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THE INFLUENCE OF NON-IDEAL PHASE FLOW ON THE EXTRACTION EFFICIENCY FOR THE CASE OF A LINEAR EQUILIBRIUM DISTRIBUTION

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Abstract

The influence of the fundamental parameters of non-ideal phase flow and the extraction parameters on the number of equilibrium stages - N_D, theoretical stages - N_T, as well as the number of stages (N_D - N_T), the existence of which is a consequence of the backflow in extractors, was investigated. The calculated number of stages (N_D - N_T) served as a measure of the influence of the denoted parameters on the extraction efficiency.

The results of the investigation indicate that the number of stages (N_D - N_T) considerably increased with increasing backmixing coefficients and that the dependence was linear. It was established that the increase of the ratio of the flowrate of the heavy and light phase and the decrease of the equilibrium distribution coefficient, as well as the increase of the total separation factor, led to an exponential increase of the number of stages in the extractor, which consequently caused a decrease in the extraction efficiency.

Keywords: extraction, separation factor, linear equilibrium distribution, backmixing, theoretical number of stages, equilibrium number of stages, stage additivity model

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1. Introduction

The influence of non-ideal phase flow in counter-current operations has been insufficiently studied despite its exceptional significance. Namely, in most extractors, when describing counter-current phase flow, the existence of axial mixing, which causes a decrease in the mass transfer in the equipment, must be taken into account, i.e. the decrease in extraction efficiency [1-3]. A modest contribution to this problem is presented in some papers of the author of this publication [4-6].

Keeping in mind that the equipment efficiency decreases during non-ideal phase flow, especially in the case of counter-current extraction equipment, the influence of the fundamental parameters of non-ideal phase flow and the extraction parameters for the case of a linear equilibrium distribution was systematically investigated in this paper.

Analytical expressions were used in the paper to calculate the number of equilibrium stages - N_D, and theoretical stages - N_T, for the case when the extraction factor $F \neq 1$, for which $F = L/(mG)$ where L and G are the flowrates of the heavy and light phase, respectively, and m is the solute distribution coefficient [7].

The calculated number of stages (N_D - N_T) was used to analyse the influence of the denoted parameters on the extraction efficiency.

2. Theoretical consideration

Starting from the stepwise backmixing model, in which the basic parameters of the non-ideality of the phase flows are expressed by the backmixing coefficient f in phase L and s in phase G, A. Tolic and V. Rod [8,9] developed a new model to calculate counter-current extractors. On the basis of this model it follows that phase flow non-ideality is defined by the difference in the number of equilibrium stages - N_D, defined by A. Tolic and T. Miyauchi [10], and the number of theoretical stages - N_T. The analytical expressions for the calculation of the number of stages ND and NT for the case of a linear equilibrium dependence, and $F \neq 1$, are given in Table 1. The case for $F = 1$, for which other analytical expressions are also required, was not included in the analysis, as the selected values of the extraction factor did not include this value.

Table 1. Analytical expressions for the calculation of the number of stages N_D and N_T for the case of a linear equilibrium distribution and $F \neq 1$

$$N_T = \frac{\ln \dfrac{1-\psi_N}{1-\psi_N \cdot F}}{\ln F} \qquad\qquad N_D = \frac{\ln \left(\dfrac{F+s+Ff}{1+s+Ff} \cdot \dfrac{1-\psi_N}{F(1-\psi_N \cdot F)} \right)}{\ln \dfrac{F+s+Ff}{1+s+Ff}}$$

The total degree of extraction, i.e. the total degree of separation - ψ_N is defined as $\psi_N = (x_{in} - x_{out})/(x_{in} - x_{in}^*)$, where x_{in} and x_{out} are the extractant concentrations in phase L at the inlet and outlet from the extractor, while x_{in}^* is the inlet equilibrium concentration.

3. Initial data

Realisation of the set goal requires, foremost, the selection of parameters of non-ideal phase flow, as well as the denoted extraction parameters by which their ranges will cover the interval of interest for extraction.

The following data were selected: a general type of linear equilibrium dependence $y = mx$, where m is the coefficient of equilibrium distribution proportional to the solvent concentration; the backmixing coefficients $f = 0 \div 5$ and $s = 0 \div 5$ with the increment 0.1; the extraction factor $F = 0.3 \div 0.9$ with the increment 0.1; the total separation factor $\psi_N = 0.90 \div 0.98$ with the increment 0.1.

4. Results and discussion

The calculated values for N_T, N_D and $(N_D - N_T)$ for a broad range of initial data are presented in Table 2. The existence of backmixing in phase L ($f = 0 \div 5$) and the absence of it in phase G ($s = 0$) were assumed.

