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Nonlinear Mirror and Weibel modes: peculiarities of quasi-linear dynamics

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Abstract. A theory for nonlinear evolution of the mirror modes near the instability threshold is developed. It is shown that during initial stage the major instability saturation is provided by the flattening of the velocity distribution function in the vicinity of small parallel ion velocities. The relaxation scenario in this case is accompanied by rapid attenuation of resonant particle interaction which is replaced by a weaker adiabatic interaction with mirror modes. The saturated plasma state can be considered as a magnetic counterpart to electrostatic BGK modes. After quasi-linear saturation a further nonlinear scenario is controlled by the mode coupling effects and nonlinear variation of the ion Larmor radius. Our analytical model is verified by relevant numerical simulations. Test particle and PIC simulations indeed show that it is a modification of distribution function at small parallel velocities that results in fading away of free energy driving the mirror mode. The similarity with resonant Weibel instability is discussed.

Keywords. Interplanetary physics (MHD waves and turbulence)

1 Introduction

Mirror mode structures have been the subject of extensive studies since the late 1950s when they were shown to be the result of a resonance instability, the so-called mirror instability (MI), displaying the interplay between the magnetic pressure, the bulk plasma pressure and the pressure of resonant ions with almost zero parallel velocity (Vedenov and Sagdeev, 1958). Subsequent observations have shown them to be ubiquitous in nature, often as solitary holes or peaks in the magnetic field (Soucek et al., 2008). Over the past half a century these observations have also raised a number of intriguing conundrums such as the occurrence of mirror dips in regions of mirror stable plasma, the lack of mirror modes between peak structures even though the plasma is mirror unstable or the violation of adiabaticity in the ion temperatures. Recently the data from the THEMIS satellites were used to resolve these dilemmas in terms of the global structure of mirror modes and the role of the trapped particles in their dynamics (Balikhin et al., 2009, 2010).

One of the first attempts of a nonlinear treatment of the mirror instability (MI) was made more than four decades ago by Shapiro and Shevchenko (1964). These authors using the random phase approximation have reduced the problem of nonlinear saturation of the MI to the study of a quasi-linear (QL) diffusion equation for the ion distribution function. Indeed, the background ion distribution function is shown to be modified which leads to saturation of MI. Shapiro and Shevchenko (1964) came to an important conclusion on the special role of ions having small parallel velocities.

Some effects related to nonlinear saturation of mirror waves in the magnetosheath have been already discussed by Pantellini (1998) and Kivelson and Southwood (1996). Recently Kuznetsov et al. (2007) suggested a new nonlinear theory of MI in bi-Maxwellian plasmas that describes the formation of magnetic holes in terms of a process known under the name of wave collapse (Kuznetsov, 1996). A further development of such scenario was offered by Califano et al. (2008). The role of trapped particles in the MI nonlinear dynamics was discussed by Pokhotelov et al. (2008), Istomin et al. (2009a) and Istomin et al. (2009b). Recently Hellinger et al. (2009) presented the model describing the matching the QL theory for the space-averaged ion distribution function with a reductive perturbative description of the mirror modes. The method was based on the numerical modelling of the diffusive equation for the ion distribution function.

The purpose of the present manuscript is to provide a further nonlinear analysis of the MI which, however, is based

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on the exact analytical solution of the diffusion equation. We will show that flattening of the ion distribution function which is inherent to the QL dynamics substantially modifies the mirror mode dispersion equation and results in two important effects. The first one is connected with the fact that the resonant interaction of the ions with the mirror modes mode rapidly vanishes and gives the place to the slower adiabatic interaction, similar to the Bernstein-Greene-Kruskal (BGK) modes (Bernstein et al., 1957). As the result the MI dispersion equation becomes of the second order equation in frequency. The second effect is associated with the additional decrease in the free energy which turns out to be much stronger than that predicted by reductive perturbation model.

The paper is organized as follows: Sect. 2 formulates the derivation of the general dispersion relation for the mirror waves. The effect of flattening of the distribution function is described in Sect. 3. The derivation of the nonlinear instability growth rate is given in Sect. 4. The similarities between mirror and Weibel instabilities are discussed in Sect. 5. Our conclusions and outlook for future research are found in Sect. 6.

