KINEMATICS OF MATERIAL POINT ON THE COCHLEOID

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Abstract. *They are shown the geometrical considerations and the properties of cochleoid, a flat curve in the spiral's family. Determine the positions, velocities and accelerations of material point in motion on the cochleoid. They are obtained diagrams, which are interpreted.*

Keywords: material point, kinematics, cochleoid

1. Introduction

The cochleoid (in English *cochleoid*, in Italian *cocleoide*, *Kochlias* in Greek) draws its name from the snail "shell". It is a flat curve of spiral type, similar to the vertical projection of the spatial curve representing the line describing the snail shell (Fig. 1). The cochleoid was researched by J. Peck in 1700, by Bernoulli in 1726 and in 1884 by Benthan and Falkenburg.

The cochleoid is a part of the spiral's family.

Fig. 1.*The snail shells*

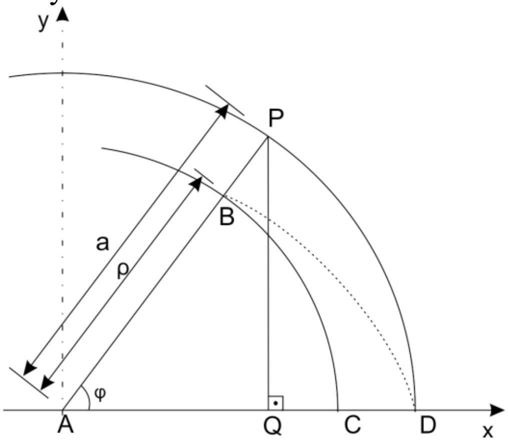

Fig. 2. *Geometry of cochleoid*

2. The cochleoid geometry. Material point positions

The BD cochleoid from Fig. 2 is described by B point on the AP radius, when P point goes on DP circle of radius AP = a = constant, with AB = ρ , the condition being that the PQ segment is equal to the BC arc, meaning:

$$\rho\,\varphi = a \sin\varphi \quad (1)$$

Are written the following relationships:

$$x_P = a\cos\varphi;\ y_P = a\sin\varphi \quad (2)$$

$$\rho = \frac{y_P}{\varphi} \quad (3)$$

$$x_B = \rho\cos\varphi;\ y_B = \rho\sin\varphi \quad (4)$$

In Fig. 3 it is shown the resulting cochleoid at one complete rotation of the AP segment of Fig. 2, to a= 50 mm. Similarly, in Fig. 4 is shown the cochleoid for two full rotations, and in Fig. 5 for three full rotations. It appears that by increasing the number of rotations, it increases the number of loops near the origin of axes.

