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An influence of static load on fatigue life of parts under combined stress

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Abstract

The paper deals with a special case of multiaxial fatigue in a plain stress possessing one component static and the other dynamic. Exponents of Haigs' limit curves were obtained experimentally both for tensile fatigue test and combined tensile/torgue tests. Errors of estimated fatigue lives are less than 20 %.

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1. Introduction

There are cases often occurring in technical practice when a structure assembly in operation is subjected to loading by a combination of statically and dynamically acting forces. As an example, we can indicate various structure assemblies of vehicle undercarriage frames, turbine blades, bridge structure assemblies and also small components axially pre-stressed by e.g. a high static force transferring a dynamically acting torsion moment. Characteristic for this type of loading is that the system is loaded by a space system of forces and moments of which some act statically and do not cause fatigue failure by themselves, others act dynamically and basically participate in the fatigue damage process and consequently in the resulting fatigue life of the monitored structural unit. It is known based on the obtained knowledge that also the above mentioned statically acting components of loading more or less, and often very significantly, influence the life.

Our contribution will focus on solving this issue in a simple structural assembly such as a tubular specimen with a transverse hole which was subjected to a combination of axial tension and torsion. One of the above components of loading always acted statically, the other dynamically with a harmonic as well as random character of loading. Patterns accompanying the above way of loading were searched for and calculation methodology, practicable when considering the patterns for application in practice, was proposed.

2. Influence of static component of loading

Already in some of our previous works [1] and [2] we pointed out some regularities by which the simultaneous acting of the static component of loading shows itself on the fatigue life of the dynamically stressed machine part. The performed works showed that the dynamic component

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is always decisive for the direction of fatigue crack propagation in the part subjected to a combination of static and dynamic components of stress. Fatigue cracks also propagate in accordance with the dynamic stress character. The static component, however, asserts in the crack propagation velocity affecting by that also the overall life. This life time decreasing shows itself both in the area of short and long cracks. A SN curve of material is influenced by the static component so that it shifts the slant branch of the curve left, to the area of lower lives and decreases the resulting value of the given component fatigue limit. Fig. 1 shows the effect of static axial load acting in combination with the dynamic torsion on the tubular specimen with a transverse hole.

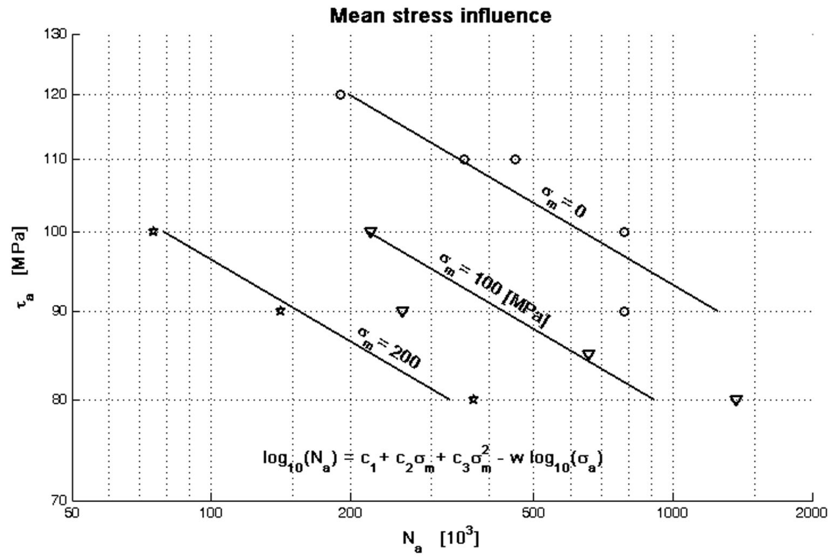

Fig. 1. Influence of static pre-stress on position of SN curve

It is obvious from fig. 1 that with the increase of the tension static component of the direct stress σ_m from the axial load, the regression lines of the SN curve slanted branch move to the area of lower lives, nevertheless, the curves slope (i.e. their exponent w) is not changed and in the given case, remains identical with the basic fatigue curve slant branch exponent for a dynamically acting component of torsional stress τ_a. The same effect was found for the case of statically acting component of torsional stress τ_m and dynamically acting axial stress σ_a from tension-pressure.

