MARKOV CHAIN MODEL FOR PROBABILITY OF DRY, WET DAYS AND STATISTICAL ANALISIS OF DAILY RAINFALL IN SOME CLIMATIC ZONE OF IRAN

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ABSTRACT- Markov chain model for probability of dry, wet days and statistical analisis of daily rainfall in some climatic zone of Iran. Water scarcity is a major problem in arid and semi-arid areas. The scarcity of water is further stressed by the growing demand due to increase in population growth in developing countries. Climate change and its outcomes on precipitation and water resources is the other problem in these areas. Several models are widely used for modeling daily precipitation occurrence. In this study, Markov Chain Model has been extensively used to study spell distribution. For this purpose, a day period was considered as the optimum length of time. Given the assumption that the Markov chain model is the right model for daily precipitation occurrence, the choice of Markov model order was examined on a daily basis for 4 synoptic weather stations with different climates in Iran (Gorgan, Khorram Abad, Zahedan, Tabriz)during 1978-2009. Based on probability rules, events possibility of sequential dry and wet days, these data were analyzed by stochastic process and Markov Chain method. Then probability matrix was calculated by maximum likelihood method. The possibility continuing2-5days of dry and wet days were calculated. The results showed that the probability maximum of consecutive dry period and climatic probability of dry days has occurred in Zahedan. The probability of consecutive dry period has fluctuated from 73.3 to 100 percent. Climatic probability of occurrence of dry days would change in the range of 70.96 to 100 percent with the average probability of about 90.45 percent.

***Keywords*:** Markov Chain, Daily Rainfall, Modeling, Occurrence probability**.**

1. INTRODUCTION

The yield of crops particularly in Dry land condition depends on the rainfall pattern. Simple criteria related to consecutive phenomena like dry and wet spells could be used for analyzing rainfall data to obtain specific information needed for crop planning and for carrying out agricultural operations. Senthilvelan et al. (2012) used Markov Chain model for probability of weekly rainfall in OrathanaduTaluk, Thanjavur District and Tamil Nadu. A week period was considered as the optimum length of time. The present study has been carried out to find the probabilities of occurrence of wet week (W) and wet week proceeded by

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wet week (W/W) at different threshold limits of 10 and 20 mm. The results showed that the 3 and $3\frac{1}{2}$ month varieties are best suited for Vettikkadu region; $3\frac{1}{2}$ and 4 months paddy varieties can be successfully grown in Neivasal Thenpathi area and 4 and $4\frac{1}{2}$ months paddy varieties are favorably grown in Orathanadu region. Moradi et al. (2011) examined the characteristics of meteorological drought in Fars Province. Their objective of this research was to examine and forecast the intensity, duration, frequency and extent of droughts in this area. For this purpose, precipitation data within the same statistical period of 32 years (1968 to 1999) in five time scales of 3, 6, 12, 24 and 48 months from some 26 stations located within and out of the province were used. The results of the drought station surveys for the mentioned characteristics showed that the highest drought intensity (in terms of % deviation from normal) was at the Gozoon station and the longest duration of drought in months was at the Polkhan station; droughts both of long duration and with the highest frequency during the statistical period were observed at the Jahrom and Darshahy stations. Also, the results showed that droughts in the south of the province were of higher intensity and longer duration than elsewhere. Lennartsson et al. (2008) were proposing a new method for modeling precipitation in Sweden. They consider a chain dependent stochastic model that consists of a component that models the probability of occurrence of precipitation at a weather station and a component that models the amount of precipitation at the station when precipitation does occur. The results showed the distribution of the modeled indices and the empirical ones show good agreement, which supports the choice of the model. Srinivasareddy et al. (2008) were used Markov Chain model probability of dry, wet weeks and statistical analysis of weekly rainfall for agricultural planning at Bangalore. The average annual rainfall of GKVK campus, Bangalore was found to be 923.9 mm and coefficient of variation (CV) was 25.4%. The data on onset and withdrawal rainy season indicated that the monsoon starts effectively from 24th SMW (11 - 17th June) and remains active up to 45th SMW (5 - 11th November). During rainy season the probability of occurrence of wet week is more than 35% except during 25th - 27th SMW and 44th - 48th SMW. During rainy season the mean weekly rainfall is found to be more than 40 mm during 36th - 41st SMW and found to be less than20 mm during 20th SMW, 25th - 27th SMW and 44th - 48th SMW. The results through analysis have been used for agricultural planning at Bangalore region. Pabitra and et al. (2000) used Markov chain models to evaluate probabilities of getting a sequence of wet and dry weeks during South-West monsoon period over the districts Purulia in West Bengal and Giridih in Bihar state and dry farming tract in the state of Maharashtra of India. Pandharinath (1991) used the Markov Chain model to study the probability of dry and wet spells. In this study, Markov chain model has been used to determine the probability of daily precipitation, for Gorgan, Khorram Abad, Zahedan and Tabeiz synoptic stations based on daily time scales of wet and dry periods.

