NEW METHOD FOR DETERMINATION OF TEMPERATURE IN SPALLATION REACTIONS

by

Dragana J. JORDANOV [1*], ***Bojana S. GRABEŽ*** [2], ***Krunoslav M. SUBOTIĆ*** [1], ***Laslo J. NADJDJERDJ*** [1]

[1] Vinča Institute of Nuclear Sciences, University of Belgrade, Belgrade, Serbia
[2] Institute of Physics, University of Belgrade, Belgrade, Serbia

Scientific paper
UDC: 536.24/.25:539.12-17
DOI: 10.2298/NTRP1102153J

We propose a new method for determination of temperature in spallation events. It is shown that temperature can be determined by applying the friction model of energy dissipation in participant-spectator model of a spallation process. First order estimate of temperature dependence of the participant zone on reaction *Q*-value is obtained from the Fermi gas model considerations. The heat diffusion process is also discussed.

Key words: spallation, nuclear temperature, heavy-ion reactions

INTRODUCTION

Determination of temperature and its distribution in the participant zone of an ion-ion interaction are of importance for the studies of the equation of state of nuclear matter. Temperature is the key parameter needed for characterization of the system.

In practice, the temperature of equilibrated nuclear systems can be determined from gamma decay on the basis of the intensities of γ-ray lines. The population distribution of excited states in statistical equilibrium depends on the temperature of the system and the spacing between the levels [1]. The fraction f in a given excited state is a simple function of temperature, related to the population ratio, R, as

$$f \quad \frac{R}{1 \quad R} \tag{1}$$

and

$$R \quad \frac{2j_{\mathrm{n}} \quad 1}{2j_{\mathrm{e}} \quad 1} \exp \frac{\Delta E}{kT} \tag{2}$$

where j_{n} and j_{e} are the spin quantum numbers of the lower and higher states, respectively, ΔE is the energy difference between the states, k – the Boltzmann constant and T – the temperature.

Statistical model calculations are based on the assumption of particle emission from equilibrated subsets of nucleons. There is a relation between the relative population of states and temperature at the point at which the particles leave the equilibrated subsystem.

In the non-equilibrated systems the temperature is determined from the slope of particle spectra. The method for determination, of nuclear temperature from the slope of emitted particle spectra is based on calculations of relative yield of the specific detached type of emitted particle at different values of kinetic energy. The measured spectra are consistent with the mean nuclear temperature [2]. Alternatively, the kinetic energy of the recoil can be measured. The influence of temperature on kinetic energy spectra is expected to manifest via a factor of the type exp $(-\varepsilon_{\mathrm{k}} \ T$.

The expression for the emission rate has the form

$$\frac{\mathrm{d}N^2}{\mathrm{d}t\mathrm{d}\varepsilon_{\mathrm{k}}} \quad (\varepsilon_{\mathrm{k}} \quad V_C)\exp \quad \frac{\varepsilon_{\mathrm{k}}}{T} \tag{3}$$

where V_{C} is the Coulomb potential [3], ε_{k} – the kinetic energy, and T – the temperature.

In the spallation reaction, fraction of particle emission occurs before thermodynamic equilibrium is established. Before relaxation of the nuclear system and eventually reaching the equilibrium, a process of heat diffusion takes place and this influences the energy and angular distribution of emitted nucleons. It is observed that energy distribution of emitted particles deviates from the equilibrium spectrum corresponding to the excitation energy [4] *i. e.* that a considerable enhancement occurs in the high-energy region. Consideration of this distribution provides direct information about heat conductivity of nuclear matter.

* Corresponding author; e-mail: djordano@vinca.rs

In this work we study the spallation reaction $^{20}_{10}\text{Ne} \quad ^{27}_{13}\text{Al}$ at the energy of 84 MeV per nucleon (intermediate energy), which is described in detail in reference [5]. The temperature of the system and the characteristics of the temperature field are obtained by applying appropriate microscopic models.

DETERMINATION OF TEMPERATURE IN SPALLATION PROCESSES

It has been shown that spallation, which results in production of a massive target-like (TL) fragment, is a pre-equilibrium process which occurs before the establishment of thermal equilibrium, with the process of thermal diffusion in nuclear matter. The spallation process cannot last longer than the relaxation time of thermal diffusion τ_r. The developing temperature field is anisotropic, producing the nucleon flux in the direction from the target towards the outside.

