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# Sharpening of the Multistage Modified Comb Filters

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**Abstract:** This paper describes the application of filter sharpening method to the modified comb filter (MCF) in the case of decimation factor, which is product of two or more positive integers. It is shown that in the case of multistage decimation with MCF, filters in each stage are also MCF. Applying the sharpening to the decimation filter in the last stage provides very good results, with savings in the number of operations comparing to the case of sharpening of the complete filter. Direct-form FIR polyphase filter structure is proposed for the filters in each stage.

**Keywords:** CIC filter, Comb-based filter, Filter sharpening, Multirate systems, Polyphase decomposition.

## 1 Introduction

The application of comb-based digital filters has become very intense in multirate systems in the last years, because of their low computational complexity and low power consumption, as well as the possibility of working at high sampling rates.

This section presents organization of the paper. Section 2 describes main characteristics of cascaded integrator-comb filter (CIC) and modified comb filter. Multistage decimation is introduced in the Section 3, while Section 4 proposes use of filter sharpening method in the last stage of multi-stage filter, which provides good compromise between computational complexity and frequency response. Section 5 introduces algorithm for obtaining filter coefficients in fixed-point arithmetic. Finally, Section 6 gives brief conclusion. References are given in Section 7.

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## 2 CIC Filter and Modified Comb Filter

The cascaded integrator comb filter is the most frequently used filter of its class, due to very low complexity [1, 2]. It is implemented by cascading the integrator and comb filter. Thus, CIC implementation does not require use of multipliers. The efficient Hogenauer implementation of the CIC filter requires only two adders and two delay elements [3]. On the other hand, CIC filter has poor frequency characteristics (shown on Fig. 1). The frequency response of the CIC filter with length *N* can be represented by following equation

$$\left|H(e^{j\omega})\right| = \frac{1}{N}\left|\frac{\sin\left(N\omega/2\right)}{\sin\left(\omega/2\right)}\right|. \tag{1}$$

CIC filter has natural zeros at the frequencies *k/N*, where $k = 1,2,\ldots,N/2$, for $N$ being even number, and $k = 1,2,\ldots,(N-1)/2$, for $N$ being odd number. In the case of decimation CIC filter, natural nulls lie at the centers of the aliasing bandwidths, but it can provide sufficient suppression of the aliasing components only in very narrow bandwidth around the nulls. Thus, decimation CIC filter cannot suppress all aliasing components in the whole signal baseband. Simple way to increase selectivity of the CIC filter consists of cascading several identical CIC filters, resulting in multi-stage CIC filter. Unfortunately, the distribution of nulls remains the same as for basic CIC filter, only the order of each null is increased.

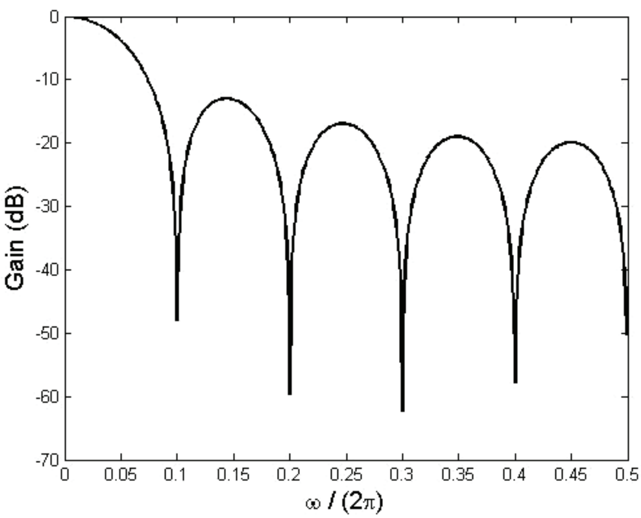



**Fig. 1 –** *Gain response of CIC filter with length* $N = 10$.

Modified comb filter (MCF) with rotated zeros has better distribution of nulls than multistage CIC filter regarding stopband attenuation in the aliasing bands, thus providing better suppression of the quantization noise [4, 5]. Therefore, it is used in implementation of sigma-delta A/D converters.