2. MATERIAL AND METHODS

Daily rainfall data recorded at the Iran Meteorological Organization for a period of 31 years (1978 – 2008) were used in Tabriz, Gorgan, Khorram Abad and Zahedan synoptic stations for the present study. Geographical and climatic information of these stations are given in Table 1. The climates base on extended Domarten classification at the stations is also specified (Khalili, 1997).

Table 1. ***Geographic information and Extended Domarten climatic classification of the study stations***

Station name	latitude		longitude		Elevation from sea level	Climate	Months of the study
Tabriz	38	05	46	17	1361	Semi-arid cold	Oct- May
Khoram Abad	33	26	48	17	1147.8	Semi-arid moderate	Oct-May
Zahedan	29	28	60	53	1370	Arid moderate	Nov-May
Gorgan	36	51	54	16	13.3	Moditerranean moderate	Jan-Dec

The dry and wet spell analysis was carried out using daily rainfall based on Markov Chain Model considering less than 0.1 mm rainfall in a day as a dry day and 0.1 mm or more as a wet day.

The different notations followed in this analysis are given. Equation 1 to 6 is shown the initial probability and conditional probabilities (Pandharinath, 1991).

$$P_D = \frac{F_D}{n} \quad (1)$$

$$P_w = \frac{F_w}{n} \quad (2)$$

$$P_{DD} = \frac{F_{DD}}{F_D} \quad (3)$$

$$P_{WW} = \frac{F_{WW}}{F_W} \quad (4)$$

$$P_{WD} = 1 - P_{DD} \quad (5)$$

$$P_{DW} = 1 - P_{WW} \quad (6)$$

P_D is probability of the day being dry, P_W is probability of the day being wet, F_D is Number of dry days, F_W is number of wet days, n is number of years of data, P_{DD} is probability (conditional) of a dry day preceded by a dry day, P_{WD} is probability (conditional) of a wet day preceded by a dry day, P_{ww} is probability (conditional) of a wet day preceded by a wet day, P_{DW} is probability (conditional) of a dry day preceded by a wet day, F_{DD} is number of dry days preceded by another dry day, F_{WW} is number of wet days preceded by another wet day.

Consecutive dry and wet day probabilities in equation 7 t0 10 is shown.

$$2D = P_{D_{W_1}} . P_{DD_{W_2}} \quad (7)$$

$$2W = P_{W_{W_1}} . P_{WW_{W_2}} \quad (8)$$

$$3D = P_{D_{W_1}} . P_{DD_{W_2}} . P_{DD_{W_3}} \quad (9)$$

$$3W = P_{W_{W_1}} . P_{WW_{W_2}} . P_{WW_{W_3}} \quad (10)$$

Where, 2D is probability of 2 consecutive dry days starting with the day, 2Wis probability of 2 consecutive wet days starting with the day, 3D is probability of 3 consecutive dry days starting with the day, 3W is probability of 3 consecutive wet days starting with the day, $P_{D_{W1}}$ is probability of the day being dry (first day), $P_{DD_{W2}}$ is probability of the second day being dry, given the preceding day dry, $P_{DD_{W3}}$ is probability of the third day being dry, given the preceding day dry, $P_{W_{W1}}$ is probability of the day being wet (first day), $P_{WW_{W2}}$ is probability of the second day being wet, given the preceding day wet, $P_{WW_{W3}}$ is probability of the third day being wet, given the preceding day wet.

The estimation of return periods m- day dry spell length in equation 11 is shown.

$$T_m = \frac{p_{01} + p_{10}}{n p_{01} p_{10} (1 - p_{01})^n} \quad (11)$$

Where, n is number of days in each month, m is dry spell length and T_m is return periods m- day dry spell length.