According to the microscopic model of Abul-Magd [6], the nucleon that is to be abraded from the target is hit by a projectile and is "kicked out" from its bound orbit in the potential well, $-U$. When the nucleon gets the four-momentum transfer $q(\boldsymbol{q}, \Delta Q)$, where $\Delta Q = q^2/2m_0 \quad 25$ MeV, the abraded nucleon leaves the potential well with momentum q' and energy $q'^2/2m_0 = \Delta Q - U$. Linear momentum gain of the recoiled fragment per momentum transferred in an elementary nucleon-nucleon collision $p = q - q'$ is defined by the friction coefficient

$$f \quad \frac{p}{q} \tag{4}$$

from which one may evaluate the potential

$$U \quad f(2 \quad f)\Delta Q \tag{5}$$

The potential becomes "shallower" at more central collisions, due to rising of the temperature T in the participant zone. The first order estimate of temperature as well as that of the flux of nucleons leaving the potential well may be obtained from the Fermi gas model [7]. Reference [7] gives the relationship between the incident energy per nucleon, the excitation energy of the source and its temperature for particles in heavy-ion reactions.

The excitation energy E and temperature T of the absolutely relaxed (equilibrated) nuclear system with mass number, A, are coupled via the "equation of state"

$$E = a\,T^2 \tag{6}$$

where

$$a \quad \frac{1}{17}A \;[\text{MeV}]^{\quad 1} \tag{7}$$

The temperature field is isotropic for the equilibrium process, which means that the temperature is equally distributed over the entire sphere of nucleus, while for a pre-equilibrium process it starts to occupy the part of the sphere in some preferential direction. The distribution of the temperature field and the correlated distributions of energies and emission angles of the nucleons leaving the nucleus in an abrasion process are not isotropic in the system of the recoiling nucleus ($\boldsymbol{q} = 0$). They are very sensitive functions of the ratio $\chi \quad \tau \quad \tau_r$ [4], *i. e.* ratio between the minimum time required for the system to reach a local equilibrium (τ) and the relaxation time of the system (τ_r). The high-energy region of the spectrum and the forward peaked angular distribution are considerably enhanced at the values χ [4] (in our case χ). Thus, one may expect that pre-equilibrium emission flux of the abraded nucleons, takes place through the base of the cone defined by a characteristic circle. As observed experimentally, emission takes place mostly through the base of the cone.

The existence of the well defined cone, in the system of the recoiling nucleus ($\boldsymbol{q} = 0$), of the preferential nucleon emission at abrasion, is consistent with the characteristics of mass transfer arising from reaction kinematics, and may be explained by characteristics of the heat diffusion process in the pre-equilibrium phase of nuclear matter.

RESULTS AND DISCUSSION

In our calculations we use the Fermi gas model [7]. In the experiment we measure the atomic numbers, angular distribution, and distribution of energy per nucleon of TL fragments in a 4 geometry by using the CR-39 plastic track detectors [5]. These data were the input data to our FORTRAN code INES (INtermediate Energy Spallation). This code was developed to calculate the important parameters for the reaction $^{20}_{10}\text{Ne} \quad ^{27}_{13}\text{Al}$ [8].

The following eqs. (8)-(15) (based on [1] and [6]) have been used in the code INES to calculate the temperature

$$\frac{q^2}{2m} \quad U \quad \frac{q'^2}{2m} \tag{8}$$

$$\Delta s \quad (A_2 \quad A_3)/\tau\Phi \tag{9}$$

$$\cos\theta \quad 1 \quad 2\Delta s/S \tag{10}$$

$$\Phi \quad 2\pi g T^2 \frac{m}{h^3} e^{\quad \beta/T} \tag{11}$$

where U is the depth of the potential well, Φ – the flux of nucleons from the target towards the projectile, β – the separation energy of the particle, ΔS – the area of the base of the emission cone, τ – the time of the rotation of the system *i. e.* the life-time of the transient system and θ – the angle of the emission cone *i. e.* the abrasion angle. Table 1 summarizes the obtained results.

