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# ROBUST CONTROL METHODS FOR A RECYCLE BIOREACTOR

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Abstract: The paper presents a robust control design strategy for bioprocesses, which are characterized by strongly nonlinear dynamics. More precisely, we present the $H_2$ methodology in order to compute the controller for a recycle Continuous Stirred Tank Bioreactor (CSTB). We consider a general method of formulating control problem, which makes use of linear fractional transformation as introduced by Doyle (1978). The formulation makes use of the general two-port configuration of the generalized plant with a generalized controller. The $H_2$ norm is the quadratic criterion used in optimal control as LQG. The overall control objective is to minimize the $H_2$ norm of the transfer matrix function from the weighted exogenous inputs to the weighted controlled outputs. The advantage of $H_2$ control technique, which uses the linearized model of the CSTB, is that it is completely automated and very flexible. Finally, we prove that the closed loop control structure has very good inner robustness.

Keywords: Biotechnology, Nonlinear Systems, Linear Fractional Transform, $H_2$ controller.

## 1. INTRODUCTION

The implementation of advanced control strategies on real bioprocesses is difficult because of absence of reliable instrumentation. The performances of the control depend on the reliability of the model. When some kinetic parameters are imprecisely known, it is necessary to design adaptive and/or robust controllers. An interesting approach has been proposed by Bastin and Dochain (1990) who designed adaptive controllers for bioreactors. The design of stable and convergent nonlinear adaptive controllers for bioprocesses is a complex task.

Another viable alternative can be the robust approach. The advantages are that one fixed controller structure apriori designed can be use for an entire class of plants nearby the nominal plant. Using a robust controller good results are obtained for parametric uncertainties or for variable delay in the plant model, which often characterize the recycle CSTB.

The paper is organized as follows. In Section 2, the general dynamical model of the Continuous Stirred Tank Bioreactor is presented. Based on this model, a general control design architecture is analysed. Section 3 deals with the design and implementation of the $H_2$-controller that makes use of the linearized model of the nonlinear plant model. In Section 4, the behaviour of the proposed controller is analysed. The robustness of the proposed control architecture is emphasized. Finally, Section 5 collects the conclusions.

## 2. MATHEMATICAL MODEL OF THE CSTB

A bioreactor is a tank in which several biological reactions occur simultaneously in a liquid medium. Bioreactors that operate in the continuous mode are

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usually known as Continuous Stirred Tank Bioreactors. In a CSTB, the substrates (the nutrients) are fed to the bioreactor continuously and an effluent stream is continuously withdrawn such that the culture volume is constant. Often, a part of the biomass is recycled. To recycle, the biomass must be separated from the substrate and yield, then travel through pipes after separation. This time of recycle introduce delays in the states and complicates the dynamic. The benefits are that the recycle increases the overall conversion and reduces the costs.

The dynamical state-space model of a biotechnological process in a CSTB expresses the mass balance of the components in the bioreactor (Bastin and Dochain, 1990):

$$\frac{dx_1}{dt} = m(x_1,x_2)\cdot x_1 - D\cdot x_1 \qquad (1)$$

$$\frac{dx_2}{dt} = -k_1 m(x_1,x_2)\cdot x_1 - D\cdot x_2 + D\cdot S_{in} \qquad (2)$$

where $x_1, x_2$ represent the biomass and the limiting substrate concentrations [g/l]. $S_{in}$ is the influent substrate concentration and $D$ is so-called dilution rate [$h^{-1}$], i.e. the specific volumetric outflow rate. In (1), (2) $m$ is the specific growth rate and $k_1>0$ the yield coefficient.