3. RESULTS AND DISCUSSION

The results pertaining to initial and conditional probabilities of dry and wet days and consecutive dry and wet days as of Khorram Abad and Zahedan synoptic stations are presented in Tables 2 to 5, respectively, for rainy season days. As probability of sequences of dry days in Tabriz synoptic station with semi-arid cold climate is 35.2 to 100 % while climatic probability of occurrence of dry days is 32.2 to 93.5 % by the mean of 72.2%. As probability of sequences of dry days in Gorgan synoptic station with the Mediterranean moderate climate is 42.1 to 100 % while climatic probabilitiy of occurrence of dry days is 41.9 to 96.7 % by the mean of 72.8%. As probability of sequences of dry days in Khorram Abad synoptic station with semi-arid moderate climate is 52.17 to 100 % while climatic probabilitiy of occurrence of dry days is 45.16 to 100 % by the mean of 72.4%. As probability of sequences of dry days in Zahedan synoptic station with arid moderate climate is 73.3 to 100 % while climatic probability of occurrence of dry days is 70.96 to100 % by the mean of 90.45%. Average consecutive 2- 5 days of dry day probabilities in Tabriz synoptic station is 0.577, 0.461, 0.368 and 0.295. Average consecutive 2- 5 days of dry day probabilities in Gorgan synoptic station is 0.586, 0.474, 0.384 and 0.313. Average consecutive 2- 5 days of dry day probabilities in Khorram Abad synoptic station is 0.594, 0.491, 0.407 and 0.342. Average consecutive 2- 5 days of dry day probabilities in Zahedan synoptic station is 0.841, 0.781, 0.728 and 0.674. Figure.1 shows the return periods m- day dry spell length decreases from Zahedan synoptic station with arid moderate climate to Gorgan synoptic station with the mediterranean moderate climate and reaches its lowest value in Gorgan synoptic station.

Table 2. ***Initial and conditional probabilities of dry and wet days of Zahedan station***

number of day julius	Conditional probability				Simple probability	
	P_{dd}	P_{dw}	P_{wd}	P_{ww}	P_d	P_w
1	0.931	0.069	1	0	0.935	0.064
2	0.966	0.034	1	0	0.968	0.032
3	0.9	0.1	1	0	0.903	0.096
⋮	⋮	⋮	⋮	⋮	⋮	⋮
108	0.966	0.034	0.5	0.5	0.935	0.064
109	0.931	0.069	1	0	0.935	0.064
110	0.828	0.172	0.5	0.5	0.806	0.193
⋮	⋮	⋮	⋮	⋮	⋮	⋮
210	0.92	0.08	0.667	0.333	0.871	0.129
211	0.926	0.074	0.75	0.25	0.903	0.097
212	0.964	0.036	0.667	0.333	0.935	0.064

Table 3. ***Consecutive probabilities dry and wet days of Zahedan station***

Number of day julius	Consecutive dry day probabilities				Consecutive wet day probabilities			
	D2	D3	D4	D5	W2	W3	W4	W5
1	0.903	0.813	0.668	0.614	0	0	0	0
2	0.871	0.715	0.658	0.631	0	0	0	0
3	0.742	0.683	0.655	0.608	0.032	0.021	0.007	0.0023
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
108	0.871	0.721	0.721	0.695	0	0	0	0
109	0.774	0.774	0.747	0.695	0.032	0.011	0.0054	0.0054
110	0.806	0.779	0.725	0.617	0.064	0.032	0.032	0.008
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
205	0.935	0.871	0.811	0.724	0	0	0	0
206	0.871	0.811	0.724	0.666	0	0	0	0
207	0.871	0.777	0.715	0.662	0.032	0.032	0.011	0.0027
208	0.806	0.742	0.687	0.662	0.096	0.032	0.008	0.0027
209	0.742	0.687	0.662	0	0.064	0.0161	0.0054	0
210	0.806	0.777	0	-	0.032	0.011	0	-
211	0.871	0	-	-	0.032	0	-	-
212	0	-	-	-	0	-	-	-

