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Radiation and mass transfer effects on an unsteady MHD free convection flow past a heated vertical plate in a porous medium with viscous dissipation

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Abstract

An unsteady, two-dimensional, hydromagnetic, laminar free convective boundary-layer flow of an incompressible, Newtonian, electrically-conducting and radiating fluid past an infinite heated vertical porous plate with heat and mass transfer is analyzed, by taking into account the effect of viscous dissipation. The dimensionless governing equations for this investigation are solved analytically using two-term harmonic and non-harmonic functions. Numerical evaluation of the analytical results is performed and graphical results for velocity, temperature and concentration profiles within the boundary layer and tabulated results for the skin-friction coefficient, Nusselt number and Sherwood number are presented and discussed. It is observed that, when the radiation parameter increases, the velocity and temperature decrease in the boundary layer, whereas when thermal and solutal Grashof increases the velocity increases.

Keywords: Radiation, viscous dissipation, heat and mass transfer

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1 Introduction

Coupled heat and mass transfer (or double-diffusion) driven by buoyancy, due to temperature and concentration variations in a saturated porous medium, has several important applications in geothermal and geophysical engineering such as the migration of moisture through the air contained in fibrous insulation, the extraction of geothermal energy, underground disposal of nuclear wastes, and the spreading of chemical contaminants through water-saturated soil. Bejan and Khair [1] investigated the vertical free convection boundary layer flow in porous media owing to combined heat and mass transfer. The suction and blowing effects on free convection coupled heat and mass transfer over a vertical plate in a saturated porous medium was studied by Raptis et al. [2] and Lai and Kulacki [3] respectively.

Hydromagnetic flows and heat transfer have become more important in recent years because of its varied applications in agriculture, engineering and petroleum industries. Raptis [4] studied mathematically the case of time varying two-dimensional natural convective flow of an incompressible, electrically conducting fluid along an infinite vertical porous plate embedded in a porous medium. Soundalgekar [5] obtained approximate solutions for two-dimensional flow of an incompressible viscous flow past an infinite porous plate with constant suction velocity, the difference between the temperature of the plate and the free stream is moderately large causing free convection currents. Takhar and Ram [6] studied the MHD free convection heat transfer of water at $4\,^{\circ}$C through a porous medium. Soundalgekar et al. [7] analyzed the problem of free convection effects on Stokes problem for a vertical plate under the action of transversely applied magnetic field with mass transfer. Elbashbeshy [8] studied heat and mass transfer along a vertical plate under the combined buoyancy effects of thermal and species diffusion, in the presence of magnetic filed.

In all these investigations, the radiation effects are neglected. For some industrial applications such as glass production and furnace design and in space technology applications, such as cosmical flight aerodynamics rocket, propulsion systems, plasma physics and spacecraft re-entry aerothermodynamics which operate at higher temperatures, radiation effects can be significant. Alagoa et al. [9] studied radiative and free convection effects on MHD flow through porous medium between infinite parallel plates with time-dependent suction. Bestman and Adjepong [10] analyzed unsteady hydromagnetic free convection flow with radiative heat transfer in a rotating fluid.

In all these investigations, the viscous dissipation is neglected. The viscous dissipation heat in the natural convective flow is important, when the flow field is of extreme size or at low temperature or in high gravitational field. Gebhart [11] shown the importance of viscous dissipative heat in free convection flow in the case of isothermal and constant heat flux at the plate. Soundalgekar [12] analyzed the effect of viscous dissipative heat on the two-dimensional unsteady, free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate. Israel Cookey et al. [13] investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction.

The objective of the present chapter is to study the radiation and mass transfer effects on an unsteady two-dimensional laminar free convective flow of a viscous, incompressible, electrically conducting fluid past vertical a heated porous plate with suction, embedded in a porous medium, under the influence of a uniform transverse magnetic field, by taking into account the effect of viscous dissipation. The equations of continuity, linear momentum, energy and diffusion, which govern the flow field are solved by using a regular perturbation method. The behaviour of the velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number has been discussed for variations in the physical parameters.

2 Mathematical analysis

An unsteady two-dimensional hydromagnetic laminar free convection boundary layer flow of a viscous, incompressible, electrically conducting and radiating fluid in an optically thin environment past an infinite heated vertical porous plate, embedded in a uniform porous medium, in the presence of thermal and concentration buoyancy effects, is considered. The x'- axis is taken along the vertical plate and the y'- axis normal to the plate. A uniform magnetic field is applied in the direction perpendicular to the plate. The fluid is assumed to be slightly conducting, and hence the magnetic Reynolds number is much less than unity and the induced magnetic field is negligible in comparison with the transverse magnetic field. It is further assumed that there is no applied voltage, so that electric field is absent. The level of concentration of foreign mass is assumed to be low, so that the Soret and Dufour effects are negligible. Since the plate is of infinite length, all the physical variables are functions of y' and t' only. Now, under the usual Boussinesq's approximation, the flow field is governed by the following equations.

$$\frac{\partial\, v'}{\partial\, y'} = 0 \tag{2.1}$$

$$\frac{\partial\, u'}{\partial\, t'} + v'\,\frac{\partial\, u'}{\partial\, y'} = \frac{\partial U'}{\partial\, t'} + \nu\frac{\partial^2\, u'}{\partial\, y'^2}$$

$$+\, g\,\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) - \left(\frac{\sigma\mu_e^2 H_0^2}{\rho} + \frac{\nu}{K'}\right)(u' - U') \tag{2.2}$$

$$\frac{\partial\, T'}{\partial\, t'} + v'\,\frac{\partial\, T'}{\partial\, y'} = \frac{k}{\rho\, c_p}\left[\frac{\partial^2 T'}{\partial\, y'^2} - \frac{1}{k}\frac{\partial\, q'}{\partial\, y'}\right] + \frac{\nu}{c_p}\left(\frac{\partial\, u'}{\partial\, y'}\right)^2 \tag{2.3}$$

$$\frac{\partial^2 q'}{\partial\, y'^2} - 3\alpha^2 q' - 16\sigma^*\alpha T'^3_\infty\frac{\partial\, T'}{\partial\, y'} = 0 \tag{2.4}$$

$$\frac{\partial\, C'}{\partial\, t'} + v'\,\frac{\partial\, C'}{\partial\, y'} = D\,\frac{\partial^2\, C'}{\partial\, y'^2} \tag{2.5}$$

where u', v' are the velocity components in x',y' directions respectively, t'- the time, p'- the pressure, ρ - the fluid density, σ - the electrical conductivity of the fluid, ν - the kinematic viscosity, μ_e- the magnetic permeability, K' - the permeability of the porous medium, g - the acceleration due to gravity, β and $\beta*$ - the thermal and concentration expansion coefficients, T' - the temperature of the fluid in the boundary layer, T'_∞ - the temperature of the fluid far away from the plate,C' - the species concentration in the boundary layer, C'_∞ - the species concentration in the fluid far away from the plate, H_0^2- the constant transverse magnetic field, k - the thermal conductivity, q'- the radiative heat flux, σ^*- the Stefan- Boltzmann constant and D - the mass diffusivity. The second and third terms on the right hand side of the momentum equation (2) denote the thermal and concentration buoyancy effects respectively. Also, the second term on right hand side of the energy equation (2.3) represents the radiative heat flux and the third term represents viscous dissipation. Equation (2.4) is the differential approximation for radiation under fairly broad realistic assumptions. In one space coordinate y', the radiative flux q' satisfies this nonlinear differential Equation [10].

The boundary conditions for the velocity, temperature, and concentration fields are

$$u' = 0, \quad T' = T_w \, C' = C_w \quad at\ y' = 0$$

$$u' = U'(t') = V_0(1 + \varepsilon\, e^{n't'})\,,\ T' \to T'_\infty, \quad C' \to C'_\infty \ as\ y' \to \infty \quad (2.6)$$

where T'_w and C'_w are the temperature and concentration of the plate respectively. Equation (2.1) asserts that, the suction velocity at the plate is either a constant or a function of time. Hence the suction velocity normal to the plate is assumed to be of the form

$$v' = -V_0(1 + \varepsilon\, A\, e^{n't'}) \quad (2.7)$$

where A is a real positive constant, ε is small such that $\varepsilon << 1$, and $\varepsilon A << 1$ and V_0 is scale of suction velocity which is non-zero positive constant. The negative sign indicates that the suction is towards the plate.