2 Mirror nonlinear dispersion relation

We consider a collisionless plasma composed of ions and electrons embedded in a magnetic field $\boldsymbol{B}_0$. We shall use a local Cartesian coordinate system where the z-axis is along the external magnetic field $\boldsymbol{B}_0$, the x-axis is along the wave vector $\boldsymbol{k}$, and the y-axis completes the triad. The total magnetic field is $\boldsymbol{B}=\boldsymbol{B}_0+\delta\boldsymbol{B}$, where $\delta\boldsymbol{B}$ corresponds to perturbation. The perturbation of the magnetic field consists of only the z- and x-components satisfying the property of solenoidality, i.e. $\nabla\cdot\delta\boldsymbol{B}\equiv\partial\delta B_{\rm x}/\partial x+\boldsymbol{\partial}\delta B_{\rm z}/\partial z=0$. The $\delta B_{\rm y}$ component corresponds to the so-called non-coplanar magnetic component, and does not enter our basic equations and thus can be set to zero. Furthermore, our analysis will be limited to the case of most importance when the ion temperature is much larger than the electron temperature.

Assuming all perturbed values to vary as $\sim\exp(-i\omega t+i\boldsymbol{k}\cdot\boldsymbol{r})$ for each $(\boldsymbol{k},\omega)$ mode the linear response of the distribution function for the mirror-type perturbations is

$$\delta F_{\boldsymbol{k}}=\frac{v_\perp^2}{2}\left(\frac{\partial F}{v_\parallel\partial v_\parallel}-\frac{\partial F}{v_\perp\partial v_\perp}\right)b_{\boldsymbol{k}} -\frac{v_\perp^2}{2}\frac{\omega}{\omega-k_\parallel v_\parallel}\frac{\partial F}{v_\parallel\partial v_\parallel}b_{\boldsymbol{k}}, \qquad (1)$$

where F is unperturbed ion distribution function and $\delta F_{\boldsymbol{k}}$ corresponds to the Fourier component of the perturbation, $v_{\perp(||)}$ is the perpendicular (parallel) particle velocity, $k_\parallel$ is the component of the wave vector $\boldsymbol{k}$ parallel to the external magnetic field $\boldsymbol{B}_0$, ω is the wave frequency, $b_{\boldsymbol{k}}=\delta B_{\rm z}(\boldsymbol{k})/B_0$ is the dimensionless amplitude of the $\boldsymbol{k}$-th harmonic of the perturbation.

The first term on the right-hand side of Eq. (1) is the so-called mirror force (due to that the instability is called mirror instability (MI)), it vanishes if plasma is isotropic. The second term refers to the kinetic contribution. The MI is found in the low-frequency limit when $\omega\ll k_\parallel v_\parallel$. In this limit the second term in Eq. (1) is negligible except for particles with $v_{||}=0$. For these particles this term is of the same order and potentially of larger magnitude than the preceding mirror term. The expansion of the resonant denominator in this case reads

$$\frac{\omega}{\omega-k_\parallel v_\parallel}\simeq-\frac{i\pi\omega}{|k_\parallel|}\delta(v_\parallel)+\frac{\omega^2}{k_\parallel^2v_\parallel^2}, \qquad (2)$$

where $\delta(x)$ is the Dirac delta function. Usually the second term in Eq. (2) is neglected as the small parameter of the order of $\omega/k_\parallel v_{T_{||}}\propto K$, where K is the instability threshold. However, in the nonlinear regime the character of the expansion is changed, instead of small parameter $\omega/k_\parallel v_{T_{||}}$ we will have $\omega/k_\parallel\Delta v^*$, where Δv^* is the width of the zone occupied by the flattening of the ion velocity distribution function and thus the second term in Eq. (2) can be important or even dominate.

