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## CONTROL OF BIO-REACTOR PROCESSES USING A NEW CDM PI-P CONTROL STRATEGY

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### Abstract

For economical point of view, it is necessary to operate a bio-reactor system at unstable steady state condition. The design of controller for such system is challengeable one. If any delay includes in the bio-reactor system then, the control methodology is found to be more difficult. In the present work, a new, Coefficient Diagram Method (CDM) based PI-P control strategy is proposed to operate the bio-reactor effectively at unstable steady state condition. The proposed control strategy, designated as CDM PI-P, is tested with the bio-reactor system which is approximated as Unstable First Order Plus Time Delay (UFOPTD) transfer function. Simulation results clearly indicate that the proposed control strategy gives an enhanced control performance in operating the bio-reactor at unstable condition. The performance of the control strategy is analyzed in terms of Integral Square Error (ISE), Integral Absolute Error (IAE) and Total Variation (TV). A comparison of the proposed strategy with other control strategy is made.

Keywords: CDM, PI-P, Unstable bio-reactor process, ISE, Total variation

### 1. Introduction

Bio-chemical reactors are used in wide variety of processes, from waste treatment to alcohol fermentation. The dynamics of a bio-reactor is highly non linear and for certain parameter values, the system exhibits output multiplicity. Some recent publications addressing the control of unstable processes from different points of view can be found in [1, 2]. However, all the suggested methods show a poor closed loop response. Hence there is a need to design a controller that gives the transient response with less overshoot and fast settling time. In the present work, a CDM PI-P control strategy is proposed for UFOPTD bio process. Explicit tuning rules for designing the CDM PI-P controller parameters are derived using Coefficient Diagram Method as a base. Closed loop simulation results with this

**Nomenclatures**

$A(s)$	Forward denominator polynomials
$B(s)$	Feedback numerator polynomials
$C(s)$	Main controller
$C_f(s)$	Pre-filter
$D(s)$	Denominator polynomials of the transfer function
$d$	External disturbance signal
$F(s)$	Reference numerator polynomials of the controller
$K_c$	Proportional gain
$K_f$	Feedback proportional gain
$K_i, l_i$	Controller parameters
$K_0, K_1$	CDM controller parameters
$K_i$ and $l_i$	Controller parameters
$N(s)$	Numerator polynomials of the transfer function
$P(s)$	Characteristic polynomial of the closed-loop system
$r$	Reference input
$t_i$	Integral time constants
$t_s$	Settling time, s
$u$	Controller signal
$y$	Output
$\%M_p$	Percentage overshoot

*Greek Symbols*

$\lambda$	Tuning factor
$\gamma_i$	Stability indices
$\tau$	Equivalent time constant, s

**Abbreviations**

CDM	Coefficient diagram method
DOF	Degree of Freedom
IAE	Integral absolute error
ISE	Integral square error
PI-P	Proportional integral – Proportional
TV	Total variation
UFOPTD	Unstable first order plus time delay

proposed control strategy are compared with the controller designed by [3].

The paper is organized as follows: Section 2 gives the basics of CDM and the CDM controller design steps. Proposed new CDM PI-P control strategy is dealt in Section 3. In Section 4, simulation result is presented to illustrate the effectiveness of the proposed control strategy. Concluding remarks are given in Section 5.

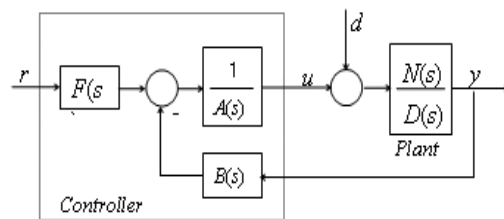
**2. Basics of CDM**

The polynomial algebraic method namely CDM was developed and introduced by Manabe [4] in 1991. The merits of the classical and modern control techniques are

integrated with the basic principles of CDM. The important features of this CDM are: the adaptation of polynomial representation for both the plant and the controller, the use of two-degree of freedom control structure, the non-existence (or) very small overshoot in the closed loop response, determination of settling time at the beginning and to continue the design accordingly. CDM is an efficient and fertile control tool with which very good control systems can be designed. It is easy to realize a controller under the conditions of stability, time domain performance and robustness. The close relations between these conditions and coefficients of the characteristic polynomial can be easily found. It means that CDM is not only effective for control system design but also for controller tuning [5].