Since the medium is optically thin with relatively low density and α (absorption coefficient) $<< 1$, the radiative heat flux given by Equation (2.4), in the spirit of Cogley et al. [14] becomes

$$\frac{\partial q'}{\partial y'} = 4\alpha^2(T' - T'_\infty) \quad (2.8)$$

where

$$\alpha^2 = \int_0^\infty \delta\lambda \frac{\partial B}{\partial T'} \quad (2.9)$$

where B is Planck's function.

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$y = \frac{V_0}{\nu} y'\ ,\ n = \frac{4\nu}{V_0^2} n',\ u = \frac{u'}{U_0},\ t\ =\ \frac{V_0^2}{4\nu} t'\ ,\ \ v = \frac{4\nu}{V_0^2} V',\ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},$$

$$C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \chi^2 = \frac{\upsilon^2}{K' V_0^2}, \Pr = \frac{\mu\, c_p}{k} = \frac{\nu}{\alpha}, \qquad Sc = \frac{\nu}{D},$$

$$M^2 = \frac{\mu_e^2 \sigma H'^2_0}{\rho V_0^2}, Gr = \frac{\nu\, \beta\, g(T'_w - T'_\infty)}{U_0\, V_0^2}\ ,\ Gm = \frac{\nu \beta^* g(C'_w - C'_\infty)}{U_0 V_0^2},$$

$$Ec = \frac{U_0^2}{c_p(T'_w - T'_\infty)}, R^2 = \frac{4\alpha^2}{\rho\, k c_p V_0^2}(T'_w - T'_\infty) \tag{2.10}$$

In view of Equations (2.4), (2.7), (2.8), (2.9) and (10), Equations (2), (2.3) and (2.5) reduce to the following dimensionless form.

$$\frac{1}{4}\frac{\partial u}{\partial t} - (1 + \varepsilon\, A\, e^{nt})\frac{\partial u}{\partial y} = \frac{1}{4}\frac{\partial U}{\partial t} + \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\, C + (M^2 + \chi^2)(u - U) \tag{2.11}$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t} - (1 + \varepsilon\, A\, e^{nt})\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\left[\frac{\partial^2 \theta}{\partial y^2} - R^2\right] + Ec\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.12}$$

$$\frac{1}{4}\frac{\partial C}{\partial t} - (1 + \varepsilon\, A\, e^{nt})\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{2.13}$$

where Gr, Gm, Pr, R, Ec and Sc are the thermal Grashof number, solutal Grashof Number, Prandtl Number, radiation parameter, Eckert number and the Schmidt number respectively.

The corresponding boundary conditions are

$$\begin{aligned} &u = 0, \quad \theta = 1, \quad C = 1 \;\; at\; y = 0 \\ &u \to 1 + \varepsilon e^{nt}, \quad \theta \to 0\ ,\ C \to 0\;\; as\, y \to \infty \end{aligned} \tag{2.14}$$

3 Solution of the problem

Equations (2.11) - (2.13) represent coupled, non-linear partial differential equations, which cannot be solved in closed-form. So, in order to solve these equations, we may represent the velocity, temperature and concentration of the fluid in the neighbourhood of the plate as

$$u(y, t) = u_0(y) + \varepsilon\; e^{nt}\, u_1(y) + o(\varepsilon^2) + ...$$

$$\theta(y,t) = \theta_0(y) + \varepsilon\, e^{nt}\,\theta_1(y) + o(\varepsilon^2) + ... \tag{3.1}$$
$$C(y,t) = C_0(y) + \varepsilon\, e^{nt}\, C_1(y) + o(\varepsilon^2) + ...$$

Substituting (3.1) in Equations (2.11) - (2.13), and equating the harmonic and non - harmonic terms, and neglecting the higher-order terms of O (ε^2), we obtain

$$u_0'' + u_0' - (\chi^2 + M^2)\, u_0 = -N - Gr\,\theta_0 - Gm\, C_0 \tag{3.2}$$

$$u_1'' + u_1' - (\chi^2 + M^2 + \frac{n}{4})\, u_1 = -(N+n) - Au_0' - Gr\,\theta_1 - Gm\, C_1 \tag{3.3}$$

$$\theta_0'' + \Pr\theta_0' - R^2\theta_0 = -\Pr Ec(u_0')^2 \tag{3.4}$$

$$\theta_1'' + \Pr\theta_1' - (R + \frac{n\Pr}{4})\theta_1 = -\Pr A\theta_0' - 2\Pr Ecu_0'u_1' \tag{3.5}$$

$$C_0'' + Sc\, C_0' = 0 \tag{3.6}$$

$$C_1'' + Sc\, C_1' - n\, Sc\, C_1' = -A\, Sc\, C_0' \tag{3.7}$$

where prime denotes ordinary differentiation with respect to y. The corresponding boundary conditions can be written as

$$u_0 = 0,\, u_1 = 0\,,\ \theta_0 = 1\,,\ \theta_1 = 0\,,\ C_0 = 1\,,\ C_1 = 0\ at\ y = 0$$

$$u_0 = 1\,,\ u_1 = 1\,,\ \theta_0 \to 0\,,\ \theta_1 \to 0\,,\ C_0 \to 0\,,\ C_1 \to 0\ \ as\, y \to \infty \tag{3.8}$$

The Equations (3.2) - (3.7) are still coupled non-linear equations, whose exact solutions are not possible. So we expand $u_0, u_1, \theta_0, \theta_1, C_0, C_1$ in terms of Ec in the following way, as the Eckert number for incompressible fluid is always very small.

$$u_0(y) = u_{01}(y) + Ec\ u_{02}(y)$$
$$u_1(y) = u_{11}(y) + Ec\, u_{12}(y)$$
$$\theta_0(y) = \theta_{01}(y) + Ec\,\theta_{02}(y)$$
$$\theta_1(y) = \theta_{11}(y) + Ec\,\theta_{12}(y)$$
$$C_0(y) = C_{01}(y) + Ec\, C_{02}(y)$$

$$C_1(y) = C_{11}(y) + Ec\,C_{12}(y) \tag{3.9}$$

Substituting (3.9) in Equations (3.2) - (3.7), equating the coefficients of Ec to zero and neglecting the terms in Ec^2 and higher order, we get the following equations.

The zeroth order equations are

$$u''_{01} + u'_{01} - (\chi^2 + M^2)\,u_{01} = -(\chi^2 + M^2) - Gr\,\theta_{01} - Gm\,C_{01} \tag{3.10}$$

$$u''_{02} + u'_{02} - (\chi^2 + M^2)\,u_{02} = -Gr\,\theta_{02} - Gm\,C_{02} \tag{3.11}$$

$$\theta''_{01} + \Pr\ \theta'_{01} - R^2\theta_{01} = 0 \tag{3.12}$$

$$\theta''{}_{02} + \Pr\,\theta'{}_{02} - R^2\theta_{02} = -\Pr u'^2_{01} \tag{3.13}$$

$$C''_{01} + Sc\,C'_{01} = 0 \tag{3.14}$$

$$C''_{02} + Sc\,C'_{02} = 0 \tag{3.15}$$

and the respective boundary conditions are

$$u_{01} = 0, u_{02} = 0, \theta_{01} = 1, \theta_{02} = 0, C_{01} = 1, C_{02} = 0 \quad at \quad y = 0 \tag{3.16}$$

$$u_{01} \to 1, u_{02} \to 0, \theta_{01} \to 0, \theta_{02} \to 0, C_{01} \to 0, C_{02} \to 0 \quad as \quad y \to \infty$$