With the help of expansion (2) the perturbation of the distribution function reads

$$\delta F_{\boldsymbol{k}}=\frac{v_\perp^2}{2}\left(\frac{\partial F}{v_\parallel\partial v_\parallel}-\frac{\partial F}{v_\perp\partial v_\perp}\right)b_{\boldsymbol{k}}-\frac{v_\perp^2}{2}\frac{\omega^2}{k_\parallel^2}\frac{\partial F}{v_\parallel^3\partial v_\parallel}b_{\boldsymbol{k}} +i\pi\frac{v_\perp^2}{2}\frac{\omega}{|k_\parallel|}\delta(v_\parallel)\frac{\partial F}{v_\parallel\partial v_\parallel}b_{\boldsymbol{k}}. \qquad (3)$$

The magnetic mirror mode is the pressure balanced structure and thus obeys the perpendicular plasma pressure condition (Pokhotelov et al., 2008)

$$\frac{\delta p_\perp}{2p_{\perp0}}+\frac{1}{\beta_\perp}\left(1+\frac{3}{4}\rho_i^2k_\perp^2\right)b_{\boldsymbol{k}}=-\frac{k_\parallel^2}{k_\perp^2\beta_\perp}\left(1+\frac{\beta_\perp-\beta_\parallel}{2}\right)b_{\boldsymbol{k}}, \qquad (4)$$

where $p_{\perp0}$ is the equilibrium perpendicular plasma pressure and $\delta p_\perp=\int(mv_\perp^2/2)\delta F_{\boldsymbol{k}}d\boldsymbol{v}$ corresponds to its variation. Using (3) and calculating $\delta p_\perp$, from Eq. (4) one obtains

$$\left(\Delta+\frac{i\pi m}{8p_{\perp0}}\frac{\omega}{|k_\parallel|}\int v_\perp^4\frac{\partial F}{v_\parallel\partial v_\parallel}\delta(v_\parallel)d\boldsymbol{v}-\frac{\omega^2}{k_\parallel^2}I_1^{\rm M}\right)b_{\boldsymbol{k}}=0, \qquad (5)$$

where

$$\Delta=\frac{m}{8p_{\perp0}}\int v_\perp^4\left(\frac{\partial F}{v_\perp\partial v_\perp}-\frac{\partial F}{v_\parallel\partial v_\parallel}\right)d\boldsymbol{v} -\frac{1}{\beta_\perp}\left(1+\frac{3}{2}\rho_i^2k_\perp^2\right)-\frac{k_\parallel^2}{k_\perp^2\beta_\perp}\chi, \qquad (6)$$

$$I_1^{\rm M}=\frac{m}{8p_{\perp0}}\int v_\perp^4\frac{\partial F}{v_\parallel^3\partial v_\parallel}d\boldsymbol{v}, \qquad (7)$$

and $\chi=1+(\beta_\perp-\beta_{||})/2$ and superscript "M" stands for the mirror instability.

Equation (5) shows that generally the MI dispersion relation is the second order equation in frequency. Of course in the linear limit the additional term, containing ω^2, is small as $\omega^2/k_{||}^2 v_{T_{||}}^2 \ll 1$ (where $v_{T_{||}}$ is the plasma characteristic parallel thermal velocity) relative to the resonant term. However, as it will be shown below in the nonlinear regime the expansion parameter $\omega/k_{||}v_{T_{||}}$ is now replaced by $\omega/k_{||}\Delta u_*$, where Δu_* is the width of the "plateau" in the ion velocity distribution, which is much smaller than the ion thermal velocity. Due to that the quadratic term can be important or even dominate.

By integrating the first term on the r.h.s. of Eq. (6) by parts one finds

$$\Delta = K - \frac{3}{4\beta_\perp}\rho_i^2 k_\perp^2 - \frac{k_\parallel^2}{k_\perp^2 \beta_\perp}\left(1+\frac{\beta_\perp - \beta_\parallel}{2}\right), \tag{8}$$

where

$$K = I_2^{\mathrm{M}} - 1 - \frac{1}{\beta_\perp}, \tag{9}$$

and

$$I_2^{\mathrm{M}} = -\frac{m}{8p_{\perp 0}}\int v_\perp^4 \frac{\partial F}{v_\parallel \partial v_\parallel} d\boldsymbol{v}. \tag{10}$$

Actually the quantity K is proportional to the amount of free energy necessary for the instability. In bi-Maxwellian plasma it reduces to the usual value of the threshold condition, $K = K_M \equiv T_\perp/T_{||} - 1 - \beta_\perp^{-1}$, where $T_{\perp(||)}$ is the perpendicular (parallel) plasma temperature.

