INDUSTRY

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Abstract

In the aviation industry, propeller motor engines have a lifecycle of several thousand hours of flight and the maintenance is an important part of their lifecycle. The present article considers a multi-resource, priority-based case scheduling problem, which is applied in a Romanian manufacturing company, that repairs and maintains helicopter and airplane engines at a certain quality level imposed by the aviation standards. Given a reduced budget constraint, the management's goal is to maximize the utilization of their resources (financial, material, space, workers), by maintaining a prior known priority rule. An Off-Line Dual Maximum Resource Bin Packing model, based on a Mixed Integer Programming model is thus presented. The obtained results show an increase with approx. 25% of the Just in Time shipping of the engines to the customers and approx. 12,5% increase in the utilization of the working area.

Keywords: *Bin packing, maximization, resources, scheduling*

JEL Classification: L93

Introduction

Scheduling in the field of Maintenance is any variety of scheduled maintenance to an object or item of equipment. Specifically, Planned Maintenance is a scheduled service visit carried out by a competent and suitable agent, to ensure that an item of equipment is operating correctly and to therefore avoid any unscheduled breakdown and downtime.[1]

Aircraft maintenance checks are periodic inspections that have to be done on all commercial/civil aircraft after a certain amount of time or usage, according to Federal Aviation Administration (FAA) in the United States,[2] Transport Canada, for Canadian carriers, or the European Aviation Safety Agency (EASA) for EU carriers. Taking into account that an airplane (or helicopter) engine must be checked and repaired in such a way that the quality of the repair should be nearly perfect and the operational scheduling involved in this field, we've decided to investigate a scheduling problem within a maintenance facility, where the engines must be priority-based scheduled function of the availability of multiple resources.

The present article aims to present a way for solving the problem of maximizing the utilization of the available resources for the maintenance and repairing operations of the

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[1] Wood, Brian,*Building care*. Wiley-Blackwell, 2003.

[2]AFS-600."Chapter 8.Inspection Fundamentals". In *Aviation Maintenance Technician Handbook*, ed. Federal Aviation Administration.2008, 129.

propeller motor engine (for helicopters and small commercial and recreational airplanes)based on a priority rule when entering the company. In order to solve it, a Mixed Integer Programming model is created, but due to the complexity of the problem we try to solve it by an analogy to the Off-line Dual Maximum Resource Bin Packing problem where three novel approaches are used. Then, the model is tested in a Romanian company and results are presented accordingly.

The Bin Packing Problem has been extensively studied in combinatorial optimization. We can state it as follows:

-for a given set of n items with a weight $w_1, w_2, \dots, w_n$ and an unlimited number of identical bins of maximum capacity c, find a way of packing all the items into the minimum number of bins so that no item is left aside and the capacity of the bin c is not exceeded.

This problem has several applications in different areas like production scheduling, manufacturing, hospitals, logistics, but however is a NP-hard problem[1]as Karp[2] showed.

In our case, the above stated problem was modelled as a Dual Bin Packing Problem (DBPP). If classical bin packing problem is about minimizing the total number of bins required to assign a given set of items, the dual one is about maximizing the number of items that can be packed in the available bins, i.e. maximizing the resources utilization for our maintenance operations.

The DBPP has been referred to in the scientific literature[3], as a problem of maximization the number of items packed in a fixed number of bins[4], but also as a problem of unlimited number of bins[5], where we should pack items in as many bins as possible so that the total weight of each bin is at least equal to its capacity. Assmann[6]studied this problem from a discrete point of view. We will however consider the first type of the DBPP in our study case.

There are a lot of applications of the DBPP but until this article was written, the authors didn't find an application in the manufacturing aviation industry where products are scheduled according to their priorities and the workers' performance on their jobs, which in the worst case scenario may increase their time working on a specific assigned product, thus the schedule would be affected.