Table 2. The calculated number of stages N_T, N_D and (N_D - N_T) based on the analytical expressions (Table 1) for selected values of the parameters f = 0 ÷ 5, s = 0, F = 0.3 ÷ 0.9 and ψ_N = 0.90 ÷ 0.98

			N_T			N_D					(N_D – N_T)			
		f	**0**	**1**	**2**	**3**	**4**	**5**	**0**	**1**	**2**	**3**	**4**	**5**
ψ_N	**F**	**s**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	0.3		1.65	2.01	2.36	2.71	3.05	3.39	0.00	0.36	0.71	1.06	1.40	1.74
	0.4		2.03	2.68	3.32	3.95	4.58	5.21	0.00	0.65	1.29	1.93	2.56	3.19
	0.5		2.46	3.49	4.52	5.53	6.55	7.56	0.00	1.04	2.06	3.07	4.09	5.10
0.90	**0.6**		2.99	4.53	6.06	7.59	9.11	10.64	0.00	1.54	3.07	4.60	6.12	7.65
	0.7		3.67	5.90	8.13	10.35	12.57	14.79	0.00	2.23	4.46	6.68	8.90	11.13
	0.8		4.61	7.85	11.08	14.30	17.53	20.76	0.00	3.23	6.46	9.69	12.92	16.14
	0.9		6.09	10.92	15.75	20.58	25.41	30.24	0.00	4.83	9.66	14.49	19.32	24.15
	0.3		1.83	2.29	2.74	3.17	3.61	4.04	0.00	0.46	0.91	1.34	1.78	2.21
	0.4		2.26	3.06	3.84	4.61	5.39	6.16	0.00	0.80	1.58	2.36	3.13	3.90
	0.5		2.75	4.00	5.23	6.45	7.67	8.89	0.00	1.25	2.47	3.70	4.92	6.14
0.92	**0.6**		3.37	5.21	7.04	8.86	10.68	12.50	0.00	1.84	3.67	5.49	7.31	9.13
	0.7		4.19	6.85	9.51	12.16	14.82	17.47	0.00	2.67	5.32	7.98	10.63	13.28
	0.8		5.35	9.24	13.13	17.01	20.90	24.78	0.00	3.89	7.78	11.66	15.55	19.43
	0.9		7.27	13.21	19.15	25.09	31.03	36.97	0.00	5.94	11.89	17.83	23.77	29.71
	0.3		2.06	2.65	3.22	3.78	4.34	4.89	0.00	0.59	1.16	1.72	2.28	2.83
	0.4		2.56	3.55	4.52	5.48	6.43	7.39	0.00	0.99	1.96	2.92	3.88	4.83
	0.5		3.14	4.66	6.16	7.66	9.15	10.64	0.00	1.52	3.02	4.51	6.00	7.49
0.94	**0.6**		3.88	6.12	8.34	10.55	12.76	14.98	0.00	2.24	4.46	6.67	8.88	11.09
	0.7		4.88	8.13	11.36	14.60	17.83	21.06	0.00	3.25	6.48	9.72	12.95	16.18
	0.8		6.36	11.15	15.94	20.73	25.51	30.30	0.00	4.79	9.58	14.37	19.15	23.94
	0.9		8.95	16.49	24.02	31.56	39.09	46.63	0.00	7.54	15.08	22.61	30.15	37.68
	0.3		2.39	3.17	3.91	4.65	5.37	6.10	0.00	0.78	1.52	2.25	2.98	3.71
	0.4		2.98	4.25	5.48	6.71	7.93	9.15	0.00	1.26	2.50	3.72	4.95	6.16
	0.5		3.70	5.62	7.51	9.39	11.27	13.14	0.00	1.92	3.81	5.69	7.57	9.44
0.96	**0.6**		4.62	7.43	10.22	13.00	15.78	18.56	0.00	2.81	5.60	8.38	11.16	13.94
	0.7		5.90	10.00	14.09	18.17	22.25	26.33	0.00	4.10	8.19	12.27	16.35	20.43
	0.8		7.88	14.03	20.17	26.32	32.46	38.60	0.00	6.15	12.30	18.44	24.58	30.72
	0.9		11.62	21.69	31.75	41.82	51.89	61.95	0.00	10.07	20.14	30.20	40.27	50.34
	0.3		2.96	4.05	5.10	6.14	7.16	8.18	0.00	1.09	2.14	3.18	4.20	5.22
	0.4		3.73	5.46	7.16	8.84	10.52	12.20	0.00	1.74	3.43	5.12	6.80	8.47
	0.5		4.67	7.28	9.85	12.41	14.96	17.51	0.00	2.61	5.18	7.74	10.29	12.84
0.98	**0.6**		5.92	9.74	13.53	17.31	21.09	24.87	0.00	3.82	7.61	11.39	15.17	18.94
	0.7		7.72	13.35	18.95	24.55	30.15	35.74	0.00	5.63	11.23	16.83	22.43	28.02
	0.8		10.66	19.31	27.94	36.57	45.20	53.82	0.00	8.64	17.28	25.91	34.53	43.16
	0.9		16.85	31.88	46.91	61.94	76.96	91.99	0.00	15.03	30.06	45.09	60.12	75.14

The calculated number of stages is presented in Table 3 for the same values of the initial data and the case of the absence of backmixing in phase L (f = 0) and its presence in phase G (s = 0 ÷ 5).