3 QL modification of the ion distribution function

For the sake of clarity we consider that due to the rapid motion of resonant particles we assume that in the vicinity of small parallel velocities the background ion distribution function would flatten and takes the shape of quasi-plateau. This to happen does not necessarily requires the assumption of random phases and is valid even in the single-mode (sinusoidal) regime. In order to take into account the effect of flattening of the distribution function we assume that the coefficients in Eq. (5) are not frozen into their initial values and are evaluated from the instantaneous distribution function given by the QL diffusion equation. In QL regime the amplitude of oscillations remains so small that perturbations of particle velocities and particle densities are linear relative to the wave amplitude. Only the averaged distribution function slowly varies under chaotic wave perturbations. The equation that governs this slow variation has been obtained by Shapiro and Shevchenko (1964) and is

$$\frac{\partial F}{\partial t} = \frac{1}{2}\sum_{\boldsymbol{k}} \gamma_{\boldsymbol{k}} |b_{\boldsymbol{k}}|^2 \left\{ \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left[v_\perp \left(\frac{v_\perp^2}{2}\frac{\partial F}{\partial v_\perp} - \frac{v_\perp^3}{v_\parallel}\frac{\partial F}{\partial v_\parallel}\right)\right] + \frac{\partial}{\partial v_\parallel}\left(\frac{v_\perp^4}{v_\parallel^2}\frac{\partial F}{\partial v_\parallel}\right)\right\}. \tag{11}$$

The QL approximation is valid if $|k_\parallel|\Delta v_\parallel \gg \gamma_{\boldsymbol{k}}$, where $\Delta v_\parallel$ is the the region occupied by diffusion. In our case $\Delta v_\parallel \simeq v_{T_\parallel}$. Since $\gamma_{\boldsymbol{k}}/|k_\parallel| v_{T_\parallel} \propto K_M$, the validity of QL approximation is satisfied when deviation of plasma parameters from the equilibria is relatively small.

Equation (11) shows that the last term on the right-hand side possesses a strong singularity in the vicinity of $v_\parallel \to 0$ and thus the most noticeable change in the shape of the distribution function arises in this region. Therefore, Eq. (11) reduces to

$$\frac{\partial F}{\partial t} = \sum_{\boldsymbol{k}} \gamma_{\boldsymbol{k}} |b_{\boldsymbol{k}}|^2 \frac{v_\perp^4}{4}\frac{\partial}{\partial v_\parallel}\left(\frac{\partial F}{v_\parallel^2 \partial v_\parallel}\right). \tag{12}$$

With the help of the relation

$$\frac{\partial |b_{\boldsymbol{k}}|^2}{\partial t} = 2\gamma_{\boldsymbol{k}} |b_{\boldsymbol{k}}|^2, \tag{13}$$

Equation (12) becomes

$$\frac{\partial F}{\partial h} = \frac{v_\perp^4}{4}\frac{\partial}{\partial v_\parallel}\left(\frac{\partial F}{v_\parallel^2 \partial v_\parallel}\right), \tag{14}$$

where

$$h = \sum_{\boldsymbol{k}} |b_{\boldsymbol{k}}|^2. \tag{15}$$

Equation (14) can easily be solved by decomposing the variables and searching the solution in the form of Fourier-Bessel integral. We assume that when $h = 0$ the distribution function $F(h, v_\parallel, v_\perp)$ reduces to bi-Maxwellian form, i.e.

$$F_0(v_\parallel, v_\perp) = \frac{n}{\pi^{\frac{3}{2}} v_{T_\perp}^2 v_{T_\parallel}} \exp\left(-v_\parallel^2/v_{T_\parallel}^2 - v_\perp^2/v_{T_\perp}^2\right), \tag{16}$$

where n is the plasma density and $v_{T_{\perp(\parallel)}}$ is perpendicular (parallel) thermal velocity.

