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APPLICATION OF THE FINITE-ELEMENT METHOD FOR DETERMINING THE STIFFNESS OF ROLLING BEARINGS

Summary. The paper presents the results of numerical tests performed with use of the FEM method, the aim of which was determining the stiffness of the outer raceway - rolling element - inner raceway system of bearing 6307. The characterization obtained has been compared with a characterization determined with analytical methods and in the next stage, it will be used to determine the total stiffness of the bearing, variable in working time. Correct modelling of bearing stiffness is one of important conditions for obtaining correct results of simulation calculations. Obtained results will allow the determination of possibilities of limiting vibroactivity of toothed gears, commonly used in transport.

ZASTOSOWANIE METODY ELEMENTÓW SKOŃCZONYCH DO WYZNACZANIA SZTYWNOŚCI ŁOŻYSK TOCZNYCH

Streszczenie. W artykule przedstawiono wyniki przeprowadzonych badań numerycznych z użyciem metody MES, których celem było określenie sztywności układu bieżnia zewnętrzna - element toczny - bieżnia wewnętrzna łożyska 6307. Otrzymana charakterystyka została porównana z charakterystyką wyznaczoną metodami analitycznymi i w kolejnym etapie zostanie użyta do wyznaczenia całkowitej, zmiennej w czasie pracy, sztywności łożyska. Prawidłowe zamodelowanie sztywności łożysk, jest jednym z warunków uzyskania poprawnych wyników obliczeń symulacyjnych, prowadzonych w celu znalezienia możliwości ograniczenia wibroaktywności przekładni zębatych, powszechnie stosowanych w transporcie.

1. INTRODUCTION

Applying the FEM method for modelling a system comprised of an inner and outer raceway of the bearing and the rolling element between them is the next step of works on the method of calculating stiffness of bearings. This method is being developed for the needs of a model stand for testing toothed gears operating in a circulating power system [1]. Correct modelling of bearing stiffness is one of important conditions for obtaining correct results of simulation calculations and so it will allow reduction of the number of laboratory tests. It is expected that the results obtained with use of the model developed will allow the determination of possibilities of limiting vibroactivity of toothed gears, being the major element of power transmission systems.

In the discussed calculation method, it was assumed that the bearing stiffness depends on the stiffness of the outer raceway - rolling element – inner raceway systems being under load [7]. These stiffnesses are nonlinear functions of load imposed on the rolling element, thereby dependent on its position in relation to the direction of the force. Such approach allows taking into consideration disfunctions of the vibration signal resulting from the changeable bearing stiffness caused by both, changes in the position of rolling elements in relation to the direction of the force and damage and wear and tear of the interacting bearing elements.

2. METHODS OF DETERMINING THE STIFFNESS OF BEARINGS

Descriptions of many methods of calculating bearing stiffness can be found in professional literature. The ones more precise require much data concerning the elements of a bearing [2], which most often are not available in manufacturers' catalogues. For this reason, approximate methods are used most frequently, which allow determining the values of bearing deformation, depending on the value of load imposed on the bearing.

On the modelled stand, eight single-row ordinary ball bearings 6307 are mounted. Radial dislocation of the journal of such bearing, determined as a function of maximal load of the rolling part and of the rolling element's diameter, is described by dependence (1) [2, 4]. If, additionally, the number of rolling elements is known, dependence (2) can be used [2, 3].

$$\delta_r = \frac{0{,}44 \cdot Q_{\max}^{\frac{2}{3}}}{D_t^{\frac{1}{3}} \cdot \cos\alpha}, \quad [\mu m] \tag{1}$$

where: Q_{max} – maximal load of the rolling part [N], D_t – rolling element diameter [mm], α – bearing operation angle [rad].

$$\delta_r = 0{,}96 \cdot \sqrt[3]{\frac{Q^2}{0{,}1 \cdot d_k}}, \ [\mu m]; \quad Q = \frac{R}{2e} \tag{2}$$

where: d_k – diameter of the bearing ball [mm], R – radial load of bearing [N], e – number of rolling elements in the bearing.