In the CSTB with recycle stream, a part of the biomass is recycled. If the recycle occurs, then the bioprocess model (1), (2) must be rewritten (Selisteanu and Petre, 2001):

$$\frac{dx_1(t)}{dt} = m(x_1(t),x_2(t))x_1(t) - Dx_1(t) + (1-q)Dx_1(t-r) \qquad (3)$$

$$\frac{dx_2(t)}{dt} = -k_1 m(x_1(t),x_2(t))x_1(t) - Dx_2(t) + F_{in} \qquad (4)$$

In (3) $(1-q)\cdot D$ is the recycle flow rate. The constant $q$ varies from 0 to 1, with zero corresponding to total recycle and 1 to no recycle. The constant $r$ is the recycle delay time and $F_{in}$ is the input flow. A compact representation of the state- space model (3), (4) is:

$$\dot{x} = f(x) \qquad (5)$$

where $x = [x_1 x_2]^T$ is the state vector and the function $f(\cdot)$ is the nonlinear vectorial function $f(x) = [f_1(x_1,x_2), f_2(x_1,x_2)]^T$ .

The equilibrium states of (3), (4) are of two types:

1. Wash-out equilibrium states (E1), defined by:

$$\text{(E1)} \qquad x_s = [x_{s1} x_{s2}]^T = [0 \quad F_{in}/D]^T \qquad (6)$$

2. Operational equilibrium states (E2), implicitly defined by:

$$\text{(E2)} \qquad \begin{cases} m(x_{s1},x_{s2}) = qD \\ k_1 m(x_{s1},x_{s2})x_{s1} + Dx_{s2} = F_{in} \end{cases} \qquad (7)$$

Equilibrium (E1) corresponds to the bioreactor wash-out, therefore only equilibrium (E2) has a technological interest. We suppose that the form of the specific growth rate is the Haldane kinetic model that takes into account substrate inhibition on the growth

$$m(x_1,x_2) = m(x_2) = m_0 \frac{x_2}{K_M + x_2 + x_2^2/K_i} \qquad (8)$$

$K_M$ is the Michaelis - Menten constant, $K_i$ the inhibition constant and $m_0$ the maxim specific growth rate.

From (7), if the specific growth rate is the model (8) we have two possibilities for the equilibrium (E2):

a).

$$x_{s1} = \frac{F_{in} - Dx_{s2}}{k_1 qD} \overset{N}{=} x_{s1,1} = \frac{F_{in} - Dx_{s2,1}}{k_1 qD}\ ; x_{s2} \overset{N}{=} x_{s2,1} \qquad (9)$$

b).

$$x_{s1} = \frac{F_{in} - Dx_{s2}}{k_1 qD} \overset{N}{=} x_{s1,2} = \frac{F_{in} - Dx_{s2,2}}{k_1 qD}\ ; x_{s2} \overset{N}{=} x_{s2,2} \qquad (10)$$

The case (a) corresponds to a stable equilibrium point (stable node). The case (b) leads to a saddle type for the equilibrium (E2) (see Selisteanu and Petre, 2001).

The phase plane corresponding to the system (3), (4) for the values of the process parameters: $m_0 = 6h^{-1}$, $K_M = 10 g/l$, $K_i = 100 g/l$, $k_1 = 1$, $D = 3.6 h^{-1}$, $F_{in}$ = 540 $h^{-1}$ g/l, $q$ = 0.8, $r$ = 0.25 h and for different initial conditions is represented in Fig. 1. From this picture it can be seen that when the substrate inhibition appears, the process can exhibit unstable or, maybe worse, the evolution leads to wash-out steady-states, for which the microbial life has disappeared and the reactor is stopped. Furthermore, a bigger value for the delay time can induce a worst behaviour. Moreover, in many cases, the stable equilibrium point corresponding to (a) is not technological operable (requires a big initial amount of biomass). The conclusion is that for the CSTB with recycle stream it is necessary to design a control strategy.

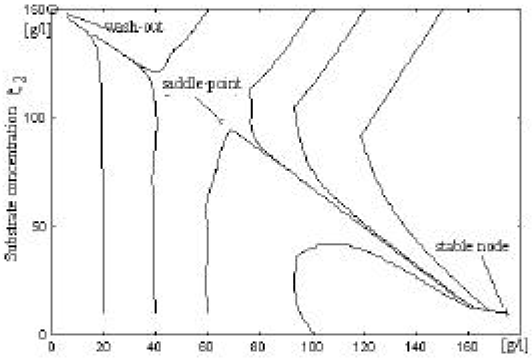


Fig. 1. Phase plane of CSTB with recycle stream

For control purposes, it is necessary to find the linear approximation of the system (3), (4) around the equilibrium point (E2). The goal of the control strategy is to stabilize the equilibrium point (10), which is interesting from technological point of view.