The result can be written as

$$F(h, v_{||}, v_\perp) = C(v_\perp)|v_{||}|^{3/2} \times \int_0^\infty \frac{e^{-t^2 x^2 (\frac{v_\perp}{v_{T_\perp}})^4 (\frac{T_\perp}{T_\parallel}) h}\, t^{\frac{1}{4}} J_{-\frac{3}{4}}(t v_{||}^2/v_{T_\parallel}^2)}{(t^2+1)^{\frac{3}{4}}} dt, \tag{17}$$

where $C(v_\perp) = \Gamma(3/4) n \exp(-v_\perp^2/v_{T_\perp}^2)/\pi^2 v_{T_\perp}^2 v_{T_\parallel}^{5/2}$, $\Gamma(x)$ and $J_\nu(x)$ are the Gamma and Bessel functions, respectively.

The plot of the distribution function F as a function of $v_{||}$ for different values of h and constant $v_\perp$ is depicted in Fig. 1. One sees that velocity diffusion leads to substantial flattening of the distribution function for small $v_{||}$. Instead of initial dependence $\propto \exp(-v_\parallel^2/v_{T_\parallel}^2)$ the distribution function now scales as $\propto \exp(-v_\parallel^4/v_{T_\parallel}^4)$ and thus the second term on the left (the term containing the δ-function) vanishes and the next term in the expansion (2) starts to play an important role.

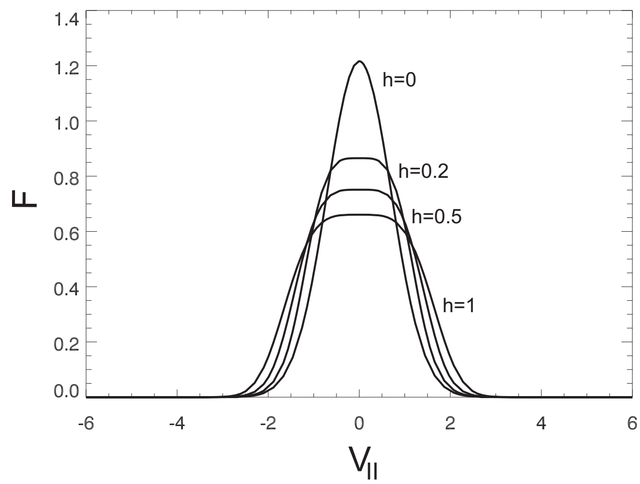

Fig. 1. Velocity distribution function F as the function of the wave amplitude h.

4 Nonlinear regime

As it follows from Eq. (5) the first nonzero term in the MI dispersion relation will be that proportional to ω^2 and the mirror dispersion relation becomes of the second order in frequency. Actually the flattening of the velocity distribution mainly affects two terms in the dispersion relation containing the integrals I_1^{M} and I_2^{M}. Using the explicit expression for the distribution function (18) one can easily calculate the value of I_1^{M}

$$I_1^{\mathrm{M}} \simeq -\frac{\alpha}{v_{T_\perp}^2}\left(\frac{T_\perp}{T_{||}}\right)^{3/2} h^{-1/4}, \tag{18}$$

where $\alpha = \frac{3}{16}\Gamma(\frac{1}{4})\Gamma(\frac{1}{2}) \simeq 1.2$.

For calculation of I_2^{M} it is more convenient to find the deviation from its equilibrium value, i.e. to find $\delta I_2^{\mathrm{M}} = I_2^{\mathrm{M}} - I_{20}^{\mathrm{M}}$, where

$$I_{20}^{\mathrm{M}} = -\frac{m}{8p_{\perp 0}}\int v_\perp^4 \frac{\partial F_0}{v_\parallel \partial v_\parallel} d\boldsymbol{v} \equiv \frac{T_\perp}{T_{||}}. \tag{19}$$

Then one finds

$$I_2^{\mathrm{M}} = \frac{T_\perp}{T_{||}} - \mu\left(\frac{T_\perp}{T_{||}}\right)^{3/2} h^{1/4}, \tag{20}$$

where $\mu = -\frac{3}{32}\Gamma(-3/4)\Gamma(\frac{1}{4}) \simeq 1.6$.