The first order equations are

$$u''_{11} + u'_{11} - N_1\,u_{11} = -N_1 - Gr\,\theta_{11} - Gm\,C_{11} - Au'_{01} \tag{3.17}$$

$$u''_{12} + u'_{12} - N_1\,u_{12} = -Gr\,\theta_{12} - Gm\,C_{12} - Au'_{02} \tag{3.18}$$

$$\theta''_{11} + \Pr\ \theta'_{11} - N_2\,\theta_{11} = -A\Pr\theta'_{01} \tag{3.19}$$

$$\theta''_{12} + \Pr\ \theta'_{12} - N_2\theta_{12} = -\Pr A\theta'_{02} - 2\Pr u'_{01}u'_{11} \tag{3.20}$$

$$C''_{11} + Sc\,C'_{11} - n\,Sc\,C_{11} = -AScC'_{01} \tag{3.21}$$

$$C''_{12} + Sc\,C'_{12} - n\,Sc\,C_{12} = -AScC'_{02} \tag{3.22}$$

and the respective boundary conditions are

$$u_{11} = 0, u_{12} = 0, \theta_{11} = 0, \theta_{12} = 0, C_{11} = 0, C_{12} = 0 \quad at \quad y = 0$$

$$u_{11} \to 1, u_{12} \to 0, \theta_{11} \to 0, \theta_{12} \to 0, C_{11} \to 0, C_{12} \to 0 \quad as \quad y \to \infty \tag{3.23}$$

where $N_1 = \chi^2 + M^2 + \frac{n}{4}$, $N_2 = R^2 + \frac{n\,\mathrm{Pr}}{4}$

Solving Equations (3.10) - (3.15) under the boundary conditions (3.16), and Equations (3.17) - (3.22) under the boundary conditions (3.23), and using Equations (3.9) and (3.1), we obtain the velocity, temperature and concentration distributions in the boundary layer as

$$\begin{aligned}
u(y,t) \quad = \quad & P_3 e^{-(m_3 y)} + P_1 e^{-m_2 y} + P_2 e^{-Scy} + 1 \quad + \\
& Ec\{J_{10} e^{-(m_3 y)} + J_1 e^{-m_2 y} + J_2 e^{-2m_3 y} + J_3 e^{-(2m_2)y} \\
& + J_4 e^{-2Scy} + J_5 e^{-(m_2+m_3)y} + J_6 e^{-(m_2+Sc)y} + \\
& J_7 e^{-(m_3+Sc)y}\} + \varepsilon e^{nt}[\{D_{10} e^{-(m_5 y)} + D_6 e^{-m_3 y} \\
& + D_7 e^{-m_2 y} + D_8 e^{-Scy} + D_9 e^{-m_4 y} + D_5 e^{-m_1 y} + 1\} \\
& + Ec\{Z_{20} e^{-m_5 y} + Z_1 e^{-m_4 y} + Z_2 e^{-m_2 y} \\
& + Z_3 e^{-2m_3 y} + Z_4 e^{-m_3 y} + Z_5 e^{-2m_2 y} + \\
& Z_6 e^{-2Scy} + Z_7 e^{-(m_2+m_3)y} + Z_8 e^{-(m_2+Sc)y} \\
& + Z_9 e^{-(m_3+Sc)y} + Z_{10} e^{-(m_5+m_3)y} + Z_{11} e^{-(m_4+m_3)y} \\
& + Z_{12} e^{-(m_2+m_5)y} + Z_{13} e^{-(m_2+m_4)y} \\
& + Z_{14} e^{-(m_3+m_4)y} + Z_{15} e^{-(m_5+Sc)y} + Z_{16} e^{-(m_1+m_3)y} + \\
& Z_{17} e^{-(m_1+m_2)y} + Z_{18} e^{-(m_1+Sc)y}\}]
\end{aligned}$$

$$\begin{aligned}
\theta(y,t) = & e^{-m_2 y} + Ec\{S_7 e^{-m_2 y} + S_1 e^{-2m_3 y} + S_2 e^{-2m_2 y} \\
& + S_3 e^{-2Scy} + S_4 e^{-(m_3+m_2)y} + S_5 e^{-(m_2+Sc)y} \\
& + S_6 e^{-(m_3+Sc)y}\} + \varepsilon e^{nt}[\{D_1 \quad e^{-m_2 y} - D_2 e^{-m_4 y}\} \\
& + Ec\{G_1 e^{-m_2 y} + G_2 e^{-2m_3 y} \\
& + G_3 e^{-2m_2 y} + G_4 e^{-2Scy} + G_5 e^{-(m_3+m_2)y} \\
& + G_6 e^{-(m_2+Sc)y} + G_7 e^{-(m_3+Sc)y} + \\
& + G_8 e^{-(m_3+m_5)y} + G_9 e^{-(m_3+m_4)y} + G_{10} e^{-(m_5+m_2)y} \\
& + G_{11} e^{-(m_4+m_2)y} + G_{12} e^{-(m_4+Sc)y} \\
& + G_{13} e^{-(m_5+Sc)y} + G_{14} e^{-(m_1+m_3)y} + G_{15} e^{-(m_2+m_1)y}
\end{aligned}$$

$$+G_{16}e^{-(m_1+Sc)y} + G_{20}e^{-m_4y}\}]$$

$$C(y,t) = e^{-Scy} + \varepsilon e^{nt}\left\{\frac{4ASc}{n}(e^{-Scy} - e^{-m_1y})\right\}$$

where the expressions for the constants are given in Appendix.

The skin-friction, the Nusselt number and the Sherwood number are important physical parameters for this type of boundary layer flow.

Knowing the velocity field, the skin-friction at the plate can be obtained, which in non-dimensional form is given by

$$C_f = \left(\mu\frac{\partial u'}{\partial y'}\right)_{y'=0} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left(\frac{\partial u_0}{\partial y} + \varepsilon e^{nt}\frac{\partial u_1}{\partial y}\right)_{y=0}$$

$$= -P_3m_3 - P_1m_2 - P_2Sc + Ec\{-J_{10}m_3 - J_1m_2 - 2J_2m_3 - 2J_3m_2 - 2J_4Sc$$
$$-J_5(m_2+m_3) - J_6(m_2+Sc) - J_7(m_3+Sc) + \varepsilon e^{nt}[\{-D_{10}m_5 - D_6m_3 - D_7m_2$$
$$-D_8Sc - D_9m_4 - D_5m_1\} + Ec\{-Z_{20}m_5 - Z_1m_4 - Z_2m_2 - 2Z_3m_3 - Z_4m_3 - 2Z_5m_2$$
$$-2Z_6Sc - Z_7(m_2+m_3) - Z_8(m_2+Sc) - Z_9(m_3+Sc) - Z_{10}(m_5+m_3)$$
$$-Z_{11}(m_4+m_3) - Z_{12}(m_2+m_5) - Z_{13}(m_2+m_4) - Z_{14}(m_3+m_4)$$
$$-Z_{15}(m_5+Sc) - Z_{16}(m_1+m_3) - Z_{17}(m_1+m_2) - Z_{18}(m_1+Sc)\}]$$

Knowing the temperature field, the rate of heat transfer coefficient can be obtained, which in the non-dimensional form, in terms of the Nusselt number, is given by

$$Nu = \frac{q'\nu}{kV_0(T'_w - T'_\infty)}, \quad where \quad q' = -k\left(\frac{\partial T'}{\partial y'}\right)_{y'=0}$$

$$= -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = -\left(\frac{\partial\theta_0}{\partial y} + \varepsilon e^{nt}\frac{\partial\theta_1}{\partial y}\right)_{y=0}$$

$$= -(-m_2 + Ec\{-S_7m_2 - 2S_1m_3 - 2S_2m_2 - 2S_3Sc - S_4(m_3+m_2) - S_5(m_2+Sc)$$
$$+\varepsilon e^{nt}[\{-D_1m_2 + D_1m_4\} + Ec\{-G_1m_2 - 2G_2m_3 - 2G_3m_2 - 2G_4Sc$$
$$-G_5(m_2+m_3) - G_6(m_2+Sc) - G_7(m_3+Sc) - G_8(m_3+m_5)$$
$$-G_9(m_3+m_4) - G_{10}(m_2+m_5) - G_{11}(m_2+m_4) - G_{12}(m_4+Sc)$$

