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NUMERICAL ANALYSES OF PLATE LOADING TEST

BY

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Abstract. A numerical simulation of plate loading test, in order to underlines the size effect on settlements and derived values of geotechnical parameters, is shown. The study is based on the comparison between the results obtained by Finite Element Method (FEM) using the Mohr-Coulomb soil model and by some observations from literature. The obtained numerical results revealed that the subgrade reaction coefficient is strictly dependent on parameters like size of the loaded area and loading magnitude, and thus completely general and generic, and not a fundamental material property of soil that can somehow be determined rationally, as often one claims to be.

Key Words: Plate Loading Test; Finite Element Method; Winkler Model; Coefficient of Subgrade Reaction; Elastic Continuum Model.

1. Introduction

The key aspect in the design of flexible structural elements in contact with bearing soils is the way in which soil reaction, referred to qualitatively as p, is assumed or accounted for in analysis. A magnitude and distribution of p might be preliminary assumed, or some mathematical relationship could be incorporated into the analysis itself, so that p is calculated as part of the analysis.

In common practice, a simple and relatively crude mathematical model for p, the well-known Winkler's Hypothesis, is (still) routinely used to eliminate the bearing soil reaction as a variable in the problem solution. In its basic form, Winkler's Hypothesis assumes that the soil medium is a system of identical, independent, closely spaced, discrete and linearly elastic springs and ratio between contact pressure, p, and settlement, w, produced by load application at an arbitrary point, i, on the contact surface, is given by the coefficient of subgrade reaction, k_s (or spring stiffness). Mathematically, this is expressed as

$$k_s = \frac{\text{pressure}}{\text{settlement}}. \tag{1}$$

One critical shortcoming is the difficulty in evaluating the coefficient of subgrade reaction, k_s, on a rational base. k_s is by no means an intrinsic property of

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the soil. Its value depends not only on soil stiffness, but also on various geometric-mechanical factors (*e.g.* geometry and stiffness of structural element/soil).Typical ranges of subgrade reaction coefficient can be found in the literature [1], but great care is required owing to the problem-dependent nature of the parameter. For a given soil, appropriate values for beams, rafts, laterally loaded piles and flexible walls are all different [2].

Another approach to eliminate p as a variable in the problem solution, is the elastic continuum idealization, were generally soil is assumed to be linearly elastic half space and isotropic for the sake of simplicity. This approach provides much more information on the stress and deformation within soil mass compared to Winkler model, and it has the important advantage of simplicity of the input parameters, the Young's modulus (and Poisson's ratio).

Both approaches, Winkler and elastic continuum idealization, requires appropriate values for the input parameters, subgrade reaction coefficient and Young's modulus (and Poisson's ratio), k_s and $E_s,\ \nu$, respectively. A direct method to estimate both E_s and k_s is plate loading test (PLT) that it is done with circular plates or equivalent rectangular plates. PLT provides a direct measurement af the compressibility and bearing capacity of soil and essentially consists in loading a rigid plate and determining the settlements corresponding to each load increment. The results of a PLT are presented as applied contact pressure *versus* settlement curves (Fig. 1). The interpretation of results (deformation properties) is usually made using isotropic elastic theory because of its convenience. Thus geotechnical parameters as Young's modulus and coefficient of subgrade reaction, may be derived as follows.

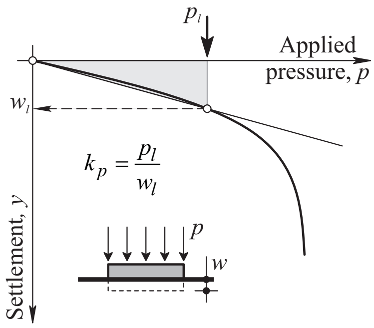

Fig. 1. – Typical presentation of results from a PLT.

