

No. 1/2010

COMPARING VARIANCE/COVARIANCE AND HISTORICAL SIMULATION IN THE CONTEXT OF THE FINANCIAL CRISIS – DO EXTREME MOVEMENTS HAVE AN INFLUENCE ONTO PORTFOLIO SELECTION?

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Abstract: Portfolio theory and the basic ideas of Markowitz have been extended in the recent past by alternative risk models as historical simulation or even copula functions. The central question of this paper is if these approaches lead to different results compared to the classical variance/covariance approach. Therefore, empirical data of the last 10 years is analysed. Both approaches are compared in the special context of the financial crisis. The worst case optimization and the Value at Risk (VaR) are defined in order to define the minimum risk portfolio before and after the financial crisis. The result is that the financial crisis has nearly no impact onto the portfolio, but the two approaches lead to different results.

Keywords: Portfolio Theory; Financial Crisis; Historical Simulation; Variance/Covariance Approach

JEL Classification: G10, G15, G30

1 Introduction and Aim of the Paper

Portfolio theory has been in existence for more than 50 years. But the financial crisis in 2008 led to the discussion of the fact that the extreme movements of diversified portfolios had not been forecasted. It is often said that this model failure led to high losses in 2008 that might have been prevented if a model that integrates worst cases scenarios would have been used. Alternative but more complex methods should have been used.

Portfolio theory in extreme market situations is therefore a topic that has to be discussed. Therefore, the historical simulation and the classical variance/covariance approach are analyzed. The central question to be answered by this paper is:

Does portfolio optimization by means of historical simulation in the times of financial crisis lead to other results than the classical variance/covariance approach? Do the results change if a high confidential level or the worst case is chosen?

The assumption lying behind this is that extreme developments of portfolios cannot be quantified by means of a normal distribution in the variance/covariance approach. This has to be proven.

The structure is as follows. After the introduction, section 2 discusses the theoretical basics according to portfolio theory. The relevant theories of portfolio management, asset allocation and risk/return are presented and discussed in the context of portfolio theory in a worst case scenario. The main aim is to define the used optimizing criteria for section 3. Section 3 describes the simulation and the parameters. Section 4 deals with the different results of the variance/covariance approach and the historical simulation. Therefore, several simulations based on historical data are done. Section 5 sums up the main results, discusses them critically and answers the critical question of this paper looking at all aspects.

2 Theoretical Aspects of Portfolio Theory

2.1 Definition of Risk and VaR

Whenever the discussion about portfolio diversification and risk minimization occurs, the first step is to define risk. In general, risk is the positive or negative unexpected difference from an expected value (Woll, 1996, p. 605). The positive difference is called chance, so only the negative difference from the expected value remains (Rolfes, 1999, p. 29; Reuse, 2006, p. 366; Reuse 2008, p. 5). This leads to the result that expected changes from the actual value of an asset are not defined as risk. They can be calculated ex ante and can be priced into the asset. An example is the so called spread in the corporate bond market. It is defined as the difference between the yield of the bond and the maturity equivalent risk free rate, for example US treasuries or German treasury bonds. The spread becomes higher, if the credit risks in the corporate bond increases.

One of methods to measure risk is the VaR (Value at Risk). It can be defined as the unexpected loss that will not be exceeded within a certain time under the assumption of a defined security level, called confidential level (e.g. Schierenbeck, 2001, p. 17; Reuse, 2006, p. 366). This definition leads to several factors that influence the VaR. In general, it can be stated that the VaR will increase, if the disposition period becomes longer and the confidential level gets higher (Reuse, 2006, p. 366). The model used to measure the risk influences the VaR as well, but this depends on the risk category and not on the basic assumptions to measure risk (Rolfes, 1999, p. 120).

In this article, three risk measurement variables will be used: the variance ($z = 84.13$), the VaR ($z = \text{e.g. } 99.00$) and the worst case ($z = 100.00$) that is defined as the highest loss than could have been stated in the historical development of an asset.

2.2 Portfolio Selection

The origin of Portfolio Theory can be found in 1952. Harry M. Markowitz published his theory in an article in the “Journal of Finance” titled “Portfolio Selection” (Markowitz, 1952, p. 77; Markowitz, 1987, p. 3). Markowitz demonstrates how an investor can reduce

the risk, or the standard deviation of the portfolio returns by choosing stocks that do not move exactly together (Discussed in Linnertová & Reuse, 2008, pp. 554–556).