4.1. The dependence of the number of (N_D - N_T) stages on the degree of backmixing

By selecting the first value of the parameter ψ_N = 0.90 and using data from

Tables 2 and 3, it is possible to graphically present the dependence of (N_D - N_T) on f, i.e. of (N_D - N_T) on s for the whole range F = 0.3 ÷ 0.9 where the coefficients k_1 and k_2 designate the slopes of the obtained linear dependencies (Fig. 1 and 2).

Table 3. The calculated number of stages N_T, N_D and (N_D - N_T) based on the analytical expressions (Table 1) for selected values of the parameters f = 0, s = 0 ÷ 5, F = 0.3 ÷ 0.9 and ψ_N = 0.90 ÷ 0.98

			N_T	N_D					$(N_D - N_T)$					
		f	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
ψ_N	**F**	**s**	**0**	**1**	**2**	**3**	**4**	**5**	**0**	**1**	**2**	**3**	**4**	**5**
	0.3		1.65	2.82	3.95	5.07	6.20	7.32	0.00	1.17	2.30	3.42	4.55	5.67
	0.4		2.03	3.64	5.21	6.78	8.35	9.92	0.00	1.61	3.19	4.76	6.33	7.90
	0.5		2.46	4.52	6.55	8.58	10.60	12.63	0.00	2.06	4.09	6.12	8.14	10.17
0.90	**0.6**		2.99	5.55	8.09	10.64	13.18	15.72	0.00	2.56	5.11	7.65	10.19	12.73
	0.7		3.67	6.86	10.03	13.21	16.38	19.55	0.00	3.19	6.36	9.54	12.71	15.89
	0.8		4.61	8.65	12.69	16.72	20.76	24.79	0.00	4.04	8.08	12.11	16.14	20.17
	0.9		6.09	11.46	16.83	22.19	27.56	32.92	0.00	5.37	10.73	16.10	21.46	26.83
	0.3		1.83	3.32	4.76	6.19	7.62	9.05	0.00	1.49	2.93	4.36	5.79	7.22
	0.4		2.26	4.23	6.16	8.08	10.00	11.92	0.00	1.97	3.90	5.82	7.74	9.66
	0.5		2.75	5.23	7.67	10.11	12.55	14.98	0.00	2.47	4.92	7.35	9.79	12.22
0.92	**0.6**		3.37	6.43	9.47	12.50	15.53	18.57	0.00	3.06	6.10	9.13	12.16	15.19
	0.7		4.19	7.99	11.78	15.57	19.36	23.15	0.00	3.81	7.60	11.39	15.18	18.97
	0.8		5.35	10.21	15.07	19.93	24.78	29.64	0.00	4.86	9.72	14.58	19.43	24.28
	0.9		7.27	13.87	20.47	27.07	33.67	40.28	0.00	6.60	13.21	19.81	26.41	33.01
	0.3		2.06	3.97	5.81	7.64	9.47	11.30	0.00	1.91	3.75	5.58	7.41	9.24
	0.4		2.56	5.00	7.39	9.77	12.15	14.53	0.00	2.44	4.83	7.22	9.60	11.97
	0.5		3.14	6.16	9.15	12.12	15.10	18.07	0.00	3.02	6.00	8.98	11.96	14.93
0.94	**0.6**		3.88	7.60	11.29	14.98	18.66	22.34	0.00	3.72	7.41	11.09	14.78	18.46
	0.7		4.88	9.51	14.13	18.75	23.36	27.98	0.00	4.63	9.25	13.87	18.48	23.10
	0.8		6.36	12.35	18.33	24.32	30.30	36.28	0.00	5.99	11.97	17.96	23.94	29.92
	0.9		8.95	17.32	25.70	34.07	42.44	50.82	0.00	8.38	16.75	25.12	33.50	41.87
	0.3		2.39	4.89	7.30	9.71	12.11	14.50	0.00	2.50	4.91	7.32	9.72	12.11
	0.4		2.98	6.10	9.15	12.19	15.22	18.26	0.00	3.11	6.16	9.20	12.24	15.27
	0.5		3.70	7.51	11.27	15.02	18.77	22.51	0.00	3.81	7.57	11.32	15.07	18.81
0.96	**0.6**		4.62	9.29	13.93	18.56	23.19	27.81	0.00	4.67	9.31	13.94	18.57	23.19
	0.7		5.90	11.75	17.59	23.41	29.24	35.07	0.00	5.85	11.69	17.52	23.34	29.17
	0.8		7.88	15.57	23.24	30.92	38.60	46.27	0.00	7.69	15.37	23.04	30.72	38.39
	0.9		11.62	22.80	33.99	45.18	56.36	67.54	0.00	11.19	22.37	33.56	44.74	55.93
	0.3		2.96	6.48	9.88	13.27	16.65	20.02	0.00	3.52	6.92	10.31	13.69	17.06
	0.4		3.73	8.00	12.20	16.37	20.54	24.71	0.00	4.28	8.47	12.65	16.82	20.98
	0.5		4.67	9.85	14.96	20.06	25.16	30.26	0.00	5.18	10.29	15.39	20.49	25.58
0.98	**0.6**		5.92	12.27	18.57	24.87	31.16	37.45	0.00	6.35	12.65	18.94	25.23	31.52
	0.7		7.72	15.75	23.75	31.75	39.74	47.73	0.00	8.03	16.03	24.03	32.02	40.01
	0.8		10.66	21.47	32.26	43.04	53.82	64.61	0.00	10.80	21.59	32.38	43.16	53.94
	0.9		16.85	33.55	50.25	66.95	83.64	100.34	0.00	16.70	33.40	50.10	66.80	83.49