The plot of I_2 as a function of h is depicted in Fig. 2, the left panel. Since I_2^{M} enters K it changes the free energy of the system. The free energy necessary for the instability decreases with the growth of the wave energy. The latter becomes clear from the Fig. 2, the right panel, where K is plotted as a function of h.

Substituting Eqs. (18) and (20) into Eq. (5) one obtains expression for the nonlinear growth rate

$$\gamma = \frac{|k_{||}|v_{T_{||}}}{\alpha^{\frac{1}{2}}}\left(\frac{T_{||}}{T_\perp}\right)^{\frac{1}{4}} h^{\frac{1}{8}} \tag{21}$$

$$\times\left(K_M - \frac{3}{4\beta_\perp}\rho_i^2 k_\perp^2 - \mu\left(\frac{T_\perp}{T_{||}}\right)^{3/2} h^{1/4} - \frac{k_{||}^2}{k_\perp^2\beta_\perp}\chi\right)^{\frac{1}{2}}.$$

Let us estimate this growth rate of the most growing mode. We consider that in the course of QL evolution the wave amplitude is so small that we do not deviate substantially from the initial most growing mode. The latter corresponds to the values of the parallel and perpendicular wave numbers given by the linear theory (cf. Pokhotelov et al., 2008)

$$(k_{||}\rho_i)^2_{\max} = \beta_\perp^2 \frac{K_M^2}{12\chi}, \tag{22}$$

and

$$(k_\perp \rho_i)^2_{\max} = \beta_\perp \frac{K_M}{3}. \tag{23}$$

Thus, the nonlinear growth rate for this mode is

$$\gamma_{\max} = \frac{\omega_{ci}}{2^{\frac{3}{2}}3^{\frac{1}{2}}\alpha^{\frac{1}{2}}\chi^{\frac{1}{2}}}\left(\frac{T_{||}}{T_\perp}\right)^{3/4} h^{\frac{1}{8}} K_M$$
$$\times\left(K_M - 2\mu\left(\frac{T_\perp}{T_{||}}\right)^{3/2} h^{1/4}\right)^{1/2}. \tag{24}$$

From Eq. (24) follows that QL saturation of the most growing mode is attained at relatively small amplitude when $K_M \simeq 2\mu\left(T_\perp/T_{||}\right)^{3/2} h^{1/4}$ or .

$$\delta B_z/B_0 \approx K_M^2 (T_{||}/T_\perp)^3/4. \tag{25}$$

At this point a few comments are in order. A further nonlinear evolution of the MI will be controlled by the effects that were excluded during our analysis. Among them are the mode coupling (incurred stresses) and nonlinear variations of the FLR effect. The departure is observed when the QL evolution tends to saturate the magnetic field fluctuations. If the mode coupling is taken into consideration the magnetic energy continues to grow and b displays a sharp increase leading to a finite-time blow-up, in accordance with the model of subcritical bifurcation (Kuznetsov et al., 2007). The termination of the singularity requires incorporation of the nonlinear variations of the FLR effect. Such a scenario is supported by Vlasov-Maxwell numerical simulations (e.g., Hellinger et al., 2009).