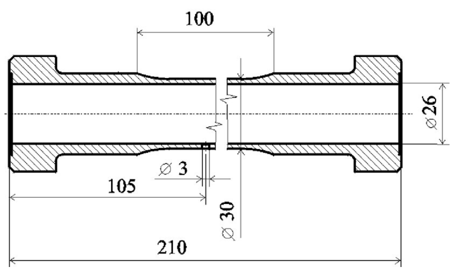

Fig. 2. Dimensions of test specimen

3. Life calculation for a given combination of loads

Let us assume a component element loaded as per the below fig. 3 In the given case, the tubular specimen was subjected to static axial stress σ_m and dynamic torsional stress τ_a. Parameters of a fatigue curve of a specimen under fig. 2 made of material 11523.1 with a transverse hole 3 mm in diameter have been determined by experiment and are gathered in tab. 1

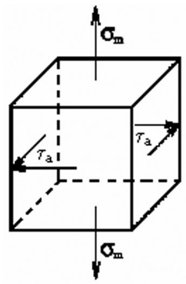

Fig. 3. Stress situation on an element

Table 1. Parameters of SN curve of the specimen for alternating torsion

SN curve	$\log_{10} N_{a\tau} = 14.7679 - 4.5226 \log_{10} \tau_a$
Fatigue limit of notched specimen	${\tau_c}^* = 80\,\text{MPa}$
Number of cycles to failure	${N_{c\tau}}^* = 1.448\,723\,10^6$
Slanted branch exponent	${w_\tau}^* = 4.522\,6$

The torsion strength value was estimated as per literature based on the tensile strength using relation $R_{\mathrm{m}\tau} = 0.6\,R_{\mathrm{m}\sigma} = 0.6 \times 550.87 = 330.5\,\text{MPa}$. When calculating the life, we start from the above experimentally verified assumptions that the life curve due to positive static stress shifts left to the area of lower number of cycles, nevertheless, it remains parallel with the basic fatigue curve determined for dynamically acting torsion. At the same time, the fatigue limit value is decreased. For the calculation of the fatigue limit $\tau_{c\sigma} = \tau_c(\sigma_m)$ and the breakpoint $N_{c\tau\sigma}$ of this new fatigue curve, we will use the below relations, already indicated in [1]

$$N_{c\tau\sigma} = N_{c\tau}\,[1 - (\sigma_m/R_{\mathrm{m}\sigma})^2] = N_{c\tau}\,g_{\tau\sigma}\,, \tag{1}$$

$$\tau_{c\sigma} = \tau_c\,(1 - \sigma_m/\sigma_F)^{k_{\mathrm{H}\tau\sigma}} = \tau_c\,h_{\tau\sigma}\,. \tag{2}$$

The first formula is parabolic and says that under $\sigma_m = 0$, the $N_{c\tau\sigma} = N_{c\tau}$, while for $\sigma_m = R_{\mathrm{m}\sigma}$ the number of cycles is zero as the σ_m has already reached the ultimate strength. The second equation describes a so called Haigh diagram and indicates how the critical amplitude of stress $\tau_{c\sigma}$ changes in our case with the mean cycle stress σ_m. The searched life (a number of cycles) $N_{a\tau\sigma}$ corresponding to an arbitrary level of torsional stress τ_a can then be calculated for the given static component of stress σ_m from the relation

$$N_{a\tau\sigma} = (\tau_{c\sigma}/\tau_a)^{w_\tau}\,N_{c\tau\sigma}\,. \tag{3}$$

However, when applying the calculation, we encounter a problem how to select a value of the Haigh diagram limit line exponent $k_{\mathrm{H}\tau\sigma}$ in the equation (2). The performed calculation result significantly depends on the correct selection of this exponent. In order to obtain further information about its possible selection in dependence on the stress, we realized a simple experimental program using the above mentioned tubular notched specimens which were subjected to life monitoring at various combinations of tensile and torsion loading where one component of this combined load was acting statically and the other dynamically. Knowing the actual life $N_{c\sigma}$ corresponding in our case to the value of the statically acting tension σ_m, we can first calculate the value of limit stress $\tau_{c\sigma}$ from the equation (3). Substituting this value in the relation (2) we obtain the value of the searched exponent $kHts$ from the relation

$$k_{\mathrm{H}\tau\sigma} = \frac{\log_{10}(\tau_{c\sigma}/\tau_c)}{\log_{10}(1 - \sigma_m/R_{\mathrm{m}\sigma})}\,, \tag{4}$$

provided that we substitute a value of tensile strength $R_{\mathrm{m}\sigma}$ for the fictitious stress σ_F. We used the same procedure also in the case when the torsional stress component τ_m was acting statically in combination with the dynamically acting tension-pressure component σ_a, or in the case of unsymmetrical uniaxial tension-pressure with values σ_m and σ_a. In such cases, a fatigue curve was used for given tubular specimens and its parameters were determined experimentally, as follows:

Table 2. Parameters of a fatigue curve for a test specimen under alternating tension-pressure

SN curve	$\log_{10} N_{a\sigma} = 23.785\,5 - 8.361\,9 \log_{10} \sigma_a$
Fatigue limit of notched specimen	$\sigma_c^* = 120\,\mathrm{MPa}$
Number of cycles to failure	$N_{c\sigma}^* = 2.509\,544\,10^6$
Slanted branch exponent	$w_\sigma^* = 8.361\,9$
Tensile strength	$R_{\mathrm{m}\sigma} = 550.87\,\mathrm{MPa}$

The equations (1)–(4) have been written as if the specimens were plain. However, in fact, the specimens were notched by a lateral hole, what caused that all quantities influenced by the notch ought to have asterisk in superscript positions just like in tab. 1 and tab. 2.

4. Test results

The fatigue life tests were performed on tubular specimens made of material 11523.1 with a transverse hole 3 mm in diameter. The specimens dimensions are shown in fig. 2. The following tab. 3 shows lives to failure for applied combinations of specimen loadings.

Table 3. Experiment results

Dynamic stress component	Static stress component	Number of cycles to failure
$\tau_a = 100\,\mathrm{MPa}$	$\sigma_m = 100\,\mathrm{MPa}$	222 042
	$\sigma_m = 200\,\mathrm{MPa}$	75 555
$\sigma_a = 140\,\mathrm{MPa}$	$\tau_m = 70\,\mathrm{MPa}$	482 885
	$\tau_m = 140\,\mathrm{MPa}$	171 062
$\sigma_a = 90\,\mathrm{MPa}$	$\sigma_m = 200\,\mathrm{MPa}$	464 000
	$\sigma_m = 250\,\mathrm{MPa}$	300 794

The results, as indicated in the table, were processed using the above mentioned procedure. Regression curves of functions $\tau_{c\sigma} = \tau_c(\sigma_m)$, $\sigma_{c\tau} = \sigma_c(\tau_m)$ and $\sigma_{c\sigma} = \sigma_c(\sigma_m)$, from which the mean values of the searched exponents k_{H} were evaluated for the assumed polytropic dependence under the formula (2) of the Haigh diagram limit line, are shown in fig. 4–6.