1. Problem definition

The Romanian company involved in our research, further "the Company", was as many other manufacturing and maintenance companies, hit by the financial crisis. Due to this unfortunate situation, the management had to let go some of the employees and the working floor was redesigned in order to better comply with customers' requirements and due-dates. A problem with the process line occurred when the delayed orders hit a record number of 300 per week (because workers were laid off and human resources together with the financial resources had to be drastically reduced). Also due to the Company's working profile, the maintenance has to be done according to the airspace regulations and ISO 9000 Quality Standards, so a middle way had to be found when scheduling the incoming orders for different types of engines with different problems.

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The Company's facilities are equipped with a tear-down area, a dismantling dedicated area and several types of equipment for measurement and NDT capabilities. Engines repairs and overhauls are performed using state-of-the art technology. All phases of the repairs are applied according to the requirements: dismantling, stripping and cleaning, visual inspection of components, measurement and/or non-destructive inspection tests. After the preliminary analysis, technical reports are filled in and the replacement of non-reparable parts is made accordingly. The engine is then reassembled and tested in another testing facility for ensuring the required performance as part of the quality assurance process. The Company's testing facilities cover a wide range of tests: test rigs for fuel/oil/hydraulic accessories of the engine, balancing and over-speed equipment, test beds for turbojets and turbo shafts. Similar procedures are applied on specific equipment for Dynamic Components for helicopters.

Assembling of components, subassemblies and final assembling are performed in a dedicated shop area, divided in several working areas (WA), using complex assembling precision specific equipment, where specific personnel works in teams, each team being run by a Team Leader (TL) and each team having assigned one engine at a time. There is also available and used equipment for balancing (shafts, rotors, rotating assemblies) and over-speed tests.

Due to the large number of airplanes and helicopter engines which have been waiting for the maintenance operation to be performed on and also due to a new lucrative contract for the next 5 years with important clients, the management decided to redesign the production and overhaul shop space in parallel with an optimization of the available resources, according to a priority-based rule.

For the production 2,000 sqm were dedicated and for overhaul 4,000 sqm shop space. The assembly shop of the new facility allows forother engines from the same family-types and sizes similar to the engines in current maintenance to be repaired. But in order to do so, the current orders had to be scheduled for maintenance and the delayed orders had to be reduced to zero.

The old and new orders had to be scheduled according to: the priority of each case (late orders had the highest priority, because of every additional day of the one week, i.e.5 working days, guaranteed maintenance time, the Company had to pay 0.01% penalties; and also new orders had high priority, i.e., the customers had the opportunity of paying an extra fee for getting their engines fixed faster than usual), the resource availability (available shop space, equipment, each TL was responsible for his/her engine and each TL had a certain affinity and thus was specialized in a certain engine type) and working time of each team. Due to the constraint imposed by the limited resources compared to the high number of engines which had to be repaired in the same time, a working schedule was implemented for the 8 hours, 5 days a week, working time. Sequencing the maintenance operations on engines and placing them at the right time slots, according to the available resources plays an important role in maximizing their utilization.

In this schedule case it was considered the case of maximizing the resource utilization– for the fixed costs to be as lower as possible, when assigned to final products. In other words, the fixed costs with the resources and the new production and overhaul shop space, are known in advance and taking in consideration that the working overtime would make the management to pay workers their overtime, the goal is to maximize the number of engines that are repaired using fixed working hours through a given set of available resources. Some of the engines which arrived for their annual maintenance procedure had complaints from their owners and had to be subjected to additional tests before the typical maintenance procedure, and so required a delayed start of the maintenance, i.e. delayed start.

The engines when entering the Company's facility are registered in the internal informational system (database) from where are scheduled on a first-come, first-served basis.

The Service department, based on this FCFS policy but also according to the priority of each engine in combination with some engines which had to be subjected to different tests (had a delayed start) had to choose the optimal solution of the sequence of the engines which entered the shop floor. Accordingly, a tool was needed in order to comply with these constraints but also to maximize the resources utilization, as part of the top management's priority.