For small values of the bearing operation angle, whose value is used in formula (1), considerable occurrence of stiffness characteristics can be obtained for a number of rolling elements equal to 5, however, the 6307 bearings installed on the stand, depending on the manufacturer, have 7 or 8 rolling elements.

The method suggested in this paper [7] requires determining the number of rolling elements under load as well as the stiffness characterization of one system: outer raceway – rolling element - inner raceway. In order to determine the number of rolling elements under load, the knowledge is necessary concerning the size of radial clearance of the bearing (for the 6307, the adequate values were taken from [5]) and the load distribution angle, ψ_ε – Table 1.

In order to determine the load distribution on individual rolling elements and the maximal load δ_{max} of the rolling part, a notion of the load distribution angle coefficient is introduced [4]:

$$\varepsilon_d = \frac{\delta_{\max}}{2\delta_{\max} + g} = \frac{1}{2}\left(1 - \frac{g}{2\delta_{\max} + g}\right) = \frac{1}{2}\left(1 - \cos\psi_\varepsilon\right) \tag{3}$$

Tab. 1

Load distribution angle ψ_ε for various values of radial clearance g and deformation δ_{max} of bearing 6307

ψ_ε [°]	C2 clearance [μm]		normal clearance [μm]		C3 clearance [μm]	
	min	max	min	max	min	max
δ_{max}	**1**	**11**	**6**	**20**	**15**	**33**
10	87,3	69,2	76,7	60,0	64,6	51,5
20	88,6	77,5	82,5	70,5	74,2	63,1
30	89,1	81,1	84,8	75,5	78,5	69,2
40	89,3	83,1	86,0	78,5	80,9	73,0
50	89,4	84,3	86,8	80,4	82,5	75,6
60	89,5	85,2	87,3	81,8	83,6	77,5
70	89,6	85,8	87,6	82,8	84,4	79,0
80	89,6	86,3	87,9	83,6	85,1	80,2
90	89,7	86,7	88,2	84,3	85,6	81,1
100	89,7	87,0	88,3	84,8	86,0	81,9
110	89,7	87,3	88,5	85,2	86,3	82,5
120	89,8	87,5	88,6	85,6	86,6	83,1
130	89,8	87,7	88,7	85,9	86,9	83,5
140	89,8	87,8	88,8	86,2	87,1	83,9
150	89,8	88,0	88,9	86,4	87,3	84,3
160	89,8	88,1	88,9	86,6	87,4	84,6

It arises from here that the load distribution area, ψ_ε, depends on the maximal deformation δ_{max} and clearance g in the bearing [4]:

$$\psi_\varepsilon = \arccos\left(\frac{g}{2\delta_{\max} + g}\right) \tag{4}$$

Load distribution on the rolling elements in the bearing, depending on the value of the load distribution angle is shown in Fig. 1 [4].