The portfolio theory analyses the situation of investment decisions considering risk using the principle of $\mu \sigma$: The volatility can be seen as a degree of risk if all shares can be described by the expected returns (μ) and the volatility (σ) of returns. The theory assumes risk-averse behaviour of investors, which means that they are willing to renounce with return in favour of reducing the risk of an investment. A portfolio is consequently risk efficient if there is no alternative to get less σ for the same μ , more μ for the same σ or both more μ and less σ .

The expected rate of return resulting from diversification corresponds to the sum of returns of particular shares weighted by their proportion within the portfolio. Therefore, the expected rate of return of a portfolio μ_p consisting of n shares i with expected returns μ_i is:

$$\mu_p = \sum_{i=1}^n x_i \mu_i \quad \text{with} \quad \sum_{i=1}^n x_i = 1 \quad (1)$$

where:

μ_p	=	expected return of portfolio
μ_i	=	expected return of share i
n	=	number of available shares
x_i	=	proportion of share i within the portfolio

The standard deviation of the portfolio diversification is not equivalent to the weighted sum of every single variance. Additionally, the stochastic dependence of particular shares measured by the covariance cov_{ij} has to be taken into account. The basic idea lying behind this is that a diversified risk is always smaller than the weighted sum of the risks if the correlations between the assets are smaller than one. Markowitz argues with covariances and variances of portfolios instead of correlation effects but can be cited as follows: “If two original Portfolios have *equal* variance then typically the variance of the resulting (compound) portfolio will be less than the variance of either original portfolios” (Markowitz, 1952, pp. 89–90). The formula can be written as follows (Schierenbeck, 2001, pp. 78–79), whereas the standard deviation is multiplied with a certain z-factor in order to achieve the relevant VaR for the portfolio risk (Hager, n.Y., p. 3).

$$\text{VaR}_p = \sqrt{\sum_{i=1}^n \sum_{j=1}^n x_i \cdot x_j \cdot \sigma_i \cdot \sigma_j \cdot \rho_{ij}} \cdot z \quad (2)$$

$$VaR_P = \sqrt{[\sigma_1 \cdot x_1; \sigma_2 \cdot x_2; \dots; \sigma_n \cdot x_n] \cdot \begin{bmatrix} 1 & \rho_{1,2} & \dots & \rho_{1,n} \\ \rho_{2,1} & 1 & \dots & \rho_{2,n} \\ \dots & \dots & 1 & \dots \\ \rho_{n,1} & \rho_{n,2} & \dots & 1 \end{bmatrix} \cdot \begin{bmatrix} \sigma_1 \cdot x_1 \\ \vdots \\ \vdots \\ \sigma_n \cdot x_n \end{bmatrix}} \cdot z \quad (3)$$

where:

- VaR_p = Value at Risk of the portfolio
- x_i = part of asset i
- x_j = part of asset j
- σ_i = standard deviation of asset i
- σ_j = standard deviation of asset j
- ρ_{i,j} = correlation coefficient between the assets i and j
- n = number of assets
- z = z factor of the normal distribution (2.326 for 99%, 1.645 for 95%, 1.000 for 84.13%)
- k_{i,j} = $\frac{COV_{i,j}}{\sigma_i \cdot \sigma_j}$

This approach will be defined as a variance/covariance or up to now classical Markowitz approach as it defines the basic idea how to optimize a portfolio – including all advantages and disadvantages of the assumption of a normal distribution (Reuse 2006, p. 367).

2.3 Historical Simulation – Extending the Variance/Covariance Approach

If historical data is used to define diversification, the historical simulation can be used as well. While the variance/covariance approach shows the inherent assumption of a standard normal distribution (Bühler/Korn/Schmidt, 1998, p. 67), the historical simulation goes one step further. It uses a “real” distribution without converting historical data into a distribution function. It defines risk as the difference from an expected value. A further advantage is that extreme changes in the history that exist above the chosen confidential level can be measured as well as they are not converted into a distribution function. The historical simulation can be modelled as follows:

$$VaR_P = [r_a - Quartile_{s\%}(r_i)] \quad (4)$$

$$r_i = \ln\left(\frac{V_i}{V_{i-1}}\right) \quad (5)$$

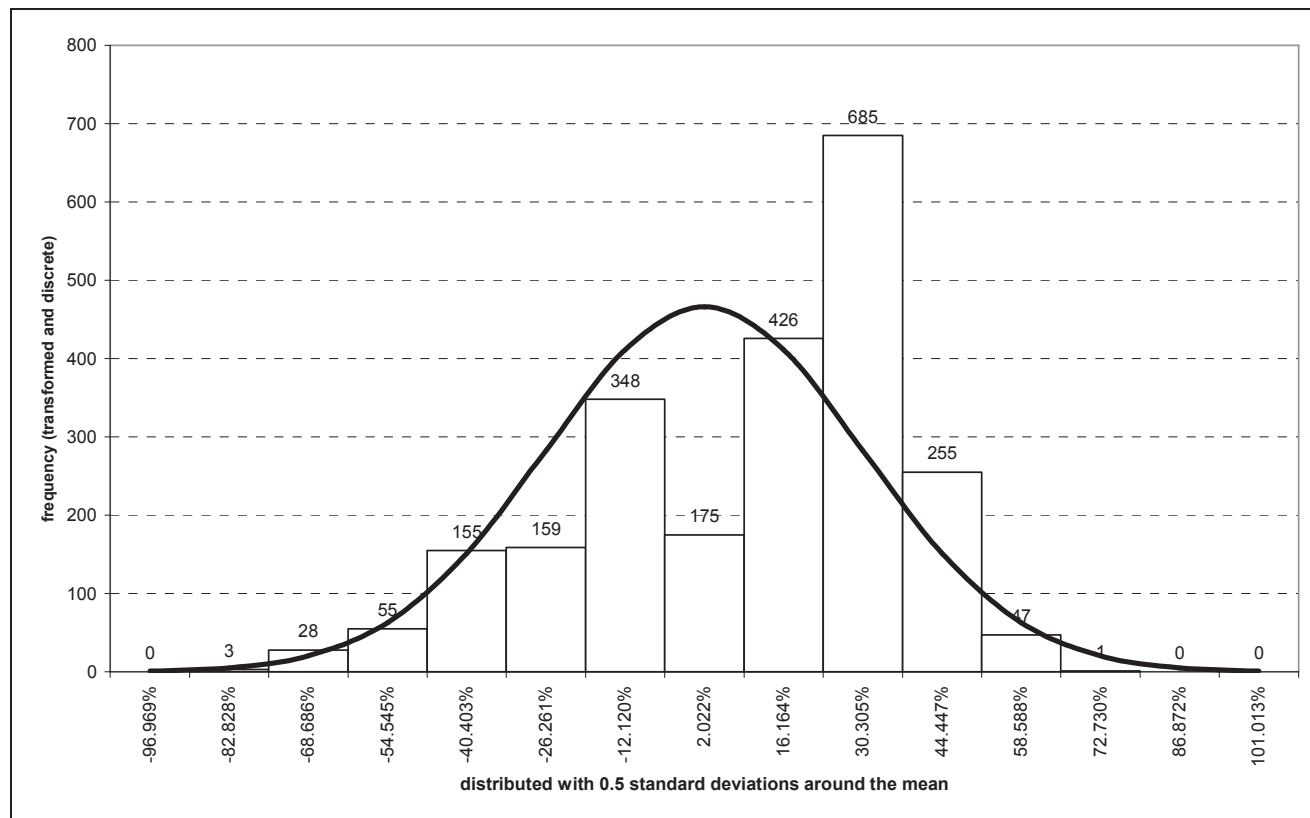
where:

- VaR_p = Value at Risk of the portfolio
- r_i = all historical yields (e.g. 1 / 10 / 250 days) of the used historical data
- r_a = average of all historical yields (e.g. 1 / 10 / 250 days) of the used historical data
- s = security level (e.g. 100%, 99%, 95%, 84.13%) for defining the z value of the normal distribution
- V_i = value of portfolio at t_i

The real distribution differs from the normal distribution function. This is shown in figure 1 that represents the lognormal daily changes in a “real” distribution compared with a transformation into a standard distribution of the German DAX (Deutscher Aktienindex).

Figure 1 shows that a standard normal distribution is only an approximation of the “real” distribution. The yields show extreme movements in other barriers than expected. So it would be the best to use the historical yields to measure risk. In addition, the procedure is less analytical than the variance/covariance approach but the amount of the needed data is much higher (A critical analysis of several VaR methods can be found in Goebel & Sievi & Schumacher, 1999, p. 236, discussed in Reuse 2006, p. 367).

Fig. 1 DAX yields – real and transformed distribution



Source: VWD, yearly returns, mean = 2.022%, standard deviation = 23.283% p.a. Data: 12/31/1998 up to 11/28/2008

It works as follows (Bühler/Korn/Schmidt 1998, p. 68): the historical price developments are transformed into lognormal yields (see equation 5). Yields of a specific history for all assets build a yield set. All yield sets that are evaluated in this way – typically several thousands – are added to the existing portfolio. Thousands of virtual portfolio changes can be defined. They are sorted by height and the needed quartile is defined by counting these changes. In the case of 100 possible portfolio changes and 99% confidential level, the second worst value is the VaR that will not be exceeded within the defined period. The worst case scenario would be defined as the worst simulated yield in order to get the most extreme historical movements of the past.