Our problem is a complex one because it takes in consideration a long term period of time, where the resources can have distinct availabilities, function of different parameters like: delayed materials needed for the repairs, different tools and/ or machines are not in handy for one team because are used by another team, one team cannot work on more than one engine at a time and so the assigned floor space is occupied until the engine is ready to go.

Another constraint is added to the actual problem, i.e., new people are needed to be specialized for the new orders (also because of the lay-offs, it was cheaper to employ and train new workers than to increase the salary of the old ones, which accordingly were laid off) and so they should be able to be trained on the spot within different maintenance operations and the maintenance could have been prolonged.

The article is further organized as follows. A MIP model is presented in the 3rdpart, while in the 4thpart a simplified example of the DBPP is solved using three heuristic approaches for finding the best possible solution: First Fit Decreasing-based (FFD), First Fit Increasing (FFI) and a new First Fit Best Randomized (FFBR) approach. In the last part of the article conclusions are drawn based on the obtained results and we show that the FFBR is the best possible solution for our example, taking in consideration that DBPP is NP-hard.

2. A Mixed Integer Programming(MIP) model

We've started from our goal of maximizing the utilization of available resources (people, space, fixed working hours) according to the cases' priority and availability. In order to do so we should maximize the number of repaired engines in a given working time-slot according to the resources' availability. Due to the priority constraint, we should maximize the sum of priorities of the scheduled engines as an objective. In this way, weights are given to each engine when entering the facility in order for a priority scale to be made, and use these weights for the model to solve first the highest priority cases and then the lower priority cases. In order to maximize the utilization of the working area and the shipment of the repaired engines, two coefficients are introduced:

$k_1 = \frac{total\ time\ the\ WAs\ are\ occupied}{total\ available\ time\ across\ all\ WAs}$ and $k_2 = \frac{scheduled\ engines}{total\ number\ of\ engines\ to\ be\ scheduled}$

and will be used is assessing the improvement of the maintenance process after implementation of the three approaches for our DBPP.

Before trying to state the MIP model, several initial conditions should be stated:

- A team can only work on one engine at a time in only one designated shop area.
- A team can repair an engine only if all the members, including the TL is present at work in that day, at that time interval (time-slot).
- Each team is specialized in one engine and when prioritizing this aspect is considered, but every team can also switch the type of engine which will repair in case of increased number of a certain engine type.
- Engines, when entering the facility are prioritized according to the customers' requirements and they are sent to testing facility or to the deposit (where will wait to be processed at a later time) or directly to the WA. Accordingly, we can give 3 levels

of priority to these engines: 1 star to the engines which are not an urgency and can still wait a little time in the deposit, 2 stars for those which should be tested and 3 stars to those, which customers need as soon as possible and for which they paid an extra fee to "be first in the row".

- Each TL has assigned the type on engine at which his/her team is the best and we can predict that the duration of each repair is known in advance.
- In order for the time to be better managed, the working day (8 hours) is divided in 30 minutes time slots obtaining a 16 time-slots for one working day. We have chosen 30 minutes, because this is the average time for checking the parameters of an engine (visual and computer aided process) by performing a basic diagnostic analysis.
- The Company works on a 1 shift, 8 hours a day, 5 day per week, working program.
- An engine can be automatically scheduled for maintenance by the Service department, but also by each of the Team Leaders according to their time availability. As part of the management's motivation scheme, a Team Leader and his/her team can enter the internal competition of "the Best Team" Award, with extra financial benefits, according to the highest number of engines repaired according to their priority. But also the number of engines assigned by the TLs cannot surpass the number of the assigned engines by the Service department.

For a clearer description of the problem, we give a mathematical formulation for the problem. We first give notation used in the formulation (and throughout the paper).

The parameters, indices and variables of the proposed model can be seen from table 1 below:

Variable	Equals to:
$x_{ewatltsd}$	1, if the engine e is scheduled for maintenance in the WA with a team, run by team leader TL, at time-slot ts on day of the week d, 0, otherwise
y_{ewad}	1, if the engine e is assigned to the WA on day of the week d, 0, otherwise
z_{etl}	1, if the engine e is scheduled for maintenance by their TL, 0, if it is not scheduled at all
w_{eS}	1, if the engine e is scheduled for maintenance by the Service department, 0, if it is not scheduled at all

Table 1 – Variables of the MIP model

A start time (St) and an end time (Et) can also be considered for the maintenance process of a given engine.