Using elastic theory, the settlement of a rigid surface plate of diameter D, with uniform load p applied on a semi-infinite isotropic soil characterised by Young's modulus E_s and Poisson's ratio ν, is given by [1], [3], …, [8]

$$w_l = \frac{\pi}{4}\frac{p_l D\left(1-\nu^2\right)}{E_s}, \tag{2}$$

from which Young’s modulus may be evaluated by [1]

$$E_s = \frac{\pi}{4}\frac{p_l}{w_l}D\left(1-\nu^2\right). \tag{3}$$

The coefficient of subgrade reaction, k_s, is the initial slope of the curve (Fig. 1) until the limit pressure, p_l, is reached. The following equation, which is produced by the theory of elasticity, comparaison of eqs. (1) and (2), may be used to determine the value of k_s [1]:

$$k_s = \frac{4E_s}{\pi D\left(1-\nu^2\right)}. \tag{4}$$

Eq. (4) clearly demonstrates that the subgrade reaction coefficient is not a soil parameter and it depends, for the same soil, primarly on the size of the loaded area . Thus, if ones uses results from a PLT to evaluating k_s for full sized footing, it is appropriate to adjust the k_s value obtained from PLT. Terzaghi [2] proposed that k_s, for full sized footings, could be obtained from PLT using the following equations:

$$k_s = k_p\frac{B_p}{B}, \quad \text{for clayey soils;} \tag{5}$$

$$k_s = k_p\left(\frac{B+B_p}{2B}\right)^2, \quad \text{for sandy soils.} \tag{6}$$

where B_p is the plate diameter (or side dimension of the square plate) used in the PLT to produce k_p (the value of k_s for bearing plate) and B – side dimension of full sized footing.

In the present paper, according to these uncertainties, with use of finite element (FE) software, the effect of side dimension of loading plate on settlements and derived values of geotechnical parameters are investigated for diameters $D = (0.1,\ldots,3.0)$ m. The plate is assummed to be rigid and smooth.

2. Finite Element Model

All FE analysis were performed with an axis-symmetric mesh, because of the problem symmetry. The domain radius and height are $5D$ [9], [10]. A total of $1,015$, 15-noded triangular elements with a fourth order interpolation for displacements and twelve Gauss points for the numerical integration were used to define the finite element mesh shown in Fig. 2. Near the edges of a loaded area were stress concentrations are expected, mesh is refined by reducing the size of the elements [10]. Analysis is performed under displacement control by a prescribed vertical displacement boundary condition applied to the soil surface below the position of the loading plate [11], [12].

In order to prevent any rigid body motions of the whole problem domain, it is assumed that both the displacement in the horizontal and vertical direction are zero for all nodes along the bottom boundary of the mesh. On the vertical side boundaries, the horizontal displacement have been assumed to be zero too [11], [12].

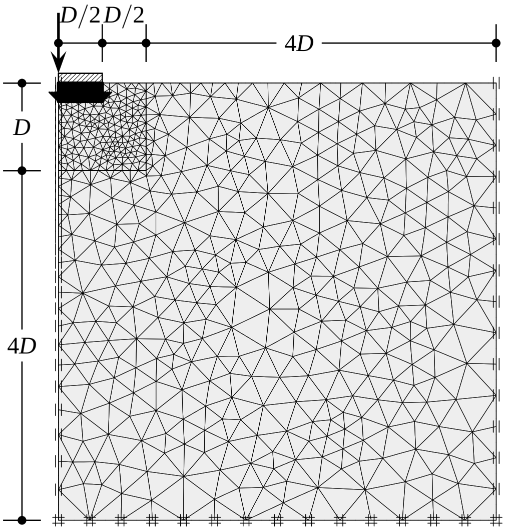

Fig. 2. – Mesh and geometry for finite element model.

Each FE calculation is divided in two phases. The behaviour of the ground depends on the current stresses and strains. It is therefore essential to prescribe the stress conditions which exist in the ground *prior* to the start of the event to be analysed. Thus in the first phase the initial soil stresses are generated [11]. In the second phase the displacement were set to zero and the loading begins. The loading is simulated by a prescribed displacement as described above.