## 2.1. CDM controller design

The standard block diagram of the CDM control system is shown in Fig. 1, where  $y$  is the output,  $r$  is the reference input,  $u$  is the controller signal and  $d$  is the external disturbance signal.  $N(s)$  and  $D(s)$  are numerator and denominator polynomials of the transfer function of the plant.  $A(s)$  is the forward denominator polynomial while  $F(s)$  and  $B(s)$  are the reference numerator and the feedback numerator polynomials of the controller transfer function respectively. Since the transfer function of the controller has two numerators, it resembles to a 2DOF (Two Degree of Freedom) system structure.  $A(s)$  and  $B(s)$  are designed as to satisfy the desired transient behavior, while pre-filter  $F(s)$  is determined as zero order polynomial and used to provide the steady-state gain.



**Fig. 1. Block Diagram of CDM Control System.**

The output of the CDM control system from Fig. 1, is given by

$$y = \frac{N(s)F(s)}{P(s)} r + \frac{A(s)N(s)}{P(s)} d \quad (1)$$

where  $P(s)$  is the characteristic polynomial of the closed-loop system. This polynomial is a Hurwitz polynomial with real positive coefficients and defined by

$$P(s) = A(s)D(s) + B(s)N(s) = \sum_{i=0}^n a_i s^i, \quad a_i > 0 \quad (2)$$

The polynomials,  $A(s)$  and  $B(s)$  appearing in the CDM control structure are given as

$$A(s) = \sum_{i=0}^p l_i s^i \quad \text{and} \quad B(s) = \sum_{i=0}^q k_i s^i \quad (3)$$

where the condition  $p \geq q$  must be satisfied for practical realization.

The CDM design parameters, namely equivalent time constant,  $\tau$ , and stability indices,  $\gamma_i$ , are chosen as follows:

$$\tau = t_s / (2.5 \approx 3), \quad (4a)$$

where  $t_s$  is the user specified settling time

$$\gamma_i = [2.5 \ 2 \ 2] \quad (4b)$$

The above  $\gamma_i$  values are from the standard Manabe form [4] and these values can be changed in order to satisfy the desired performance.

The controller polynomials defined in Eq. (3) are replaced in

$$P(s) = A(s)D(s) + B(s)N(s) = \sum_{i=0}^n a_i s^i, \quad a_i > 0 \quad (5)$$

Hence the coefficients of this characteristic polynomial  $P(s)$  are expressed in terms of  $K_i$  and  $l_i$ , i.e.,  $P(s)$  is expressed in terms of the coefficients of the controller polynomials.

Using the design parameters  $\tau$  and  $\gamma_i$ , a target characteristic polynomial,  $P_{\text{target}}(s)$ , is determined as

$$P_{\text{target}}(s) = a_0 \left[ \sum_{i=2}^n \left( \prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}} \right) (\tau s)^i \right] + \tau s + 1 \quad (6)$$

Equating the two polynomials represented in Eqs. (5) and (6), a Diophantine equation [6] of

$$A(s)D(s) + B(s)N(s) = P_{\text{target}}(s) \quad (7)$$

is obtained. The controller parameters  $K_i$  and  $l_i$  are computed by solving this equation easily.

### 3. Proposed New CDM PI-P Control Strategy

#### Part – I: Formulation of modified CDM blocks

The CDM block diagram shown in Fig. 1 is modified [7] as shown in Fig. 2, where the main controller  $C(s)$  and pre-filter  $C_f(s)$  are expressed by  $B(s)/A(s)$  and  $F(s)/B(s)$  respectively. Here, the CDM controller polynomials are chosen as follows

$$A(s) = s, \quad (8a)$$

$$B(s) = K_1 s + K_0 \quad (8b)$$

and the numerator polynomial

$$F(s) = P(s)/N(s)|_{s=0} = P(0)/N(0) = 1/K = K_0 \quad (8c)$$

The controller polynomials defined in Eqs. (8a) and (8b) is substituted in Eq. (2) and the characteristic polynomial  $P(s)$  is obtained. A target characteristic polynomial,  $P_{\text{target}}(s)$ , is determined only by specifying the stability index  $\gamma_i$  because the equivalent time constant,  $\tau$ , has been defined implicitly. Equating  $P(s)$  to  $P_{\text{target}}(s)$  and solving the Diophantine equation, the CDM controller parameters  $K_I$  and  $K_0$  are computed.