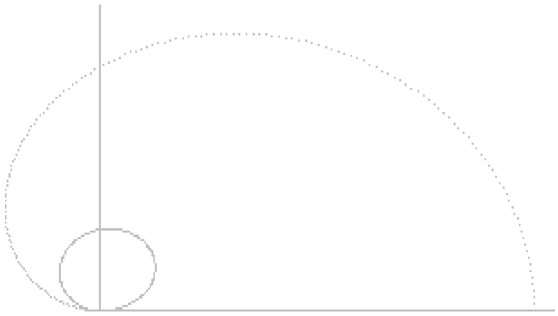

Fig. 3. *The cochleoid for one rotation*

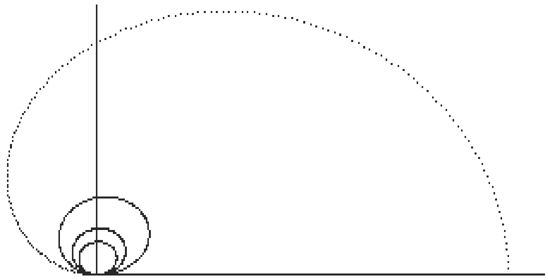

Fig. 4. *The cochleoid for two rotations*

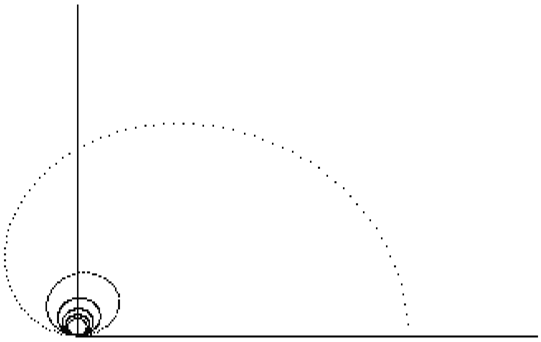

Fig. 5. The cochleoid for three rotations

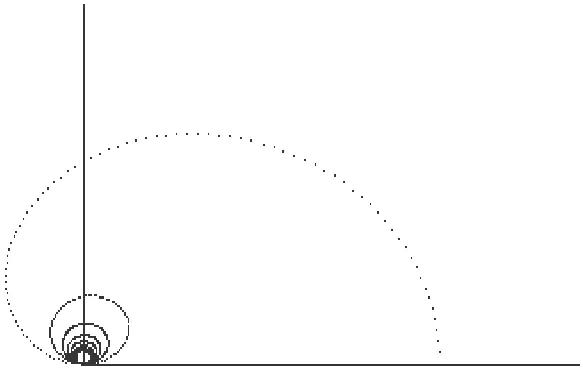

Fig. 6. The cochleoid for four rotations

The cochleoid branches, in a rotation case, are shown in Fig. 7 (φ=0…180), in fig. 8 (φ=180…270) and in fig. 9 (φ=270…360).

In Fig. 10 are given the variation diagrams in relation to φ, of the curvature radius ρ and of the tracer point coordinates, B (Fig. 2). It appears that ρ is maximumin D point, then it decreases, reaching a minimum for φ = 270 degrees, then increases slightly, never reaching the original position. x_B decreases, rises and falls again, and y_B increases, decreases, increases and then decreases again, reaching the initial value. All curves have the same end point, at zero value.

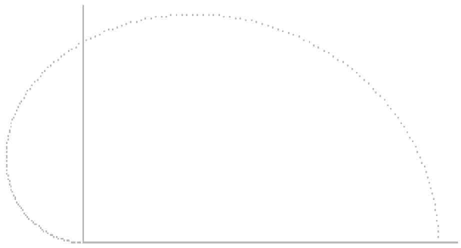

Fig. 7. *φ =0…180*

Fig. 8. *φ =180…270*

Fig. 9. *φ =270…360*

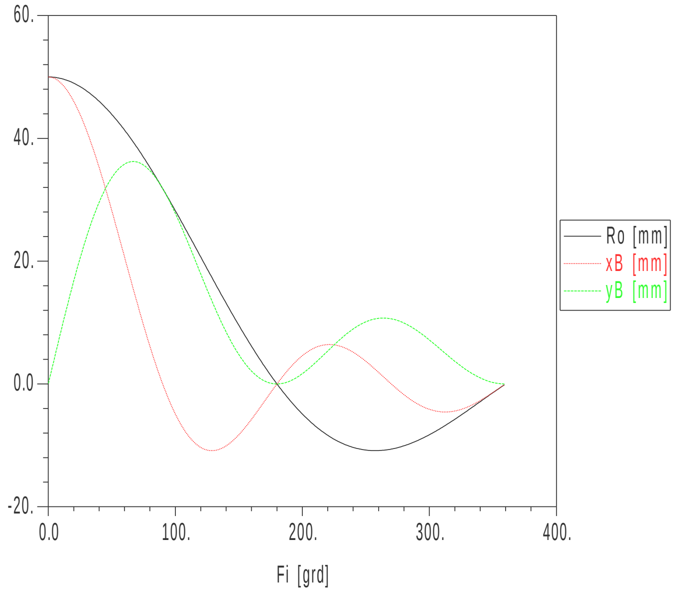

Fig. 10. *Material point positions*

3. Speed

By derivation the cochleoid geometry relations with respect to time, we obtain the speed of P and B points and radial velocity $\dot{\rho}$.

For φ'=2 and φ''=0,5 rad/sec/secwereobtainedresults intable1:

Radial velocity $\dot{\rho}$ decreases in the negative domain, then increases, becoming positive. The velocity components of tracer material point decrease, then rise, fall and rise again, being both negative and positive .