The soil behavior it is assumed to be described by the Mohr-Coulomb model, having Young's modulus, $E_s = 30$ MPa, Poisson's ratio, $\nu = 0.3$, cohesion, $c =$ $= 1$ kPa and angle of shearing resistance, $\phi = 30°$.

3. Results and Discussions

Results from sixteen finite element analyses, using the mesh shown in Fig. 2 and with properties given above, are shown in Fig. 3. Dry condition were assumed and the soil had a bulk unit weight $\gamma = 17$ kN/m^3.

The results from PLT can be used to directly estimate the settlement of a footing and some geotechnical parameters may be derived too. Among them the stress–strain modulus (Young's modulus), E_s, and the subgrade reaction coefficient k_s, are of most interest. These values are commonly used in computing estimates of foundation settlements.

Making the assumption that the plate settlement is the same of an elastic half-space, until the limit pressure is reached, the stress–strain modulus, E_s, can be expressed from results of a plate load test in terms of the ratio of bearing pressure to plate settlement, as stated in eq. (3). This assimilation is not truly justified because under the edges of the loaded area a local punch failure may occur and thus no more being an elastic equilibrium in all points beneath plate. Therefore Boussinesq's solution may lead to erroneous outcomes especially in case of cohesionless soils with low punch strength.

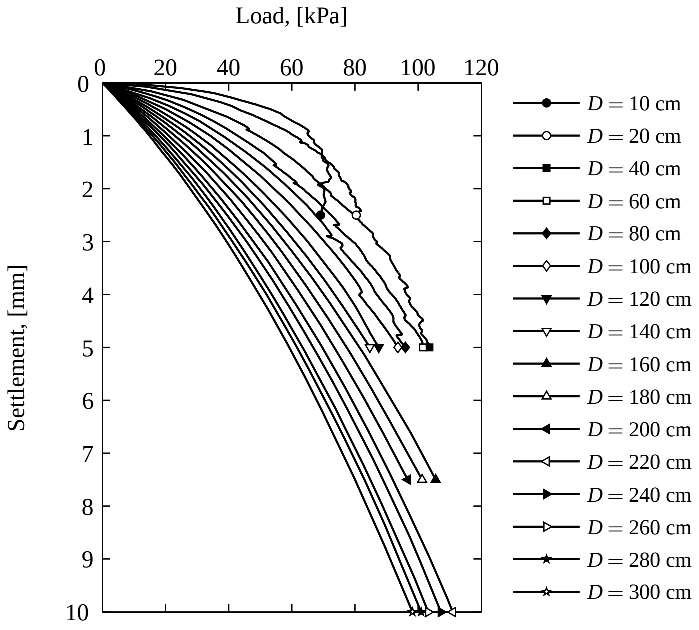

Fig. 3. – Numerical load *vs.* settlement curves.

To underline the foregoing, Fig. 4 shows the plastified zone by means of relative shearing stress, developed in bearing soil for the case of plate with diameter $D = 100$ cm that corresponds to an aapplied load by only 4.7 kPa (prescribed vertical displacement by 0.01 mm). The relative shear stress is defined as

$$\tau_{\text{rel}} = \frac{\tau}{\tau_{\max}}, \tag{7}$$

were τ is the maximum value of shear stress (*i.e.* the radius of the Mohr stress circle). The parameter $\tau_{\max}$ is the maximum value of shear stress for the case were the Mohr's circle is expanded to touch the Coulomb failure envelope keeping the intermediate principal stress constant.