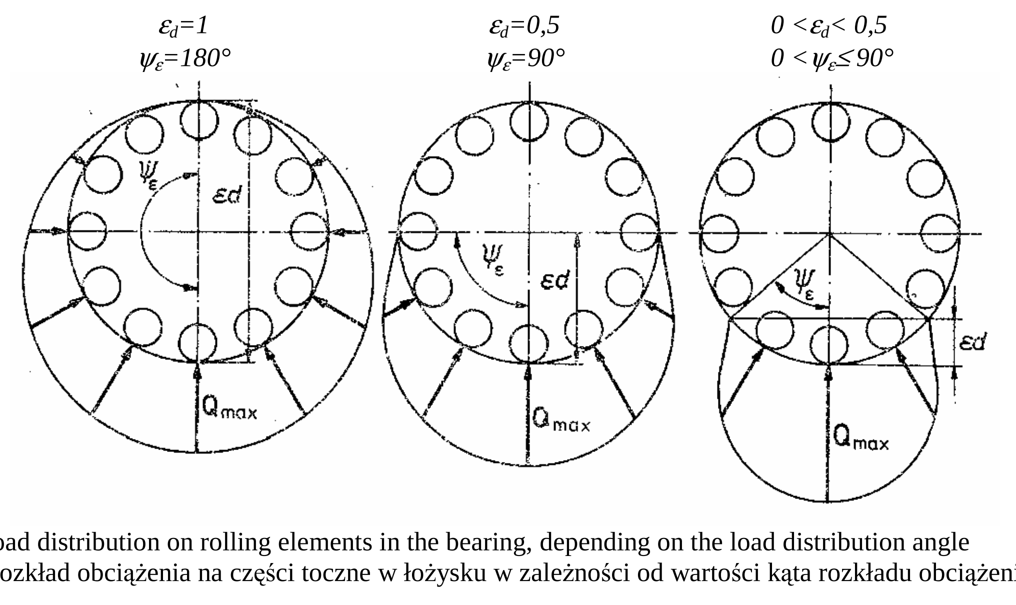

Fig. 1. Load distribution on rolling elements in the bearing, depending on the load distribution angle
Rys. 1. Rozkład obciążenia na części toczne w łożysku w zależności od wartości kąta rozkładu obciążenia

An aspect of the bearings' modelling method described in [7], more difficult to solve, is correct determination of stiffness of the raceways – rolling element system. A characterization of stiffness of a rolling element – raceways system, determined by analytical methods based on geometrical dependencies in the bearing and with use of the above formulas, is shown in Fig. 2 [7].

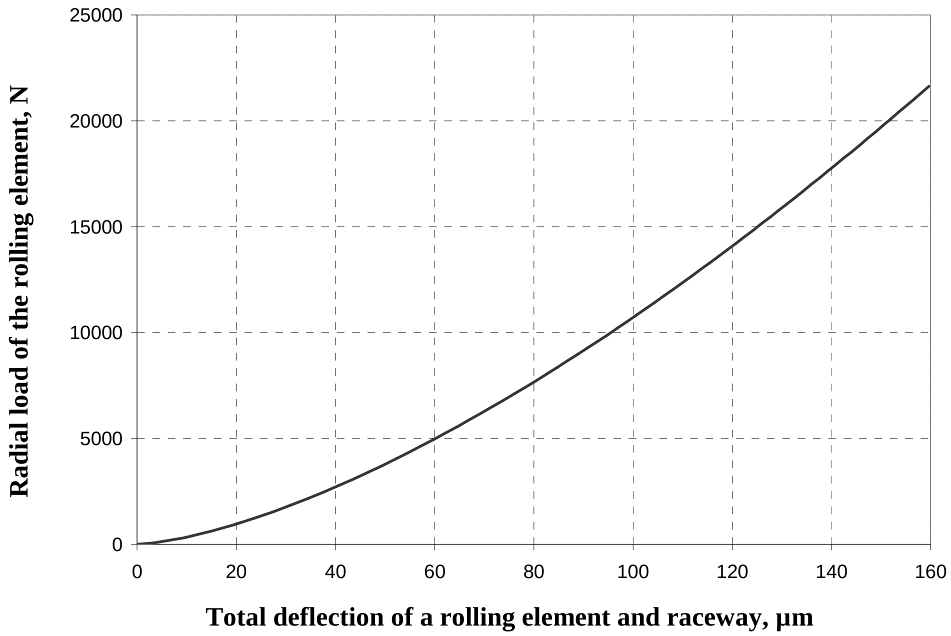

Fig. 2. Stiffness of a rolling element – bearing's raceways system determined analytically
Rys. 2. Wyznaczona analitycznie sztywność układu element toczny - bieżnie łożyska

3. DETERMINING THE STIFFNESS OF A ROLLING ELEMENT – BEARING RACEWAYS SYSTEM BY FEM

In order to verify the stiffness characterization obtained analytically, the researchers decided to model the rolling element – bearing raceways system and determine the stiffness with the FEM method. Correct modelling of this system is a determinant of preparing a correct model of the whole bearing, in which the determined characterization may be one of the input data. Stiffness characterization of such system is non-linear and, since the connection transmits only compression, unilateral [6].

