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A SINGLE PERIOD INVENTORY MODEL OF A DETERIORATING ITEM SOLD FROM TWO SHOPS WITH SHORTAGE VIA GENETIC ALGORITHM

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Abstract: Inventory of differential units of a deteriorating item purchased in a lot and sold separately from two shops under a single management is considered. Here deterioration increases with time and demands are time- and price-dependent for fresh and deteriorated units respectively. For the fresh units, shortages are allowed and later partially-backlogged. For the deteriorated units, there are two scenarios depending upon whether initial rate of replenishment of deteriorated units is less or more than the demand of these items. Under each scenario, five sub-scenarios are depicted depending upon the time periods of the two-shops. For each sub scenarios, profit maximization problem has been formulated and solved for optimum order quantity and corresponding time period using genetic Algorithm (GA) with Roulette wheel selection, arithmetic crossover and uniform mutation and Generalized Reduced Gradient method (GRG). All sub-scenarios are illustrated numerically and results from two methods are compared.

Keywords: Deteriorating item, two shops problem, time dependent demand, single period inventory model, genetic algorithm.

1. INTRODUCTION

Since the development of EOQ model by Harris (1915), the researchers have formulated and solved the different types of inventory models. Detailed reviews on the development in this area can be obtained in Hadley and Whitin (1963), Naddor (1966), etc. In reality, there are many situations where the demand rate depends on time. The demand of some items especially seasonable products like garments, shoes, mangoes, tomatoes etc., is low at the beginning of the season but increases as the season progresses i.e., changes with time. Donaldson (1991), Wee and Wang (1999), Chang and Dye (1999), Bhunia and Maiti (1998) and others developed their inventory models with time varying demand.

One of the most important assumptions in classical inventory models is that the lifetime of an item is infinite while it is in storage. But, in inventory management, the decay of the items plays an important role. In reality, some of the items are either damaged or decayed or vaporized or affected by some other factors, i.e., they do not remain in a perfect condition to satisfy the demand. The rate of deterioration of an item may be constant, time dependent or stock dependent. Some items, which are made of glass, china clay or ceramic, are often broken during their storage period and in this case, the deterioration rate depends upon the size of the total inventory. The decaying items such as photographic film, electronic goods, fruits and vegetables etc. gradually lose their utility with time. In the existing models, it is generally assumed that the deteriorated units are complete loss to the inventory management. But, in reality, it is not always true. There are some perishable items (e.g., fruits, vegetables, food grains, etc), which have a demand to some particular customers even after being partially deteriorated. This phenomenon is very common in the developing countries where majority of people live under poverty line. In business, the partially affected items are being immediately and continuously separated from the lot to save the fresh ones, otherwise the good ones will be affected by getting in contact with the spoiled ones. These damaged units are sold from the adjacent secondary shop. Here, the fresh/good units may be sold with a profit while the deteriorated ones are usually sold at a lower price, even incurring a loss, in such a way that the management makes a profit out of the total sales from the two shops.

For the solution of decision-making problems, there are some inherent difficulties in the traditional direct and gradient-based optimization techniques used for this purpose. Normally, these methods (i) are initial solution dependent, (ii) get stuck to a sub optimal solution, (iii) are not efficient in handling problems having discrete variables, (iv) can not be efficiently used on parallel machines and (v) are not universal, rather problem dependent. To overcome these difficulties, recently genetic algorithms (GAs) are used as optimization techniques for decision making problems. GAs[Goldberg(1989), Davis(Ed)(1991), Michalewicz(1992)] are adaptive computational procedures modeled on the mechanics of natural genetic systems. They exploit the historical information to speculate on new offspring with expected improved performance [Goldberg(1989), Pal et al(1997)].These are executed iteratively on a set of coded solutions (called population) with three operators: selection/reproduction, crossover and mutation. An iteration of these three operators is known as a generation in the parlance of GAs. Since a GA works simultaneously on a set of coded solutions, it has very little chance to get stuck at local optima. Here, the resolution of the possible search space is increased by operating on potential solutions and not on the solutions themselves. Further, this search space needs not to be continuous. Recently, GAs have been applied in different areas like neural network [Pal et al (1997)], travelling salesman [Forrest (1993)], scheduling [Davis (Ed) (1991)], numerical optimization [Michalewicz (1992)], pattern recognition [Gelsema (1995)], etc.

In this paper, an inventory model for a deteriorating item, especially fruits, vegetables, etc., comprising both good and damaged products and purchased in a lot is formulated under the assumption that the demand of the good units is time dependent whereas the deteriorated ones having only selling price dependent demand. Demand of the good unit linearly increases with time till the shortages occur and after that, gradually decreases during the shortage period. It is also assumed that at the beginning, a lot of the item including fresh and damaged units are received at the primary shop and the damaged ones are spotted, separated and transferred to other place known as secondary shop. Only good products are sold from the primary shop. Also during the sale at the primary shop, as the time progresses, some fresh units are damaged and these spoiled ones are spotted and transferred to the secondary shop continuously. These damaged units are sold at the reduced price. Shortages are allowed and fully backlogged at the primary shop but not in the secondary shop. In the primary shop, shortages are met by the fresh units specially purchased at higher price at the end of the cycle. There may be two cases for the present model depending upon the rate of initial replenishment of deteriorated units to the secondary shop being greater than or less than its demand rate. Again under each case, there may be five scenarios in the secondary shop depending upon the time periods of the two shops. The time period of the secondary shop may be equal, less than and greater than the time period of the primary shop. When it is less, it may occur before, after or exactly at the time of occurrence of the shortages at the primary shop. The time period of the secondary shop may be equal, less than or greater than that of the primary shop. Depending upon all these criteria, five different scenarios are observed for each case. For each scenario, inventory model has been formulated taking both primary and secondary shops into account. To achieve the maximum profit out of the total proceeds from two shops, the problem has been solved for optimum order quantities and the corresponding time periods using Genetic Algorithm and a gradient based optimization method (GRG). All the sub-scenarios of the model have been numerically illustrated and results are compared.