Indices and parameters:

E Index for engines requiring maintenance e = 1, 2, … , E

WA Index for working areas: wa = 1, 2, … , WA

TL Index for team leaders: tl = 1, 2, … , TL

T Index for teams: t=1, 2, …, T

D Index for days, used for scheduling the maintenance operations: d = 1, 2, …, D

TS Index for time-slots during a working day: ts = 1, 2, … , TS. In our case TS=16, for one shift working day, but we consider a general approach.

I Index for repairing operation: i = 1, 2, … , I

J Index for engine type which will be repaired: j = 1, 2, … , J

S Set of all engines which must be repaired / pass maintenance operation

S^T Set of engines which require additional tests
S_i Set of engines requiring the repairing operation i
S_j Set of engines of type j
S_{tlj} Set of TLs specialized in repairing the engine type j
P_{ed} Priority of engine e in the day d: $P_{ed} \in \{1, 2, \dots, k\}$ $stars$ as a priority scale
T_e Total time for maintenance / repair for engine e, required by the best TL and his / her team (we assume that T_e = 1 time slot = 30min)
T_{tl} Time needed by the TL for fixing an engine according to his/ her abilities (skills)
T_T Time for the transportation of the materials, people, engine to and from the WA, before and after the maintenance
A_{tltsd} 1, for the availability of the TL (with his / her team) at time slot ts on day d; 0, otherwise.
E_{tsd} Total equipment (tools, machines, parts, consumables) available for time slot ts on day d
N_{ee} Number of equipment e required for fixing an engine e

Although each TL has his / her own team, on some engines which are in better conditions than others, there is no need for all of the team members to participate in the repairing process, thus we consider that the number of team members is variable and we introduce:

M_{tsd} Total number of team members available at time slot ts on day d
N_{me} Number of team members required for fixing an engine e

Our goal is to maximize the number of engines scheduled in all the WAs during all given working days. This can be noted as follows:

$\sum y_{ewad} P_{ed} = maximum$ subject to :

$\sum_{watld} x_{ewatltsd} \leq 1, \forall\, e, ts$ (1)

This states that an engine can be assigned at any time slot to at most one TL and WA on a certain day.

$\sum_{etl} x_{ewatltsd} \leq 1, \forall\, wa, ts, d$ (2)

That is, at most one maintenance operation can be performed in a WA in a given day and time slot.

$\sum_{ewa} x_{ewatltsd} \leq 1, \forall\, tl, ts, d$ (3)

Any TL can work on at most one engine at a given time slot and day.

$\sum_{wad} y_{ewad} \leq 1, \forall\, e$ (4)

An engine will be scheduled to at most one WA across all days.

$\sum_{tlts} x_{ewatltsd} \leq TS\, y_{ewad} \forall\, e, wa, d$ (5)

A TL works on an engine in a given WA at a time slot on a day no more than the assigned TS for engine e to be repaired in the WA on day d.

$\sum_{wad} y_{ewad} = \sum_{tl} (z_{etl} + w_{eS})\ \forall\, e$ (6)

The scheduled engine will be assigned with its TL to a specific WA on a day d.

$\sum_{watsd} x_{ewatltsd} = (T_e + T_{tl})(z_{etl} + w_{eS}) + T_T\ \ \forall\, e, tl$ (7)

The total time slots an engine is worked on, equals to the sum of total time for maintenance required by the best TL and the time of the actual TL according to his/ her skills, where the T_e is considered a reserve time slot for additional complications which may appear during the repair.

$\sum_{wa} x_{ewatltsd} \leq A_{tltsd} z_{etl} + A_{tltsd} w_{eS}\ \forall\, e, ts, d, tl$ (8)

The scheduled engines by their TL and the Service department should be repaired only by the Team Leaders who are available.