Since the FEM models allow free representation of the raceway surface, it is possible to [6]:

- determine the real characterization of force – deflection,
- determine the dependencies between the force and the state of stress in the rolling element and in raceways,
- determine plastic deformations of the raceway,
- taking into account the rolling expansion (plastic deformation) and wear of the raceway.

When determining stiffness of the outer raceway – rolling element – inner raceway system, it is possible to use a discrete 2-D circular symmetric model through linearization from the radial direction to the circumferential direction of the bearing [6]. In the study, however, the authors decided to apply three-dimensional models, which allow direct usage of the results obtained. For their creation, a geometric model was used, consisting of a fragment of bearing circumference with a symmetrically situated (between the raceways) rolling element (Fig. 3).

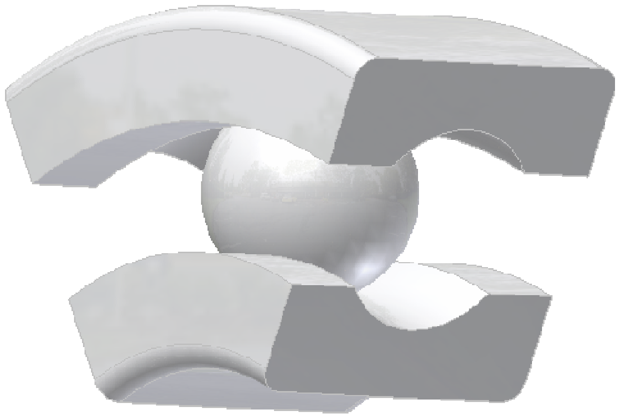

Fig. 3. Geometric model of the system: rolling element – bearing raceways
Rys. 3. Model geometryczny układu element toczny - bieżnie łożyska

A grid of finite elements was created in HyperMesh programme, with use of C3D8I and C3D6 hexahedral elements from the finite elements library of the Abaqus application and taking into consideration two symmetry planes of the model. In the contact place of the rolling element with the raceway, the researches decided to compact the lattice (Fig. 4).