Modified comb filter of the third order (MCF3) can be obtained from third order CIC filter, by rotating its zeros. Instead of third order zeros at $\omega_k = 2k\pi/N$, MCF3 has group of three zeros at frequencies: $2k\pi/N$, $2k\pi/N+\alpha$, $2k\pi/N-\alpha$, for $k=1,2,\ldots,N-1$. In the further text, we'll denote $\mathrm{MFC3}(N,\alpha)$ as modified comb filter of the third order used for decimation with factor *N*. The transfer function of the $\mathrm{MFC}(N,\alpha)$ filter can be represented as follows

$$H_{MCF3}(z) = H_{CIC}(z)H_{rot+}(z)H_{rot-}(z), \tag{2}$$

where the factors in (2) are:

$$\begin{aligned} H_{CIC}(z) &= \frac{1}{N}\frac{1-z^{-N}}{1-z^{-1}}, \\ H_{rot+}(z) &= \frac{1}{N}\frac{1-e^{\mathrm{j}N\alpha}z^{-N}}{1-e^{\mathrm{j}\alpha}z^{-1}}, \\ H_{rot-}(z) &= \frac{1}{N}\frac{1-e^{-\mathrm{j}N\alpha}z^{-N}}{1-e^{-\mathrm{j}\alpha}z^{-1}}. \end{aligned} \tag{3}$$

Fig. 2 shows distribution of zeros for $\mathrm{MFC3}(10, 0.02\pi)$.

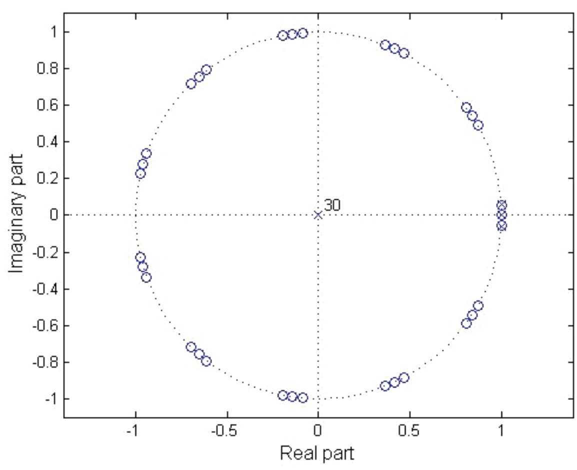



**Fig. 2 –** *Zero distribution of* MCF3 *filter* ( $N=10$, $\alpha = 0.02\pi$ ).

Gain response of $\text{MFC3}(10, 0.02\pi)$ and third order CIC filter is represented on Fig. 3.

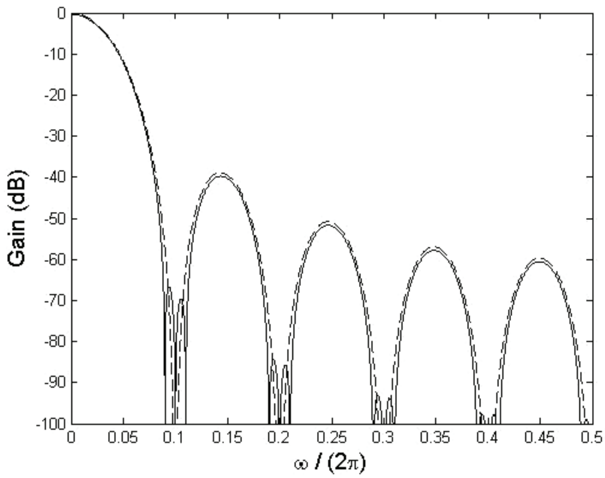



**Fig. 3 –** *Gain responses of MCF3 and third-order CIC filter, for* $N = 10$ *and* $\alpha = 0.02\pi$ (*solid line –* MCF3*, dashed line – third-order CIC filter*).

## 3 Multistage Decimation with MCF3 Filter

If decimation factor can be represented by the product of two or more integers, it is possible to perform multistage decimation. Using a multistage decimation relaxes the design requirements for decimation filters in each stage, and all operations, except in the first stage, are carried out at lower sampling rate.

Let us consider the case of two-stage decimation, where the decimation factor can be represented as $N = N_1 N_2$. We consider the case in which the resulting decimation filter is MCF3 filter. Let us denote with $H_{\text{MCF3}}(z, N, \alpha)$ transfer function of decimation filter $\text{MFC3}(N, \alpha)$. The following identity can be proven

$$H_{MCF3}\left(z, N, \alpha\right) = H_{MCF3}\left(z, N_1, \alpha\right) H_{MCF3}\left(z^{N_1}, N_2, N_1\alpha\right), \qquad (4)$$

by proving similar identity for each factor in (2). It can be concluded from identity (4) that equivalent $\text{MFC3}(N, \alpha)$ can be replaced by two MCF3 filters, where $\text{MFC3}(N_1, \alpha)$ is in the first stage, and $\text{MFC3}(N_2, N_1\alpha)$ is in the second stage [6].