Following the arguments of Weiner and Westrom [4], after deposition of energy ΔQ in the nucleon-nucleon collision in the participant zone, the projectile nucleon creates a local excitation ("hot

Table 1. Mass number of target-like fragments produced in the reaction (A_3), the corresponding Q-values, temperature of equilibrium (T_{eq}), temperature of the participant zone (T), the ratio T/T_{eq}, and the cosine of the emission angle of target-like fragments (cos θ), as calculated by the computer code INES

A_3	Q	T_{eq}	T	T/T_{eq}	$\cos\theta$
6.00	869.613	12.17	36.934	3.036	0.804
6.00	867.818	12.15	36.934	3.039	0.804
6.00	873.650	12.19	36.934	3.029	0.804
6.00	869.565	12.17	36.934	3.036	0.804
6.00	876.916	12.22	36.934	3.023	0.804
6.00	868.458	12.16	36.934	3.038	0.804
8.00	799.461	11.67	34.698	2.974	0.744
8.00	804.696	11.7	34.698	2.965	0.744
8.00	802.189	11.69	34.698	2.969	0.744
8.00	832.419	11.9	34.698	2.915	0.744
8.00	840.859	11.96	34.698	2.900	0.744
8.00	830.790	11.89	34.698	2.918	0.744
8.00	835.495	11.93	34.698	2.910	0.744
8.00	831.309	11.9	34.698	2.917	0.744
10.00	786.607	11.57	31.851	2.753	0.671
10.00	752.423	11.32	31.851	2.814	0.671
10.00	787.217	11.58	31.851	2.752	0.671
10.00	798.352	11.66	31.851	2.732	0.671
10.00	780.264	11.52	31.851	2.764	0.671
10.00	790.914	11.6	31.851	2.745	0.671
12.00	728.813	11.14	28.165	2.529	0.582
12.00	741.533	11.23	28.165	2.507	0.582
12.00	730.533	11.15	28.165	2.526	0.582
12.00	732.244	11.16	28.165	2.523	0.582
14.00	674.106	10.71	23.335	2.178	0.471

spot") in the target nucleus. In the heat diffusion process the developed temperature field results in anisotropic distribution and the abraded nucleons leave the target under the effect of gradient of this field in the cone defined by the solid angle Ω $\Delta S/r^2$. The nucleons leave the "abraded" nucleus through the surface element ΔS, which equals the drift velocity (number of nucleons which leave this surface from the target to the projectile) divided by flux of the nucleons $\Delta S = V_d/\Phi$, the drift velocity itself being equal $V_d = \Delta A/t_{im}$, with t_{im} being the lifetime of the transient system. The temperature of the "hot spot" is experimentally determined from the depth of the abraded nucleon potential, and temperature of the equilibrated system T_{eq} is determined via the aforementioned equation of state.

According to Weiner and Westrom [4] the ratio of the temperature of the participant zone to the temperature of the system when equilibrium is attained is the function of the relaxation time $T/T_{eq} = F(\tau_r)$. On the other side, the relaxation time depends on the thermodynamic parameters of nuclear matter and can be determined from the angular distribution of the temperature field. The ratio T/T_{eq} is presented in fig. 1 as a function of the cosines of the angle of emission cone. The two representative values of the experimentally

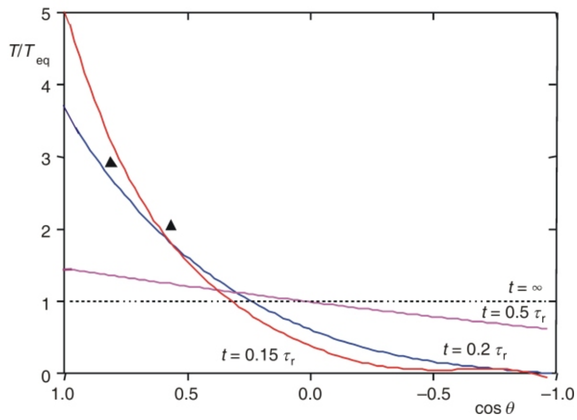

Figure 1. Angular dependence of the temperature field of the participant zone, T, in the units of temperature of equilibrium, T_{eq}, for ratio χ τ τ_r at various times during relaxation (for times $t = 0.15\ \tau_r$, $t = 0.2\ \tau_r$ and $t = 0.5\ \tau_r$); at equilibrium ($t = \infty$) the distribution is isotropic.

obtained dependence T/T_{eq} on the cosine abrasion angle θ (tab. 1) are shown in the figure (triangles) (χ) and are compared with theoretical values calculated for different times in units of the relaxation time [4]. It is seen that our results agree with strongly anisotropic distribution, resulting from the process far from equilibrium.