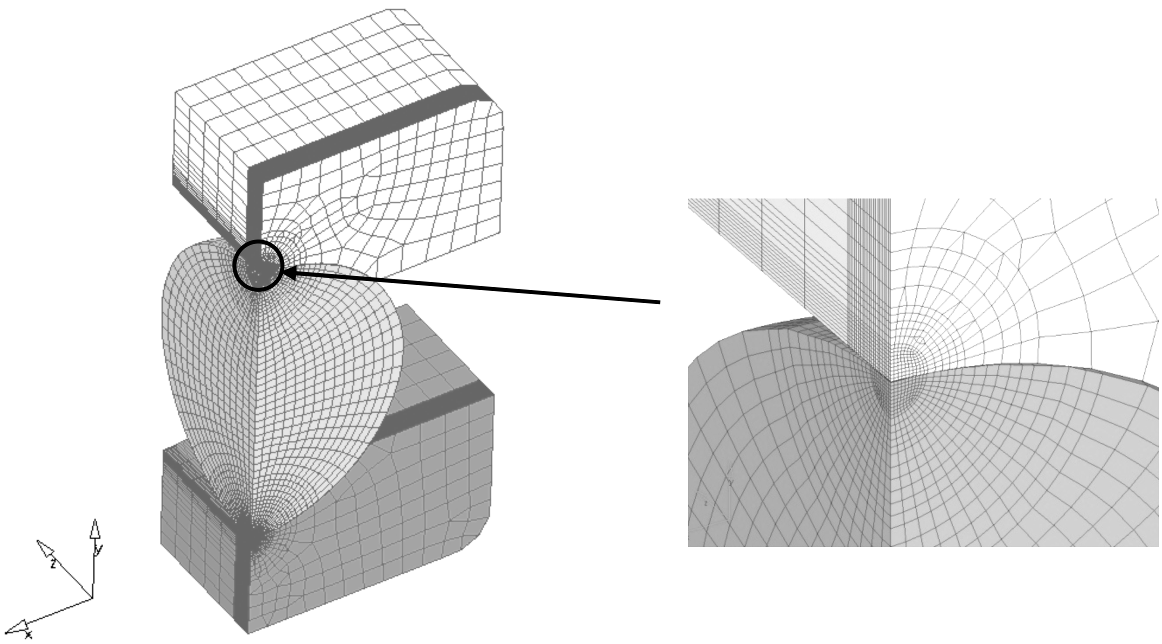

Fig. 4. FEM model of the system: rolling element – bearing raceways with a finite element grid
Rys. 4. Model MES układu element toczny - bieżnie łożyska z nałożoną siatką elementów skończonych

The amount of all elements was 41 132 with 42 789 grid points. The points located on the outer surface of one of the raceways were taken all degrees of freedom away. Taking the symmetry planes of the model into consideration, tantamount to simplifying the FEM method to one fourth of the original geometric model, allowed significant shortening of the calculation time, but, simultaneously extorted the introduction of slip planes. In the YZ plane, the tested bearing element was deprived of the possibility of moving towards X, while in the XY plane, towards Z. For the declared dislocations of the inner raceway towards the outer raceway along axis Y, the load values were read out, i.e. the reaction forces in the place of fastening the bearing.

In the model created by means of FEM, individual parts of the bearing interact. This necessitated entering additional contact elements in the Abaqus programme's solver. They are defined through an additional contact surface superimposed on the external surfaces of solid elements. In the case under consideration, those elements were placed on the inner sides of the raceway and on the rolling element (Fig. 5). The direction of the contact elements was defined as well as the type of interaction between the elements (by introducing the friction coefficient into the simulation) and the interacting contact surfaces.

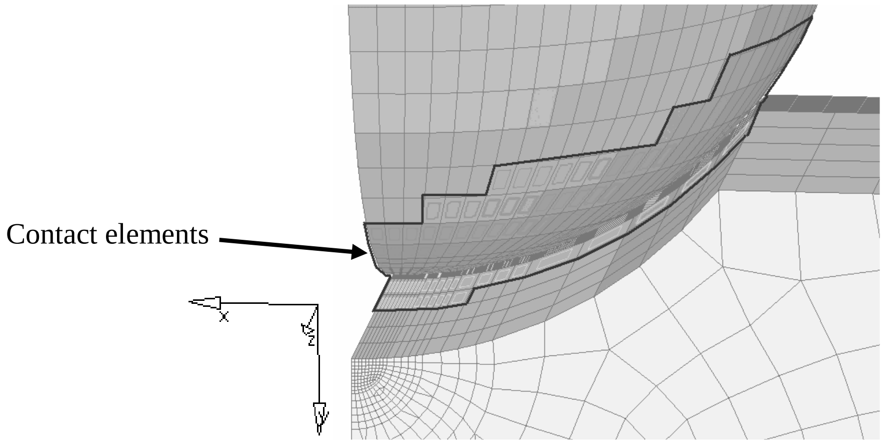

Fig. 5. Contact surface of raceway and ball
Rys. 5. Powierzchnia kontaktowa bieżni i kulki

Material properties were set for the model made of bearing steel described with Young modulus, E=210 000 MPa, and Poisson's ratio, ν=0.3. The calculations were performed with use of four models of a bearing, differing in the value of congruence coefficient *w*, whose values amounted to 0.95, 0.96, 0.97 and 0.98, respectively. This coefficient, according to [8], is defined as a ratio of the ball radius to the raceway radius.