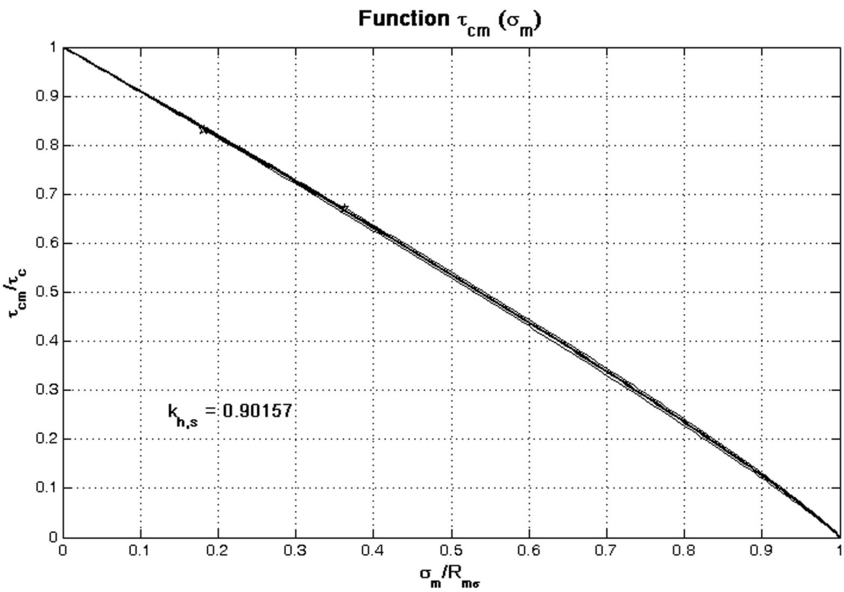

Fig. 4. Standardized Haig diagram for $\tau_{c\sigma} = \tau_c(\sigma_m)$

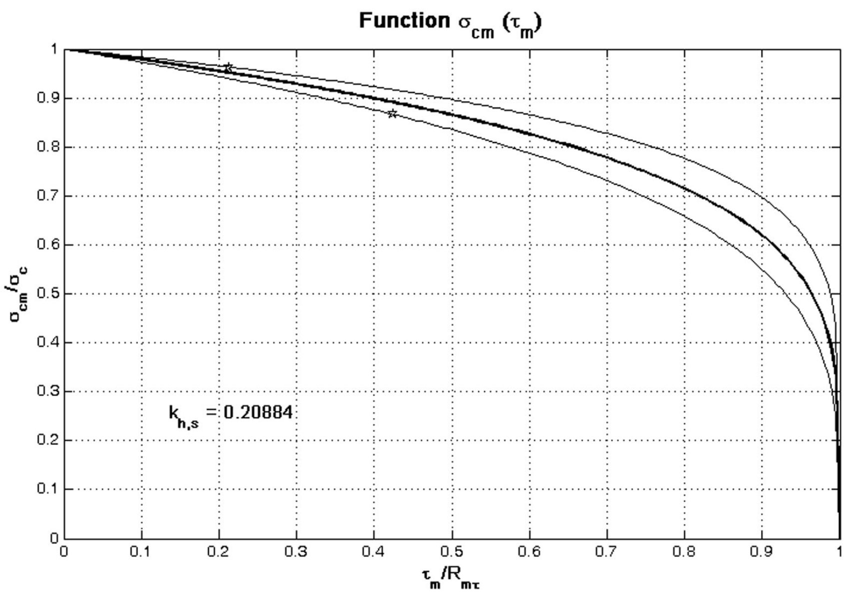

Fig. 5. Standardized Haig diagram for $\sigma_{c\sigma} = \sigma_c(\tau_m)$

It results from the comparison of those figures that the regression curves in fig. 4 and fig. 6 do not differ too much while the curve in fig. 5 differs substantially. The values of respective exponents k_{H} of the limit line in the Haigh diagram are also related to that. In the first two combinations of loading, the exponents kH are about a value of $0.9 \div 0.95$, while in the other around a value of 0.2. It seems from the above that the exponent size is decided by the character of the static load component σ_m, possibly τ_m. If the static load component is normal, the exponent k_{H} shall be selected higher, around a value of 0.9; if the component is torsional, the exponent k_{H} shall be selected lower, around a value of 0.2. The character of dynamic component of loading is irrelevant.

