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Satellite orbit determination using satellite gravity gradiometry observations in GOCE mission perspective

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Abstract. Between the years 2004 and 2005 the launch of the first gradiometric satellite is planned. This satellite will be an important element of the Gravity Field and Steady – State Ocean Circulation Explorer Mission (GOCE). This mission is one of the reasons for performing the simulation research of the Satellite Gravity Gradiometry. Our work contains the theory description and simulation results of the satellite orbit determination using the gravity tensor observations. In the process of the satellite orbit determination the initial dynamic state vector corrections are obtained. These corrections are estimated by means of the gravity gradiometry measurements. The performed simulations confirm the possibility of satellite orbit determination by means of the gravity tensor observations.

Key words. satellite geodesy, satellite gradiometry, satellite orbits

1 Introduction

In the fifties, the idea of Satellite Gravity Gradiometry (SGG) was proposed. This technics allows to determine the gravity tensor components by means of the gradiometer located on the satellite board (Colombo and Kleusberg, 1983; Drożyner et al., 1989). To obtain the gravity gradients (i.e. gravity tensor components), the contribution of the gravitational force must be separated from the acceleration measurements given by the accelerometers in the spaceborne gradiometer (Sollazzo, 1987; Rummel, 1988). The gravity tensor components are the second spatial derivatives of geopotential. They are the elements of following matrix:

$$\mathbf{T} = \begin{bmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{bmatrix} \tag{1}$$

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where $\mathbf{T}$ is the gravity tensor, $T_{kl}(k, l = x, y, z)$ is its component, for example: $T_{xy} = \frac{\partial^2 V}{\partial x \partial y}$, V is the gepotential and x, y, z are rectangular coordinates.

An important application of Satellite Gravity Gradiometry observations is the gravity field determination (Colombo and Kleusberg, 1983; Rummel and Colombo, 1985). But, the gravity gradients do not contain only gravitational information. They depend also on the position of gradiometer (i.e. gradiometric satellite too). Thus, the purpose of this work is to analyse the possibilities of satellite orbit determination by means of the SGG measurements (i.e. gravity tensor components), especially in the context of the GOCE satellite mission.

2 Satellite orbit improvement by means of the gravity tensor components

Determination of the GOCE satellite orbit will be based on the GPS observations. But the question of helpfulness of gravity tensor measurements in the orbit determination will be considered here.

In the first step of performed research the observation equation system was made. The particular observation equations result from the general relation in the following form:

$$T_{ij}^{obs} - T_{ij}^{c} = \frac{\partial T_{ij}^{c}}{\partial \boldsymbol{r}} \frac{\partial \boldsymbol{r}}{\partial(\boldsymbol{r}_0, \dot{\boldsymbol{r}}_0)} \begin{bmatrix} \Delta \boldsymbol{r}_0 \\ \Delta \dot{\boldsymbol{r}}_0 \end{bmatrix} . \tag{2}$$

In this formula:
T_{ij}^{obs} – is the measured gravity tensor component (at time t, simulated by means of the gravity field coefficients degree and order up to 360 – EGM96 model; Lemoine et al., 1998),
T_{ij}^{c} – is the approximated gravity tensor component (at time t, obtained in the orbit improving process using the truncated EGM96 model; degree and order of the coefficients less than 360),
$\boldsymbol{r}, \boldsymbol{r}_0, \dot{\boldsymbol{r}}_0$ – are the satellite position vector (at time t), satellite initial position vector and satellite initial velocity vector (at time t_0), respectively,

$\frac{\partial T_{ij}^c}{\partial \boldsymbol{r}}$ – are third spatial derivatives of the geopotential (at time t, computed in the orbit improving process by means of the truncated EGM96 model),

$\frac{\partial \boldsymbol{r}}{\partial(\boldsymbol{r}_0,\dot{\boldsymbol{r}}_0)}$ – are partial derivatives of the satellite position vector with respect to the initial dynamic state vector of satellite (at time t, determined using the variational equations in the orbit improving process), and the estimated quantities:

$\Delta\boldsymbol{r}_0 = [\Delta x_0\ \Delta y_0\ \Delta z_0]^T$ – corrections to the components of initial position vector of satellite,

$\Delta\dot{\boldsymbol{r}}_0 = [\Delta \dot{x}_0\ \Delta \dot{y}_0\ \Delta \dot{z}_0]^T$ – corrections to the components of initial velocity vector of satellite.