Table 1

Fi	ro'	xB'	yB'
10	-5.800058	-22.98872	96.97439
20	-11.49436	-44.31281	88.14125
30	-16.97945	-62.4511	74.20964
40	-22.15641	-76.15586	56.28984
50	-26.93297	-84.55726	35.79354
60	-31.22555	-87.23251	14.30764
70	-34.96095	-84.23375	-6.546013
80	-38.07794	-76.07236	-25.25167
100	-42.27863	-48.22663	-51.43443
110	-43.30928	-31.18154	-57.43785
120	-43.61625	-14.0019	-58.44763
130	-43.21036	1.91144	-54.80315
140	-42.11688	15.3538	-47.22421
150	-40.37488	25.41627	-36.72743
160	-38.03618	31.55328	-24.51834
170	-35.16392	33.61338	-11.86993

180	-31.83104	31.83104	-1.462-04
190	-28.1185	26.78206	10.03951
200	-24.11331	19.308	17.45438
210	-19.90645	10.41865	21.76734
220	-15.59077	1.182759	22.84546
230	-11.25887	-7.381344	20.89117
240	-7.000944	-14.40439	16.40044
250	-2.902874	-19.2445	10.09367
260	.9557193	-21.5383	2.827393
280	7.676899	-18.51274	-11.05955
290	10.42534	-13.88044	-16.14641
300	12.70813	-7.969998	-19.27549
310	14.49713	-1.527531	-20.20634
320	15.77664	4.687604	-18.95755
330	16.54349	9.986381	-15.78997
340	16.80669	13.82177	-11.16439
350	16.58689	15.84123	-5.679902

Omega=2 rad/s; epsilon = 0,5 rad/s/s

4. Accelerations

By derivation the speeds from relations with respect to time, we obtain the accelerations of P and B points and $\ddot{\rho}$.

Table2showsthe resultsfor ρ '', x_B''andy_B'' in relation to φ. Sinceat φ =0, sin φ isinfinite, we obtain veryhighaccelerationsnear φ =0. It appearsthatthese valuesincrease, decrease, increaseanddecreaseagain,withdifferentsizesanddomains, bothinpositiveandnegative area. Valuesarelargeat the beginning ofthe movementandsmallat the end ofthe movement.

Table 2

Fi	ro''	xB''	yB''
10	-26186.84	-25985.25	-4580.199
20	-6395.718	-6186.807	-2274.672
30	-2718.757	-2497.89	-1493.014
40	-1429.797	-1194.18	-1087.679
50	-834.0666	-583.2628	-828.5643
60	-512.5046	-248.6886	-639.1949
70	-321.199	-49.12882	-487.6339
80	-199.8741	73.42921	-359.1448
100	-65.17083	183.5662	-148.4004
110	-27.75072	194.2633	-63.00017
120	-2.055363	184.5161	8.663633
130	15.31739	159.4968	65.68124
140	26.64342	123.9555	107.3224
150	33.50514	82.42626	133.3815
160	37.04057	39.20113	144.3825
170	38.09296	-1.815992	141.6601
180	37.30334	-37.3029	127.3242

190	35.16977	-64.70713	104.1289
200	32.08591	-82.39183	75.2621
210	28.36703	-89.71284	44.08644
220	24.26767	-87.01424	13.85911
230	19.99408	-75.53848	-12.53823
240	15.713	-57.25962	-32.83126
250	11.55818	-34.65546	-45.52295
260	7.635183	-10.44133	-49.98557
280	.7883297	32.41508	-36.01056
290	-2.034423	46.82875	-20.30532
300	-4.41833	54.77201	-1.473031
310	-6.353242	55.82835	18.17367
320	-7.841673	50.34072	36.38278
330	-8.897075	39.33333	51.19588
340	-9.542161	24.36587	61.13935
350	-9.807299	7.33881	65.35541
360	-9.72898	-9.728541	63.6621

Omega=2 rad/s; epsilon = 0,5 rad/s/s

5. Movement modeling

They sought successive positions of AB vector radius (Fig. 2), yielding the image in Fig. 11.In Fig. 12 were represented the components of tracer point speed.