5 Mirror and Weibel instabilities

The mathematical description of MI has much in common with another instability theoretically identified by Fried (1959) and Weibel (1959) and usually termed Weibel instability. The Weibel instability results in the breakdown of the plasma into current filaments. Recently, this instability has attracted considerable attention for both astrophysical and laboratory plasmas. For example, it is considered that this instability can be driven in strong collisionless shock waves

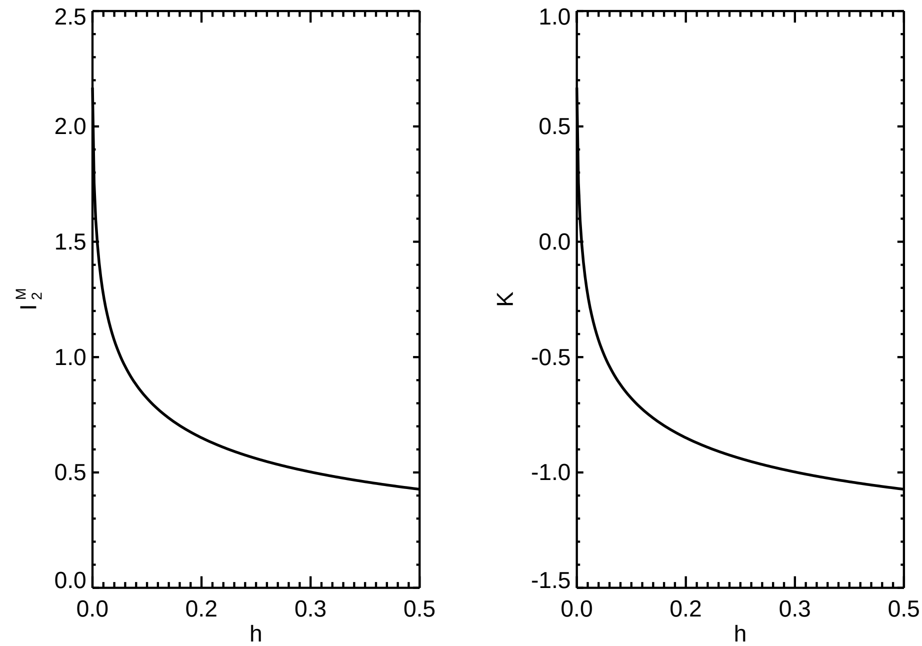

Fig. 2. The plot of I_2^{M} (left) and K (right) as the function of the wave amplitude h. The plasma parameters are: $T_\perp/T_{||}=2$ and $\beta_\perp=2$.

associated with various astrophysical phenomena, e.g., pulsar winds (Kazimura et al., 1998), gamma-ray bursts, and/or their afterglows (Medvedev and Loeb, 1999) or gravitational collapse of large-scale structures in the universe (Schlickeiser and Shukla, 2003). The Weibel instability generates magnetic fields on a very small spatial scale of the order of plasma skin depth. In contrast to MI the Weibel instability is driven by the electrons with anisotropic velocity distribution. The linear response of the electron distribution function for the Weibel-type perturbations is

$$\delta F=-\frac{ie}{m}\frac{\delta B_x}{k}v_y\left(\frac{\partial F}{v_\perp\partial v_\perp}-\frac{\partial F}{v_{||}\partial v_{||}}+\frac{\omega}{\omega-kv_{||}}\frac{\partial F}{v_{||}\partial v_{||}}\right), \quad (26)$$

where F corresponds to the unperturbed electron velocity distribution function, $v_y=v_\perp\sin\varphi$ and φ is angle between $\boldsymbol{v}$ and the wave vector which is directed along the z-axis.

Comparing Eqs. (1) and (26) one sees that they are very similar. The two first terms on the r.h.s. of Eq. (26) in the round brackets plays the role of the mirror force and the last terms describes the contribution of resonant particles. Due to that one can expect that linear growth rate and also the nonlinear dynamics of both instabilities might have similar features.

Calculating the y-component of the electric current and substituting it into the Ampére's law one obtains the linear dispersion relation for the Weibel instability

$$I_2^W-1-\frac{k^2c^2}{\omega_p^2}-\frac{i\pi\omega}{|k|n}\int\frac{v_\perp^2}{2}\frac{\partial F}{v_{||}\partial v_{||}}\delta(v_{||})d\boldsymbol{v}+\frac{\omega^2}{k_{||}^2}I_1^W=0, \quad (27)$$

where c is the velocity of light, ω_p is the Langmuir frequency, n is the plasma number density and superscript W stands for the Weibel instability. The quantities I_1^W and I_2^W, entering dispersion relation (27), are given by (cf. Eqs. 7 and 9)

$$I_1^W=n^{-1}\int\frac{v_\perp^2}{2}\frac{\partial F}{v_{||}^3\partial v_{||}}d\boldsymbol{v}, \quad (28)$$

and

$$I_2^W=-n^{-1}\int\frac{v_\perp^2}{2}\frac{\partial F}{v_{||}\partial v_{||}}d\boldsymbol{v}. \quad (29)$$