To compute the second (T_{ij}^{obs}, T_{ij}^c) and the third partial derivatives of geo-potential, the expressions given by Métris et al. (1999) were used. These partial derivatives were simulated along the computed satellite orbit.

To the satellite motion modelling only the Earth's gravity field was taken. The EGM96 model was truncated to obtain the approximated satellite orbit. Then the approximated gravity tensor measurements were computed using the truncated EGM96 model and approximated satellite orbit.

Taking into account the observation equation system and the least squares method, the corrections to the components of initial state vector of satellite were determined. Next, the improved satellite orbit was obtained by means of the Cowell numerical integration based on the corrected initial state vector. It was repeated in the successive iterations.

3 Solution variants of the orbit determination

All computations were performed by means of the modified TOP-SGG package (Drożyner, 1995), which realizes the orbit improving process. A solution of the orbit improving process contains the corrected components of initial state vector, their mean square errors and the RMS parameters (the fitness errors of the obtained orbit to the reference orbit). Thanks to some modifications of the initial data of orbit improving process, the various solution variants were obtained. These modifications included:

– changes of the length of processed orbital arc,
– taking into account different values of the sampling interval (i.e. time interval between the succesive measurements of gravity tensor),
– changes of the measured components of gravity tensor,
– taking into account two reference frames where the observations were simulated. The first used frame is the Conventional Inertial System (at standard epoch J2000.0). The second frame is called the orbital frame. It has the origin in the satellite mass center. Its Z axis is always pointing along radial direction (opposing with respect to the direction towards Earth), the Y axis is perpendicular to the orbital plane and the X axis – completes the frame to the right-handed frame.
– adding to the simulated observations the random errors with the normal distribution and different standard deviations: 10^{-1}, 10^{-2}, 10^{-3} E.U. (1 E.U. = 1 Eötvös Unit = 10^{-9} s^{-2}).

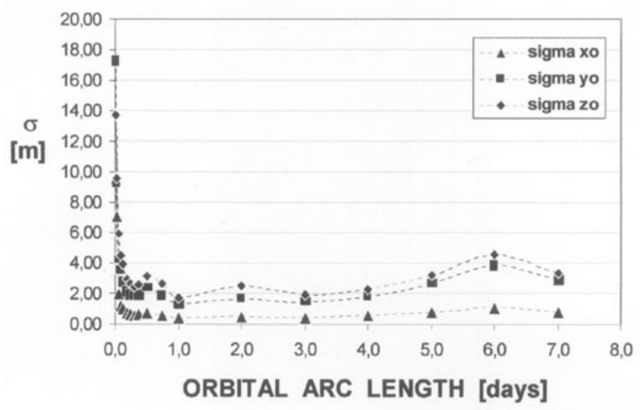

Fig. 1. Errors of the improved components of initial position vector depending on the orbital arc length. The points present the accuracy of solution variants of satellite orbit improvement.

In the last phase of research the analysis and accuracy comparison of computed variants were made. The mean square errors of corrected components of initial state vector were taken into account in that comparison.

4 Results

Table 1 shows a typical solution of the orbit improving process. This solution was obtained for the following initial conditions: the full gravity tensor with respect to the inertial frame was simulated, one – day orbital arc of almost circular and polar satellite orbit (altitude equals about 200 km) was used, the sampling interval was equalled 60 s and the maximal degree and order of gravity field coefficients for the satellite motion model were equalled 150. From Table 1 it is seen, that the absolute values of corrections to the initial position components are correlated with the absolute values of the initial velocity components. For the growing absolute value of initial velocity component the absolute value of correction to the correspoding initial position component increases. The similar dependence can be noticed taking into account the absolute values of initial position components and the absolute values of corrections to the initial velocity components.

In Fig. 1, the accuracy of orbit improvement solutions according to the orbital arc length is shown. The initial data of presented solutions are almost the same as for the solution from Table 1. However, there are two differences. In this case the maximal degree and order of gravity field coefficients for the satellite motion model equals 110 and the orbital arc length is changing.

From this figure results an optimal value of the one day orbital arc length for the orbit improving process. The computation error cumulation causes the decrease of solution precison for the orbital arc lengths above one day.