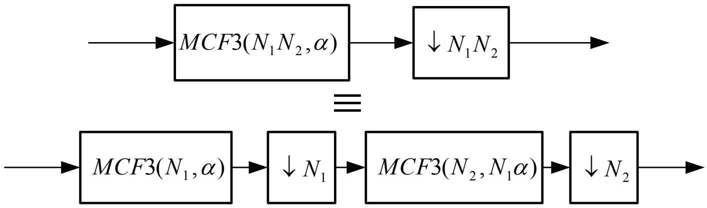



**Fig. 4 –** *Two-stage* MCF3 *decimator.*

Fig. 4 shows scheme of two-stage MCF3 decimator. It is not difficult to prove the similar equivalence stands for any multistage MCF3 decimator.

## 4 Application of the Filter Sharpening Method

Filter sharpening is a method which provides enhancement of filter characteristics in both passband and stopband [7]. In the other words, by implementing the filter sharpening method, passband error is decreased and the stopband attenuation is increased at the same time. The method is based on using several replicas of the original filter, which frequency response should be improved. The filter sharpening is applicable to the linear phase FIR filters. If we denote $H(z)$ as transfer function of linear phase FIR filter with delay $D$, the transfer function of the simplest sharpened filter can be obtained by following equation [7]

$$H_{sh}(z) = H^2(z)\left[3z^{-D} - 2H(z)\right]. \tag{5}$$

It is obvious from the equation (5) that the first component $H^2(z)$ is responsible for increasing of stopband attenuation, while the other component, $3z^D - 2H(z)$, reduces variation in the passband attenuation. The sharpened filter requires three copies of original filter, a delay line of $D$ samples, and two multipliers. As we can see, the improvement is paid by increasing computational complexity of resulting filter. Filter sharpening technique can be applied to both CIC and MCF filters [8, 9].

This paper proposes the use of the filter sharpening in the last stage of multistage MCF3 decimator. There are several reasons which can justify the use of the filter sharpening in such manner. First of all, it is obvious that the computational complexity of such implementation is less than if filter sharpening was applied to the equivalent one-stage MCF3 filter. Second, the filter sharpening is introduced in the last stage, at the lowest possible sampling frequency. We’ll consider the two-stage MCF3 decimator in further text, as the most interesting case.

Let us denote with $H_1(z)$ and $H_2(z)$ transfer functions of MCF3 filters in the first and second stage of two-stage decimator, respectively. Fig. 5 shows two-stage decimator with filter sharpening applied to MCF3 filter in the second stage. $H_{2,sh}(z)$ denotes transfer function of the sharpened filter in the second stage.

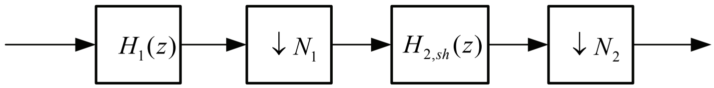



**Fig. 5 –** *Two-stage* MCF3 *decimator with filter sharpening in the second stage.*

Both filters $H_1(z)$ and $H_{2,sh}(z)$ can be implemented with direct implementation structure. Polyphase decomposition should be also used in implementation, thus providing the arithmetic operations to be performed at the lowest possible sampling rate. All operations of the first-stage filter will be performed at $f_S/N_1$, while the second-stage filter operates at $f_S/(N_1N_2)$, where $f_S$ is the sampling rate of the input signal [10]. Filter coefficients $\{h_{2,sh}[n]\}$ can be calculated as follows:

$$h_{2,sh} = conv\{conv\{h_2, h_2\}, h_2'\}, \tag{6}$$

where *conv* denotes convolution operator, and $h_2'$ can be represented in a following way:

$$h_2'[n] = \begin{cases} -2h_2[n], & n \neq D, \\ 3 - 2h_2[n], & n = D. \end{cases} \tag{7}$$