2. NOTATIONS AND ASSUMPTIONS

To develop the inventory model of a deteriorating item for both primary and secondary shops under a single management, the following notations and assumptions are used:

For the primary shop:

(i) Lead time is considered to be negligible.

(ii) Replenishment rate is infinite.

(iii) c is the purchasing cost per item .

(iv) Shortages are allowed at the primary shop but backlogged by the specially purchased goods at a higher price, c' per unit item at the end of the cycle, where $c' = m'c \quad (m' > 1)$.

(v) t_1 is the time of shortage point.

(vi) The demand rate, $D(t)$ for good units is linear function of time t i.e.,

$$D(t) = \begin{cases} d_1 + d_0 t, & \text{during no shortage period} \\ D(t_1) - \delta(t - t_1), & \text{during shortage period} \end{cases}$$

where $d_0, d_1, \delta > 0$.

(vii) The rate of deterioration, $\theta(t)$ is linearly dependent on time, t i.e., $\theta(t) = at, \quad a > 0$.

(viii) The inventory holding and shortage costs per unit per unit time are C_{1p} and C_{2p} respectively and the replenishment cost is C_{3p} per period.
(ix) t_2 is the time period of the primary shop.
(x) p_1 is the selling price per unit item of the primary shop.

For the Secondary shop:
(i) Lead time is considered to be negligible.
(ii) Shortages are not allowed.
(iii) The demand rate, λ of deteriorating units is dependent on the selling price, $p_2 = r_1 c,\ 0 < r_1 < 1$, where p_2 is the selling price of the deteriorating units and $\lambda = \alpha - \beta p_2,\ \alpha > 0$ and $\beta > 0$.
(iv) C_{1s}, C_{2s} are the holding cost per unit item per unit time and the set up cost per replenishment period respectively.
(v) The rate of deterioration θ' is assumed to be constant.
(vi) t_3 is the time period of the secondary shop.

3. MODEL FORMULATION

It is assumed that initially after the arrival of a lot of S units, deteriorated units that are a certain fraction (say μ) of the initial lot size are separated and transferred to the secondary shop. Therefore, the on-hand inventory level in the primary shop is $(1-\mu)S$ at t=0 and up to $t = t_2$, it gradually declines mainly to meet up the demand of fresh units and partially due to deterioration of the units which are continuously transferred to secondary shop for sale. The stock level reaches zero at time $t = t_1$ and then shortages are allowed and continue up to the time $t = t_1$ when next lot arrives. At $t = t_2$, the maximum shortage level let is S_1. These S_1 fresh units are specially purchased at a higher price at the end of the cycle. The geometrical representation of the model is given in Figure 1.

At the secondary shop, the deteriorating units are sold and shortages are not allowed. In this shop, initially the amount of stock is μS. Depending upon the rate of deterioration, here two cases may arise. In the first case, rate of replenishment of deteriorating units is initially less than the demand per unit time and the inventory level gradually declines up to the time $t = t_1$ with the stock level S_2. During this period, demand is met up partly from the current deteriorating units received from the primary shop and partly from the stock. After $t = t_1$, demand is met up fully from the stock as the primary shop goes to shortages at $t = t_1$ and inventory level gradually declines to zero at $t = t_3$. In this situation, three separate sub-scenarios may arise in the secondary shop depending upon the cases when the time period t_3 of the secondary shop is equal, less than, or greater than t_2. When $t_3 > t_2$, the stock at t_2 in the secondary shop is S_3. When

$t_3 < t_2$, there may be another three different sub-scenarios depending upon $t_3 < t_1$, $t_3 > t_1$ and $t_3 = t_1$. The geometrical representations of the sub-scenarios are given in Figures 2-6.

In the second case, replenishment rate is initially greater than the demand rate and gradually increases up to the time $t = t'$ when the stock attains a level S_4. As the amount of replenishment gradually declines, we assume that after time $t = t'$, the demand rate is greater than the rate of replenishment and demand is met up partly from currently deteriorated units received from the primary shop and partly from the stock. This process continues up to $t = t_1$ when stock attains a level S_2. After $t = t_1$, supply of the deteriorated items stops as shortages start by that time at the primary shop. The inventory level S_2 gradually declines to zero at time $t = t_3$ (say). As before, in this case also, there may be five sub-scenarios that are depicted in the Figures 7-11.

3.1. Primary Shop:

The differential equations governing the instantaneous state of inventory $q(t)$ (Figure 1) at the primary shop are:

$$\frac{dq(t)}{dt} = \begin{cases} -\{\theta(t)q(t) + D(t)\} & \text{if} \quad 0 \le t \le t_1 \\ -\{D(t_1) - \delta(t - t_1)\} & \text{if} \quad t_1 \le t \le t_2 \end{cases} \tag{1}$$

with boundary conditions:

$$q(t) = \begin{cases} (1-\mu)S & \text{if} \quad t = 0 \\ 0 & \text{if} \quad t = t_1 \\ -S_1 & \text{if} \quad t = t_2 \end{cases} \tag{2}$$

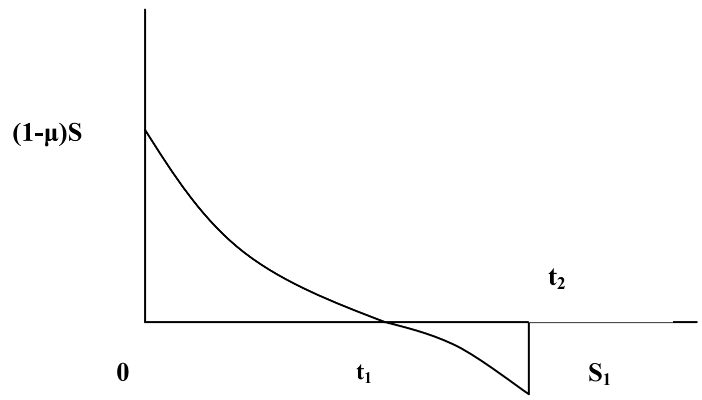

Figure 1.