## 2. $H_2$ CONTROLLER DESIGN

A general method it is considered for formulating $H_2$ control problem, which makes use of linear fractional transformation as introduced by Doyle and Stein (1981). The formulation makes use of the general two-port configuration of the generalized plant presented in Fig. 2.

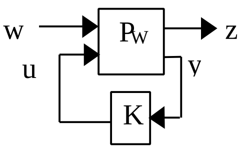


Fig. 2. Two-port representation of control loop

The overall control objective is to minimize the norm of the transfer matrix function from the weighted exogenous inputs *w* to the weighted controlled outputs *z*.

The $H_2$ optimal problem can be stated as

$$\inf_{K\in RH_\infty} \left\| H_{w}^{z} \right\|_2 \qquad (11)$$

and is suitable for linear systems. Generally, if we consider w = {r, d, n} the exogenous input of the system (r for reference signals, d for disturbances and n for measurement noises), z = {$z_1$, $z_2$, $z_3$} the quality output of the system and u the control input (controller output) and (y = e) the controller input (system error), we have the representation from Fig.3. $W_1$, $W_2$ and $W_3$ are appropriate weighting functions, used in the controller design process (see Skogestad and Postlethwaite, 1995). We consider a general method of formulating control problem, which makes

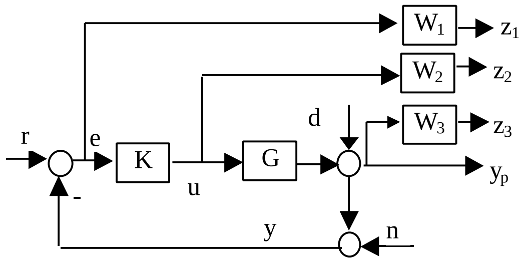


Fig. 3. The $H_2$ controller design architecture

use of linear fractional transformation as introduced by Doyle and Stein (1981). The formulation uses the general two-port configuration of the generalized plant with a generalized controller. The classical control loop from Fig. 2 is equivalent with the so-*called two-port representation* of the generalized plant from Fig. 3.

In the next sections is used only the linearized model G of the nonlinear CSTB around the unstable saddle point (10), which has a reasonable biomass concentration $x_1$. Beginning with this linear model G, we shall derive the generalized plant $P_W$ from Fig. 3. (called also the augmented plant) using the following general formulas:

$$P_W(s) = H_{\left[\begin{smallmatrix} z \\ y \end{smallmatrix}\right]\left[\begin{smallmatrix} w \\ u \end{smallmatrix}\right]}(s) = \left[\begin{array}{c|c} W_1 & -W_1 G \\ 0 & W_2 \\ 0 & W_3 G \\ \hline I & -G \end{array}\right] \qquad (12)$$

The design of the $H_2$ controller is based on this augmented plant transfer matrix $P_W(s)$. From (12) we can see that the augmented plant can be represented in state-space form as:

$$P_W(s) \stackrel{s}{=} \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \qquad (13)$$

