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Bright and Dark Solitons in Optical Fibers with Parabolic Law Nonlinearity

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Abstract: This paper utilizes the ansatz method to obtain bright and dark 1-soliton solution to the nonlinear Schrodinger's equation with parabolic law nonlinearity in birefringent fibers. There are a few Hamiltonian type perturbation terms taken into account. The exact soliton solution comes with baggages that are referred to as constraint conditions that must hold in order for these solitons to exist.

Keywords: Birefringence; Integrability; Solitons.

1 Introduction

Optical solitons is one of the most important topics of research in nonlinear fiber optics [1 – 10]. Solitons form the basic fabric of fiber optic communications across trans-continental and trans-oceanic distances. It is therefore imperative to take a deeper look at optical solitons from a different perspective.

This paper will study the dynamics of optical solitons when the optical fiber maintains cubic-quintic law of nonlinearity that is also known as the parabolic law nonlinearity. The focus will be on extracting exact bright and dark 1-soliton solution in birefringent fibers in presence of perturbation terms. While the norm is to look into optical fibers with Kerr law nonlinearity, this paper studies parabolic law fibers so that the results will stand on a generalized setting as compared to Kerr law that was studied earlier [2].

The ansatz approach will be the integration tool applied to obtain exact 1-soliton solution to the governing coupled nonlinear Schrodinger's equation (NLSE) with parabolic law nonlinearity in birefringent fibers. Using this approach both bright and dark soliton solutions will be obtained. There are several constraint conditions that will fall out. These constraints will be necessary conditions for bright and dark solitons to exist.

The perturbation terms that will be taken into account are inter-modal dispersion, nonlinear dispersion and self-steepening. These are all Hamiltonian

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type perturbations [4, 6, 7, 9]. Therefore the integrability aspect of the governing NLSE will not be hampered.

2 Governing Equation

The governing equation for the propagation of solitons through optical fibers is the NLSE. However, in presence of birefringence when the pulse splits into two parts, the corresponding model is the coupled NLSE. For parabolic law nonlinearity, in birefringent fibbers, this coupled equation reads

$$\begin{aligned} & iq_t + a_1 q_{xx} + \left(c_1 |q|^2 + d_1 |r|^2\right) q + \left(\xi_1 |q|^4 + \eta_1 |q|^2 |r|^2 + \zeta_1 |r|^4\right) q \\ & \quad + i\left\{\alpha_1 q_x + \lambda_1 \left(|q|^2 q\right)_x + \nu_1 \left(|q|^2\right)_x q + \theta_1 |q|^2 q_x\right\} = 0, \end{aligned} \tag{1}$$

$$\begin{aligned} & ir_t + a_2 r_{xx} + \left(c_2 |r|^2 + d_2 |q|^2\right) r + \left(\xi_2 |r|^4 + \eta_2 |r|^2 |q|^2 + \zeta_1 |q|^4\right) r \\ & \quad + i\left\{\alpha_2 r_x + \lambda_2 \left(|r|^2 r\right)_x + \nu_2 \left(|r|^2\right)_x r + \theta_2 |r|^2 r_x\right\} = 0. \end{aligned} \tag{2}$$

In (1) and (2) $q(x, t)$ and $r(x, t)$ are complex valued functions that represent the soliton profiles for the two components in birefringent fibers.

For $l = 1, 2$, a_l represents the group velocity dispersions GVD, c_l and d_l represents the self-phase modulation (SPM) and cross-phase modulation (XPM) terms respectively. From the perturbation terms α_l represents the inter-modal dispersion, λ_l is the self-steepening terms to avoid the formation of shock waves, v_l and θ_l are nonlinear dispersions. The terms with ξ_l, η_l and ζ_l are associated with the quintic terms of the cubic-quintic law of nonlinearity.

In order to obtain an exact bright and dark 1-soliton solution to previous equations we use the ansatz method. At the starting point, the solitons are considered in the phase-amplitude format as [6, 7]

$$q(x,t) = P_1(x,t)e^{\varphi_1(x,t)} = P_1(x,t)e^{i(-\kappa_1 x + \omega_1 t + \sigma_1)}, \tag{3}$$

$$r(x,t) = P_2(x,t)e^{\varphi_2(x,t)} = P_2(x,t)e^{i(-\kappa_2 x + \omega_2 t + \sigma_2)}, \tag{4}$$

where P_l for $l = 1, 2$ are the amplitude components of the solitons and $\phi_1(x, t)$ are its phase components, that are defined as

$$\varphi_l(x,t) = -\kappa_l x + \omega_l t + \sigma_l\,. \tag{5}$$

Here κ_l are the frequencies of the solitons in two components, ω_l are the wave numbers, while σ_l are the phase constants. Substituting (3) and (4) into (1) and (2) and decomposing into real and imaginary parts lead to