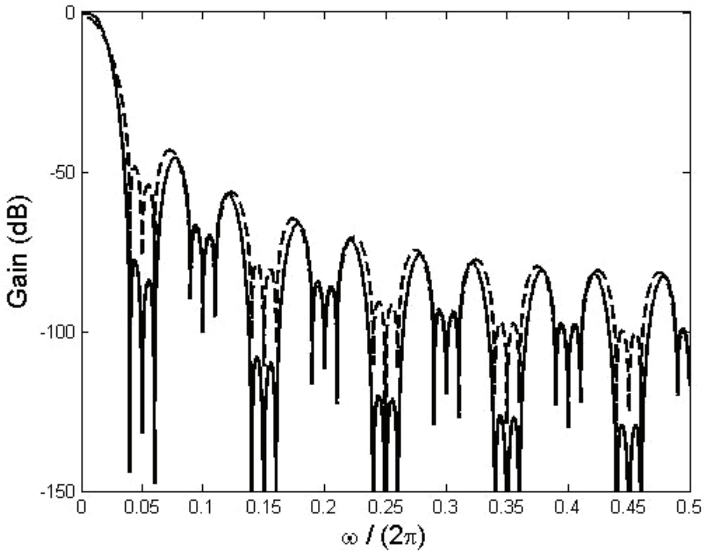
The filter sharpening introduced in the second stage does not affect the number of operations in the first stage. On the other hand, the number of additions and multiplications in the second stage, at sampling frequency, is increased by $6(N_2 - 1)$ due to filter sharpening. If the total decimation factor is constant, by increasing decimation factor of the first stage we can get smaller number of operations. The number of operation is summarized in **Table 1**.

**Table 1**

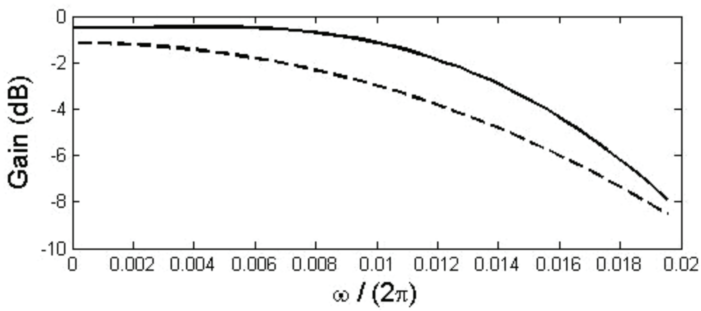
*Number of operations in the second stage of two-stage* MCF3 *decimator.*

| | Number of multiplications | Number of additions |
|---|---|---|
| without filter sharpening | $3N_2 - 2$ | $3N_2 - 3$ |
| with filter sharpening | $9N_2 - 8$ | $9N_2 - 9$ |

The following example should illustrate procedure introduced in this section. In this example, the decimation factors of two stages are $N_2 = 10$ and $N_2 = 2$, while $\alpha = 0.02\pi$. Fig. 6 represents frequency response of resulting decimation filter, with and without filter sharpening in the second stage. The filter sharpening provides significant reduction of the passband droop. At the same time, frequency characteristics of the resulting filter is improved in stopband, comparing to the case without filter sharpening.




(a)




(b)

**Fig. 6** – *Frequency response of two-stage* MCF3 *decimation filter* (*solid line – filter sharpening in the second stage, dashed line – without filter sharpening*): (a) *complete frequency band*; (b) *passband*.

Better suppression of aliasing bands can be achieved with greater value of $N_2$, but such choice will increase variation of the passband attenuation and the number of operations.

## 5 Implementation in Fixed-Point Arithmetic

The previous analysis did not consider quantization effects in MCF3 filter. This section is dedicated to the implementation of MCF3 filter in fixed-point arithmetic. Let us denote with $\{\Delta h[n]\}$ quantization error of MCF3 filter coefficients. The filter length is $L$, while the digital word length (number of bits) is $B$. Frequency response error can be obtained as follows:

$$\Delta H(e^{\mathrm{j}\omega}) = \sum_{n=0}^{L-1} \Delta h[n] e^{-\mathrm{j}\omega n} . \tag{8}$$

The maximum coefficient quantization error can be represented as:

$$\left|\Delta h[n]\right| \le \frac{q}{2} = 2^{-B} , \tag{9}$$

where $q$ is quantization step.

Starting from expressions (8) and (9) it is possible to determine upper limit for frequency response error.

$$\left|\Delta H\left(e^{\mathrm{j}\omega}\right)\right| \le \sum_{n=0}^{L-1} \left|\Delta h[n]\right| \left|e^{-\mathrm{j}\omega n}\right| \le \frac{L}{2^B} . \tag{10}$$

If $\Delta H_{\max}$ is maximum tolerable frequency response error in each stage, then the minimum digital word length can be calculated by following equation:

$$B_{\min} = \left\lceil \log_2 \frac{L}{\Delta H_{\max}} \right\rceil . \tag{11}$$

The length of MCF3 filter to be used in decimation with decimation factor $N$, is $3N-2$, and after the filter sharpening, filter length will be increased by factor of 3. It is not difficult task to calculate digital word length for given $\Delta H_{\max}$.