Applying this on a portfolio leads to the conclusion that this procedure must be done for all asset combinations for the given historical yields. The advantage is that extreme movements can be analyzed specifically and that only changes that appeared in the past are interpolated into the future.

3 Modelling the Simulation on the Basis of Historical Data

3.1 Asset Classes and Data used

There are many assets, but the most interesting ones are the five assets that are presented in figure 2.

Fig. 2 Used asset classes

Asset Class	Description	Reason why the asset class is used
RexP (Rentenindex Performance index)	This index represents the German covered bond market and is the main asset class a risk averse investor can invest in. For the asset class “yields”, it is a representative index as every maturity is implemented.	Bonds are the basic investments an investor has. This asset class has to be diversified.
DAX	Represents the biggest 30 shares in the German market and is a typical synonym for the asset class “shares”.	Correlations between shares and bonds are the oldest histories that can be analyzed. A diversified portfolio without shares would not embrace all aspects.

Deka Real Estate Europa	Estates and the investment in this asset class are becoming more popular. The presented investment fund is one of the biggest in the German market and most diversified in Europe. It may be taken as a synonym for the asset class “estates”.	The correlation between estates and all other asset classes is said to be about 0. This is the reason why it is used in this analysis.
Money Market 3M	This index consists of the 3M Euribor rate and transforms the yields into a performance. It is a synonym for “risk free yield”.	As risk is defined as the difference to the expected or secure value, a risk free rate in order to define the zero risk point has to be used in the analysis as well.
Dow Jones AIG	This rollover index consists of 19 physical commodities traded on the U.S. Exchange with the exception of aluminium, nickel, zinc which is traded on the London Metal Exchange. It may be taken as a synonym for the asset class “commodities”.	Commodities are often used to diversify a portfolio as the correlations are near 0 or negative.

Source: author's own figure

Data is available from 12/31/1998 up to 11/28/2008 on a daily basis (Source: Datastream 2008 and VWD 2008). A 10 year history shall be enough to use reliable data for the calculation. Further, the beginning of the financial crisis in 2008 and the subprime crisis can be found in the historical data.

3.2 Parameters of the Simulations

It shall be found out whether the optimization on the basis of the historical simulation leads to other results than the classical Markowitz approach. Therefore, several simulations shall be done. As only a free portfolio optimization can prove the central assumption of this assignment, no volume restrictions are entered. But all other parameters can be changed. Therefore, the following sets are calculated (Fig. 3).

Fig. 3 Used parameter sets

Parameter	Choice	Reason why these parameters are used
Disposition horizon	1-day-yield 10-day-yield 250-day-yield	These parameters are the typically used disposition horizons in practice. The overnight risk and the 10 day (two weeks) risk period are useful especially for measuring trading book risk (Schierenbeck, 2001, pp. 358, 447). The 250-day-yield reflects one year, which is the classical period in the Markowitz approach. A longer period does not make sense as the maximal used period for measuring market price risk is one year.
History	12/31/1998–12/31/2007 12/31/1998–11/28/2008	The DAX reached its high at the end of 2007. An interesting question is whether a worst case optimization on the basis of 2007 would have led to other results.
Confidential Level	99.00% 84.13% 100.00% (Worst Case in history)	The difference between the classical standard deviation and the VaR shall be analyzed critically. Further, the worst case is analyzed as well.

Source: author's own figure

The confidential level that has to be defined leads to the definition of risk. In the classical variance/covariance approach the standard deviation is used to quantify risk. In this simulation, the confidential level of 99% is chosen. This leads to a different z-factor in the variance/covariance approach when defining risk – and to another risk level in the nonnormal distributed historical simulation. This is the first way to define risk in extreme situations – even though the 99% confidential level might not be enough.

As a consequence, there are 12 scenarios that have to be calculated. As the used histories and time horizons change, expected yields and correlations change as well for the 12 scenarios. The results are distinguished into those before and into those after the financial crisis in 2008. Therefore, there are two data periods that lead to 24 scenarios in the end.

3.3 Optimization Problem – Minimizing Risk

This analysis focuses on the risk minimizing portfolios calculated by several confidential levels in the historical simulation and the variance/covariance approach. In order to solve the central questions of this work, an Excel based tool is created that is able to simulate portfolio optimization under different circumstances. The classical variance/covariance approach as well as the described historical simulation are implemented, including an

optimization on the basis of the worst case scenario (confidential level 100%). Therefore, the mathematical framework of section 2.1–2.3 is used.