$S_t \leq ts \sum_{watld} x_{ewatltsd} + TS\, (1 - \sum_{watld} x_{ewatltsd})\ \ \forall\, e, ts$ (9)

The start of the maintenance operation should not exceed the time slot allowed for the start of its repair.

$$E_t \geq (ts+1)\sum_{watld} x_{ewatltsd} \quad \forall \; e, ts \tag{10}$$

The maintenance of an engine with eventual repair should not exceed the maximum time slot allowed by the timetable and should be longer than the calculated start of the operation, with at least one time slot.

$$\sum_{watltsd} x_{ewatltsd} = E_t - S_t \;\; \forall e \tag{11}$$

The time slot allowed for the engine e should be equal to the difference from the end and the start of the maintenance and repair.

$$\sum_{ewatl} N_{me} x_{ewatltsd} \leq M_{tsd} \;\forall\, ts, d \tag{12}$$

A certain number of team members are necessary to be present at the time slot tsin the day d, for the maintenance of the engine e.

$$\sum_{ewatl} N_{ee} x_{ewatltsd} \leq E_{tsd} \;\forall\, ts, d \tag{13}$$

A certain number of equipment should be available for the maintenance of the engine e to be performed in the allowed time slot ts on the day d.

Due to the conditions of the bin packing problem, we must state that the variables can have only two values according to their specific conditions, i.e., 0 and 1.

We cannot solve the problem due to the NP-hard situation, but we can solve a simplified example where we show our three approaches.

We consider a number of8 engines with different priorities (as given by the customers), but note that the same engines have priorities given by the available time and resources' constraints within the company.

Our methodology for solving this problem can be stated as follows:

1. Assign priorities to the set of engines according to the outside constraints (from the customers who paid an extra fee).
2. Assign priorities according to the estimated repair time, the availability of the TLs and their teams in the certain days and time-slots (inside constraints).
3. Search the local optimum for both of the cases.
4. Compare and combine the local optimums in order to satisfy the constraint of maximizing the resources' utilization, i.e., to find a global optimum.
5. Send the engine e to maintenance to the assigned WA with the available TL in the day d on the time slot ts.
6. Repeat the 1st to 5th step until all the engines are repaired and shipped to the end users.

The constraints for the 8 engines' example for the day d are as follows:

Engine no.	1	2	3	4	5	6	7	8
Customers' priority	3	1	3	2	2	2	3	1
Expected maintenance time in time slots	1	1	1	3	3	3	2	2
Available Team Leaders specialized in these engines*	YES=TL1	YES=TL2	YES=TL3	NO=TL1	NO =TL2	NO =TL3	YES =TL4	NO =TL4
Special equipment available for each specific engine	NO	YES	NO	YES	NO	YES	NO	YES
Available WA in ts	YES	YES	YES	YES	NO	NO	NO	NO
Company's priority**	2	3	2	2	1	1	1	1

Table 2 – An example for 8 engines with their constraints

*) If the TLs specialized in engines 4, 5, 6, 8 are not available; other TLs available will repair these engines.
**) Take in consideration that the company's priority is solved in a very simplistic way, which doesn't correspond to the reality! However in practice we cannot find the priority according to the company's resources and time constraint so easily!
If we try to solve this MIP model we won't be able to get feasible solutions due to the complexity of the model. Also we can make an analogy with the dual bin packing problem. We are able to solve it only for a restricted number of engines which are to be scheduled for maintenance, because this relates to a NP-hard problem.

3. A new approach for solving the DBPP

The DBPP can be solved with different algorithms, but due to the fact that we must choose a way of prioritizing the incoming engines, we've decided to use the First Fit Decreasing, First Fit Increasing and a new First Fit Best Randomized approach. Our solution tries to find a way for repairing the incoming engines and according to our constraints x, y, z, w, which can take only 2 values {0,1}, our DBP problem transforms into an Off-Line Dual Maximum Resource Bin Packing (ODMRBP).