Take a look at two-stage decimation filter introduced in Section 4. If we presume the maximum frequency response error $\Delta H_{\max} = 10^{-3}$, then required coefficient bit-width for the MCF3 filters is 15 in the first stage, and 14 in the second stage. Frequency response of resulting decimation filter is shown on Fig. 7, in both cases, with exact and quantized filter coefficients.

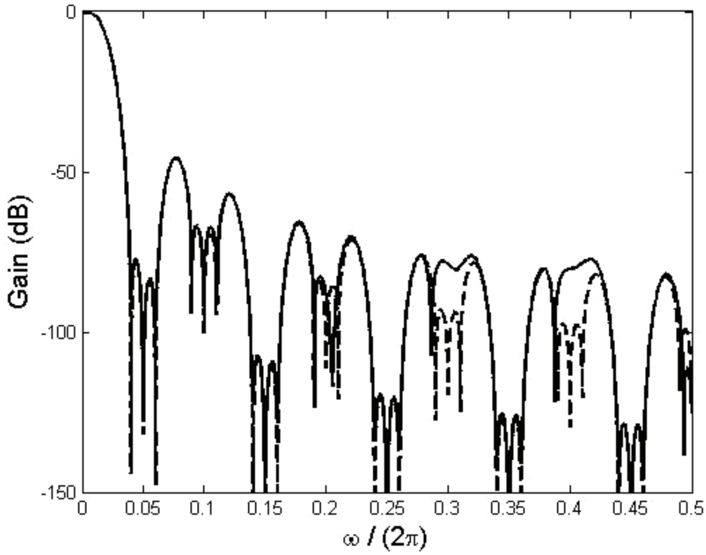



**Fig. 7 –** *Frequency response of two-stage* MCF3 *filter with filter sharpening in the second stage, for* $B_1 = 15$ *and* $B_2 = 14$ (*solid line – case with quantized coefficients, dashed line – case with exact coefficients*).

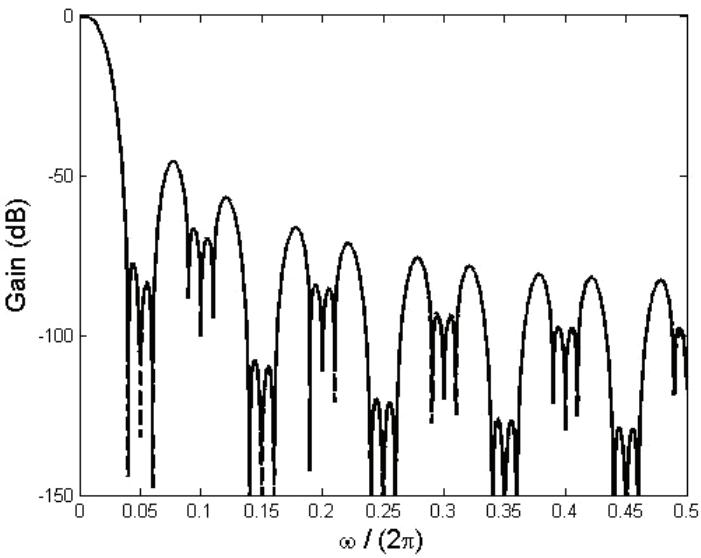



**Fig. 8 –** *Frequency response of two-stage* MCF3 *filter with filter sharpening in the second stage, for* $B_1 = 19$ *and* $B_2 = 17$ (*solid line – case with quantized coefficients, dashed line – case with exact coefficients*).

Complete procedure for the filter coefficients calculation can be divided in several steps.

**Step 1:** The exact value of MCF3 coefficients are calculated using equation (2) and (3), for the given values of decimation factor $N$ and rotation angle $\alpha$.

**Step 2:** Filter sharpening method is applied to the MCF3 filter in the second stage of decimator. Sharpened filter coefficients are obtained using (6) and (7).

**Step 3:** Starting from the given maximum frequency response error, filter coefficient bit-width is calculated (11).

**Step 4:** Quantization of filter coefficients is performed.

**Step 5:** Frequency response of resulting decimation filter is calculated. If frequency response significantly deviates from the ideal case with exact coefficients, $\Delta H_{\max}$ is reduced, and procedure is repeated again from Step 3. Otherwise, procedure is finished.

With increasing coefficient bit-width, frequency response strives to required characteristics, as shown on Fig. 8.

## 6 Conclusion

This paper illustrates the application of filter sharpening technique to the decimation modified comb filter with rotated zeros in the case of integer decimation factor, which can be factorized. Filter sharpening is used only in the last decimator stage. Such an implementation is a good compromise between computational complexity of the resulting filter and its frequency characteristics.

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