The systematic interdependence of the reaction Q-value in the excitation of the transient system and the lifetime of the transient system, suggests that the projectile nucleus describes a well-defined trajectory while crossing the collision partner. Energy deposition is developed through individual nucleon-nucleon collisions, with a constant fraction of energy loss, in participant zone.

To calculate the Q-values using the expression

$$Q \quad E_3 \quad 1 \quad \frac{M_3}{M_4} \quad E_1 \quad 1 \quad \frac{M_1}{M_4}$$
$$\frac{2\sqrt{M_1 E_1 M_3 E_3}}{M_4}\cos\theta \tag{12}$$

we put it in a form suitable for programming and replace the corresponding masses with mass numbers

$$Q \quad \frac{A_1 A_3 E_1}{(A_1 \quad A_2)A_4}$$
$$\frac{1 \quad \delta_3^2 \quad 2\delta_3\cos\theta_3}{\delta_3^3} \quad \frac{A_2 A_4}{A_1 A_3} \tag{13}$$

where

$$\delta_3 \quad \sqrt{\frac{A_1 A_3}{A_1 \quad A_2} {}^2 \frac{E_1}{E_3}} \tag{14}$$

A_1 is the mass number of the projectile ($^{20}_{10}$Ne), A_2 – the mass number of the target $^{27}_{13}$Al, A_3 – the mass number of the target-like fragment, $A_4 = A_1 + A_2 - A_3$ – the mass number of the projectile-like fragment, E_1 = = 1680 MeV – the kinetic energy of the projectile, E_3 – the kinetic energy of target-like fragment in units of MeV, and θ_3 – the scattering angle of target-like fragments in the laboratory frame of reference.

Q-value is calculated in the INES code for each ejectile accordingly set out by the experimental energy and angle of ejectile with mass A_3.

The temperature of the participant zone is defined by

$$T \quad E_{\rm f}\sqrt{1 \quad \frac{U \quad B}{E_{\rm f}}} \tag{15}$$

where $E_{\rm f}$ – the Fermi energy, *B* – the bounding energy, and *U* is nucleon potential in the potential well.

Figure 2 presents the temperature of the participant zone as a function of the *Q*-value of the interaction divided by the total number of the nucleons emitted in the interaction *i. e.* per the difference between the mass numbers of the target and the heavy spallation residue.

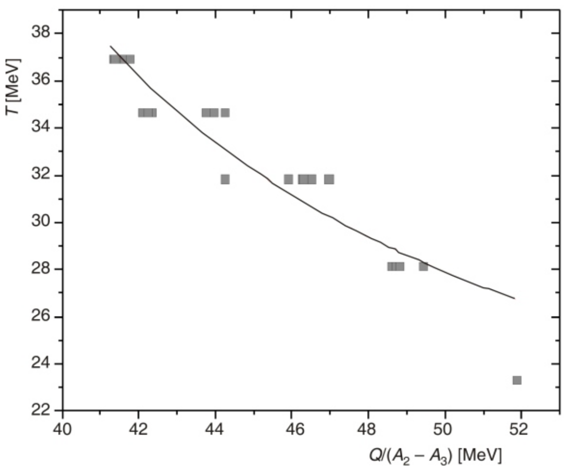

Figure 2. Temperature in the participant zone *vs.* the *Q*-value per emitted nucleon

From our results presented in fig. 1 and fig. 2 we obtain the values of the temperature at the time when the system would reach equilibrium, $T_{\rm eq}$. The plot of $T_{\rm eq}$ *vs*. *Q*-value of the interaction is presented in fig. 3.

The remarkable result is that, according to fig. 3, $T_{\rm eq}$ turns out to be a linear function of the *Q*-value.