$-G_{13}(m_5+Sc)-G_{14}(m_1+m_3)-G_{15}(m_1+m_2)-G_{16}(m_1+Sc)-G_{20}Sc]$

Knowing the concentration field, the rate of mass transfer coefficient can be obtained, which in the non-dimensional form, in terms of the Sherwood number, is given by

$$Sh = \frac{C^*\nu}{DV_0(T'_w - T'_\infty)}, \quad where \quad C^* = -D\left(\frac{\partial C'}{\partial y'}\right)_{y'=0}$$

$$= -\left(\frac{dC}{dy}\right)_{y=0} = -\left(\frac{\partial C_0}{\partial y} + \varepsilon e^{nt}\frac{\partial C_1}{\partial y}\right)_{y=0}$$

$$= -\left[-Sc + \varepsilon e^{nt}\left\{\frac{4ASc}{n}(-Sc+m_1)\right\}\right]$$

4 Results and discussion

In the preceding sections, the problem of MHD unsteady free convective flow of a viscous, incompressible, radiating and dissipating fluid past an infinite porous plate embedded in a porous medium was formulated and solved by means of a perturbation method, by applying Cogley et al. [14] approximation for the radiative heat flux. The expressions for the velocity, temperature and concentration were obtained. To illustrate the behaviour of these physical quantities, numerical values were computed with respect to the variations in the governing parameters viz., the thermal Grashof number *Gr*, the solutal Grashof number *Gm*, Prandtl number *Pr*, Schmidt number *Sc*, the radiation parameter R and the Eckert number *Ec*.

Fig.1 presents the typical velocity profiles in the boundary layer for various values of the thermal Grashof number. It is observed that an increase in *Gr*, leads to a rise in the values of velocity due to enhancement in buoyancy force. Here the positive values of *Gr* correspond to cooling of the surface. In addition, the curve show that the peak value of the velocity increases rapidly near the wall of the porous plate as Grashof number increases and then decays the free stream velocity. For the case of different values of the solutal Grashof number, the velocity profiles in the boundary layer are shown in Fig.2.The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach a free stream value. As expected, the fluid velocity increases and the peak value becomes more distinctive due to increase in the buoyancy force represented by Gm. Figs.3(a) and 3(b) display the effects of Schmidt number on the velocity and concentration profiles respectively. As the Schmidt number increases, the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. Reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers. These behaviors are evident from Figs.3 (a) and 3(b).

For different values of the radiation parameter R, the velocity and temperature profiles are plotted in Figs.4(a) and 4(b). It is obvious that an increase in the radiation parameter results in decreasing velocity and temperature within the boundary layer, as well as decrease the thickness of the velocity and temperature boundary layers. For various values of the magnetic parameter, the velocity profiles are plotted in Fig.5. It is obvious that existence of the magnetic field decreases the velocity. Fig.6 shows the velocity profiles for different values of the permeability parameter χ. Clearly as χ increases, the velocity tends to decrease.

Figs.7(a) and 7(b) illustrate the velocity and temperature profiles for different values of Prandtl number. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity. From Fig.7(b), the numerical results show that an increase in the Prandtl number results a decrease of the thermal boundary layer and in general lower average temperature with in the boundary layer. The reason is that smaller values of Pr are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher values of Pr. Hence in the case of smaller Prandtl numbers the thermal boundary layer is thicker and the rate of heat transfer is reduced.

The effects of the viscous dissipation parameter i.e., the Eckert number on the velocity and temperature are shown in Figs.8(a) and 8(b). Greater viscous dissipative heat causes a rise in the temperature as well as the velocity profiles.

Tables 1-5 show the effects of the thermal Grashof Number, solutal Grashof number, radiation parameter, the Schmidt number and Eckert number on the skin-friction C_f, the Nusselt number Nu, and the Sherwood numberSh. From Tables 1 and 2, it is observed that as Gr or Gm increases, the skin-friction coefficient increases. However, from Table 3, it can be easily seen that as radiation parameter increases, the skin-friction decreases and the Nusselt number increases. From Table 4, it is noticed that an increase in the Schmidt number reduces the skin-friction and increases the Sherwood number. Finally, Table 5 shows that as Eckert number increases, the skin-friction increases, the Nusselt number decreases.

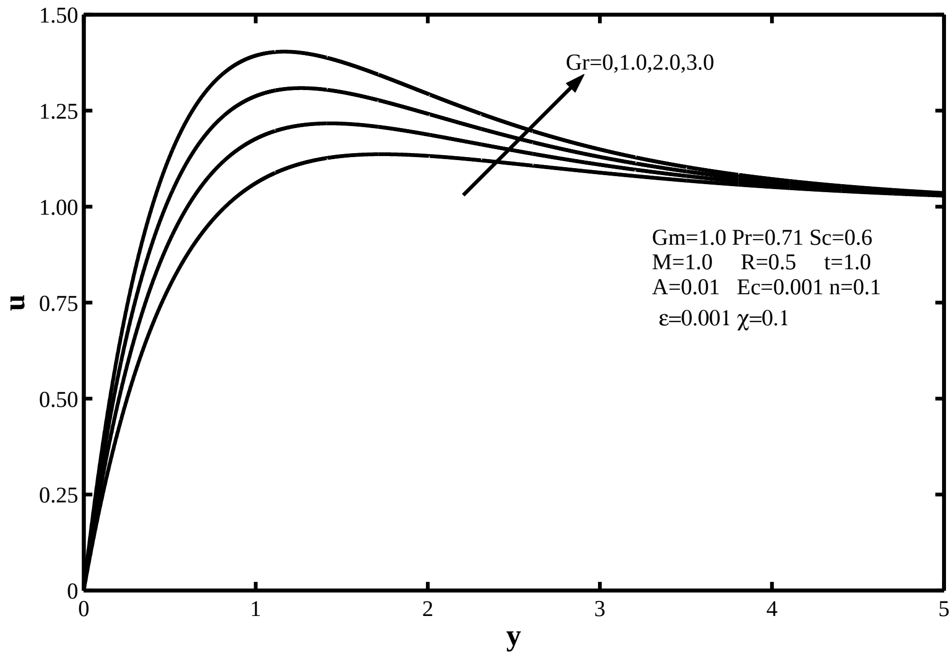

Figure 1: Effect of Gr on velocity.

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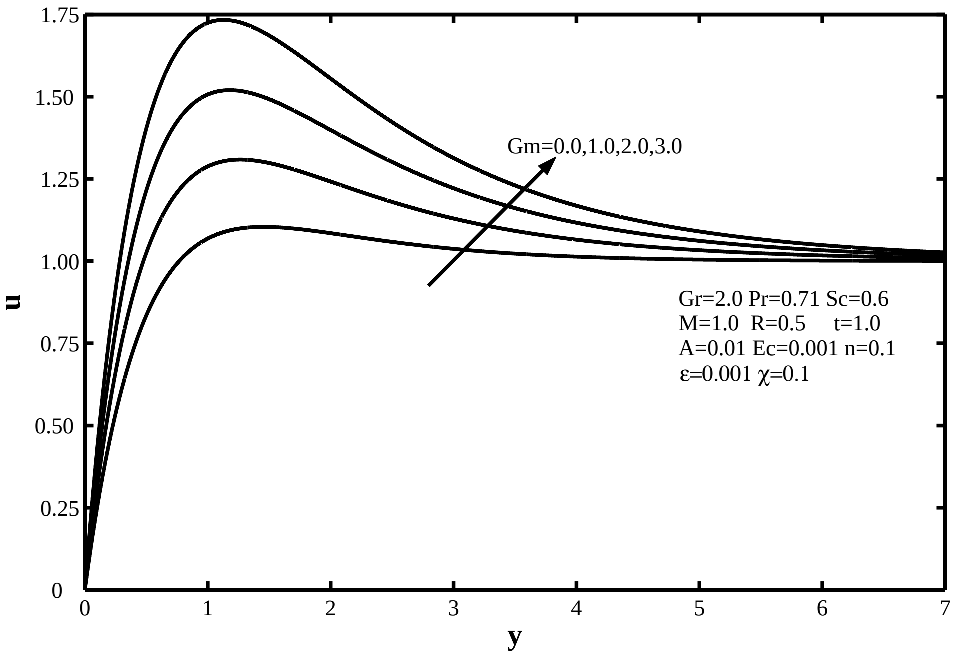