For both approaches (variance/covariance and historical simulation), an optimization problem has to be solved. This is done by means of the SOLVER in MS Excel, consisting of the following framework:

- Iterations: 100
- Exactness: 0.000001
- Tolerance: 5%
- Convergence: 0.0001
- Linear optimization of the Newton approach

For the variance/covariance approach and the historical simulation, the combination with the lowest risk is chosen in this setup in order to define the minimum risk portfolio resulting from historical data. Consequently, two minimum risk portfolios for each of the 12 scenarios have to be modelled. These problems can be set up as follows (e.g. Perridon & Steiner, 2007, p. 244):

Variance /covariance approach

$$\text{Min!} \sqrt{\sum_{i=1}^n \sum_{j=1}^n x_i \cdot x_j \cdot \sigma_i \cdot \sigma_j \cdot \rho_{ij} \cdot z} \quad (6)$$

$$\text{with: } \sum_{i=1}^n x_i = 1; 0 \leq x_i \leq 1; \mu_p = \sum_{i=1}^n x_i \mu_i$$

Historical simulation

$$\text{Min!} \left[r_a - \text{Quartile}_{s\%} \left(\sum_{i=1}^n x_i \cdot r_i \right) \right] \quad (7)$$

$$\text{with: } \sum_{i=1}^n x_i = 1; 0 \leq x_i \leq 1; \mu_p = \sum_{i=1}^n x_i \mu_i; r_i = \ln \left(\frac{V_i}{V_{i-1}} \right)$$

4 Results of the Simulations

4.1 Analyzing the Minimum Risk Portfolios

The results are summed up in figure 4. The first result is that the different history – before and after the financial crisis – does not lead to a different portfolio optimum in both models. The risk minimized portfolios show nearly the same percentages. Further, the assumption that the worst case leads to significant different results must be rejected as well. If the historical simulation is calculated at a high confidential level, the worst case portfolio combination does not differ too much from e.g. a 99% confidential level result. Comparing the three historical simulations, it is interesting to see that the combination on the basis of the standard deviation leads to different results compared to the worst case and the 99% confidential level setup. The result is that especially in the case of market price risks a historical simulation has to be done at a high confidential level in order to measure the risk in an adequate way. Low confidential levels underestimate risk and shall be omitted.