The solution of the equation (1) is:

$$q(t)=\begin{cases}\dfrac{(d_1+d_0t)\{F(t_1)-F(t)\}}{f(t)} & \text{if} \quad 0\le t\le t_1\\ d_2(t_1-t)-\dfrac{\delta}{2}(t_1^2-t^2) & \text{if} \quad t_1\le t\le t_2\end{cases} \tag{3}$$

where $f(t)$, $F(t)$ and d_2 are given by

$$f(t)=(d_1+d_0t)e^{\frac{a}{2}t^2} \tag{4}$$

$$F(t)=\int_0^t f(u)du$$

$$d_2=d_1+(d_0+\delta)t_1 \tag{5}$$

From the boundary conditions, we get:

$$S=\frac{F(t_1)}{1-\mu} \tag{6}$$

$$S_1=d_2(t_2-t_1)-\frac{\delta}{2}(t_2^2-t_1^2) \tag{7}$$

Total number of deteriorating items during $(0,t_2)$ is

$$S_d=\int_0^{t_1}\theta(t)q(t)dt=(1-\mu)S-(d_1t_1+\frac{d_0}{2}t_1^2) \tag{8}$$

The holding cost over the period $(0,t_2)$ is $C_{hp}=C_{1p}\int_0^{t_1}q(t)dt$ (9)

The shortage cost during the period $(0,t_2)$ is

$$C_{sp}=-C_{2p}\int_{t_1}^{t_2}q(t)dt$$
$$=-C_{2p}\{(d_2t_1-\frac{\delta}{2}t_1^2)(t_2-t_1)-\frac{d_2}{2}(t_2^2-t_1^2)+\frac{\delta}{6}(t_2^3-t_1^3)\} \tag{10}$$

Hence, the total profit of the inventory system for the period $(0,t_2)$ is given by

$$Z_p(t_1,t_2)=p_1(1-\mu)S+p_1S_1-cS-c'S_1-C_{hp}-C_{sp}-C_{3p}-p_1S_d \tag{11}$$

3.2. Secondary Shop:

Scenario-1: Initially, $\theta(t)q(t)<\lambda$.

Sub-scenario 1a:

In this case, the time period of the secondary shop is equal to the time period of the primary shop, i.e., $t_3=t_2$. The differential equations describing the inventory level *I(t)* (Fig. 2) are given by

$$\frac{dI(t)}{dt}+\theta' I(t)=\begin{cases}\theta(t)q(t)-\lambda & \text{if} \quad 0\le t\le t_1,\ \ \theta(t)q(t)<\lambda\\ -\lambda & \text{if} \quad t_1\le t\le t_2\end{cases} \tag{12}$$

with boundary conditions:

$$I(t)=\begin{cases}\mu S & \text{if} \quad t=0\\ S_2 & \text{if} \quad t=t_1\\ 0 & \text{if} \quad t=t_2\end{cases} \tag{13}$$

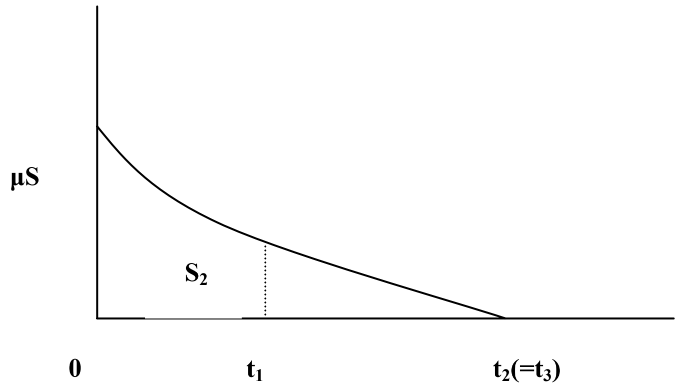

Figure 2: (sub-scenario-1a)

The solution of the equation (12) is:

$$I(t)=\begin{cases}aR(t)e^{-\theta' t}+\dfrac{\lambda}{\theta'}(e^{-\theta' t}-1)+\mu S e^{-\theta' t} & \text{if} \quad 0\le t\le t_1\\ \dfrac{\lambda}{\theta'}\{e^{\theta'(t_2-t)}-1\} & \text{if} \quad t_1\le t\le t_2\end{cases} \tag{14}$$

where $R(t)$ is given by $R(t)=\int_0^t uq(u)e^{\theta' u}du$ (15)

From the boundary conditions (13), we get

$$S_2=aR(t_1)e^{-\theta' t_1}+\frac{\lambda}{\theta'}(e^{-\theta' t_1}-1)+\mu S e^{-\theta'' t_1} \tag{16}$$

Total number of deteriorating items during $(0,t_2)$ is

$$S'_d=\int_0^{t_2}\theta' I(t)dt=\theta'(I_{11}+I_{12}) \tag{17}$$

where

$$I_{11}=\int_0^{t_1}I(t)dt=\frac{\mu S}{\theta'}(1-e^{-\theta' t_1})-\frac{\lambda}{\theta'}\{t_1+\frac{1}{\theta'}(e^{-\theta' t_1}-1)\}+aV(t_1) \tag{18}$$

$$I_{12} = \int_{t_1}^{t_2} I(t)dt = \frac{\lambda}{\theta'}\left[\frac{1}{\theta'}\{e^{-\theta'(t_2-t_1)} - 1\} - (t_2 - t_1)\right] \quad (19)$$

Here $V(t)$ is given by $V(t) = \int_0^t R(u)e^{-\theta' u} du$ (20)

The holding cost over the period $(0, t_2)$ is $C_{hs} = C_{1s}\int_0^{t_2} I(t)dt = C_{1s}(I_{11} + I_{12})$ (21)

The cycle length t_2 is given by $t_2 = t_1 + \frac{1}{\theta'}\log(1 + \frac{\theta' S_2}{\lambda})$ (22)

Hence, the return from the secondary shop during the period $(0, t_2)$ is given by

$$Z_s(t_1, t_2) = (\mu S + S_d)p_2 - S'_d p_2 - C_{hs} - C_{3s} \quad (23)$$

where the expressions of S, S_d and $S_d^{'}$ are substituted from (6), (8) and (17) respectively.