We can solve the ***H***$^2$*-norm optimal control problem* by observing that it is equivalent to a conventional *Linear-Quadratic Gaussian optimal control problem*. The **H**$^2$ optimal controller K(*s*) is thus realizable in the usual LQG manner as a full-state feedback $K_c$ and a Kalman filter with residual gain matrix $K_f$ with following relations:

a) Kalman Filter:

$$\dot{\hat{x}} = A\hat{x} + K_f\left(y - C_2\hat{x} - D_{22}u\right)$$

$$K_f = \left(SC_2^T + B_1D_{21}^T\right)\left(D_{21}D_{21}^T\right)^{-1}$$

$$SA^T + AS - \left(SC_2^T + B_1D_{21}^T\right)\left(D_{21}D_{21}^T\right)^{-1}\left(SC_2^T + B_1D_{21}^T\right)^T$$

b) Full-state feedback:

$$u = K_c\hat{x}$$

$$K_c = \left(D_{12}^TD_{12}\right)^{-1}\left(B_2^TP + D_{12}^TC_1\right)$$

$$A^TP + PA - \left(B_2^TP + D_{12}^TC_1\right)^T\left(D_{12}^TD_{12}\right)^{-1}\left(B_2^TP + D_{12}^TC_1\right)$$

$$K(s) \stackrel{s}{=} \left[ \begin{array}{c|c} A - K_f C_2 - B_2 K_c + K_f D_{22} K_c & K_f \\ \hline -Kc & 0 \end{array} \right] \qquad (14)$$

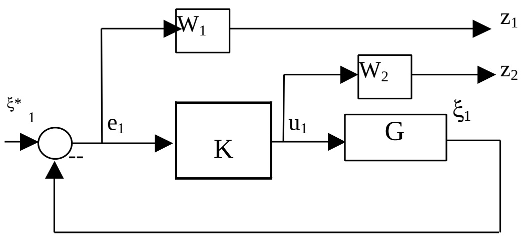


Fig. 4. Specific $H_2$ design architecture

From (13) and (14) it can be observed that the controller order is the sum of the original plant order and the orders of each weighting functions. The controller and the augmented (generalized) plant have the same order. It is our interest, in order to obtain a low-order controller, to use scalar weighting functions or to use only $W_1$ and $W_2$ as in the specific case from Fig. 4.

## 4. COMPUTER SIMULATION AND RESULTS

### 4.1. *Stabilization of the unstable saddle point*

For design the linearized plant model around the saddle point is used, which is the intermediate point between the wash-out and operational stable node. The saddle point used for linearization has the coordinates:

$$[x_1, x_2] = [64.8197, 98.1442] \qquad (15)$$

and the operational stable node has the coordinates:

$$[x_1, x_2] = [174.7636, 10.1891] \qquad (16)$$

The weighting functions have a great importance in the $H_2$ controller design, being in fact the only design parameters. For example, using

$$W_1 = \frac{10^3}{s}, \; W_2 = 10^2$$

good robustness behaviour of the controlled system is obtained.

The main purpose of the control system is to conduct the operational point from the initial point to the final point, the stable node. From technological and economical reasons, the initial point has to be near to the origin in the phase plane from Fig. 1. In such case, without a control system, the plant will go into wash-out state. Using the designed $H_2$ controller, we can start from the initial point

$$[x_1^*, x_2^*] = [14.8197, \; 98.1442]$$

and obtain the following state response:

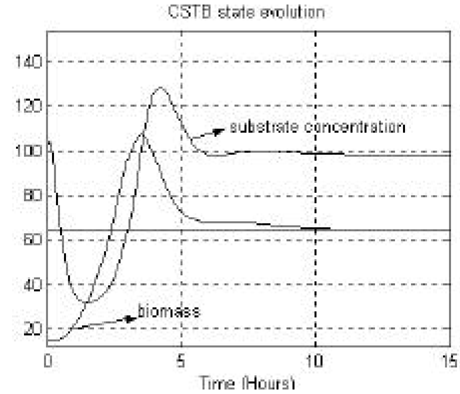


Fig. 5. State response of CSTB

It is clear that the plant functions in the unstable saddle point, starting with wash-out initial conditions. From this non-economical operational point, we can conduct the plant in another desirable operational point, applying steps on biomass reference.