In this off-line variant, we have a limited number of unit sized bins, the working areas, and a sequence of items with sizes in [0; 1], and the goal is to maximize the number of bins used to pack all the items subject to our constraints. A set of items fits in a bin if the sum of the sizes of the items is at most one, where number one can be associated with a fixed engine. In the off-line variant, there must be an ordering of the bins such that no item in a later bin fits in an earlier bin.

Thus we must first sort the items, and in our case this is done with the help of the "stars" scale, where for the engine with the highest priority the engine receives 3 stars and the one with lowest 1 star. Then the first engine e_1 should be sent to the first WA where "fits best" and the maintenance will start according to the available resources and time slot.
We consider and further solve the problem for 4 Team Leaders and 8 engines which must be scheduled on 2 Working Areas in 8 time slots from Monday, i.e, 4 hours.

Team Leader	Availability
TL 1	1-8
TL 2	3-7
TL 3	1-8
TL 4	4-8

Table 3 - Team Leaders availability on Monday

We use the following approaches:

- *First-Fit-Increasing* (*FFI*) allocates engines to WA (bins) in non-decreasing order with respect to their "sizes" (priority and resource availability).

In this case, we must first sort the engines according to the increasing priority as can be seen from Tab.4. With the time availability of the Team Leaders and their teams we find the best way for utilizing the available resources for scheduling the engines on Monday morning.

Engine no.	2	8	4	5	6	1	3	7
Customers' priority	1	1	2	2	2	3	3	3
Expected maintenance time in time slots	1	2	3	3	3	1	1	2

Table 4 – Engines sorted according to increasing priority

The results can be seen from the table below, where "?" signifies an empty time slot, an extra cost the company had to pay if this approach was chosen:

30 minutes time slots	TLs working on e_x engine in WA1	TLs working on e_x engine in WA2
1st	?	?
2nd	?	?
3rd	TL2 on e_2	TL2 on e_5
4th	TL4 on e_8	TL2 on e_5
5th	TL4 on e_8	TL2 on e_5
6th	TL1 on e_4	TL3 on e_6
7th	TL1 on e_4	TL3 on e_6
8th	TL1 on e_4	TL3 on e_6

Table 5 – Results for the FFI approach

- *First-Fit-Decreasing* (*FFD)* allocates engines to WA (bins) in non-increasing order with respect to their "sizes" (priority and resource availability).

For this approach the engines are sorted in a decreasing priority manner, according to Tab.6 and results are in Tab.7.

Engine no.	1	3	7	4	5	6	8	2
Customers' priority	3	3	3	2	2	2	1	1
Expected maintenance time in time slots	1	1	2	3	3	3	2	1

Table 6 – Engines sorted according to decreasing priority

30 minutes time slots	TLs working on e_x engine in WA1	TLs working on e_x engine in WA2
1st	TL1 on e_1	?
2nd	TL3 on e_3	?
3rd	?	TL2 on e_5
4th	TL4 on e_7	TL2 on e_5
5th	TL4 on e_7	TL2 on e_5
6th	TL1 on e_4	TL3 on e_6
7th	TL1 on e_4	TL3 on e_6
8th	TL1 on e_4	TL3 on e_6

Table 7 – Results for the FFD approach

- *First-Fit-Best-Randomized (FFBR)*allocates engines to WA (bins) in a purely randomized, corresponding to the optimistic scenario, where each TL wants to work on a specified engine and they make their schedule accordingly.