Sub-scenario-1b:
In this case, the time period of the secondary shop is less than the time period of the primary shop, i.e., $t_3 < t_2$. Differential equations and expressions can be obtained by replacing t_2 by t_3 in sub-scenario-1a. The instantaneous state of inventory is shown in Figure 3.

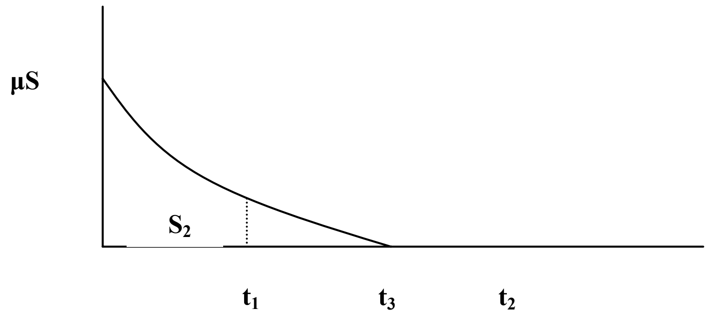

Figure 3. (sub-scenario-1b)

Sub-scenario-1c:
In this case, the time period of the secondary shop is greater than the time period of the primary shop, i.e., $t_3 > t_2$. Here the differential equations describing the inventory level $I(t)$ (Fig. 4) are given by equation (12) and the boundary conditions are the same as (13) except at $t = t_2$. At $t = t_2$, $I(t) = S_3$. The solutions of the differential equations are the same as equations (14) when $0 \le t \le t_1$ and another solution for $t_1 \le t \le t_2$ is as follows

$$I(t) = \frac{\lambda}{\theta'}\{e^{\theta'(t_2 - t)} - 1\} + S_3 e^{\theta'(t_2 - t)} \tag{24}$$

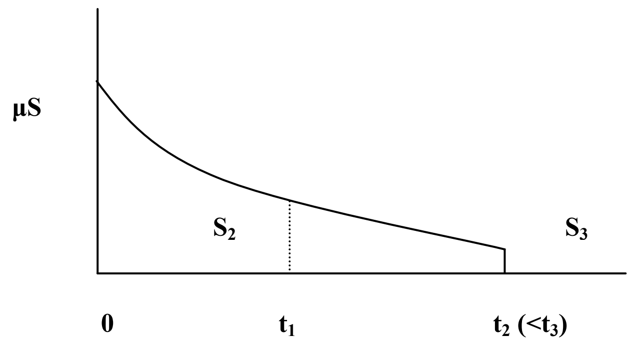

Figure 4: (sub-scenario-1c)

Here S_2 can be obtained by equation (16). Total number of deteriorating items during $(0, t_2)$ is $S'_d = \theta'(I_{11} + I_{12})$ where I_{11} is obtained by equation (18) and I_{12} is as follows

$$I_{12} = \frac{\lambda}{\theta'}\left[\frac{1}{\theta'}\{e^{-\theta'(t_2 - t_1)} - 1\} - (t_2 - t_1)\right] + \frac{S_3}{\theta'}\{e^{\theta'(t_2 - t_1)} - 1\} \tag{25}$$

The holding cost over the period $(0, t_2)$ is $C_{hs} = C_{1s}(I_{11} + I_{12})$. The cycle length t_2 is given by

$$t_2 = t_1 + \frac{1}{\theta'}\log(1 + \frac{\lambda + \theta' S_2}{\lambda + \theta' S_3}) \tag{26}$$

Hence, the return from the secondary shop during the period $(0, t_2)$ is given by

$$Z_s(t_1, t_2) = (\mu S + S_d - S_3)p_2 - S'_d p_2 - C_{hs} - C_{3s} + p'_2 S_3 \tag{27}$$

where $p'_2 = m_1 p_2, 0 < m_1 < 1$.

Sub-scenario-1d:
In this case, the time period of the secondary shop is equal to the time of shortage point of the primary shop, i.e., $t_3 = t_1$. The differential equations describing the inventory level *I*(*t*) (Fig. 5) are given by

$$\frac{dI(t)}{dt} + \theta' I(t) = \theta(t)q)t) - \lambda \text{ if } 0 \le t \le t_1 \tag{28}$$

with boundary conditions:

$$I(t) = \begin{cases} \mu S & at \;\; t = 0 \\ 0 & at \;\; t = t_1 \end{cases} \tag{29}$$

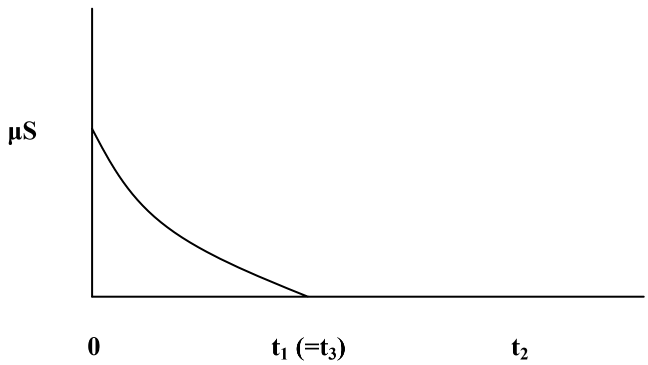

Figure 5: (sub-scenario-1d)

The solution of the equation (28) is the same as the first equation of (14).
From the boundary conditions, we get

$$aR(t_1)e^{-\theta' t_1} + \frac{\lambda}{\theta'}(e^{-\theta' t_1} - 1) + \mu S e^{-\theta'' t_1} = 0 \qquad (30)$$

Total number of deteriorating items during $(0, t_1)$ is $S'_d = \theta' I_{11}$ where I_{11} is same as (18) and the holding cost over the period $(0, t_1)$ is $C_{hs} = C_{1s} I_{11}$. Hence, the return from the secondary shop during the period $(0, t_1)$ is given by equation (23).