Fig. 4 Results of the simulations

Simulation 2007							Simulation 2008							
1 day disposition horizon							1 day disposition horizon							
Approach	Variance/Covariance 99.00% VaR	Historical Simulation 84.13% VaR	Historical Simulation 99.00% VaR	Historical Simulation Worst Case	Approach	Variance/Covariance 99.00% VaR	Historical Simulation 84.13% VaR	Historical Simulation 99.00% VaR	Historical Simulation Worst Case	Approach	Variance/Covariance 99.00% VaR	Historical Simulation 84.13% VaR	Historical Simulation 99.00% VaR	Historical Simulation Worst Case
Assets					Assets					Assets				
RexP	0.28%	10.34%	2.86%	2.98%	RexP	0.26%	8.54%	5.59%	2.82%	RexP	0.20%	5.88%	4.97%	6.22%
DAX	0.01%	1.73%	1.22%	0.97%	DAX	0.14%	1.73%	2.87%	2.00%	DAX	0.01%	2.24%	1.52%	0.61%
Deka Immo Europe	1.55%	24.00%	14.03%	5.80%	Deka Immo Europe	0.00%	28.73%	14.17%	14.81%	Deka Immo Europe	1.68%	25.98%	6.83%	9.70%
Money Market 3M	98.13%	61.04%	81.20%	89.24%	Money Market 3M	99.50%	53.79%	73.97%	75.80%	Money Market 3M	98.08%	64.29%	85.23%	80.51%
Dow Jones AIG	0.03%	2.90%	0.68%	1.01%	Dow Jones AIG	0.06%	2.94%	2.47%	2.76%	Dow Jones AIG	0.03%	1.60%	1.44%	2.96%
Risk (VaR / WC)	0.30%	0.06%	0.08%	0.09%	Risk (VaR / WC)	0.42%	0.10%	0.15%	0.16%	Risk (VaR / WC)	0.32%	0.06%	0.10%	0.14%
Return	3.14%	1.45%	2.14%	2.50%	Return	3.13%	1.29%	1.81%	1.90%	Return	3.28%	1.64%	2.40%	2.18%
10 day disposition horizon							10 day disposition horizon							
Approach	Variance/Covariance 99.00% VaR	Historical Simulation 84.13% VaR	Historical Simulation 99.00% VaR	Historical Simulation Worst Case	Approach	Variance/Covariance 99.00% VaR	Historical Simulation 84.13% VaR	Historical Simulation 99.00% VaR	Historical Simulation Worst Case	Approach	Variance/Covariance 99.00% VaR	Historical Simulation 84.13% VaR	Historical Simulation 99.00% VaR	Historical Simulation Worst Case
Assets					Assets					Assets				
RexP	0.17%	10.09%	6.60%	4.66%	RexP	0.26%	8.54%	5.59%	2.82%	RexP	0.26%	8.54%	5.59%	2.82%
DAX	0.10%	4.45%	2.79%	1.97%	DAX	0.14%	1.73%	2.87%	2.00%	DAX	0.14%	1.73%	2.87%	2.00%
Deka Immo Europe	0.00%	28.73%	14.17%	14.81%	Deka Immo Europe	0.00%	28.73%	14.17%	14.81%	Deka Immo Europe	0.00%	31.44%	13.45%	13.99%
Money Market 3M	99.67%	53.79%	73.97%	75.80%	Money Market 3M	99.67%	53.79%	73.97%	75.80%	Money Market 3M	99.50%	53.33%	75.24%	79.14%
Dow Jones AIG	0.06%	2.94%	2.47%	2.76%	Dow Jones AIG	0.06%	2.94%	2.47%	2.76%	Dow Jones AIG	0.10%	4.95%	2.85%	2.05%
Risk (VaR / WC)	0.42%	0.10%	0.15%	0.16%	Risk (VaR / WC)	0.42%	0.10%	0.15%	0.16%	Risk (VaR / WC)	0.44%	0.11%	0.17%	0.18%
Return	3.13%	1.29%	1.81%	1.90%	Return	3.13%	1.29%	1.81%	1.90%	Return	3.26%	1.38%	1.94%	2.13%
250 day disposition horizon							250 day disposition horizon							
Approach	Variance/Covariance 99.00% VaR	Historical Simulation 84.13% VaR	Historical Simulation 99.00% VaR	Historical Simulation Worst Case	Approach	Variance/Covariance 99.00% VaR	Historical Simulation 84.13% VaR	Historical Simulation 99.00% VaR	Historical Simulation Worst Case	Approach	Variance/Covariance 99.00% VaR	Historical Simulation 84.13% VaR	Historical Simulation 99.00% VaR	Historical Simulation Worst Case
Assets					Assets					Assets				
RexP	15.01%	22.08%	18.57%	19.38%	RexP	15.99%	25.37%	19.59%	18.90%	RexP	15.99%	25.37%	19.59%	18.90%
DAX	2.17%	7.38%	6.95%	6.94%	DAX	2.49%	8.64%	6.94%	7.06%	DAX	2.49%	8.64%	6.94%	7.06%
Deka Immo Europe	0.00%	31.06%	30.94%	30.20%	Deka Immo Europe	0.00%	31.06%	30.94%	30.20%	Deka Immo Europe	0.00%	18.30%	30.17%	29.70%
Money Market 3M	82.07%	30.50%	35.87%	35.84%	Money Market 3M	82.07%	30.50%	35.87%	35.84%	Money Market 3M	81.15%	39.63%	35.38%	37.16%
Dow Jones AIG	0.74%	8.98%	7.68%	7.63%	Dow Jones AIG	0.37%	8.05%	7.92%	7.18%	Dow Jones AIG	0.37%	8.05%	7.92%	7.18%
Risk (VaR / WC)	1.49%	0.17%	0.26%	0.28%	Risk (VaR / WC)	1.55%	0.20%	0.28%	0.30%	Risk (VaR / WC)	1.55%	0.20%	0.28%	0.30%
Return	3.31%	0.98%	1.00%	0.99%	Return	3.31%	0.98%	1.00%	0.99%	Return	3.38%	1.00%	1.02%	1.02%

Source: author's own figure

Normally, portfolio optimization is done to create a strategic position. Nevertheless, the investor often faces the situation that the portfolio loses very much in a short time even though the long term risk is lower. Therefore, the three different time horizons were chosen. The results differ both in the variance/covariance approach and in the historical simulation. While the first approach always achieves more than 98% of money market in the 1 day and 10 day scenario, the historical simulation only leads to about 80% money market. At least a small part (5–15%) shall be invested in real estates. When calculating a 250 day scenario, both variance/covariance approach and historical simulation lead to a smaller part of money market – but the difference in the approaches can be seen most clearly in the 250 day setup. While the variance/covariance approach offers about 80% of money market and 15% of RexP, the historical simulation only invests 35% in money markets but about 30% in real estates, 20% in RexP and about 5–10% in Dow Jones AIG and DAX.