Based on Fig. 12, are written the relations:

$$tg\gamma = \frac{\dot{y}_B}{\dot{x}_B}; \quad \cos\gamma = \frac{\dot{x}_B}{v_B}; \ \sin\gamma = \frac{\dot{y}_B}{v_B} \tag{5}$$

$$x_E = \rho\cos\varphi + v_B\cos(\varphi + 180 - \varphi - \gamma) \tag{6}$$

$$x_E = \rho\cos\varphi - v_B\cos\gamma; \qquad y_E = \rho\sin\varphi - v_B\sin\gamma \tag{7}$$

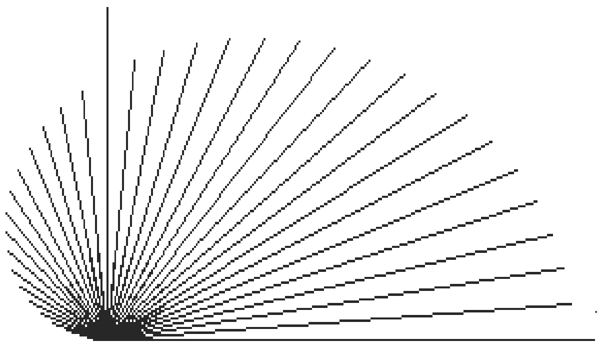

Fig. 11. *Vector radius*

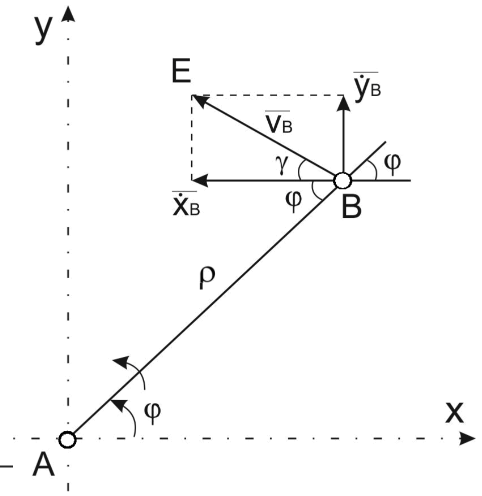

Fig. 12. *Velocity components of tracer point*

Resulting velocity directions were sought for the tracer point. In Fig. 13 are shown these directions. Directions are accurate, but velocity vector directions require discussions, in conjunction with the variations of velocity components in Fig. 9, requiring correlation with the trigonometric circle dials.

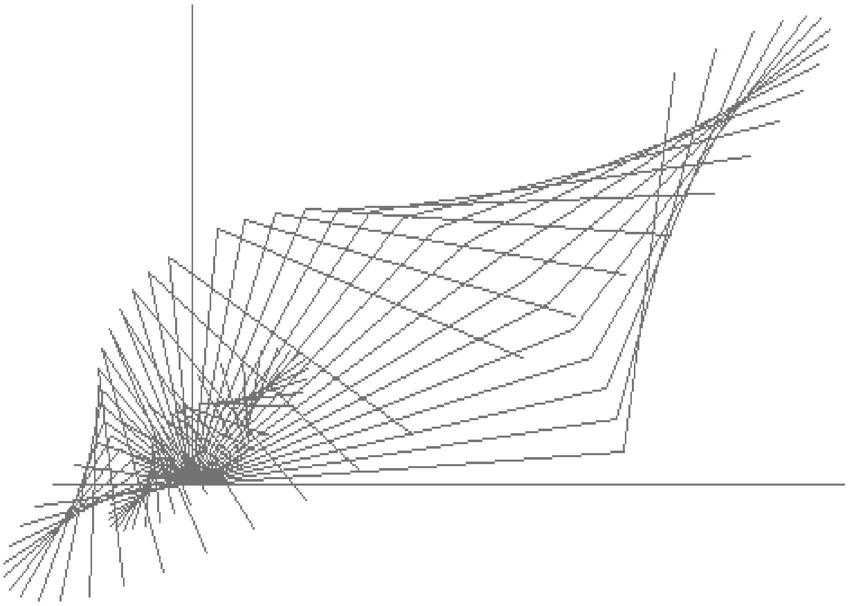

Fig. 13. *Resultant velocity directions*

It thus appears that velocity vectors v_B are not perpendicular to the vector rays, which is known for other curves also, except the circle [4].

6. Conclusions

The cochleoid is a curve of spirals family, similar to plan projection of space curve which is specific to snail shell.

The cochleoid branches are still being increasingly smaller with increasing the current angle.

Speed and acceleration of material point in moving on a cochleoid vary by nonlinear laws, with elevated values at first and becoming smaller towards the end of the range.

Velocity vectors are not perpendicular to the vector radius.

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