Sub-scenario-1e:
In this case, the time period of the secondary shop is less than the time of shortage point of the primary shop, i.e., $t_3 < t_1$. The differential equation describing the inventory level *I*(*t*) (Fig. 6) and boundary conditions are the same as equations (28) and (29) respectively only replacing t_1 by t_3. The solution of the differential equation, the boundary equation, number of deteriorating items and holding cost are the same as the sub-scenario-1d only replacing t_1 by t_3. Hence, the return from the secondary shop during the period $(0, t_3)$ is given by (23).

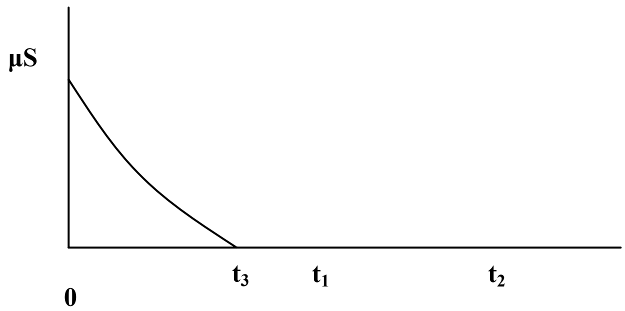

Figure 6: (sub-scenario-1e)

Scenario-2: Initially, $\theta(t)q(t) > \lambda$

Sub-scenario-2a:
In this case, the time period of the secondary shop is equal to the time period of the primary shop, i.e., $t_3 = t_2$. The differential equations describing the inventory level *I(t)* (Fig.-7) are given by

$$\frac{dI(t)}{dt} + \theta' I(t) = \begin{cases} \theta(t)q(t) - \lambda & if \quad 0 \le t \le t',\ \theta(t)q(t) > \lambda \\ \theta(t)q(t) - \lambda & if \quad t' \le t \le t_1,\ \theta(t)q(t) < \lambda \\ -\lambda & if \quad t_1 \le t \le t_2 \end{cases} \tag{31}$$

with boundary conditions :

$$I(t) = \begin{cases} \mu S & if \quad t = 0 \\ S_4 & if \quad t = t' \\ S_2 & if \quad t = t_1 \\ 0 & if \quad t = t_2 \end{cases} \tag{32}$$

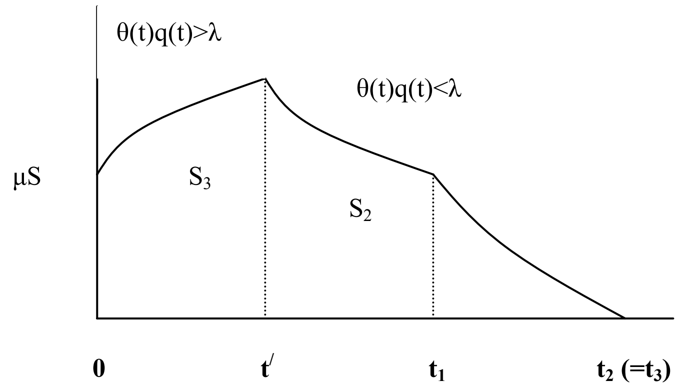

Figure 7: (sub-scenario-2a)

The solutions of the equations (31) are

$$I(t) = \begin{cases} aR(t)e^{-\theta' t} + \dfrac{\lambda}{\theta'}(e^{-\theta' t} - 1) + \mu S e^{-\theta' t} & if \quad 0 \le t \le t' \\ a\{R(t) - R(t')\}e^{-\theta' t} + \dfrac{\lambda}{\theta}\{e^{\theta'(t'-t)} - 1\} + S_4 e^{\theta'(t'-t)} & if \quad t' \le t \le t_1 \\ \dfrac{\lambda}{\theta'}\{e^{\theta'(t_2-t)} - 1\} & if \quad t_1 \le t \le t_2 \end{cases} \tag{33}$$

From the boundary conditions, we get

$$S_2 = a\{R(t_1) - R(t')\}e^{-\theta' t_1} + \frac{\lambda}{\theta'}\{e^{\theta'(t'-t_1)} - 1\} + S_3 e^{\theta'(t'-t_1)} \tag{34}$$

$$S_4 = aR(t')e^{-\theta' t'} + \frac{\lambda}{\theta'}(e^{-\theta' t'} - 1) + \mu S e^{-\theta' t'} \tag{35}$$

$$\lambda = at'(d_1 + d_0 t')\{F(t_1) - F(t')\} / f(t') \tag{36}$$

Total number of deteriorating items during $(0, t_2)$ is

$$S'_d = \int_0^{t_2} \theta' I(t)dt = \int_0^{t'} I(t)dt + \int_{t'}^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt = \theta'(I_{11} + I_{12} + I_{13})$$

where I_{11} is obtained from equation (18) only replacing t_1 by t' and I_{13} have same expression as in (19) and I_{12} can be obtained by

$$\begin{aligned} I_{12} &= a\{V(t_1) - V(t')\} + \frac{a}{\theta'}R(t')(e^{-\theta' t_1} - e^{-\theta' t'}) \\ &+ \frac{\lambda}{\theta'}\Big[\frac{1}{\theta'}\{1 - e^{\theta'(t'-t_1)}\} - (t_1 - t')\Big] + \frac{S_4}{\theta'}\{1 - e^{\theta'(t'-t_1)}\} \end{aligned} \tag{37}$$

The holding cost over the period $(0, t_2)$ is $C_{hs} = C_{1s}(I_{11} + I_{12} + I_{13})$ and the cycle length t_2 is given by (22). Hence, the return from the secondary shop during the period $(0, t_2)$ is given by (23).