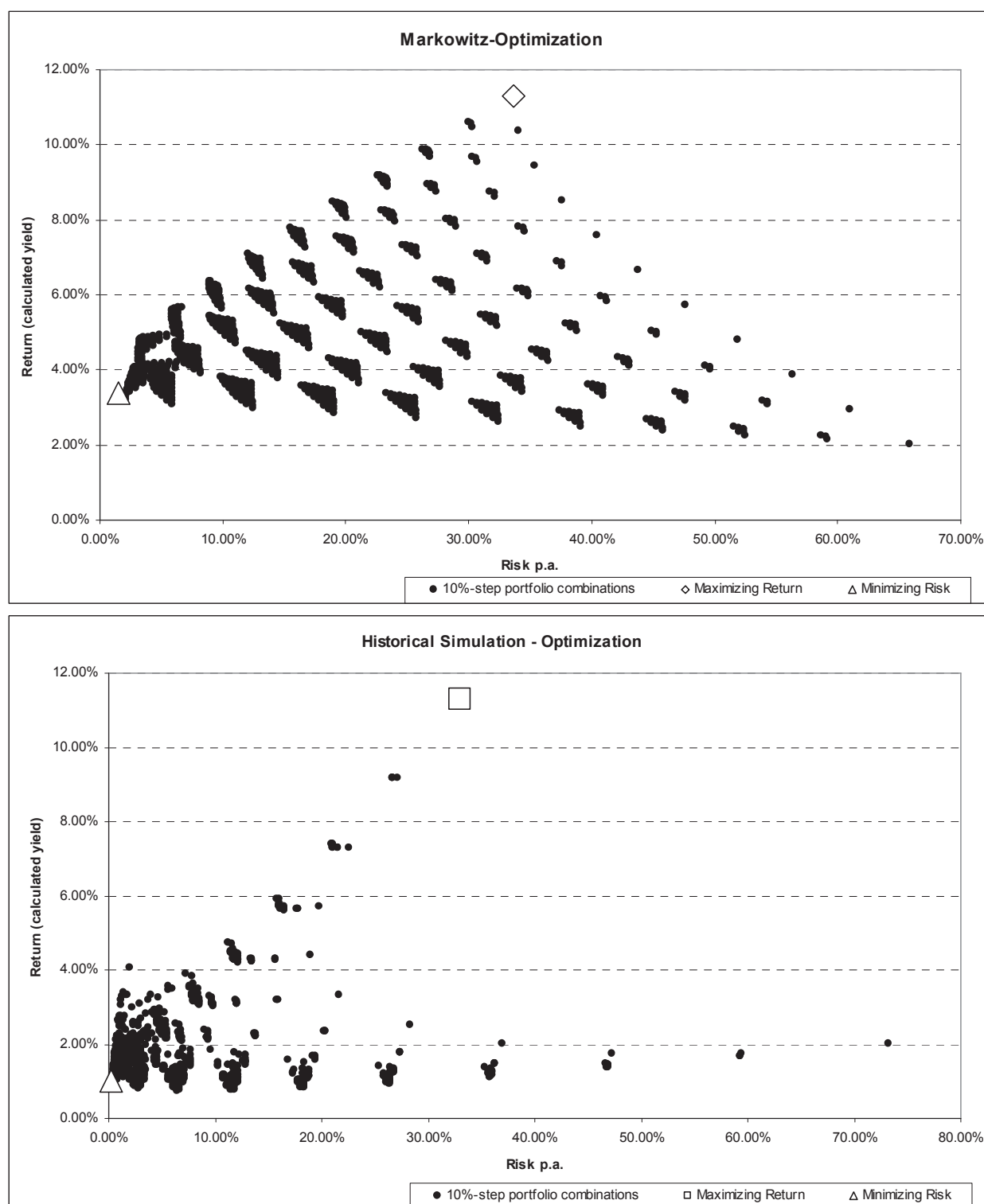
This difference in the models has to be explained. First, the historical simulation defines risk as a difference from the expected or calculated yield. The longer the disposition horizon, the higher is the different definition of risk. But the second argument is more important: the standard normal distribution cannot be assumed especially in extreme situations of a market movement. So the diversification effect between real estates and money market that is quantified with the correlation of 0.8639 in the variance/covariance approach is much higher when choosing the historical simulation approach.