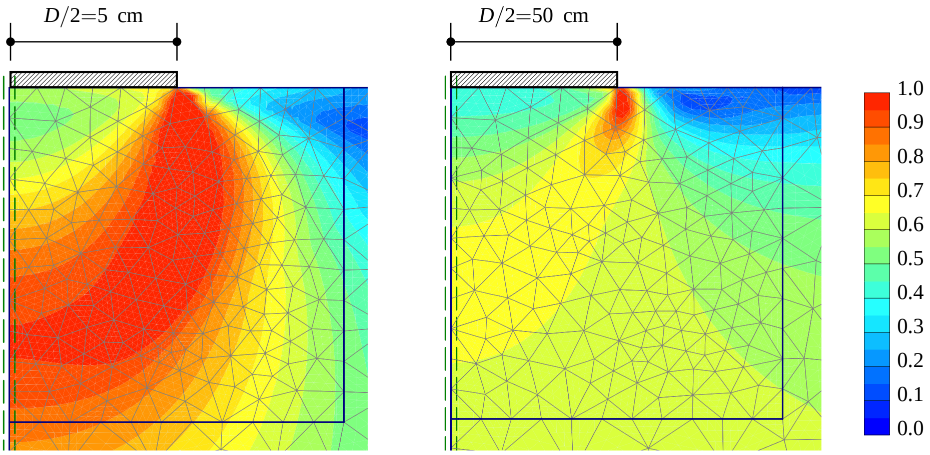

Fig. 4. – Relative shearing stress τ_{rel}.

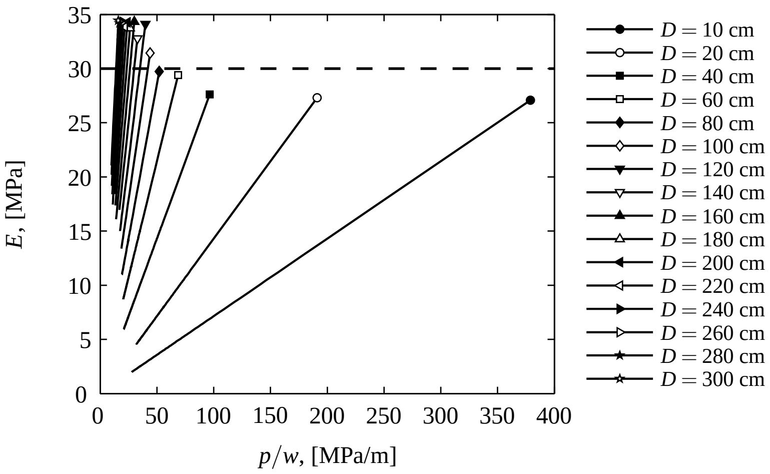

Fig. 5. – Stress–strain modulus *vs.* p/w ratio.

Applying the relation (3) for each one load-settlement curve shown in Fig. 3, the result's dependency *vs.* p/w ratio is shown in Fig. 5; one can easily observe that the error in evaluation of stress–strain modulus, E_s, by PLT is larger for plates with diameter less than 100 cm. The explanation is that the bearing soil under the loaded area consume its elasic strain more quickly (almost instantaneously) then in case of the plates with larger diameter ($D \geq 100$ cm) because of small contact area. For example, in case of plate with diameter $D = 10$ cm, to an applied load by 10 kPa (prescribed vertical displacement by only 0.05 mm), the soil beneath the plate is almost completely plasticized (Fig. 4).

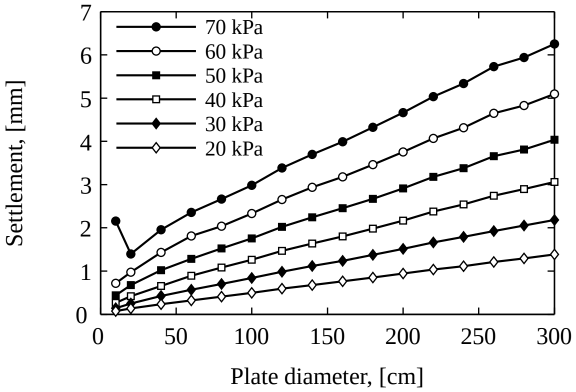

Fig. 6. – Relation between plate diameter and settlement under same load per unit area.