Sub-scenario-2b:
In this case, the time period of the secondary shop is less than the time period of the primary shop, i.e., $t_3 < t_2$. Differential equations and expressions can be obtained from Sub-scenario-2a only replacing t_2 by t_3. The instantaneous state of inventory is shown in Figure 8.

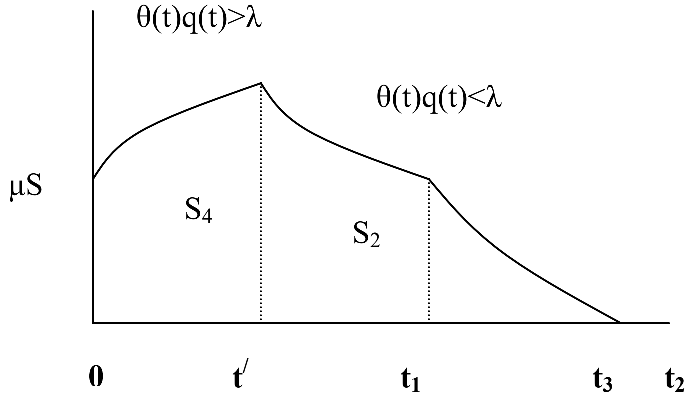

Figure 8: (sub-scenario-2b)

Sub-scenario-2c:
In this case, the time period of the secondary shop is greater than the time period of the primary shop, i.e., $t_3 > t_2$. Here the differential equations describing the inventory level *I(t)* (Fig.-9) are given by equations (31) and the boundary conditions are same as (32) except at $t = t_2$. At $t = t_2$, $I(t) = S_3$. The solutions of the differential equations are same as equations (33) when $0 \le t \le t'$, $0 \le t \le t_1$ and other solution for $t_1 \le t \le t_2$ is as follows

$$I(t) = \frac{\lambda}{\theta'}\{e^{\theta'(t_2-t)} - 1\} + S_3 e^{\theta'(t_2-t)}$$

All boundary conditions are same as equations (34), (35) and (36). The total number of deteriorating items during $(0, t_2)$ is $S'_d = \theta'(I_{11} + I_{12} + I_{13})$ where I_{11} is obtained from equation (18) only replacing t_1 by t' and I_{12} have same expression as in (37) and I_{13} is same as the expression of (25) only changing S_4 by S_3. The holding cost over the period $(0, t_2)$ is $C_{hs} = C_{1s}(I_{11} + I_{12} + I_{13})$. The cycle length t_2 is obtained from equation (26). Hence, the return from the secondary shop during the period $(0, t_2)$ is given by the equation (27).

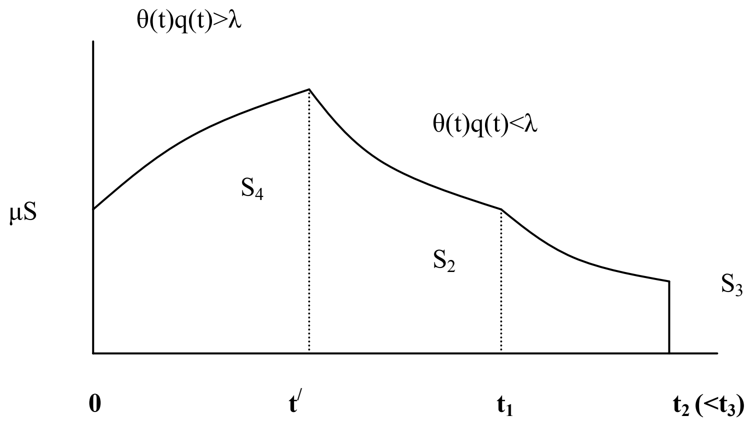

Figure 9: (sub-scenario-2c)

Sub-scenario-2d:
In this case, the time period of the secondary shop is equal to the time of shortage point of the primary shop, i.e., $t_3 = t_1$. The differential equations describing the inventory level *I*(*t*) (Fig.-10) are given by the first two equations of (31) with boundary equations

$$I(t) = \begin{cases} \mu S & if \quad t = 0 \\ S_4 & if \quad t = t' \\ 0 & if \quad t = t_1 \end{cases} \tag{38}$$

The solutions of these equations are same as the first two equations of (33). The boundary equations are same as (34)-(36). The number of deteriorating items and total holding cost are $S_d' = \theta'(I_{11} + I_{12})$ and $C_{hs} = C_{1s}(I_{11} + I_{12})$ where I_{11} is same as (18) only replacing t_1 by t' and I_{12} is same as (37). The expression for return is same as (23).

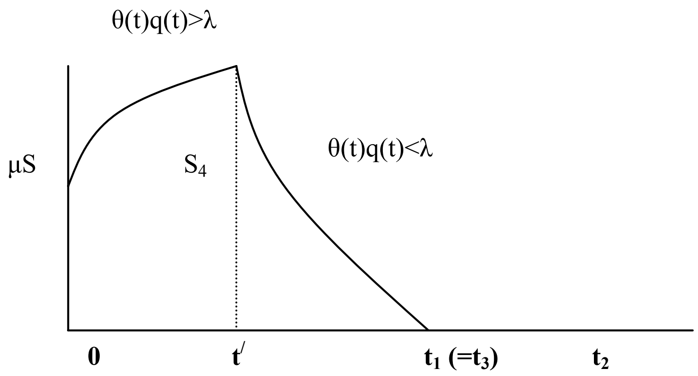

Figure 10: (sub-scenario-2d)

Sub-scenario-2e:
In this case, the time period of the secondary shop is less than the time shortage point of the primary shop, i.e., $t_3 < t_1$. Differential equations and expressions can be obtained from Sub-scenarion-2d only replacing t_1 by t_3. The instantaneous state of inventory is shown in Figure 11.