4.2 Analyzing all Combinations of One Scenario

Besides the optimization presented above, the tool is able to optimize a portfolio consisting of 5 assets in 10% steps of a portfolio mixture. So the part of every asset class is between 0 and 100% that makes 11 possible characteristics. This leads to 1,001 possible portfolio combinations.

The results are visualized in figure 5 that compares the optimization of variance/covariance approach and historical simulation for 99% confidential level, 250-day-yield, 12/31/1998–11/28/2008.

Fig. 5 Minimum risk portfolio by both approaches



Source: author's own figure. 99% confidential level, 250-day-yield, 12/31/1998–11/28/2008

The variance/covariance approach offers several portfolio combinations that did not appear in the chosen history. This model risk results from the transformation into a standard normal distribution. The real distribution differs completely from the standardized one. The portfolio combinations shown in the middle of the variance/covariance approach have a lower risk in the historical simulation approach. Further, it can be stated that the extreme values cannot be seen as often as in the

variance/covariance approach. The result is a different minimum risk portfolio combination in the historical simulation. The combination suggested by variance/covariance would have the risk of 1.55%, while the one of the historical simulation only has 0.28%. A similar result can be stated if the analysis is done with expected yields. This might be necessary as the DAX yield is too low compared to its long-term history. The yield should be at least about 8–10%. Even though there are many portfolio combinations which have nearly the same risk, the historical simulation is better at optimizing a portfolio.

5. Conclusion

Both variance/covariance approach and historical simulation have been analyzed in this article. The results can be summed up as follows:

- (1) There is nearly no difference in the portfolio mixes including and excluding the year 2008. The financial crisis has nearly no impacts on the portfolio optimization.
- (2) An optimization on the basis of the worst case (100% confidential level) does not lead to other results, if the confidential level of the chosen simulation is set high enough, e.g. 99%.
- (3) The disposition horizons (1, 10, 250 days) lead to completely different results in both approaches.
- (4) The variance/covariance approach and the historical simulation lead to different results. These differences increase with the disposition horizon. Two arguments can be given therefore: first, risk is defined as the difference from the expected yield in the historical simulation while variance/covariance defines risk as the difference from the actual value. Second, the transformation into a standard normal distribution does not reflect the real historical movements of the data.
- (5) Therefore, several risk/return combinations offered by variance/covariance do not exist in the historical simulation. The risk and return that were evaluated by the historical simulation are lower than in the variance/covariance approach.
- (6) The historical simulation leads to a better portfolio mix. The combination suggested by variance/covariance leads to a higher risk.
- (7) The diversification effects of real estates and commodities are quantified better in the historical simulation as both assets get nearly no percentage in the variance/covariance approach.

As a result, the answer to the central question of this assignment whether the historical simulation leads to other results than the variance/covariance approach can be seen as affirmative. The results differ. On the other hand, an optimization according to the worst case scenario does not lead to other results if the confidential level is set high enough.

Both approaches are used in practice and lead to reliable results. The final result of this article is that the historical simulation is better than the variance/covariance approach. But

both approaches have to be seen critically, as they are based on historical data. The choice of the history influences especially the yield to a high degree. In the used history, the DAX has a historical 1 year yield of about 2%. This is too low compared to its long term history. But the choice of the “right” horizon remains difficult.

Further, it has to be stated that both approaches led to optimized portfolios that were not able to survive the financial crisis in 2007/2008. Alternative assets as ABS and commodities were used to realize correlation effects that disappeared in the crisis. The failure of the models has numerous reasons. First, a simple historical market price analysis was done in order to define correlations – assuming that an index reflects the development of a complete asset class. Second, alternative asset classes only have a short term history. E.g. ABS indices do not exist for a very long time. Third, nearly no investor looked into the structure of the ABS investments. A simple market price analysis is not enough, if the assets are very complex in their modulation. The model risk in these asset classes is at least as high as the market price risk as the development of 2007/2008 has shown (Frère/Reuse/Svoboda, 2008, p. 16). Fourth, the classical variance/covariance approach leads to other portfolios than the historical simulation. Without knowing the reasons why this takes place or under which circumstances which model offers which combination, an investor uses a tool without understanding how it works.

The main lesson to be learned from the financial crisis is that models should be simple enough to be understood by the user – as variance/covariance or historical simulation. Even though more complex approaches as copulas (discussed e.g. Beck & Lesko, 2006; Beck & Lesko & Schlottmann & Wimmer, 2006; Lesko, 2006; Mashal & Zeevi, 2002) may lead to results that fulfil statistical requirements better, the collateral damage of using methods that are not completely understood is higher than the model inexactness.

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