$$(\omega_l + a_l\kappa_l^2 - \alpha_l\kappa_l)P_l - d_lP_lP_{\bar{l}}^2 - \xi_lP_l^5 - \eta_lP_l^3P_{\bar{l}} - \zeta_lP_lP_{\bar{l}}^4$$
$$-(\lambda_l\kappa_l^2 + \theta_l\kappa_l + c_l)P_l^3 - a_l\frac{\partial^2 P_l}{\partial x^2} = 0 \tag{6}$$

and

$$\frac{\partial P_l}{\partial t} - (2a_l\kappa_l - \alpha_l + 3\kappa_l^2)\frac{\partial P_l}{\partial x} + (3\lambda_l + 2\nu_l + \theta_l)P_l^2\frac{\partial P_l}{\partial x} = 0 \tag{7}$$

respectively for $l = 1, 2$ and $\bar{l} = 3 - l$.

The rest of the section will study two different types of solitons, namely bright solitons and dark solitons. The ansatz approach will be our integration architecture.

2.1 Bright solitons

For bright solitons, the assumption is [1]

$$P_l(x,t) = \frac{A_l}{(D_l + \cosh\tau)^{p_l}}, \tag{8}$$

where

$$\tau = B(x - vt). \tag{9}$$

Here, A_l ($l = 1, 2$) represents the amplitude of the solitons for the two components and B is the inverse width of the solitons in both components and v is the speed of the solitons in both components. The two new parameters introduced are D_l for $l = 1, 2$. Substituting this hypothesis into the imaginary part equation (7) leads to

$$\left\{(v + 2a_l\kappa_l - \alpha_l)\right\} - \frac{(3\lambda_l + 2\nu_l + \theta_l)A_l^2}{\left(D_l + \cosh\tau\right)^{2p_l}} = 0. \tag{10}$$

Setting the coefficients of the linearly independent functions from (10) implies

$$v = \alpha_l - 2a_l\kappa_l \tag{11}$$

and

$$3\lambda_l + 2\nu_l + \theta_l = 0. \tag{12}$$

Equation (11) is the velocity of the soliton for two components and equation (12) represents the constraint condition in order for the soliton to exist. From (11) equating the velocity of the solitons in the two components leads to the constraint condition given by

$$2a_1\kappa_1 - \alpha_1 = 2a_2\kappa_2 - \alpha_2. \tag{13}$$

Next, the real part equation reduces to

$$(-\omega_l - a_l\kappa_l^2 + \alpha_l\kappa_l + p_l^2 a_l B^2) - \frac{a_l p_l (2p_l+1) D_l B^2}{D_l + \cosh\tau} + \frac{a_l p_l (2p_l+1)(D_l^2-1) B^2}{\left(D_l + \cosh\tau\right)^2}$$
$$+\frac{\left(\lambda_l\kappa_l + \theta_l\kappa_l + c_l\right) A_l^2}{\left(D_l + \cosh\tau\right)^{2p_l}} + \frac{d_l A_{\bar{l}}^2}{\left(D_l + \cosh\tau\right)^{2p_{\bar{l}}}} + \frac{\xi_l A_l^4}{\left(D_l + \cosh\tau\right)^{4p_l}} \tag{14}$$
$$+\frac{\eta_l A_l^2 A_{\bar{l}}}{\left(D_l + \cosh\tau\right)^{2p_l}\left(D_{\bar{l}} + \cosh\tau\right)^{p_{\bar{l}}}} + \frac{\zeta_l A_{\bar{l}}^4}{\left(D_{\bar{l}} + \cosh\tau\right)^{4p_{\bar{l}}}} = 0.$$

By using balancing principle, equating the exponents ($2p_l = 2p_{\bar{l}} = 1$) gives

$$p_l = p_{\bar{l}} = \frac{1}{2}\,. \tag{15}$$

Setting the coefficients of the linearly independent functions to zero leads to

$$\omega_l = \frac{a_l B^2 - 4\kappa_l (a_l\kappa_l - \alpha_l)}{4}, \tag{16}$$

$$B = A_l\left[\frac{\lambda_l\kappa_l + \theta_l\kappa_l + c_l}{D_l a_l}\right]^{\frac{1}{2}}, \tag{17}$$

and

$$D_l = \frac{1}{B}\left[\frac{3B^2 a_l - 4\xi_l A_l^4}{3a_l}\right]^{\frac{1}{2}}, \tag{18}$$

which poses the constraint condition

$$D_l a_l (\lambda_l\kappa_l + \theta_l\kappa_l + c_l) > 0 \tag{19}$$

and

$$a_l (3B^2 a_l - 4\xi_l A_l^4) > 0 \tag{20}$$

for $l = 1, 2$. Hence, finally the bright 1-soliton solution for parabolic law, in birefringent fibers is given by

$$q(x,t) = \frac{A_1}{\sqrt{D_1 + \cosh\left[B(x - vt)\right]}} e^{i(-\kappa_1 x + \omega_1 t + \sigma_1)} \tag{21}$$

and

$$r(x,t) = \frac{A_2}{\sqrt{D_2 + \cosh\left[B(x - vt)\right]}} e^{i(-\kappa_2 x + \omega_2 t + \sigma_2)}, \tag{22}$$

where the parameter specifications are just discussed in details above.