4. CALCULATION RESULTS

As a result of the calculations, values of stresses and loads were obtained as a function of displacement of the tested system's elements. Fig. 6 presents contour lines of reduced stresses for the model whose congruence coefficient equalled 0.98. A diagram of characteristics of the joint stiffness for various values of congruence coefficient is shown in Fig. 7.

Fig. 6. Results of FEM analysis – contour lines of reduced stresses for congruence coefficient of 0.98
Rys. 6. Wyniki analizy metodą elementów skończonych – warstwice naprężeń zredukowanych dla współczynnika przystawania 0,98

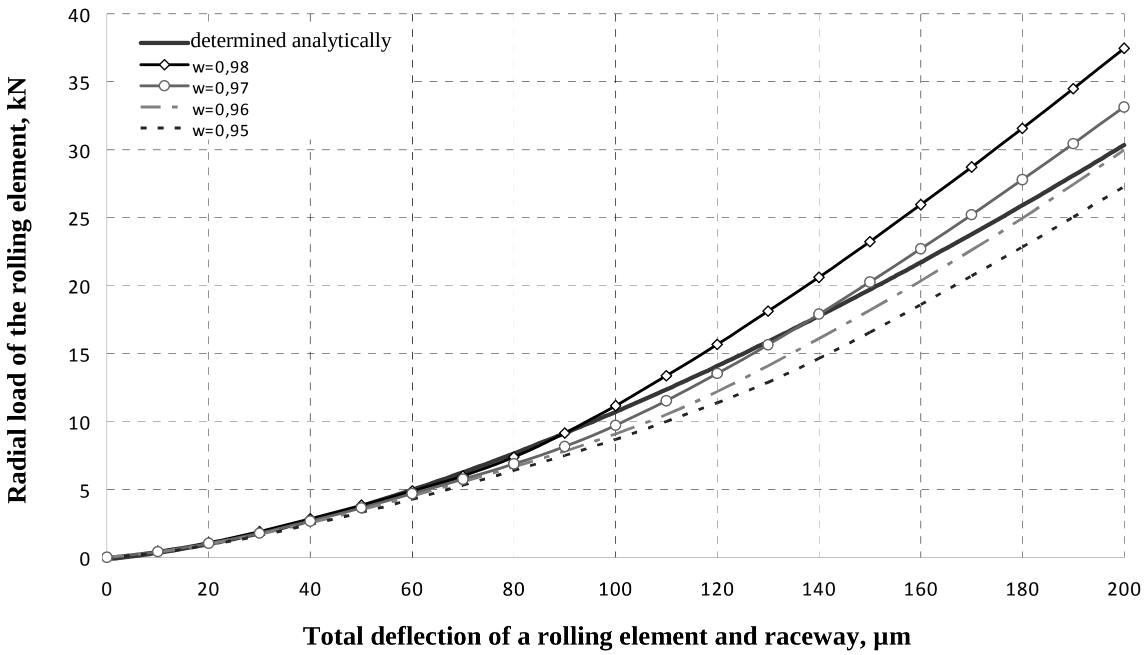

Fig. 7. Stiffness characteristics of the outer raceway - rolling element - inner raceway system determined with use of FEM for various congruence coefficients and characterization determined analytically
Rys. 7. Charakterystyki sztywności układu bieżnia zewnętrzna - element toczny - bieżnia wewnętrzna określone z użyciem MES dla różnych współczynników przystawania oraz charakterystyka wyznaczona analitycznie

It can be noted from the diagram that the characterization obtained for the congruence coefficient w=0.97 turned out to be the most similar to the results of analytical calculations.

5. CONCLUSIONS

It appears from the tests that the congruence coefficient has a significant influence on the stiffness values of the modelled system obtained by means of numerical analysis. For this reason, it is necessary to accurately reflect the bearing's geometry in the model in order to obtain correct results. Therefore, precise knowledge of the dimensions, including first of all the raceway radius and the rolling element diameter, is required. Further calculations have also shown that it is possible to create a correct FEM model of a bearing, allowing the determination of its stiffness in taking into consideration any possible damage and wear of its elements.

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