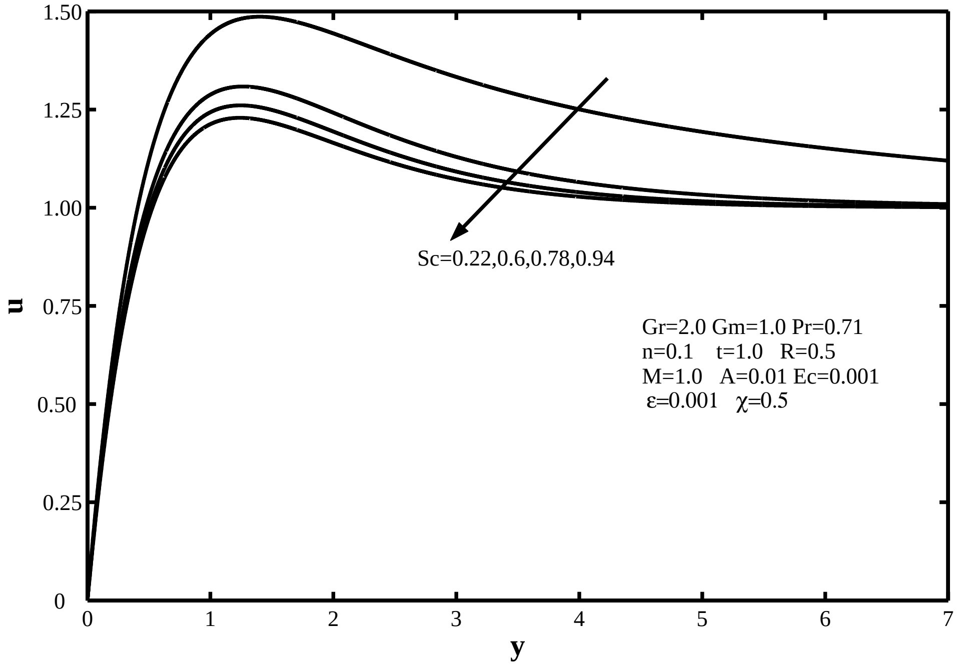
Figure 2: Effect of Gm on velocity

Gr	C_f
0	2.5278
1	3.0197
2	3.4775
3	3.8868

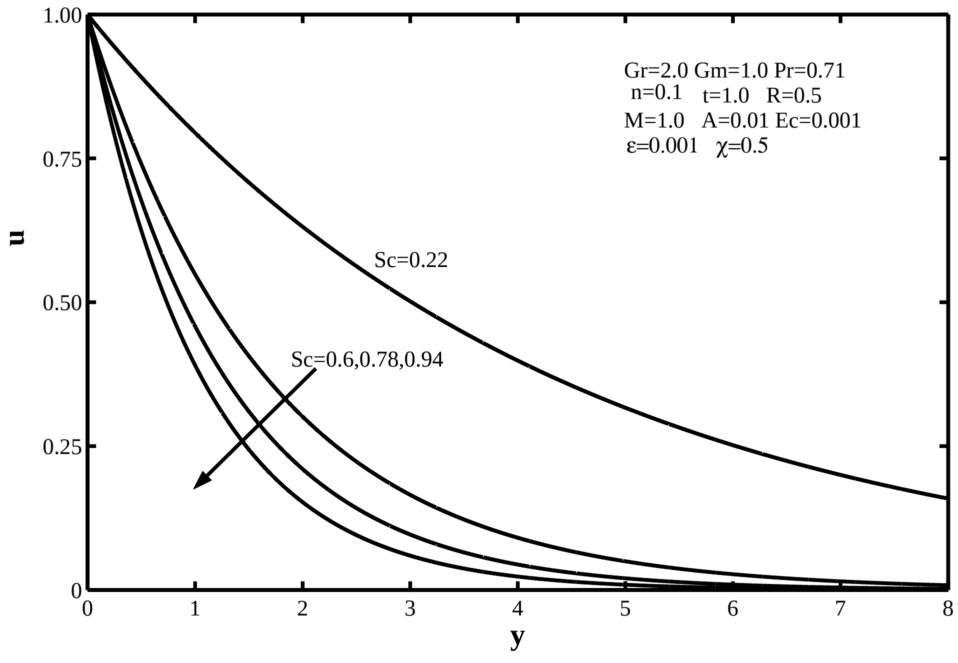
Table 1: Effects of Gr on skin-friction for reference values in Fig.1

Gm	C_f
0	2.8012
1	3.4775
2	4.1484
3	4.8138

Table 2: Effects of Gm on skin-friction for reference values in Fig.2.

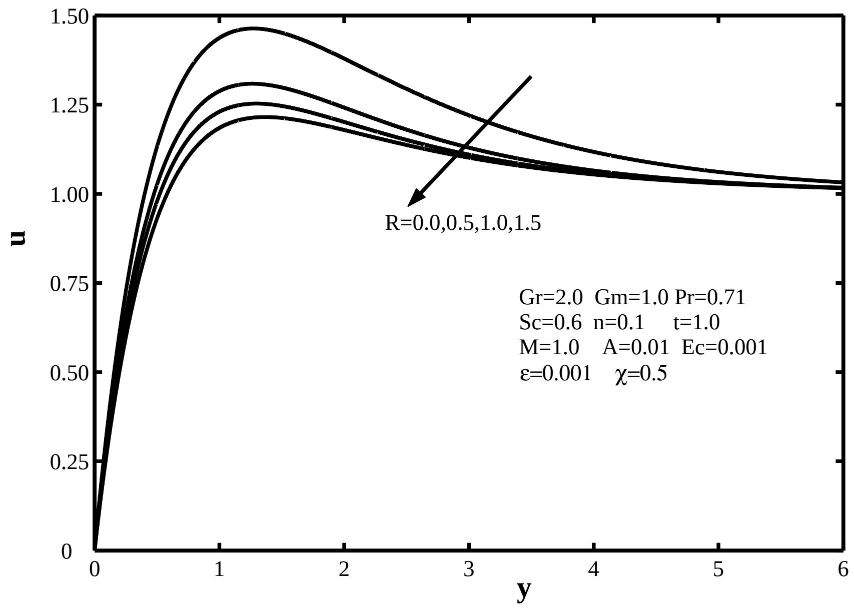


(a) Effect of Sc on velocity

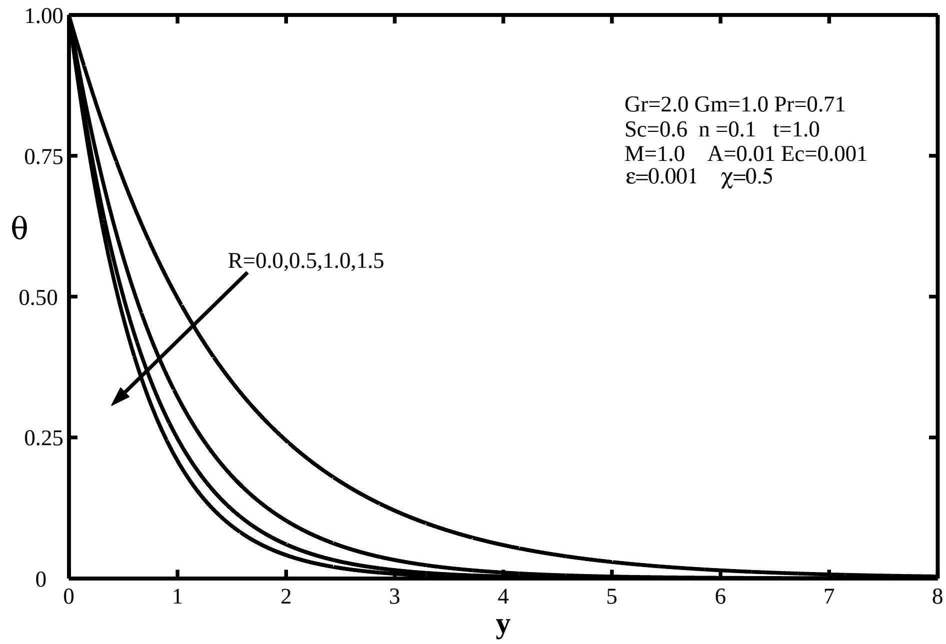


(b) Effect of Sc on concentration

Figure 3:



(a) Effect of radiation on velocity profiles



(b) Effect of radiation on temperature

Figure 4:

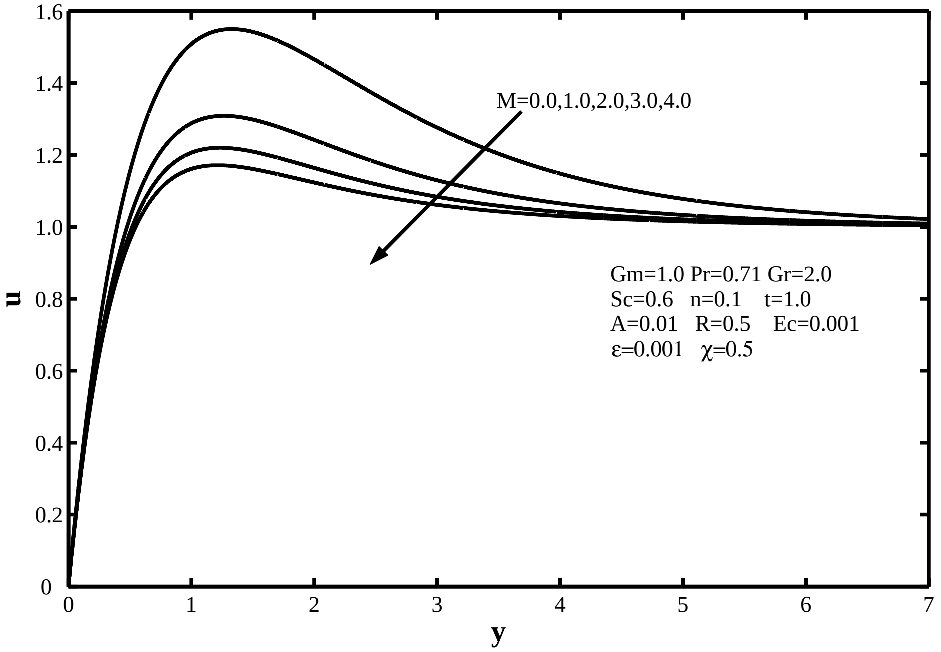


Figure 5: Effect of M on velocity

R	C_f	Nu
0.0	3.7670	0.6721
0.5	3.4775	1.1064
1.0	3.3291	1.3599
1.5	3.0956	1.4836

Table 3: Effects of radiation on skin-friction and Nusselt number for reference values in Fig.4(a) and 4(b).

Sc	C_f	Sh
0.22	3.7316	0.2201
0.60	3.4775	0.6018
0.78	3.3980	0.7804
0.94	3.3400	0.9403

Table 4: Effects of Sc on skin-friction and Sherwood number for reference values in Fig.3 (a) and 3(b).

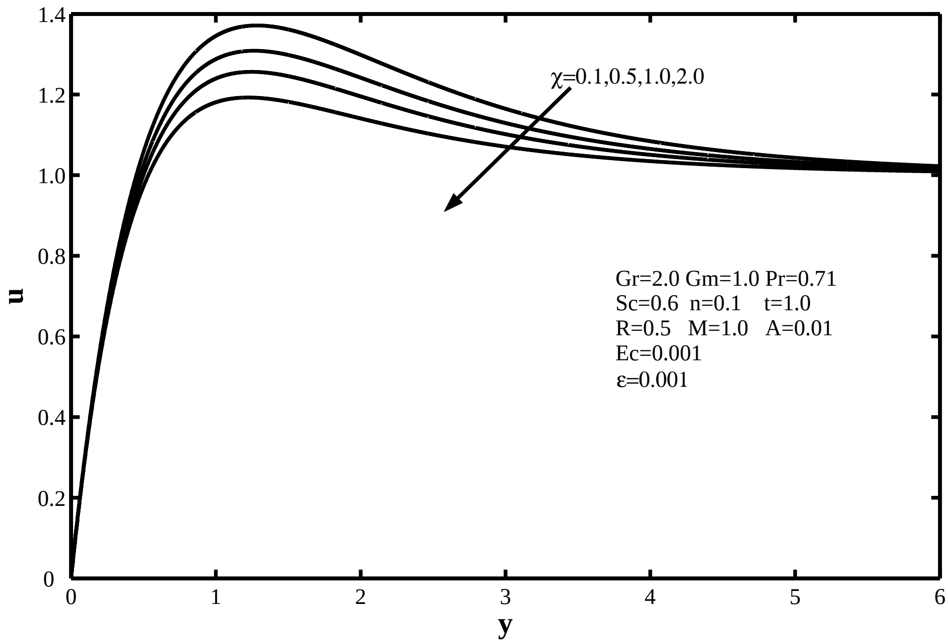
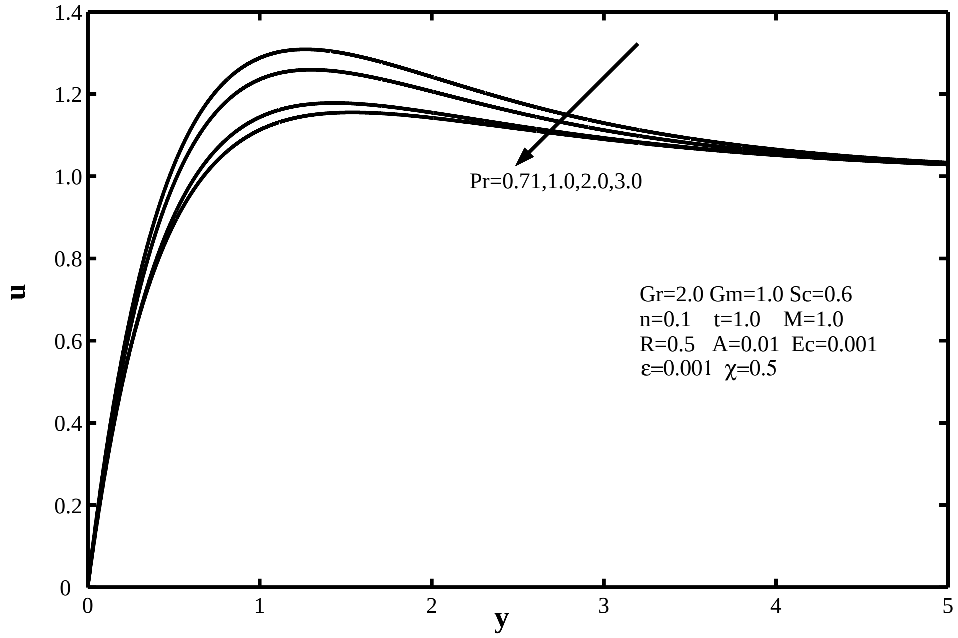