The accuracies of six variants of orbit improvement are given in Fig. 2. To obtain these solutions the 70 × 70 grav-

Table 1. Typical solution of the orbit improving process

Corrected components of the initial state vector of satellite		Corrections to the components of initial state vector of satellite*	Mean square errors of the components of initial state vector of satellite
[m], [m/s]	[m], [m/s]	[m], [m/s]	
x_0	6577345.67	−0.07	0.26
y_0	14.72	0.18	0.81
z_0	497.17	3.50	1.25
$\dot{x}_0$	−0.58802	−0.00413	0.00140
$\dot{y}_0$	271.69744	0.00005	0.00096
$\dot{z}_0$	7780.37805	0.00010	0.00031

* These corrections are referred to the components of initial state vector from the last but one iteration of orbit improving process.

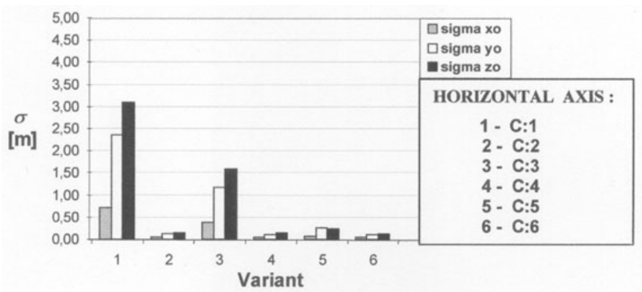

Fig. 2. Precisions of the corrected initial position components depending on the various cases of one component measurement of the tensor with respect to the orbital frame (orbit improvement variants from 1 to 6). Notation: C: $i\,(i = 1, 2, 3, 4, 5, 6)$ – solution variant with the particular component observations: 1 corresponds to T_{xx}, 2 – to T_{xy}, 3 – to T_{xz}, 4 – to T_{yy}, 5 – to $T_{yz,6}$ – to T_{zz}.

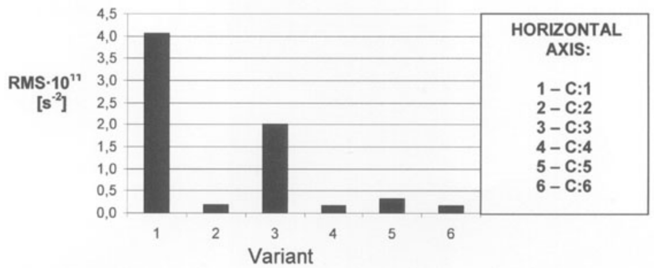

Fig. 3. The fitness errors (RMS) of the obtained orbit to the reference orbit for the various cases of one component measurement of the tensor with respect to the orbital frame. Notation: C: $i\,(i = 1, 2, 3, 4, 5, 6)$ – solution variant with the particular component observations: 1 corresponds to T_{xx}, 2 – to T_{xy}, 3 – to T_{xz}, 4 – to T_{yy}, 5 – to T_{yz}, 6 – to T_{zz}.

ity field model (70 × 70 is the maximal degree and order of the coefficients of truncated EGM96 model) was taken. The measurements of one component of the gravity tensor were simulated with respect to the orbital frame. The remaining initial data are the same as for the variant given in Table 1.

The errors of most accurate solutions presented in Fig. 2 reach the order of a few decimeters. The sixth variant with the simulation of T_{zz} radial component measurement is the best solution.

In Fig. 3 the fitness errors (RMS) of the obtained orbit to the reference orbit for the same solutions as in Fig. 2 are given. The RMS values were computed by means of the following formula:

$$\mathrm{RMS} = \sqrt{\frac{\sum_{i=1}^{n}\left(T_{kl}^{c} - T_{kl}^{obs}\right)^2}{n}}\,, \tag{3}$$

where: n – is the total number of observations along given orbital arc, T_{kl}^{c} – is the component of the gravity tensor computed taking into account the obtained orbit, T_{kl}^{obs} – is is the observed component of the gravity tensor for the reference orbit and $k, l = x, y, z$.

In Fig. 3 the sixth variant with the radial component (T_{zz}) observation has the smallest value of the RMS error.

In Fig. 4 the solutions for the observations with respect to the inertial frame are presented. The variants from 1 to 6 include the measurement of various combinations of five gravity tensor components. The seventh variant corresponds the full gravity tensor observation. The remaining initial data are the same as for the variants from Fig. 2. The accuracy of the presented variants reaches the order of a few meters. In this figure the seventh variant with the full gravity tensor observation has the best precision.

Some modifications of the initial data were performed for the solutions from Fig. 5. Three almost cirular and polar orbits were taken into account. Their altitudes were equal about 160, 200 and 240 km. The different values of orbital arc lengths (0.5, 1 and 2 days) and of sampling intervals (30, 60 and 120 s) were used. For the satellite motion model the 70 × 70 gravity field model (truncated EGM96 model) was used. The full tensor measurement simulation (with respect to the inertial frame) was performed.