2.2 Dark solitons

For dark solitons, the assumption is [8]

$$P_l(x,t) = (A_l + B_l \tanh\tau)^{p_l}, \tag{23}$$

where the τ is defined as in (9). Here, A_l and B_l are free parameters. Substituting this hypothesis into the imaginary part equation (7) leads to

$$\begin{gathered}\left[B_l^2\left\{-v-2a_l\kappa_l+\alpha_l\right\}+\right](A_l+B_l\tanh\tau)^2+\\+(3\lambda_l+2\nu_l+\theta_l)(A_l+B_l\tanh\tau)^{2p_l+2}=0.\end{gathered} \tag{24}$$

From the linearly independent functions, one recovers (13) and

$$3\lambda_l + 2\nu_l + \theta_l = 0, \tag{25}$$

which is a constraint condition between the parameters. Then, the real part equation given by (6) simplifies to

$$\begin{gathered}B^2p_la_l(p_l+1)(p_l-2)(A_l+B_l\tanh\tau)^2-2B^2A_lp_la_l(2p_l+1)(A_l+B_l\tanh\tau)^3+\\+\left[B_l^2\left\{-\omega_l-a_l^2\kappa_l+\alpha_l\kappa_l\right\}+2p_l^2B^2a_l(3A_l^2-B_l^2)\right](A_l+B_l\tanh\tau)^2+\\+2p_l(2p_l-1)B^2A_la_l(B_l^2-A_l^2)(A_l+B_l\tanh\tau)+\\+B^2p_l(p_l-1)a_l(B_l^2-A_l^2)^2+(\lambda_l\kappa_l+\theta_l\kappa_l+c_l)(A_l+B_l\tanh\tau)^{2p_l+2}+\\+d_l(A_l+B_l\tanh\tau)^2(A_{\bar{l}}+B_{\bar{l}}\tanh\tau)^{2p_{\bar{l}}}+\xi_l(A_l+B_l\tanh\tau)^{4p_l+2}+\\+\eta_l(A_l+B_l\tanh\tau)^{2p_l+2}(A_{\bar{l}}+B_{\bar{l}}\tanh\tau)^{2p_{\bar{l}}}+\\+\zeta_l(A_l+B_l\tanh\tau)^2(A_{\bar{l}}+B_{\bar{l}}\tanh\tau)^{4p_{\bar{l}}}=0.\end{gathered} \tag{26}$$

The balancing principle yields the same value of p_l as given by (15) for bright solitons. The other parameter values that are obtained from the real part are

$$\omega_l = \frac{1}{8}\left(a_lB_l^3 - 8\alpha_l\kappa_l - 8a_l^2\kappa_l - 9a_l\right), \tag{27}$$

$$B_l = \left(\frac{\lambda_l\kappa_l + \theta_l\kappa_l + c_l + d_l}{2a_l}\right)^{\frac{1}{3}}, \tag{28}$$

$$\xi_l + \eta_l + \zeta_l = 0, \tag{29}$$

$$A_l = B_l. \tag{30}$$

Hence, finally, the dark 1-soliton solution in birefriengent fibers is given by

$$q(x,t) = \sqrt{A_1\left\{1+\tanh\left[B(x-vt)\right]\right\}}e^{i(-\kappa_1x+\omega_1t+\sigma_1)}, \tag{31}$$

$$r(x,t) = \sqrt{A_2\left\{1+\tanh\left[B(x-vt)\right]\right\}}e^{i(-\kappa_2x+\omega_2t+\sigma_2)}. \tag{32}$$

These are the exact dark 1-soliton solutions of the two components of the solitons in a birefringent fiber with parabolic law nonlinearity. The parameter definitions and the necessary constraints are in place.

3 Conclusion

This paper obtained the bright and dark 1-soliton solution to the NLSE with parabolic law nonlinearity. Several constraint conditions are in place in order for the solitons to exist. These constraints are necessary conditions.

In future, one can extend these results to the case of Thirring solitons, DWDM systems and other aspects. The results of those researches will be reported in future. These form a tip of the iceberg.

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