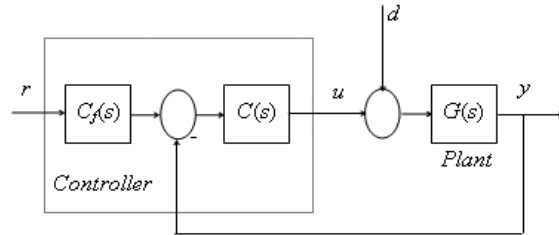


Fig. 2. CDM Control System - Equivalent Block Diagram.

### Part – II: Representation of PI-P control structure

In general, a conventional PI-P control structure is represented as shown in Fig. 3.

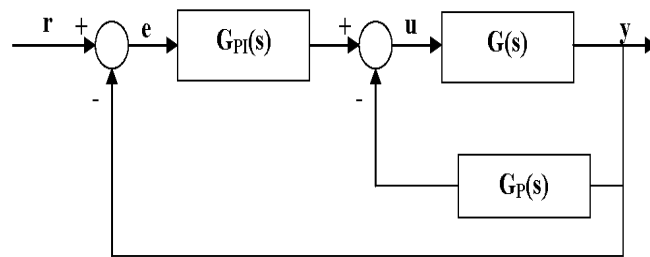


Fig. 3. PI-P Control Structure.

Here  $G(s)$  is the plant transfer function model.  $G_{PI}(s)$  and  $G_P(s)$  represent the PI and P controller transfer function models. Both models are defined as

$$G_{PI}(s) = K_c \left( 1 + \frac{1}{T_i s} \right) \quad (9)$$

$$G_P(s) = K_f \quad (10)$$

### Part – III: Design of CDM PI-P controller

Using the block diagram reduction rule, the PI-P control structure shown in Fig. 3 is reduced to equivalent structure as given in Fig. 4. In this figure,

$G_{PIP}(s) = G_{PI}(s) + G_P(s)$ . Substituting the Eqs. (9) and (10) and rearranging, we have

$$G_{PIP} = \frac{(K_c + K_f)T_i s + K_c}{T_i s} \tag{11}$$

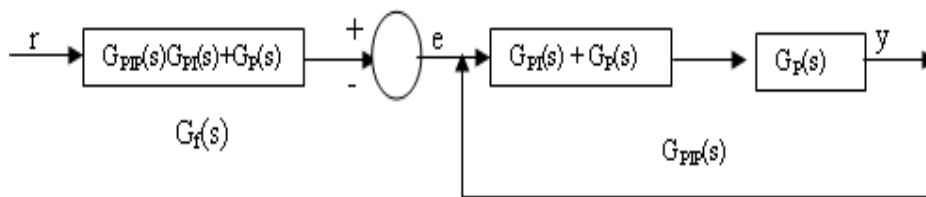
On connecting Fig. 4 with Fig. 2, the  $G_{PIP}(s)$  is equated to  $C(s)$  and the three CDM PI-P controller parameters;  $K_c$ ,  $T_i$ , and  $K_f$ , are found to be

$$K_c = \frac{\lambda K_1}{(1 + \lambda)} \tag{12}$$

$$T_i = \frac{\lambda K_1}{(1 + \lambda)K_0} \tag{13}$$

$$K_f = \frac{K_1}{(1 + \lambda)} \tag{14}$$

Note: The relation  $K_c = \lambda K_f$  is used here [8].



**Fig. 4. Equivalent PI-P Control Structure.**

The parameter  $\lambda$  is said to be a tuning factor and its value can be changed to satisfy the desired performance.

The pre-filter  $G_f(s)$  is expressed as 
$$G_f(s) = \frac{K_c(1 + T_i s)}{(K_c + K_f)T_i s + K_c} \tag{15}$$

Note: Since the  $G_f(s)$  depends on the CDM PI-P parameters directly, the designer needs not do any extra calculations.

**4. Simulation Results**

In this section, the performance of the proposed CDM PI-P control strategy is evaluated. A bio-reactor system [9], represented as

$$G(s) = \frac{-5.859}{5.888s + 1} e^{-\lambda s} \tag{16}$$

is simulated with the proposed control strategy and also with the conventional PI controller suggested by Padma Sree et al. [3].