CONCLUSION

The new method for determination of relevant temperatures in spallation process was developed. It was assumed that the reaction obeys the participant-spectator scenario. The friction model of energy dissipation and the Fermi gas model were used to calculate the temperatures. We applied our considerations to the 84 MeV per nucleon, $^{20}_{10}$Ne $^{27}_{13}$Al interaction. The obtained results for dependences of the temperature of the hot zone and of the temperature of the system in equilibrium ($T_{\rm eq}$) on the *Q*-value of the interaction are presented. The equilibrium temperature turns out to be a linear function of the *Q*-value.

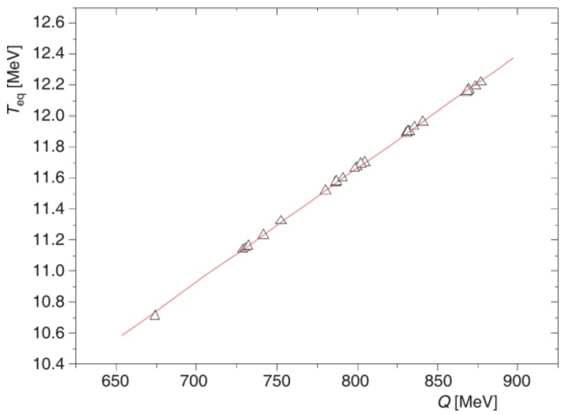

Figure 3. Temperature of equilibrium *vs.* *Q*-value of interaction

ACKNOWLEDGEMENT

The authors are grateful to the Ministry of Education and Science of the Republic of Serbia for the financial support of the present work, Project No ON171028.

REFERENCES

[1] Lee, J. H., Benenson, W., Bloch, C., Chen, Y., Radtke, R. J., Kashy, E., Mohar, M. F., Morrissey, D. J., Blue, R., Roningen, R. M., New Test of the Excited State Population Method for Measurements of Nuclear Temperatures, *Physical Review C, 41* (1990), 5, pp. 2406-2409

[2] Pochodzalla, J., Friedman, W. A., Gelbke, C. K., *et al*, Nuclear Temperatures and the Population of Particle-Unstable States of ^{6}Li in ^{40}Ar-Induced Reactions on ^{197}Au at *E*/*A* = 60 MeV, *Physical Review Letters, 55* (1985), 2, pp.177-180

[3] Morrissey, D. J., *et al.*, Excited State Production and Temperature Measurement in a Heavy Ion Reaction, *Physical letters B, 148* (1984), 6, pp. 423-427

[4] Weiner, R., Westrom, M., Diffusion of Heat in Nuclear Matter and Preequilibrium Phenomena, *Nuclear Physics A, 286* (1977), 2, pp. 282-296

[5] Jordanov, D., Mechanism of Spallation Induced by ^{20}Ne Beam at Intermediate Energies, M. Sc. thesis, Faculty of Physics, University of Belgrade, Belgrade 2008

[6] Abul-Magd, A., Hüfner, J., Schürmann, B., Friction in Heavy Ion Reactions at Relativistic Energies, *Physics Letters B*, *60* (1976), 4, pp. 327-330

[7] Nifenecker, H., The Hot Fermi Gas Model, *Zeitschrift für Physik A, Hadrons and Nuclei*, *323* (1986), 4, pp. 387-390

[8] Jordanov, D., Subotić, K., Dragosavac, D., Ujić, P., Čeliković, I., INES (CNtermediate Energy Spallation)

code for Simulation of Intermediate Energy Spallation, *Journal of Research in Physics, 31* (2007), 2, pp. 133-136

Received on February 18, 2011
Accepted on September 8, 2011

**Драгана Ј. ЈОРДАНОВ, Бојана С. ГРАБЕЖ,
Крунослав М. СУБОТИЋ. Ласло Ј. НАЂЂЕРЂ**

НОВА МЕТОДА ОДРЕЂИВАЊА ТЕМПЕРАТУРЕ У РЕАКЦИЈАМА СПАЛАЦИЈЕ

Приказана је нова метода одређивања температуре у спалационим догађајима. Показано је да температура може бити одређена применом фрикционог модела енергије дисипације на партиципант-спектатор моделу спалационих процеса. Прва процена зависности температуре партиципант зоне од реакционе Q-вредности добијена је применом Ферми гас модела. Разматран је и процес топлотне дифузије.

Кључне речи: спалација, нуклеарна температура, тешкојонске реакције