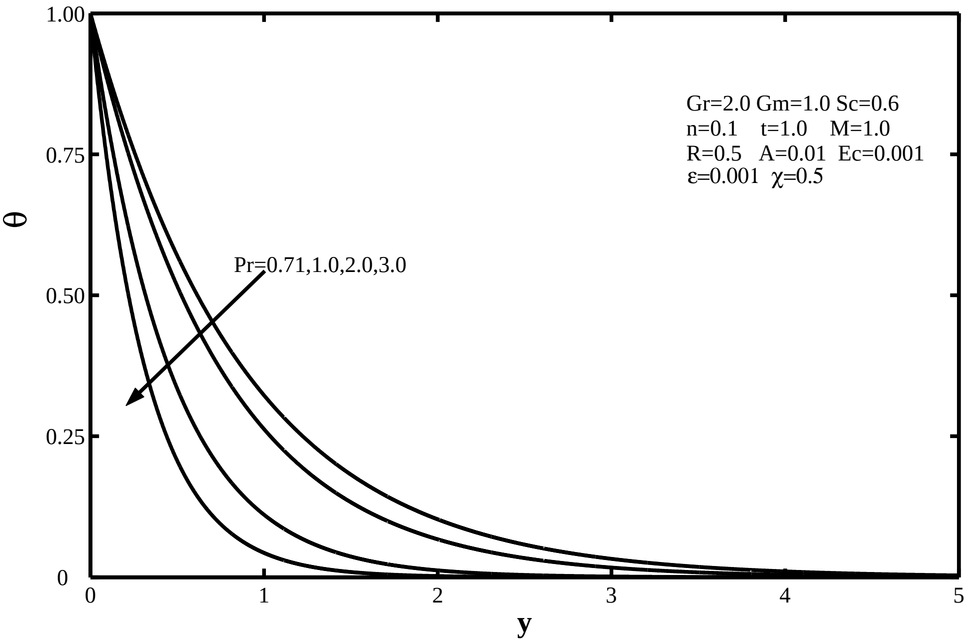
Figure 6: Effect of χ on velocity.

Ec	C_f	Nu
0	1.6302	3.3396
0.001	1.4836	3.0956
0.01	0.1644	0.8996

Table 5: Effects of Ec on skin-friction and Nusselt number for reference values in Fig.8(a) and 8(b).

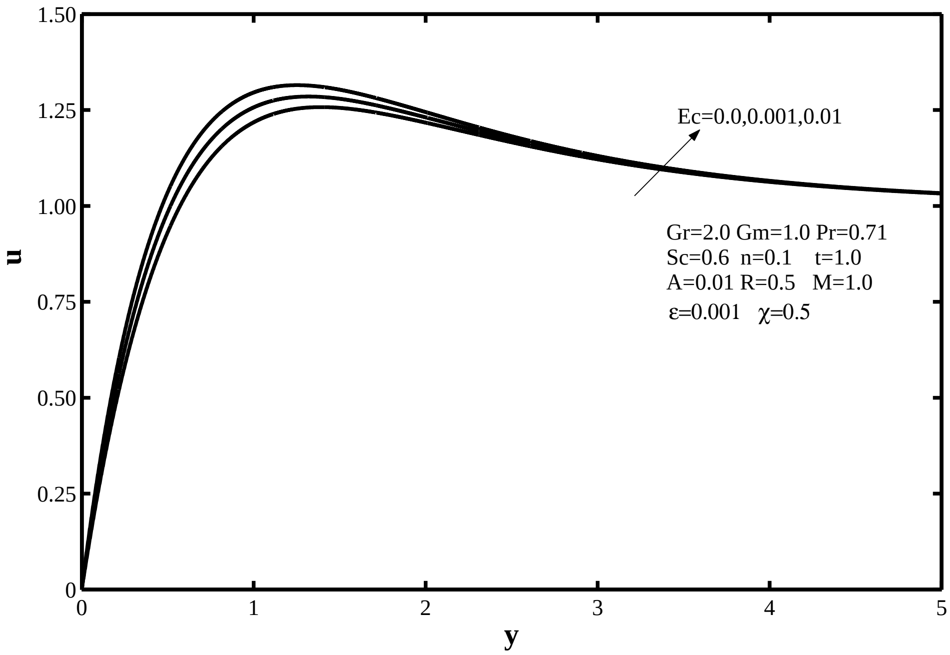


(a) Effect of Pr on velocity

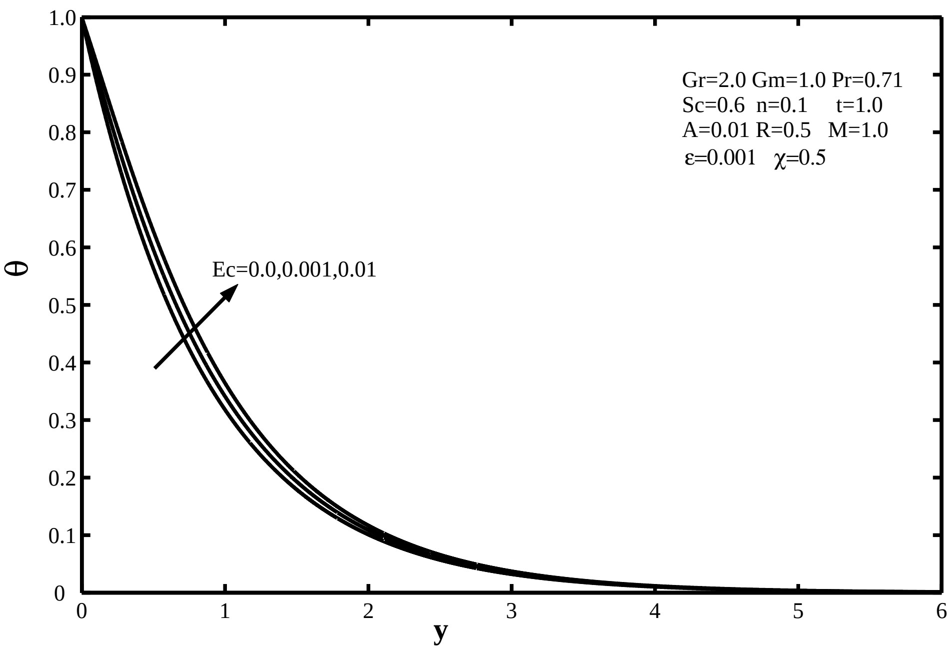


(b) Effect of Pr on temperture

Figure 7:



(a) Effect of Ec on vleocity



(b) Effect of Ec on temperature

Figure 8:

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Appendix

$$m_1 = \frac{Sc + \sqrt{Sc^2 + 4nSc}}{2}, \quad m_2 = \frac{\Pr + \sqrt{\Pr^2 + 4R^2}}{2},$$

$$m_3 = \frac{1 + \sqrt{1 + 4(\chi^2 + M^2)}}{2}, \quad m_4 = \frac{\Pr + \sqrt{\Pr^2 + 4N_2}}{2},$$

$$m_5 = \frac{1 + \sqrt{1 + 4N_1}}{2}.$$

$$P_2 = \frac{-Gm}{Sc^2 - Sc - (\chi + M)}, \quad P_3 = P_1 + P_2 - 1.$$

$$S_1 = \frac{-\Pr m_3^2 P_3^2}{4m_3^2 - 2\Pr m_3 - R}, \quad S_2 = \frac{-\Pr m_2^2 P_1^2}{4m_2^2 - 2\Pr m_2 - R},$$

$$S_3 = \frac{-\Pr Sc^2 P_2^2}{4Sc^2 - 2\Pr Sc - R}, \quad S_4 = \frac{2\Pr m_3 P_3 P_1 m_2}{(m_3 + m_2)^2 - \Pr(m_3 + m_2) - R},$$

$$S_5 = -\frac{-2\Pr m_3 P_2 P_1 Sc}{(m_2 + Sc)^2 - \Pr(m_2 + Sc) - R},$$

$$S_6 = \frac{2\Pr m_3 P_3 P_2 Sc}{(m_3+Sc)^2 - \Pr(m_3+Sc) - R},$$

$$S_7 = -(S_1+S_2+S_3+S_4+S_5+S_6).$$

$$D_1 = \frac{m_2 A \Pr}{m_2^2 - \Pr m_2 - N_2}, \quad D_5 = \frac{\frac{GmASc}{n}}{m_1^2 - m_1 - N_1},$$