It may be concluded that the number of (N_D - N_T) stages significantly depends on the values of f and s and that it linearly increases with increasing backmixing coefficient where:

$$(N_D - N_T) = k_1 \cdot f \qquad (\text{for } f \neq 0 \text{ and } s = 0) \tag{1}$$

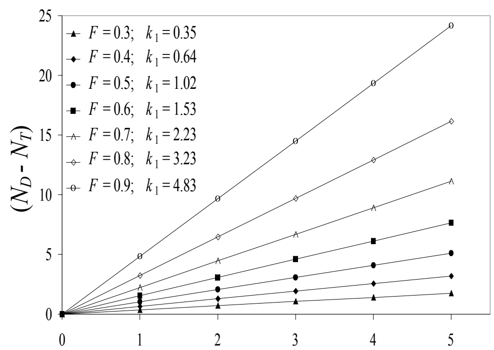


Fig. 1. Dependence of ($N_D – N_T$) on f for a selected case of extraction (ψ_N = 0.90)

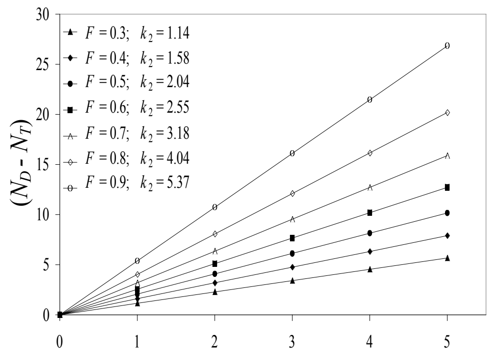


Fig. 2. Dependence of ($N_D – N_T$) on s for a selected case of extraction (ψ_N = 0.90)

$$(N_D - N_T) = k_2 \cdot s \qquad (\text{for } f = 0 \text{ and } s \neq 0) \tag{2}$$

for the case when f ≠ 0 and s ≠ 0, the following expression holds when:

$$(N_D - N_T) = k^1 \cdot f + k_2 \cdot s \tag{3}$$

Linear dependencies are obtained for all the other values of the parameter ψN, with the corresponding slopes (Table 4.) with a high value of the correlation coefficient (R ≅ 1).

4.2. The dependence of the number of (N_D - N_T) stages on the extraction factor

On the basis of the data in Table 4 it is possible to analyse the dependence of the slope of the linear dependencies k_1 and k_2 and the extraction factor, which is defined as the ratio of the phase flow and coefficient of the extractant distributions F = Q/m = L/Gm, at constant values of the total separation factor - ψ_N (Fig. 3 and 4). It is obvious that coefficients k_1 and k_2 significantly

Table 4. The slopes k_1 and k_2 of the obtained linear dependencies for selected values of the parameters F and ψ_N

		k_1					k_2				
F	ψ_N	**0.90**	**0.92**	**0.94**	**0.96**	**0.98**	**0.90**	**0.92**	**0.94**	**0.96**	**0.98**
0.3		0.35	0.44	0.57	0.75	1.05	1.14	1.45	1.85	2.43	3.42
0.4		0.64	0.78	0.97	1.24	1.70	1.58	1.94	2.40	3.06	4.21
0.5		1.02	1.23	1.50	1.89	2.57	2.04	2.45	2.99	3.77	5.12
0.6		1.53	1.83	2.22	2.79	3.79	2.55	3.04	3.70	4.64	6.31
0.7		2.23	2.66	3.24	4.09	5.61	3.18	3.80	4.62	5.84	8.01
0.8		3.23	3.89	4.79	6.15	8.63	4.04	4.86	5.98	7.68	10.79
0.9		4.83	5.94	7.54	10.07	15.03	5.37	6.60	8.37	11.19	16.70