We will sort the engines first according to the willingness of the TLs and second according to the customers' priority as can be seen from Table 8. The results one can see from the Table 9 below:

Engine no.	1	3	2	4	5	6	7	8
Customers' priority	3	3	1	2	2	2	3	1
Expected maintenance time in time slots	1	1	1	3	3	3	2	2

Table 8 – Engines sorted according to the optimistic scenario

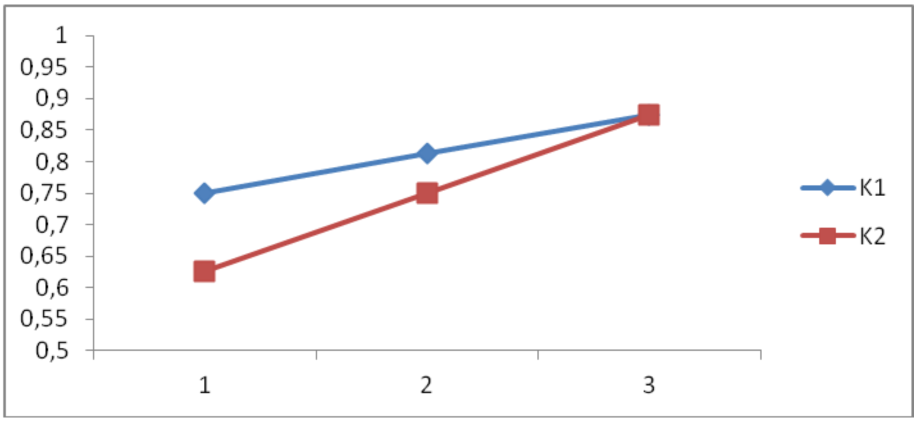
30 minutes time slots	TLs working on e_x engine in WA1	TLs working on e_x engine in WA2
1^{st}	TL1 on e_1	?
2^{nd}	TL3 on e_3	?
3^{rd}	TL2 on e_2	TL2 on e_5
4^{th}	TL1 on e_4	TL2 on e_5
5^{th}	TL1 on e_4	TL2 on e_5
6^{th}	TL1 on e_4	TL3 on e_6
7^{th}	TL4 on e_8	TL3 on e_6
8^{th}	TL4 on e_8	TL3 on e_6

Table 9 – Results for the FFBR approach

Another way of measuring the success rate of our approaches for solving the ODMRBP problem was by computing the two coefficients presented at the beginning of the paper. Also due to the solutions presented above the best values of these 2 coefficients are for the FFBR approach, where workers together with their TLs and Service department schedule their own tasks and try to optimize every day the WA and time slots' utilization.

The obtained results show an increase with approx. 25% of the Just in Time shipping of the engines to the customers and a decrease of the penalties the company had to pay for sending engines with delays and an approximate 12,5% increase in the utilization of the working area, which also increase the resources' utilization and minimizes the fixed costs of the working area when assigned to the total costs of the engines' repair.

The coefficients can be seen from the Graph below:



Graph 1 –Values of the coefficients K_1 and K_2 for FFI, FFD and FFBR approach

Conclusions

In this paper we have offered a possible solution to a multi-resource, priority-based case scheduling problem, which the authors have encountered in a Romanian manufacturing company, where helicopter and airplane engines were repaired. A Mixed Integer Programming model was developed, but due to the complexity of the problem, which is NP-hard, a solution was described only on a small batch of engines which had to be scheduled on the Monday morning. An analogy with the Dual Bin Packing problem was noticed, more exactly with the Off-Line Dual Maximum Resource Bin Packing problem and three new approaches were used in order to find the best possible solution. As one can observe the best results are when we use the new FFBR approach, as a number of 7 out of 8 engines are scheduled according to the priority imposed by the customers, the working space availability and Team Leaders' working hours. For the previous approaches a number of 5 out of 8 engines are scheduled for FFI and 6 out of 8 for FFD. Even if 7 out of 8 still misses one engine to be scheduled, the authors consider that FFBR approach can be successfully used for finding the optimum solution (with all the engines scheduled), because on a longer time interval, the complexity of the problem increases, but with the help of computational software a good solution can be found.

Due to the intellectual property rights the data presented in this paper were altered and the company's name cannot be stated, but the authors tried to present the ratios, coefficients and results as close to reality as possible.

Future studies are aimed at improving the upper presented approaches, with the goal of developing new software for solving this exact type of problem, where other constraints can be added or removed according to the specific profile of the company where the optimization of the resources is needed on a priority-based rule.

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