In the case of bi-Maxwellian velocity distribution Eq. (27) recovers the expression for the linear growth rate of the Weibel instability

$$\gamma=\frac{|k|v_{T_{||}}}{\pi^{1/2}}\frac{T_{||}}{T_\perp}\left(\frac{T_\perp}{T_{||}}-1-\frac{c^2k^2}{\omega_p^2}\right). \quad (30)$$

Equation (30) shows that it is identical to the MI growth rate if one replaces the ion quantities by the electron quantities and the role of ion Larmor radius here is now played by the plasma skin depth. Furthermore, the QL equations that govern nonlinear evolution of the electron distribution function have a similar to MI form (14). It can be easily obtained from the Vlasov equation and is

$$\frac{\partial F}{\partial h_e}=\frac{v_\perp^2}{2}\frac{\partial}{\partial v_z}\left(\frac{1}{v_z^2}\frac{\partial F}{\partial v_z}\right), \quad (31)$$

where we took into account that the dominant contribution to the r.h.s. is provided by the electrons having small parallel velocities. Furthermore, the quantity h is replaced by its electron analog

$$h_e = \frac{e^2}{m_\mathrm{e}^2} \sum_{\boldsymbol{k}} \frac{|B_{\boldsymbol{k}}|^2}{k^2}. \tag{32}$$

where m_e is the electron mass.

Thus, all conclusions that have been made for the MI can also be applicable to the Weibel instability. After some straightforward calculations one finds the expression for the nonlinear growth rate of the Weibel instability. Similar to MI, using the solution of QL equation (14) one can calculate the quantities I_1^W and I_2^W. Then, after substitution them into dispersion equation (27) one can easily obtain the nonlinear growth rate of the Weibel instability which has the form similar to Eq. (24).

6 Discussion and conclusions

A local analysis of the MI in a high-β non-Maxwellian plasma taking into account the effect of flattening of the ion velocity distribution function near the instability threshold is presented. QL evolution of the MI was investigated by direct integration of the corresponding diffusion equation. It has been shown that due to the fattening of the ion distribution function the resonant interaction of the ions with $v_{||} \simeq 0$ is rapidly "switched off" and then is replaced by a weaker adiabatic interaction with mirror mode. At this stage the mirror mode behaves similar to the BGK mode (Bernstein et al., 1957). This fact was not appreciated in the previous analyses. The MI dispersion relation which in the linear regime is a differential equation of the first order in time derivative now becomes of the second order. Furthermore, it has been shown that the main decrease in the free energy, necessary for the instability, is due to the modification of the ion velocity distribution which is very subtle near the instability threshold. It was shown that during linear and QL stage the MI evolution mathematically is similar to another instability, the Weibel instability (Weibel, 1959), which can be described by similar differential equations. The differences arise when one incorporates the higher order nonlinearities. For the MI they are quadratic in the wave amplitude whereas for the Weibel instability they are cubic. This results in different saturated states. In the first case they appear as solitary waves whereas in the case of the Weibel instability they form the filamentary structures (Palodhi et al., 2009).

The model developed in our paper still remains oversimplified. For example, in the case of the MI it has been restricted by consideration of isotropic electrons and when $b<1$. The case when $b \simeq 1$ was described by Jovanović and Shukla (2009). Furthermore, the effect of bistability of mirror modes revealed in recent observations and discussed by Califano et al. (2008) and Jovanović and Shukla (2009) was also outside the scope of this study. However, our analysis has provided a deeper insight into the physics of nonlinear dynamics of mirror modes in high-β space plasmas.

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