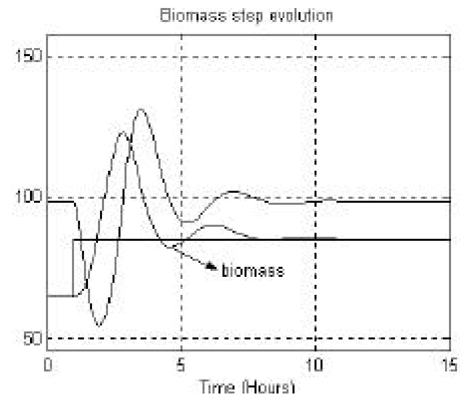


Fig. 6. Time evolution to the operational point

### 4.2. *Robustness against parametric uncertainty*

In the above simulation results, we considered no parameter uncertainty, so we assumed that the model parameters are identical with the real ones. We have to test the control system behaviour in presence of parametric uncertainty. For example, let's take a variation of the parameter $k_1$ (the yield coefficient), with nominal value 1. The following evolutions are obtained:

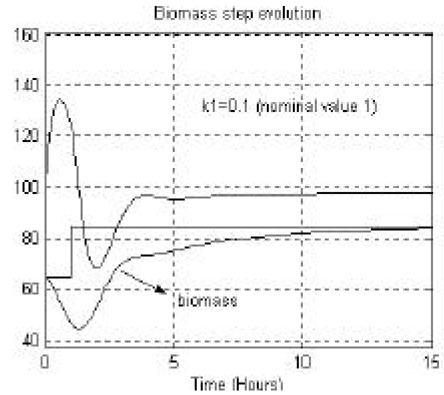


Fig. 7. Time evolution for $k_1$=0.1

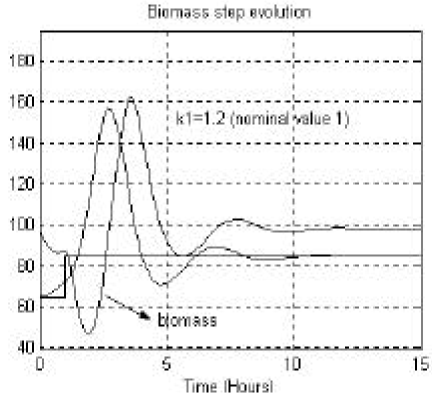


Fig. 8. Concentrations profile for $k_1$=1.2

We may simulate multiple parameters variation, as in the next figure, first with two modified parameters and after with three modified parameters (Fig.9 and Fig. 10).

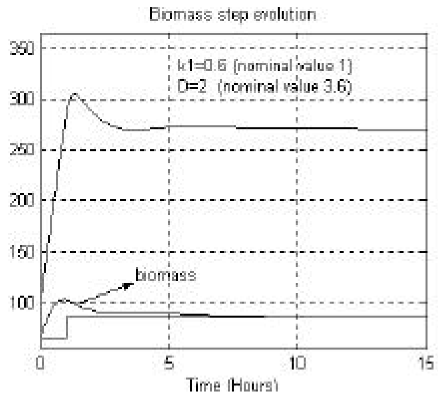


Fig. 9. Time evolution for 2 modified parameters

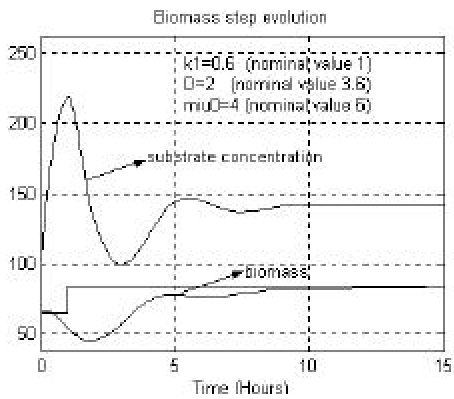


Fig. 10. Time evolution for 3 modified parameters

4.3. *Robustness against time delay*

The above simulations results used relations (1) and (2) for the plant model with no recycle stream. The last simulation results are focused on the time delay compensation, so we will use for simulation the mathematical model (3) and (4) of the CSTB with recycle stream.