The effect of tuning parameter  $\lambda$  of the CDM PI-P control strategy is studied here. Robustness of the proposed strategy is diagnosed by comparing the transient response of the system for a set point tracking; error response and the control signal response with the strategy suggested by Padma Sree et al. [3].

#### 4.1. Effect of tuning parameter $\lambda$

To illustrate the effect of tuning parameter  $\lambda$  present in the CDM PI-P controller scheme, simulation runs with different  $\lambda$  [ $\{0.1, 0.6, 1, 3, 5\}$ ] for unit set point tracking is carried out in the given bio-reactor system.

From Fig. 5, it is observed that the system time response reaches the desired value without overshoot for small  $\lambda$  value. But sluggishness is present.

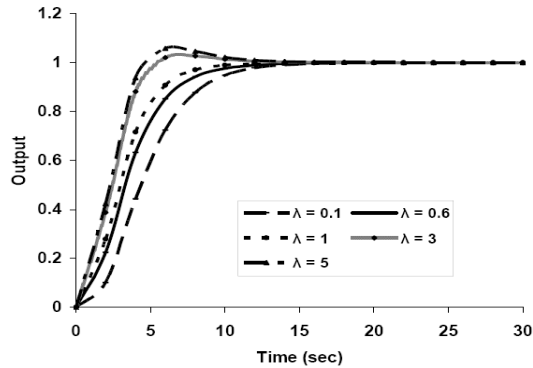


Fig. 5. Effect of  $\lambda$  on the Shape of the Time Response.

On contrary, if  $\lambda$  is increased, the response is accelerated to attain the desired value along with considerable overshoot with out sluggishness. A trade-off between the values of  $\lambda$  and the system performance is left out for the designer.

#### 4.2. Performance analysis of CDM PI-P controller

By specifying the stability index  $\gamma_1 = 3$ ,  $\gamma_2 = 2.8$  and the tuning factor  $\lambda = 0.6$ , the proposed CDM PI-P controller parameters;  $K_c = -0.1672$ ,  $T_i = 1.4266$ ,  $K_f = -0.2788$ , and  $G_f(s) = \left( \frac{-0.19672(1 + 1.4266s)}{-0.6363s - 0.1672} \right)$  are computed for the bio-rectors system

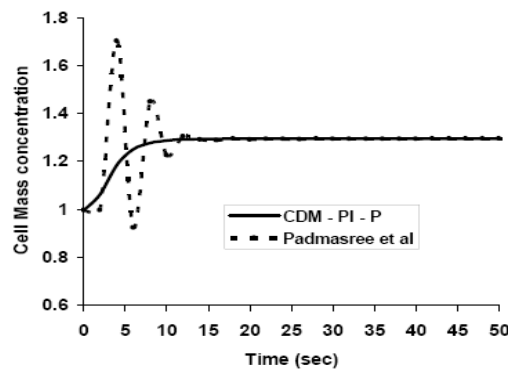


Fig. 6. Comparison of Step Response for CDM PI-P Controller with Padma Sree et al. [3].

given in Eq. (16). A simulation run is carried out with CDM PI-P controller in the system for a set point tracking of cell mass concentration from 0.9951 to 1.2936. Closed loop simulation response is recorded in Fig. 6. Similar way a simulated closed loop response for the same step change in set point with conventional PI controller settings ( $K_c = -1.23$ ,  $T_i=13.099$ ) as suggested by Padma Sree et al. [3], is also recorded in Fig. 6. It is obvious from this figure, that the proposed CDM PI-P control strategy gives enhanced performance over the other strategy.

In addition, error signal and control signal for both cases are recorded in Figs. 7 and 8. Figure 7 clearly indicates that the CDM PI-P controller brings the error value to zero at a faster rate. To evaluate the control effort, the total variation (TV) [10] of the manipulated input,  $u$ , is calculated using  $TV = \sum_{k=1}^{\infty} |u(k+1) - u(k)|$ .