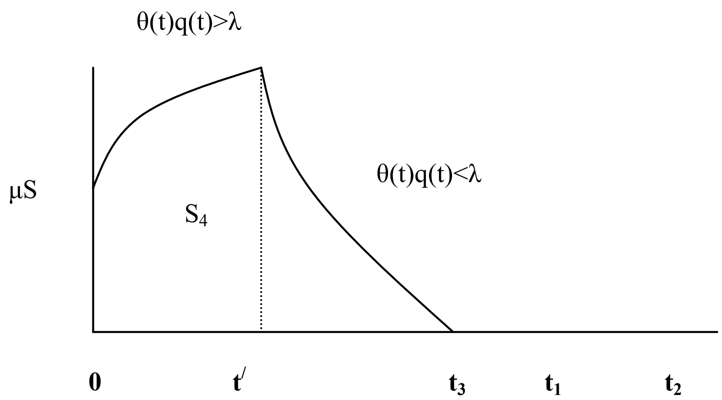

Figure 11: (sub-scenario-2e)

Total Average Profit:
Therefore, the total average profit (Z) of the system from two shops for Sub-scenarios (1a)-(2e) is given by

$$Z = \frac{Z_p + Z_s}{t_2} \quad (39)$$

where Z_p and Z_s have different expressions for different Sub-scenarios, Z_p is a function of t_1, t_2 and Z_s is a function of (i) t_1, t_2 (ii) t_1, t_2, t_3 (iii) t_1, t_2 (iv) t_1, t_2 (v) t_1, t_2, t_3 (vi) t_1, t_2, t' (vii) t_1, t_2, t_3, t', (viii) t_1, t_2, t' (ix) t_1, t_2, t' and (x) t_1, t_2, t_3, t' for the sub-scenarios-1a-2e respectively.

4a. Implementing GA

Total average profit Z given by (39) is maximized through the implementation of GA in the following way.

4a.1 Parameters:

Firstly, we set the different parameters on which this GA depends. All these are the number of generation (*MAXGEN*), population size (*POPSIZE*), probability of crossover (*PXOVER*), probability of mutation (*PMU*). There is no clear indication as to how large should a population be. If the population is too large, there may be difficulty in storing the data, but if the population is too small, there may not be enough string for good crossovers. In our experiment, *POPSIZE = 50*, *PXOVER = 0.2*, *PMU = 0.2*, *MAXGEN = 5000*.

4a.2 Chromosome representation:

An important issue in applying a GA is to design an appropriate chromosome representation of solutions of the problem together with genetic operators. Traditional binary vectors used to represent the chromosome are not effective in many physical non-linear problems. Since the proposed problem is non-linear, hence to overcome the difficulty, a real - number representation is used. In this representation, each chromosome V_i is a string of *n* number of genes G_{ij} (i=1,2,..,*POPSIZE* and *j*=1,2,..,*n*) where these *n* number of genes respectively denote the number of decision variables among t', t_1, t_2 and t_3, the value of *n* depends on the sub-scenarios(1a-2e)given in section 3.2.

4a.3 Initial population production:

To initialize the population, we first determine the dependent and independent variables and then their boundaries. Here, the dependency and independency of the variables are different for different sub-scenarios. Since all variables are related to time, here the boundaries of all independent variables are assumed to be (0,12.0). For each chromosome V_i, every gene G_{ij}, which represents the independent variable, is randomly generated between its boundary (LB_k, UB_k) where LB_k and UB_k are the lower and upper bounds of that variable and the gene G_{ij} which is the dependent variable, are generated from different conditions for different sub-scenarios given in section 3.2, until it is feasible, $i=1,2,..,POPSIZE$.

4a.4 Evaluation:

Evaluation function plays the same role in GA as that which the environment plays in natural evolution. The evaluation function *EVAL* for each chromosome V_i is defined as

$EVAL(V_i)$=objective function value for V_i.

4a.5 Selection:

This selection process is based on spinning the roulette wheel *POPSIZE* times, each time we select a single chromosome for the new population in the following way:

(a) Calculate the fitness value $EVAL(V_i)$ for each chromosome V_i (i=1,2,..,*POPSIZE*).

(b) Find the total fitness of the population as $f = \sum_{i=1}^{POPSIZE} EVAL(V_i)$.

(c) Calculate the probability of selection p_i^b for each chromosome V_i as $p_i^b = EVAL(V_i)/f$.

(d) Calculate the cumulative probability q_i for each chromosome V_i as $q_i = \sum_{j=1}^{i} p_i^b$.

(e) Generate a random real number *r* in (*0, 1*).

(f) If $r < q_1$ then the first chromosome is V_i otherwise select the *i*-th chromosome V_i ($2 \le i \le POPSIZE$) such that $q_{i-1} < r \le q_i$.

(g) Repeat steps (e) and (f) *POPSIZE* times and obtain *POPSIZE* copies of chromosomes.

4a.6 Crossover operation:

The exploration and exploitation of the solution space is made possible by exchanging genetic information of the current chromosomes. Crossover operates on two parent solutions at a time and generates offspring solutions by recombining both parent solution features. After selection of chromosomes for new population, the crossover operation is applied. Here, the whole arithmetic crossover operation is used. It is done in the following way:

(a) Firstly, we generate a random real number *r* in (0, 1).

(b) Secondly, we select two chromosomes V_k and V_l randomly among population for crossover if $r < PXOVER$.

(c) Then two offspring $V_k^{'}$ and $V_l^{'}$ are produced as follows:

$$V_k^{'} = c * V_k + (1-c) * V_l$$
$$V_l^{'} = (1-c) * V_k + c * V_l$$

where $c \in [0,1]$.

(d) Repeat the steps (a),(b) and (c) *POPSIZE/2* times.

4a.7 Mutation operation:

Mutation operation is used to prevent the search process from converging to local optima rapidly. Unlike crossover, it is applied to a single chromosome V_i. Here, the uniform mutation operation is used, which is defined as follows:

G_{ij}^{mut} =random number from the range (0, *UPB*)

where *UPB* is upper boundary to the corresponding gene.