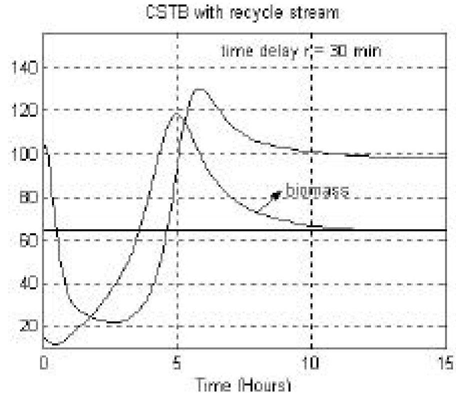


Fig. 11. Evolution of CSTB with recycle stream

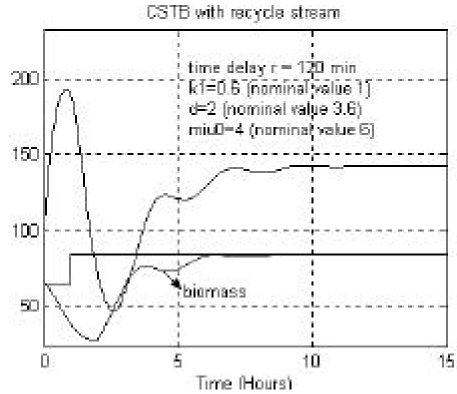


Fig. 12. Evolution for modified parameters.

## 5. CONCLUSIONS

In this paper a simple but efficient strategy for a CSTB with recycle stream control is proposed: the $H_2$ methodology for controller design. Starting with the nonlinear second-order model of the CSTB plant, the following design steps are proposed:

- Obtain the linear model from the set of nonlinearly differential equation around the unstable saddle point

$$[x_1, x_2] = [64.8197, 98.1442]$$

- Compute the $H_2$ controller using appropriate weighting functions. In our case, we do not use $W_3$ and $W_2$ is chosen a scalar only for methodological reasons.
- Test the control system behaviour for evaluating robustness performances.

Following these steps, a third order SISO $H_2$ controller is founded, with very good robustness performances, which transformed the unstable saddle point into ordinary operational point. Using successive biomass reference steps, the operational point is conducted from the starting wash-out zone to the final stable node.

In order to prove the robustness capabilities, the following tests are proposed:

- non-zero initial conditions response is stable in a large area around the linearization point (saddle point).
- The $H_2$ controller can compensate large simultaneous variations for different model parameters.
- In presence of parametric uncertainties, the $H_2$ controller still ensures a good control for a large time-delay in the recycle stream.

Finally, we have to explain why we choose the $H_2$ design procedure instead of *LTR* or $H_\infty$, methods (see Stein and Athans, 1987; Chiang and Safonov, 1992) recognized as robust techniques (Fig. 13).

$$\|H\|_2^2 = \sum_{i=1}^{n}\int_0^{\infty} s_i^2(w)dw$$

$$\|H\|_\infty^2 = \sup_w s_1^2(w)$$

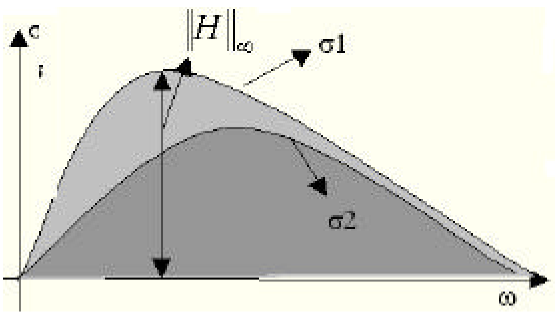


Fig. 13. $H_2$ and $H_\infty$ optimization

To understand the difference between the $H_2$ and $H_\infty$, we note that the $H_2$-norm is the Frobenius norm in terms of singular values. We see that minimizing the $H_\infty$-norm corresponds to minimizing the peak of the largest singular value, whereas minimizing the $H_2$-norm corresponds to minimizing the square of the sum of all singular values over all frequencies.

Minimizing $H_2$-norm often lead to a decrease of the peak of the largest singular value, which guarantees good robustness of stabilization and performances.

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