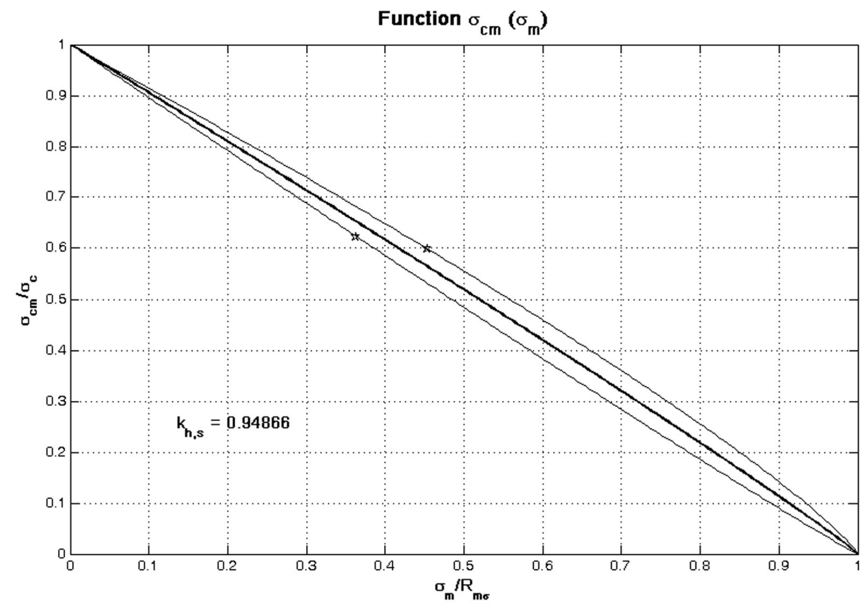

Fig. 6. Standardized Haig diagram for $\sigma_{c\sigma} = \sigma_c(\sigma_m)$

The above mentioned calculation procedure applied for the uniaxial harmonic loading combined with additional static component of force allows, using a relatively simple method, to determine a share of the static component in the decrease of life of a component subjected to stress by harmonic loading. At that, it does not matter whether this static component is a part of the dynamic component of loading under uniaxial stress (unsymmetrical loading) or whether it acts independently as a component of multiaxial combined loading. In both the cases, the calculation always results from the knowledge of fatigue curve parameters applicable for the dynamically acting load and a given material. This curve can be obtained experimentally, in that case we can assume a very good agreement of the calculation with the reality, or synthetically in cases the experiment would be too expensive and hence unfeasible. The advantage of the described approach is that the calculation is not application-demanding and does not require to make complicated stress analyses.

It is obvious from the tab. 4 that as far as we reliably know the fatigue curve parameters for a component symmetrically loaded by uniaxial stress of any character, then the effect of the additional static component on the resulting life as determined by the above calculation would not show an error higher than 40 % when using the above values of the Haigh diagram exponents $k_{\rm H}$, with the biggest errors occurring in case when the static and dynamic components are normal.

Table 4. Comparison of calculated lives with the experiment at the mean values of $k_{\rm H}$

Dynamic stress component	Static stress component	Number of cycles to failure		Error
		Experiment	Calculation	%
$\tau_a = 100\,\text{MPa}$	$\sigma_m = 100\,\text{MPa}$	222 042	225 642	1.6
	$\sigma_m = 200\,\text{MPa}$	75 555	72 867	−3.6
$\sigma_a = 140\,\text{MPa}$	$\tau_m = 70\,\text{MPa}$	482 885	435 885	−9.8
	$\tau_m = 140\,\text{MPa}$	171 062	216 823	+21.1
$\sigma_a = 90\,\text{MPa}$	$\sigma_m = 200\,\text{MPa}$	464 000	674 394	+31.2
	$\sigma_m = 250\,\text{MPa}$	300 794	182 182	−39.4

The calculated number of cycles to failure in tab. 4 has been obtained by running the MATLAB program bellow. At the beginning, it collects necessary data and then evaluates formulae given by equations (1)–(3). The function inp.m, used in the program, is available at the MathWorks collection of users' functions [3].

```
%   Flife.m     Fatigue life
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%   Miroslav Balda
%   2008-08 27
%

%      sig    tau
Rm  = 550.87*[1, 0.6];
sc  = [120, 80 ];
Nc  = [2.509544e6, 1.448723e6];
w   = [8.3619, 4.5226];
kH  = [0.94866, 0.20884
       0.90157, NaN];

format long

while 1
    Sa = inp('typ   sa','s');
    if Sa=='s'          %   sigma_a
        i = 1;
    elseif Sa=='t'
        i = 2;          %   tau_a
    else
        break
    end

    Sm = inp('typ   sm','s');
    if Sm=='s'
        j = 1;
    elseif Sm=='t'
        j = 2;
    else
        break
    end
```

```
    sa = inp([Sa '_a    '],0);
    while 1
        sm = inp([Sm '_m    '],0);
        if sm==0, break, end
        Ncam = Nc(i)*(1-(sm/Rm(j))^2)
        scam = sc(i)*(1-sm/Rm(j))^kH(i,j)
        Naam = Ncam*(scam/sa)^w(i)
    end
end

format short
```

5. Conclusion

The contribution is focused on solving the issue of the influence of static pre-stress on the life of components subjected to a combination of tension and torsion. A knowledge that the positive static stress component shifts the slant branch of the common fatigue curve left to the area of lower lives without affecting its slope was used for the solution. Relations were derived for the calculation of this new curve the accuracy of which significantly depends on the correct selection of the exponent kH of the Haigh diagram limit line. A procedure for the life time calculation was proposed at the same time. It was found that the size of exponent k_{H} depends on the static component type in the testing on tubular specimens with a transverse hole made of material ČSN 41 1523.1. In case of the normal static component σ_m, the value of k_{H} is about in a range of 0.9 to 0.95. In case of torsion (component τ_m) it is suitable to select $k_{\mathrm{H}} = 0.2$. The error in calculation is lower than 20 %.

Acknowledgements

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