As it is known, the bearing capacity of cohesionless soils decreases with the increase in size of the loading area and thus is essentially depenedent of the size of the loading area. Therefore the scale effect is another explanation for the larger error in evaluation of stress–strain modulus, E_s, by PLT with (relatively) small plates. In Fig. 6 this is illustrated by plotting the settlement *versus* plate diameter relationship for various loading magnitude. As it can be seen, only for large diametres ($D \geq 100$ cm) the settlement increases proportional with the size of the loading surface.

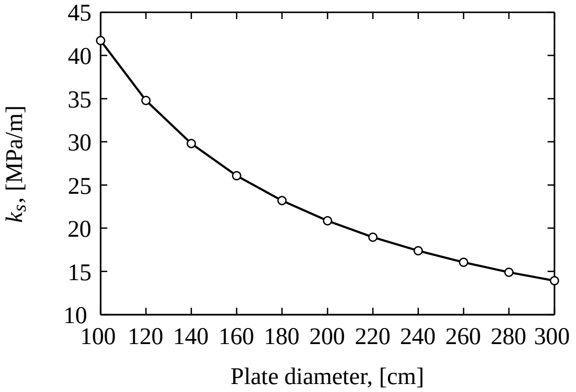

Fig. 7. – Variation of subgrade reaction coefficient *vs.* plate diameter.

As it shown forward, subgrade reaction coefficient, k_s, can be obtained from PLT results by means of elastic half-space solution. Therefore, applying eq. (4), the derived values for subgrade reaction coefficient are plotted in Fig. 7 for plates having diameters $D = 100, \ldots, 300$ cm.

It is evident from Fig. 7 that the value of the coefficient of subgrade reaction, k_s, varies according to the size of the plate used in PLT. Thus k_s has no unique value and depends on the size of the loaded area, it decreases with increasing size of plate. The use of values for k_s, usually recommended in literature (*e.g.* [3]), seems to be, therefore, meaningless.

4. Conclusions

Results of an numerical analyses of plate loading test to evaluate settlements and derived values of geotechnical parameters are presented. A total of sixteen finite element analyses were performed using rigid and smooth circular plates having diameters $D = (0.1, \ldots, 3.0)$ m.

Due to the fact that soils under loading exhibit elastoplastic behavior, the use of derived stress–strain modulus, E_s, through the PLT, can lead to misleading outcomes.

The obtained relation between plate diameter and settlement under same load per unit area is in good agreement with some observation presented in literature [1], [5], [8].

A common question asked by a structural engineer to a geotechnical engineer is “What is the subgrade reaction coefficient (k_s) at this particular site?”. Unfortunately, it has no direct, let alone a simple answer. As indicate the obtained results k_s is not a intrinsic soil property. Is just a response to a given load over a given area and depends not only on the deformation characteristics of the soil but also on the size of contact area between plate and subgrade.

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MODELAREA NUMERICĂ A ÎNCERCĂRII CU PLACA

(Rezumat)

Este prezentată o simulare numerică a încercării cu placa, cu scopul evidenţieirii influenţei dimensiunilor asupra tasărilor şi parametrilor geotehnici derivaţi. Studiul face comparaţie între rezultatele obţinute prin metoda elementului finit, utilizând pentru teren modelul de comportare elasto-plastică Mohr-Coulomb, şi unele observaţii din literatură. Rezultatele numerice obţinute arată că valoarea coeficientului de pat este strict dependentă de parametri ce ţin de forma şi dimensiunile suprafeţei de încărcare şi intensitatea încărcării. Astfel, coeficientul de pat este o mărime generică şi nu o proprietate mecanică a masivelor de pământ, aşa cum se pretinde adesea.