Two interesting properties can be noticed in Fig. 5. From comparing the accuracy of third, of fourth and of fifth vari-

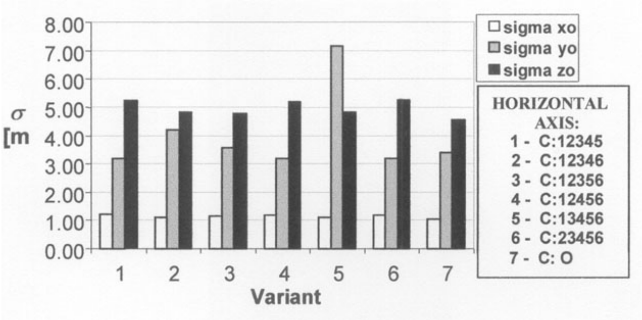

Fig. 4. Acccuracy of the corrected initial position components depending on the various cases of five component measurement of the tensor with respect to the inertial frame (variants from 1 to 6). The seventh variant of satellite orbit improvement was obtained using the full tensor measurement (with respect to the inertial frame too). Notation: C: $ijklm(ijklm = 1, 2, 3, 4, 5, 6)$ – solution variant with the particular component observations: 1 corresponds to T_{xx}, 2 – to T_{xy}, 3 – to T_{xz}, 4 – to T_{yy}, 5 – to T_{yz}, 6 – to T_{zz} , C: O – full tensor measurement.

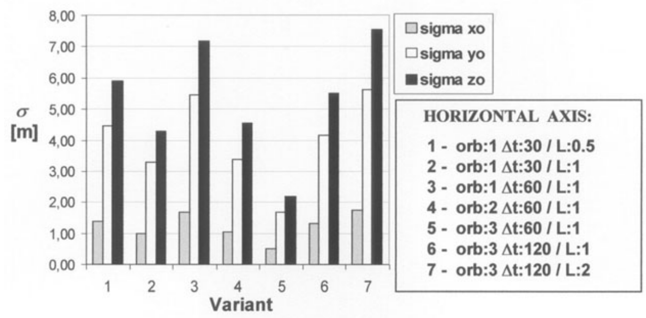

Fig. 5. Errors of the improved components of initial position vector for various variants of orbit improvement solution with the changes of the improving orbit, of sampling interval and of orbital arc length. The altitudes of chosen orbits are about 160 km (orb:1), 200 km (orb:2) and 240 km (orb:3), respectively. The sampling intervals are 30 s (Δt : 30), 60 s (Δt : 60) and 120 s (Δt : 120). Lengths of the orbital arcs have the following values: 0.5 day (L : 0.5), 1 day (L : 1) and 2 days (L : 2).

ant it is clear, that for the increasing orbit altitude the orbit improving errors decrease. This is right, but if the sampling intervals of compared variants are equal. The accuracy comparison of first and third variant shows the decrease of improved orbit precision for the increasing sampling interval (from 30 to 60 s). The total numbers of used measuremenrs in both compared variants are equal. This was achieved by using the half-day orbital arc (for first variant) and one-day orbital arc (for third variant).

5 Conclusions

The preformed research indicates the possibility of the satellite orbit determination using the real observations of gravity tensor. The accuracy of improved orbit reaches the order of meters (taking into account the gravity tensor measurements with respect to the inertial frame) and the order of decimeters (taking into account the gravity tensor measurements with respect to the orbital frame). To give the possible best solution of orbit improvement using the gravity tensor observations the value of sampling interval must be possible smallest and the orbital arc length should be equal about one day (for the 60 s value of sampling interval and for about 200 km value of the orbit altitude).

However at the real conditions the GOCE satellite orbit determination using the gravity tensor observations only can be ineffective due to the finiteness bandwidth of gradiometer. Besides the obtained acccuracy (the order of dm) is less than the expected one for the GPS observations (the order of cm). Therefore the measurements of gravity tensor components could be treated as the additional measurements with respect to the based GPS observations in the orbit determination process. In this process the correlations between the gravity tensor components should be taken into acccount. The GPS observations and the measurements of gravity tensor components could be weighted. In such case the weights of gradiometric observations (gravity tensor components) will be less than the GPS measurements ones. It will be the subject of future research.

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