increase with increasing F. These dependencies are not linear; they are exponential (the correlation coefficient R ranges from 0.98 to 0.99). They may generally be analytically expressed as:

$$k_1 = \theta_1(\psi_N)\cdot e^{\theta_2(\psi_N)F} \quad \text{and} \quad k_2 = \theta_3(\psi_N)\cdot e^{\theta_4(\psi_N)F} \qquad (4)$$

where θ_1, θ_2, θ_3 and θ_4 are coefficients the value of which depends on the value of the total separation factor - ψ_N.

The following equation is obtained by combining equations (3) and (4):

$$(N_D - N_T) = f \cdot \theta_1(\psi_N)\cdot e^{\theta_2(\psi_N)F} + s \cdot \theta_3(\psi_N)\cdot e^{\theta_4(\psi_N)F} \qquad (5)$$

However, equation (5) is only valid for one value of the parameter ψN, so it is also necessary to determine the dependence of $(N_D - N_T)$ on the separation factor.

4.3. The dependence of the number of $(N_D - N_T)$ stages on the total separation factor

The exponential dependencies presented in Fig.5 and 6 are obtained if the change of the slopes of the linear dependencies k_1 and k_2 on the total separation factor ψ_N is analysed at constant values of the extraction factor F.

In this case there is also an exponential increase of the slopes k_1 and k_2 with increasing total separation factor - ψ_N (correlation coefficient $R \cong 0.99$). The

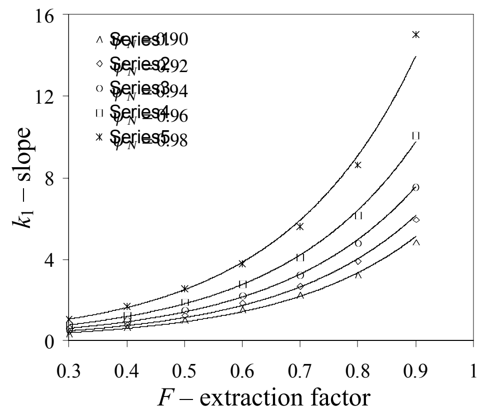


Fig. 3. The dependence of the slope k_1 on the extraction factor F for selected values of ψ_N

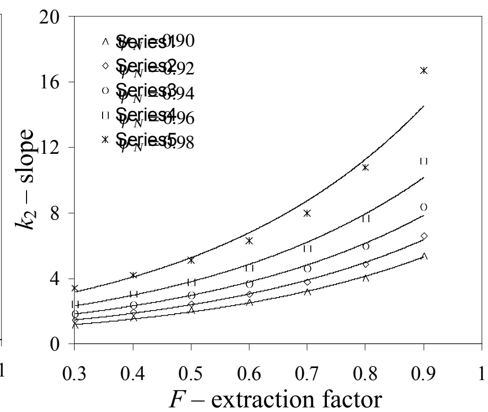


Fig. 4. The dependence of the slope k_2 on the extraction factor F for selected values of ψ_N

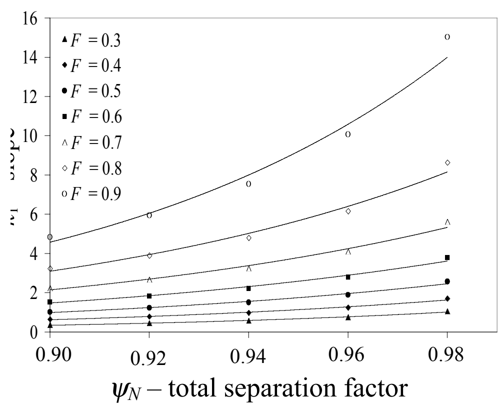


Fig. 5. The dependence of the slope k_1 on the total separation factor ψ_N for selected values of F

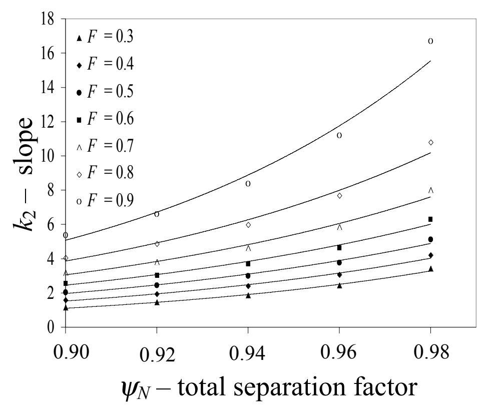


Fig. 6. The dependence of the slope k_2 on the total separation factor ψ_N for selected values of F

general analytical expression may be defined as

$$k_1 = \omega_1(F)\cdot e^{\omega_2(F)\psi_N} \quad \text{and} \quad k_2 = \omega_3(F)\cdot e^{\omega_4(F)\psi_N} \qquad (6)$$

By combining equations (3) and (7), one obtains

$$(N_D - N_T) = f \cdot \omega_1(F) \cdot e^{\omega_2(F)\psi_N} + s \cdot \omega_3(F) \cdot e^{\omega_4(F)\psi_N} \quad (7)$$

In equations (6) and (7) ω_1, ω_2, ω_3 and ω_4 are coefficients that change as the value of the extraction parameter F changes.