4a.8 Termination:

If number of iteration is less than or equal to MAXGEN then the process continues, otherwise it terminates.

The basic structure of GA is described as follows:

$Genetic\ Algorithm()$
$begin$
$\quad t \leftarrow 0$
$\quad initialize\ Population(t)$
$\quad evaluate\ Population(t)$
$\quad while(not\ \ termination\ \ condition)$
$\quad\quad begin$
$\quad\quad\quad t \leftarrow t+1$
$\quad\quad\quad select\ Population(t)\ from\ Population(t-1)$
$\quad\quad\quad alter\ (by\ crossover\ and\ mutation)\ Population(t)$
$\quad\quad\quad evaluate\ \ Population(t)$
$\quad\quad end$
$\quad write\ the\ optimum\ result$
end

4b. Advantages of GA

The advantages of GA [Goldberg (1989)] include the followings:

(i) Simple: The algorithm is easy to develop and validate.

(ii) Efficient: The algorithm is parallel, using the resource of a whole population instead of a single individual. During the evolution, the different individuals can exchange information by crossover. External information is introduced by mutation. Hence the algorithm is efficient. Even if it begins with a very poor original population, it will progress rapidly towards satisfactory solutions.

(iii) Global optimum: Use of population, crossover and mutation leads the results toward the global optimum instead of trapping into local peaks.

(iv) Domain independent: The algorithm is a parametric method, suitable in a wide range of applications. It does not require pre-knowledge about data distribution, continuity or the existence of derivative.

5. NUMERICAL ILLUSTRATION

To illustrate the model, we consider:

$$r_1 = 0.81, C_{1s} = 0.5, C_{3s} = 40, \theta' = 0.16, \alpha = 16, \beta = 0.22, \mu = 0.01, c = 5.0, p = 9.1,$$
$$C_{1p} = 0.85, C_{2p} = 4.5, C_{3p} = 100, a = 0.2, d_1 = 75, d_0 = 40, \delta = 0.8, m' = 1.24, m_1 = 0.8.$$

The optimal values of $t', t_1, t_2, t_3, S, S_1, S_2, S_3$ and S_4 along with the average maximum average profit have been calculated for different sub-scenarios by GA and GRG (using standard software package **"Student LINGO/PC release 3.1 version**") and results are displayed in Table 1.

6. DISCUSSION

Table-1 gives the optimum values using genetic algorithm. From the table-1, it is observed that in the first case, when replenishment rate of deteriorated items is less than the demand rate, the scenario-1e gives more profit than the other four scenarios. The next preferable scenarios are 1d, 1b, 1a and 1c respectively. In the second case, when the replenishment rate is initially more and then gradually reduces to an amount less than the demand rate, the scenario-2e is better than the others. The next preferable scenarios are 2d, 2b, 2a and 2c respectively.

Table 1: Result for the model

Sub-scenarios	Methods	t' t_1	t_2 t_3	S S_1	S_2 S_3	S_4	$Z(\$)$
1a	GA	- 1.84	2.05 2.05	238.18 32.39	3.35 -	-	259.88
	GRG	- 1.83	2.02 2.02	236.29 29.04	3.01 -	-	259.80
1b	GA	- 1.75	2.03 1.77	220.14 40.89	0.23 -	-	261.45
	GRG	- 1.75	2.04 1.7	220.70 41.36	0.32 -	-	261.39
1c	GA	- 1.91	2.20 -	252.72 44.25	6.13 1.53	-	257.02
	GRG	- 1.91	2.19 -	253.15 42.64	6.21 1.77	-	256.93
1d	GA	- 1.74	1.98 1.74	218.74 34.50	0.00 -	-	261.75
	GRG	- 1.74	1.98 1.74	218.74 35.02	0.00 -	-	261.75
1e	GA	- 1.90	2.16 1.74	251.44 38.73	0.00 -	-	263.24
	GRG	- 1.90	2.16 1.74	251.44 39.90	0.00 -	-	263.24
2a	GA	1.54 1.86	2.15 2.15	243.63 42.24	4.36 0.00	7.31	259.21
	GRG	1.45 1.80	1.93 -	230.44 18.95	1.97 0.00	4.93	258.48
2b	GA	1.45 1.80	2.09 1.93	230.44 43.11	1.97 -	4.93	260.68
	GRG	1.54 1.86	2.15 2.15	243.63 42.56	4.36 -	7.31	259.19
2c	GA	1.54 1.86	2.02 -	243.63 22.90	4.36 1.97	7.31	256.72
	GRG	1.58 1.91	2.27 -	252.71 54.97	6.12 0.44	9.10	256.06
2d	GA	1.36 1.74	1.98 1.74	218.74 35.02	0.0 -	3.00	261.75
	GRG	1.37 1.74	1.99 1.74	218.74 36.61	0.00 -	2.99	261.73
2e	GA	1.31 1.71	1.95 1.65	213.02 34.15	0.00 -	2.13	262.27
	GRG	1.29 1.75	1.97 1.66	220.71 31.73	1.60 -	3.29	262.26

7. CONCLUSION

Here, for a retailer, after purchasing in a lot, the sale of both good and deteriorated items from two shops under a single management has been considered and solved via genetic algorithm. This phenomenon is very common in the developing countries like INDIA, BANGLADESH, and NEPAL etc. In these countries, there is a market for both fresh and deteriorated units. Hence, a realistic and common problem faced by the retailers has been investigated and optimum decisions are presented. These results are applicable for the products like fruits, vegetables etc. which are sold to the retailers in a lot. Present methodologies can be extended to other inventory models with All Unit Discount (AUD), Incremental Quantity Discount (IQD), fixed time horizon etc. These models can also be formulated and solved in probabilistic, fuzzy and fuzzy-stochastic environments and solved via genetic algorithm.

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