Table 4. ***Initial and conditional probabilities of dry and wet days in Khorram Abad station***

number of day julius	Conditional probability				Simple probability	
	P_{dd}	P_{dw}	P_{wd}	P_{ww}	P_d	P_w
1	0.72	0.28	0.667	0.333	0.71	0.290
2	0.591	0.409	0.778	0.222	0.645	0.355
3	0.7	0.3	0.454	0.545	0.613	0.387
⋮	⋮	⋮	⋮	⋮	⋮	⋮
120	0.682	0.318	0.556	0.444	0.645	0.355
121	0.8	0.2	0.545	0.454	0.71	0.290
122	0.818	0.182	0.444	0.556	0.71	0.290
⋮	⋮	⋮	⋮	⋮	⋮	⋮
241	0.87	0.130	0.75	0.25	0.839	0.161
242	0.769	0.231	0.6	0.4	0.742	0.258
243	0.783	0.217	0.75	0.25	0.774	0.226

Table 5. ***Consecutive probabilities dry and wet days in Khorram Abad station***

Number of day julius	Consecutive dry day probabilities					Consecutive wet day probabilities		
	D2	D3	D4	D5	W2	W3	W4	W5
1	0.419	0.293	0.216	0.159	0.064	0.035	0.021	0.0085
2	0.451	0.333	0.245	0.175	0.193	0.113	0.047	0.032
3	0.451	0.333	0.237	0.211	0.226	0.094	0.065	0.035
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
120	0.516	0.422	0.307	0.234	0.161	0.089	0.039	0.023
121	0.580	0.422	0.322	0.225	0.161	0.072	0.043	0.035
122	0.516	0.393	0.275	0.241	0.129	0.077	0.063	0.046
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
236	0.581	0.429	0.361	0.312	0.129	0.097	0.048	0.026
237	0.548	0.461	0.399	0.347	0.193	0.096	0.053	0.013
238	0.516	0.446	0.388	0.298	0.193	0.107	0.026	0.010
239	0.613	0.533	0.409	0.321	0.161	0.040	0.0161	0.004
240	0.645	0.496	0.388	0	0.0645	0.026	0.0064	0
241	0.645	0.505	0	-	0.064	0.016	0	-
242	0.581	0	-	-	0.064	0	-	-
243	0	-	-	-	0	-	-	-

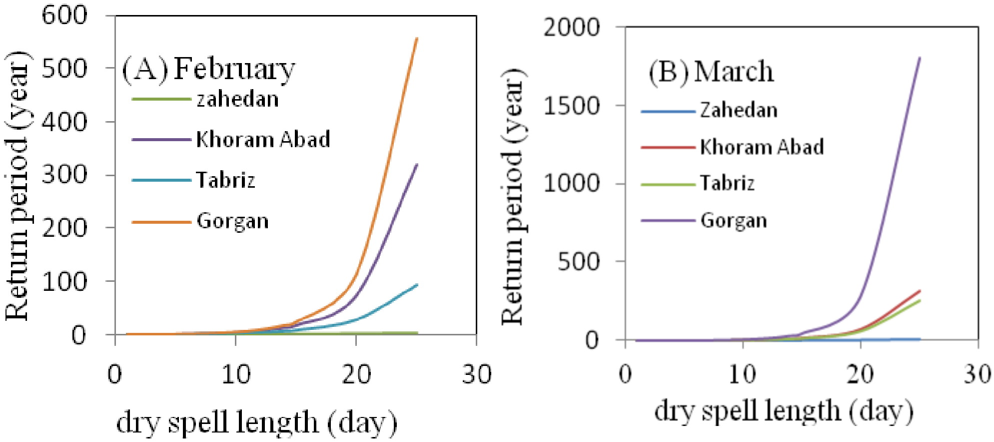

Fig. 1. ***Return period's m- day dry spell length***

4. CONCLUSIONS

Markov Chain model has been fitted to daily rainfall data to obtain sequences of dry and wet spells during the monsoon season. These sequences of wet and dry spells can be an aid to understand drought-proneness. The results of this paper showed that the maximum probability of sequences of dry days in Zahedan synoptic station with arid moderate climate is 73.3 to 100 % while climatic probability of occurrence of dry days is 70.9 to 100 % by the mean of 90.45%. The probability minimum of sequences of dry days in Tabriz synoptic station with semi-arid cold climate is 35.2 to 100 % while climatic probability of occurrence of dry days is 32.2 to 93.5 % by the mean of 72.2%. Average consecutive 2- 5 days minimum of dry day probabilities in Tabriz synoptic station is 0.577, 0.461, 0.368 and 0.295 and average consecutive 2- 5 days maximum of dry day probabilities in Zahedan synoptic station is 0.841, 0.781, 0.728 and 0.674.

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