In order to obtain the relationship to calculate the number of stages (N_D - N_T), which would include all the parameters the influence of which was investigated: f, s, F and ψ_N, simultaneous fitting was performed.

For the case when $f \neq 0$ and $s = 0$, equations (5) and (7) may be written in the form:

$$(N_D - N_T) = f \cdot \theta_1(\psi_N) \cdot e^{\theta_2(\psi_N)F} = f \cdot e^{\lambda_2(\psi_N)F} \quad (8)$$

$$(N_D - N_T) = f \cdot \omega_1(F) \cdot e^{\omega_2(F)\psi_N} = f \cdot e^{\eta_2(F)\psi_N} \quad (9)$$

A unique presentation of these two relations would be:

$$(N_D - N_T) = f \cdot e^{\varphi_1(F,\psi_N)} \quad (10)$$

After taking the logarithm of the above equation one obtains:

$$\ln(N_D - N_T) = \varphi_1(F,\psi_N) + \ln f \quad (11)$$

Using the data in Table 2 and assuming that f = 1, the values of $\varphi_1(F,\psi_N)$ may be easily calculated:

$$\ln(N_D - N_T) = \varphi_1(F,\psi_N) \quad (12)$$

Assuming that the dependence of φ_1, of the form $\varphi_1(F,\psi_N) = \alpha_1 + \beta_1 F + \gamma_1 \psi_N$ and by using the least squares method to fit the two-parameter equation (12) with the aid of the developed program (Matlab), the following values were obtained for: α_1 = -13.22; β_1 = 4.17 i γ_1 = 12.27, which means that equation (10) takes on the form:

$$(N_D - N_T) = f \cdot e^{12,21 \cdot \psi_N + 4,17 \cdot F - 13,22} \quad (13)$$

The following equations are obtained by using the same procedure and assuming that f = 0 and s ≠ 0:

$$(N_D - N_T) = s \cdot \theta_3(\psi_N) \cdot e^{\theta_4(\psi_N)F} = s \cdot e^{\lambda_2(\psi_N)F} \quad (14)$$

$$(N_D - N_T) = s \cdot \omega_3(F) \cdot e^{\omega_4(F)\psi_N} = s \cdot e^{\eta_2(F)\psi_N} \quad (15)$$

$$(N_D - N_T) = s \cdot e^{\varphi_2(F,\psi_N)} \quad (16)$$

where $\varphi_2(F,\psi_N) = \alpha_2 + \beta_2 F + \gamma_2\psi_N$. The results obtained are: α_2 = -11.58; β_2 = 2.42 and γ_2 = 12.25, so equation (16) may be written in the form:

$$(N_D - N_T) = s \cdot e^{12,25\cdot\psi_N + 2,42\cdot F - 11,58} \quad (17)$$

As the backmixing effect is most commonly present in both phases (f ≠ 0 and s ≠ 0), the total number of stages $(N_D - N_T)$ is, in accordance with the stage additivity model, equal to the sum of the number of stages defined by equation (13) when backmixing is present only in phase L and the number of stages is defined by equation (17) when this phenomenon is present only in phase G. The integral equation takes on the form:

$$(N_D - N_T) = f \cdot e^{12,21\cdot\psi_N + 4,17\cdot F - 13,22} + s \cdot e^{12,25\cdot\psi_N + 2,42\cdot F - 11,58} \quad (18)$$

A comparison of the number of stages obtained on the basis of the developed analytical expressions (Table 1), $(N_D - N_T)_{analyt}$, and the number of stages calculated based on relation (18), $(N_D - N_T)_{rel}$, was performed for all combinations of parameters f and s and some combinations of ψ_N and F within the investigated range in order to check the obtained relationship (Table 5).

The agreement of the obtained results is exceptionally good which is indicated by the very high value of the correlation coefficient (R ≅ 1) for any combination of parameters ψ_N and F within the denoted boundaries.