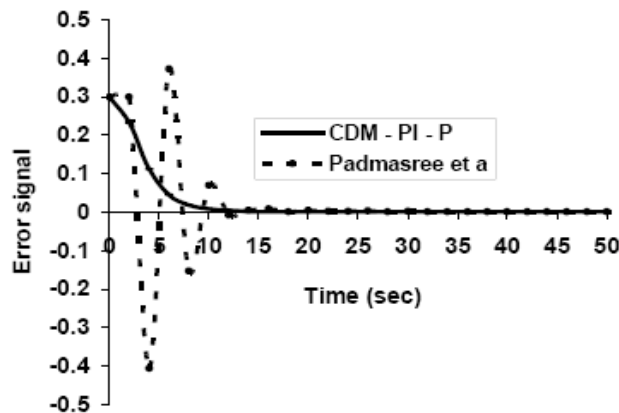


Fig. 7. Comparison of Error Signal for CDM PI-P Controller with Padma Sree et al. [3].

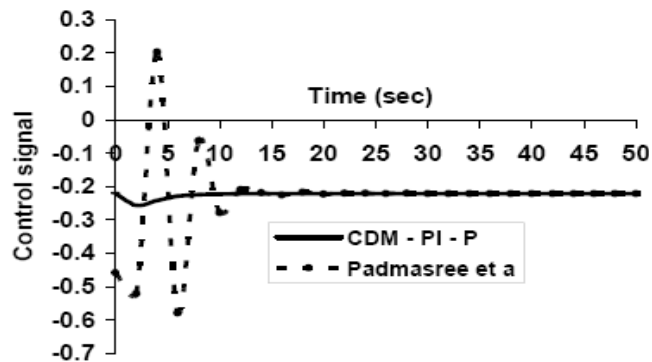


Fig. 8. Comparison of Control Signal for CDM PI-P Controller with Padma Sree et al. [3].



Performance analysis of the two control strategies in terms of ISE, IAE, TV, settling time,  $t_s$  and percentage overshoot,  $\%Mp$ , are computed and reported in Table 1.

Together with the above mentioned figures and the results in Table 1, it is concluded that the proposed CDM PI-P control strategy shows its supremacy over the other tuning method.

**Table 1. Performance Analysis of Control Strategies.**

Tuning method	ISE	IAE	TV OP	$\%M_p$	$t_s$ (s)
CDM PI - P	0.1	0.7	0.3	Nil	20
Padma Sree et al. [3] PI	0.5	1.6	2.3	31.4	60

## 5. Conclusions

In this paper, a new CDM PI-P control strategy has been proposed for unstable bio-reactor process. Using Coefficient Diagram Method as a base, new CDM PI-P controller scheme were derived. The proposed control strategy is very simple and is simulated with a bio-reactor system. The results indicate that the CDM PI-P control strategy gives excellent performance in controlling the unstable processes than the other technique. The proposed control scheme can be applied to class of processes also.

## References

1. Valentine, C.C.; and Chidambaram, M. (1997). PID control of unstable time delay systems. *Chemical Engineering Communications*, 162(1), 63 – 74.
2. Park, J.H.; Sung, S.W.; and Lee, I. (1998). An enhanced PID control strategy for unstable processes. *Automatica*, 34(6), 751 – 756.
3. Padma Sree, P.; Srinivas, M.N.; and Chidambaram, M. (2004). A simple method of tuning PID controllers for stable and unstable FOPTD systems. *Computers & Chemical Engineering*, 28(11), 2201 – 2218.
4. Manabe, S. (1998). Coefficient diagram method. *Proceedings of the 14<sup>th</sup> IFAC Symposium on Automatic Control in Aerospace*, Japan.
5. Somasundaram, S.; and Bhaba, P.K. (2009). A New CDM PI – PD Control strategy for unstable processes. *International Journal of Engineering Simulation*, 10(1), 19 – 24.
6. Kucera, V. (1993). Diophantine equations in control - a survey. *Automatica*, 29(6), 1361- 1375.
7. Hamamci, S.E.; Bhaba, P.K.; Somasundaram, S.; and Karunanithi, T. (2007). Two degree of freedom transient response control for unstable processes. *Proceedings of TIMA*, Tiruchirapalli, India, 145–150.

8. Astrom, K.J.; and Haggund, T. (1984). Automatic tuning of simple regulators with specifications on phase and gain margins. *Automatica*, 20(5), 645–651.
9. Chidambaram, M. (1998). *Applied process control*. New Delhi: Allied Publishers Ltd.
10. Chen, D.; and Seborg, D.E. (2002). PI/PID controller design based on direct synthesis and disturbance rejection. *Indian Engineering Chemistry Research*, 41, 4807 – 4822.