$$D_6 = \frac{-Am_3P_3}{m_3^2 - m_3 - N_1}, \quad D_8 = \frac{-AP_2Sc - \frac{GmASc}{n}}{Sc^2 - Sc - N_1},$$

$$D_9 = \frac{AP_1m_2 + GrD_1}{m_2^2 - m_2 - N_1}, \quad D_9 = -(D_5+D_6+D_7+D_8+D_9).$$

$$J_1 = \frac{-GrS_7}{m_2^2 - m_2 - (\chi+M)}, \quad J_2 = \frac{GrS_1}{4m_3^2 - 2m_3 - (\chi+M)},$$

$$J_3 = \frac{GrS_2}{4m_2^2 - 2m_2 - (\chi+M)}, \quad J_4 = \frac{GrS_3}{4Sc^2 - 2Sc - (\chi+M)},$$

$$J_5 = \frac{-GrS_4}{(m_2+m_3)^2 - (m_2+m_3) - (\chi+M)},$$

$$J_6 = \frac{GrS_5}{(m_2+Sc)^2 - (m_2+Sc) - (\chi+M)},$$

$$J_7 = \frac{-GrS_6}{(m_3+Sc)^2 - (m_3+Sc) - (\chi+M)},$$

$$J_{10} = -(J_1+J_2+J_3+J_4+J_5+J_6+J_7).$$

$$G_1 = \frac{RAS_7m_2}{m_2^2 - \Pr m_2 - N_2},$$

$$G_3 = -\frac{2\Pr AS_2m_2 + 2\Pr m_2^2P_1D_7}{4m_2^2 - 2\Pr m_2 - N_2},$$

$$G_4 = -\frac{2\Pr AS_3Sc + 2\Pr Sc^2P_2D_8}{4Sc^2 - 2\Pr Sc - N_2},$$

$$G_5 = \frac{2\Pr m_3P_3D_7m_2 + 2\Pr P_1D_6m_2m_3 + \Pr AS_4(m_2+m_3)}{(m_2+m_3)^2 - \Pr(m_2+m_3) - N_2},$$

$$G_6 = -\frac{2\Pr m_2 P_2 D_7 Sc + 2\Pr P_1 D_7 m_2 Sc + \Pr A S_5 (m_2 + Sc)}{(m_2 + Sc)^2 - \Pr(m_2 + Sc) - N_2},$$

$$G_7 = \frac{2\Pr m_3 P_2 D_6 Sc + 2\Pr P_3 D_8 m_3 Sc + \Pr A S_6 (m_3 + Sc)}{(m_3 + Sc)^2 - \Pr(m_3 + Sc) - N_2},$$

$$G_8 = \frac{-2\Pr m_3 P_3 D_{10} m_5}{(m_3 + m_5)^2 - \Pr(m_3 + m_5) - N_2},$$

$$G_9 = \frac{-2\Pr m_3 P_3 D_9 m_4}{(m_3 + m_4)^2 - \Pr(m_3 + m_4) - N_2},$$

$$G_{10} = \frac{2\Pr m_2 P_1 D_{10} m_5}{(m_2 + m_5)^2 - \Pr(m_2 + m_5) - N_2},$$

$$G_{11} = \frac{2\Pr m_2 P_1 D_9 m_4}{(m_2 + m_4)^2 - \Pr(m_2 + m_4) - N_2},$$

$$G_{12} = \frac{2\Pr Sc P_2 D_9 m_4}{(m_4 + Sc)^2 - \Pr(m_4 + Sc) - N_2},$$

$$G_{14} = \frac{-2\Pr m_1 P_3 D_5 m_3}{(m_1 + m_3)^2 - \Pr(m_1 + m_3) - N_2},$$

$$G_{15} = \frac{2\Pr m_2 P_1 D_5 m_1}{(m_1 + m_2)^2 - \Pr(m_1 + m_2) - N_2},$$

$$G_{16} = \frac{2\Pr Sc P_2 D_5 m_1}{(m_1 + Sc)^2 - \Pr(m_1 + Sc) - N_2},$$

$$G_{20} = -(G_1 + G_2 + G_3 + G_4 + G_5 + G_6 + G_7 + G_8 + G_9 + G_{10} + G_{11} + G_{12} + G_{13} + G_{14} + G_{15} + G_{16}).$$

$$Z_1 = \frac{-GrG_{20}}{m_4^2 - m_4 - N_1}, \quad Z_2 = \frac{GrG_1 - AJ_1 m_2}{m_2^2 - m_2 - N_1},$$

$$Z_3 = \frac{2AJ_2 m_3 - GrG_2}{m_3^2 - m_3 - N_1}, \quad Z_4 = \frac{m_3 AJ_{10}}{m_3^2 - m_3 - N_1},$$

$$Z_5 = \frac{2AJ_3 m_2 - GrG_3}{4m_2^2 - m_2 - N_1}, \quad Z_6 = \frac{2AJ_4 Sc - GrG_4}{4Sc^2 - 2Sc - N_1},$$

$$Z_7 = -\frac{AJ_5(m_2 + m_3) + GrG_5}{(m_2 + m_3)^2 - (m_2 + m_3) - N_1},$$

$$Z_8 = \frac{AJ_6(m_2+Sc)+GrG_6}{(m_2+Sc)^2-(m_2+Sc)-N_1},$$

$$Z_9 = -\frac{AJ_7(m_3+Sc)+GrG_7}{(m_3+Sc)^2-(m_3+Sc)-N_1},$$

$$Z_{10} = \frac{GrG_8}{(m_3+m_5)^2-(m_3+m_5)-N_1},$$

$$Z_{11} = \frac{GrG_9}{(m_3+m_4)^2-(m_3+m_4)-N_1},$$

$$Z_{12} = \frac{-GrG_{10}}{(m_2+m_5)^2-(m_2+m_5)-N_1,}$$

$$Z_{13} = \frac{-GrG_{11}}{(m_2+m_4)^2-(m_2+m_4)-N_1},$$

$$Z_{14} = \frac{-GrG_{12}}{(m_4+Sc)^2-(m_4+Sc)-N_1},$$

$$Z_{15} = \frac{-GrG_{13}}{(m_5+Sc)^2-(m_5+Sc)-N_1},$$

$$Z_{16} = \frac{GrG_{14}}{(m_1+m_3)^2-(m_1+m_3)-N_1},$$

$$Z_{17} = \frac{-GrG_{15}}{(m_1+m_2)^2-(m_1+m_2)-N_1},$$

$$Z_{18} = \frac{-GrG_{16}}{(m_1+Sc)^2-(m_1+Sc)-N_1},$$

$$Z_{20} = -(Z_1+Z_2+Z_3+Z_4+Z_5+Z_6+Z_7+Z_8+Z_9$$
$$+Z_{10}+Z_{11}+Z_{12}+Z_{13}+Z_{14}+Z_{15}+Z_{16}+Z_{17})$$

Submitted on July 2007.

Uticaj radiajacije i prenosa mase na nestacionarno MHD slobodno konvekciono tečenje preko zagrejane uspravne ploče u poroznoj sredini sa viskoznom disipacijom

Posmatra se nestacionarno dvodimenziono hidromagnetsko laminarno slobodno konvekciono tečenje u graničnom sloju nestišljivog Njutnovskog elektroprovodnog i zračećeg fluida preko beskonačne zagrejane uspravne ploče. Pri analizi prenosa mase i toplote uzima se u obzir viskozna disipacija. Bezdimenzione jednačine problema su rešene analitički korišćenjem dvočlanih harmonijskih i neharmonijskih funkcija. Numeričko dobijanje rezultata je izvedeno, te su grafički prikazani rezultati za brzinu, temperaturu i profile koncentracije unutar graničnog sloja kao kao i tabulirani rezultati za koeficijent "skin-trenja", Nuseltov i Šervudov broj. Primećeno je da sa porastom parametra zračenja brzina i temperatura u graničnom sloju opadaju, dok sa porastom termičkog i mešavinskog Grashofovog broja brzina raste.

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