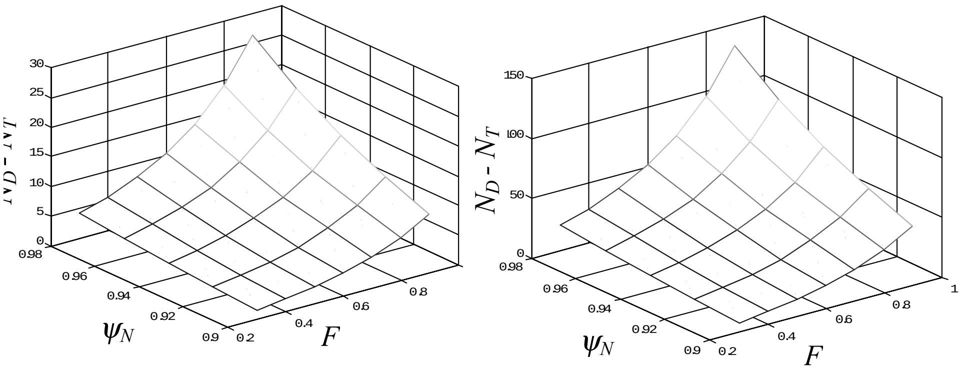
On the basis of the obtained relationship (18) it may be stated that the number of stages $(N_D - N_T)$ linearly increases with increasing backmixing coefficient in the L and G phases. With increasing value of the extraction factor, i.e. with increasing ratio of the phase flows and decreasing coefficient of equilibrium distribution (as well as solvent concentration), the necessary number of stages increases exponentially. If larger values of the total

Table 5. Comparison of the number of stages $(N_D - N_T)_{analyt}$ obtained on the basis of analytical expressions (Table 1) to the number of stages $(N_D - N_T)_{rel}$ calculated based on expression (18)

			$(N_D - N_T)_{analit}$ on the basis of expressions in Table1						$(N_D - N_T)_{rel}$ on the basis of equation (18)					
No		***f*** / ***s***	**0**	**1**	**2**	**3**	**4**	**5**	**0**	**1**	**2**	**3**	**4**	**5**
1)	$\psi_N = 0.90$, $F = 0.80$	**0**	0.00	3.23	6.46	9.69	12.92	16.14	0.00	3.19	6.37	9.56	12.75	15.93
		1	4.04	7.27	10.50	13.72	16.95	20.17	3.98	7.17	10.35	13.54	16.73	19.91
		2	8.08	11.30	14.53	17.75	20.98	24.21	7.96	11.14	14.33	17.52	20.70	23.89
		3	12.11	15.34	18.56	21.79	25.01	28.24	11.94	15.12	18.31	21.50	24.68	27.87
		4	16.14	19.37	22.59	25.82	29.05	32.27	15.92	19.10	22.29	25.48	28.66	31.85
		5	20.17	23.40	26.63	29.85	33.08	36.31	19.89	23.08	26.27	29.45	32.64	35.83
2)	$\psi_N = 0.90$, $F = 0.30$	**0**	0.00	0.36	0.71	1.05	1.40	1.74	0.00	0.40	0.79	1.19	1.58	1.98
		1	1.17	1.51	1.85	2.19	2.52	2.86	1.19	1.58	1.98	2.37	2.77	3.17
		2	2.30	2.64	2.97	3.31	3.65	3.99	2.37	2.77	3.17	3.56	3.96	4.35
		3	3.42	3.76	4.10	4.43	4.77	5.11	3.56	3.96	4.35	4.75	5.14	5.54
		4	4.55	4.88	5.22	5.56	5.89	6.23	4.75	5.14	5.54	5.93	6.33	6.73
		5	5.67	6.00	6.34	6.68	7.01	7.35	5.93	6.33	6.72	7.12	7.52	7.91
3)	$\psi_N = 0.94$, $F = 0.50$	**0**	0.00	1.52	3.02	4.51	6.00	7.49	0.00	1.49	2.98	4.47	5.96	7.45
		1	3.02	4.51	6.00	7.49	8.98	10.47	3.14	4.63	6.12	7.61	9.10	10.59
		2	6.00	7.49	8.98	10.47	11.96	13.44	6.28	7.77	9.26	10.75	12.25	13.74
		3	8.98	10.47	11.96	13.44	14.93	16.41	9.43	10.92	12.41	13.90	15.39	16.88
		4	11.96	13.44	14.93	16.41	17.90	19.39	12.57	14.06	15.55	17.04	18.53	20.02
		5	14.93	16.41	17.90	19.39	20.87	22.36	15.71	17.20	18.69	20.18	21.67	23.16
4)	$\psi_N = 0.98$, $F = 0.85$	**0**	0.00	11.12	22.23	33.34	44.44	55.55	0.00	10.48	20.95	31.43	41.90	52.38
		1	13.08	24.19	35.30	46.40	57.51	68.61	11.97	22.44	32.92	43.39	53.87	64.35
		2	26.15	37.26	48.36	59.47	70.57	81.68	23.93	34.41	44.88	55.36	65.83	76.31
		3	39.22	50.32	61.43	72.53	83.64	94.74	35.90	46.37	56.85	67.32	77.80	88.28
		4	52.28	63.39	74.49	85.60	96.70	107.8	47.86	58.34	68.81	79.29	89.77	100.2
		5	65.35	76.45	87.56	98.66	109.8	120.9	59.83	70.30	80.78	91.25	101.7	112.2
5)	$\psi_N = 0.98$, $F = 0.30$	**0**	0.00	1.09	2.14	3.18	4.20	5.22	0.00	1.06	2.11	3.17	4.23	5.29
		1	3.52	4.54	5.56	6.58	7.60	8.62	3.16	4.22	5.28	6.33	7.39	8.45
		2	6.92	7.94	8.95	9.97	10.98	12.00	6.32	7.38	8.44	9.49	10.55	11.61
		3	10.31	11.32	12.34	13.35	14.36	15.38	9.48	10.54	11.60	12.66	13.71	14.77
		4	13.69	14.70	15.71	16.73	17.74	18.75	12.65	13.70	14.76	15.82	16.87	17.93
		5	17.06	18.08	19.09	20.10	21.11	22.13	15.81	16.86	17.92	18.98	20.04	21.09

Correlation coefficient R = 1.0000, mean square deviation σ = 0.27; 2) R = 0.9995, σ = 0.29; 3) R = 0.9998, σ = 0.47; 4) R = 0.9999, σ = 4.86; 5) R = 0.9995, σ = 0.71

separation factor are to be achieved during extraction, an exponential increase of $(N_D - N_T)$ must be expected. A graphic presentation of the obtained number of stages $(N_D - N_T)$ on the basis of equation (18) for boundary conditions of the backmixing coefficients f = s = 1 and f = s = 5 are presented in Fig. 7.



a) $f = s = 1$ b) $f = s = 5$

Fig. 7. Graphical presentation of the number of stages (N_D - N_T) obtained based on relationship (19)

Equation (18) may also be used to calculate the number of stages (N_D - N_T) that are formed due to the occurrence of backmixing in the case of a linear equilibrium distribution, if the values of the initial extraction parameters, as well as the non-ideality parameters of the phase flows, are known. This application is illustrated by several examples for counter-current extraction (Table 6).

5. Conclusion

The number of stages (N_D - N_T), the existence of which is the consequence of the occurrence of backmixing in counter-current extractors, was calculated on the basis of analytical expressions developed for the case of a linear equilibrium distribution for selected values of the backmixing coefficient, extraction factor and total separation factor.

Starting from the calculated values of the number of stages (N_D - N_T), as a measure of the influence of the backmixing effect, it was demonstrated that the value of (N_D - N_T) significantly increased with increasing backmixing coefficient and that the relationship was linear. This increase evidently indicates the damaging effect of backmixing, i.e. the decrease of the extraction efficiency.

The influence of the ratio of phase flows was included in the calculation of the number of stages (N_D - N_T) via the extraction factor, as were the extractant distribution coefficient and total separation factor, which led to the establishment of relationship (18). It may be concluded on the basis of the obtained relationship that the number of stages (N_D - N_T) exponentially increased with increasing ratio of the phase flows and total separation factor, i.e. it exponentially decreased with increasing distribution coefficient.

Equation (18) represents a tool for the estimation of the number of stages in extractors in which there is non-ideal phase flow. This is especially important in practice in the selection of extractors and the denoted parameters for certain extraction operations.

Table 6. Calculation of the number of stages (N_D - N_T)rel for various values of the initial data; y_{in} - the inlet concentration of the substance in phase G; x_{in} = y_{in}/m - the equilibrium inlet concentration of the substance in stage L; d = $[(N_D-N_T)_{rel}$ - $(N_D-N_T)_{anayit}]\times 100$ / $(N_D-N_T)_{analyt}$ - relative error in %*

Initial data							Calculated values					
x_{in} mol·dm^{-3}	x_{out} mol·dm^{-3}	y_{in} mol·dm^{-3}	Q	m	f	s	x_{in}* mol·dm^{-3}	ψ_N	F	$(N_D-N_T)_{rel}$	$(N_D-N_T)_{anayit}$	δ (%)
1.00	0.10	0.00	0.25	0.30	4	0	0.00	0.900	0.833	14.65	14.69	0.27
4.00	0.25	0.00	0.50	1.00	2	4	0.00	0.938	0.500	15.08	14.54	3.71
4.00	0.40	0.05	0.70	0.85	1	5	0.06	0.913	0.824	28.97	28.31	2.35
10.00	1.00	0.10	1.00	1.50	3	2	0.07	0.906	0.667	12.11	12.43	2.53
14.00	0.30	0.00	0.45	1.20	0	3	0.00	0.979	0.375	11.17	11.70	4.52

Note: The calculated values of ψ_N and F must lie in the interval in which relation (18) is valid.

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