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a latent likelihood approach**

Hanke, Michael; Poulsen, Rolf; Weissensteiner, Alex

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Where would the EUR/CHF exchange rate be without the SNB's minimum exchange rate policy?*

Michael Hanke

Institute for Financial Services
University of Liechtenstein, Vaduz, Liechtenstein

Rolf Poulsen[†]

Department of Mathematical Sciences
University of Copenhagen, Copenhagen, Denmark

Alex Weissensteiner

Department of Management Engineering
Technical University of Denmark, Lyngby, Denmark
and

School of Economics and Management
Free University of Bolzano, Bolzano, Italy

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Abstract

Since its announcement made on Sept. 6, 2011, the Swiss National Bank (SNB) has been pursuing the goal of a minimum EUR/CHF exchange rate of 1.20, promising to intervene on currency markets to prevent the exchange rate from falling below this level. We use a compound option pricing approach to estimate the latent exchange rate that would prevail in the absence of the SNB's interventions, together with the market's confidence in the SNB's commitment to this policy.

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[†]Corresponding author. Universitetsparken 5, DK-2100 Copenhagen. E-mail: rolf@math.ku.dk; Tel.: +45 3532 0685

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1 Introduction

In the early 2000s, the EUR/CHF exchange rate hovered around 1.6, with a low of 1.444 in September 2001 and a high of 1.679 in October 2007. From 2008 onwards, the euro depreciated markedly and almost reached parity on August 11, 2011. This strength of the Swiss franc posed a severe problem especially for Swiss and Liechtenstein exporters,¹ which spurred calls for interventions on the part of the Swiss National Bank (SNB). The SNB acted by announcing on Sept. 6, 2011, to enforce a minimum exchange rate of 1.20 Swiss francs per euro, together with a commitment to buy euros in unlimited quantity in order to reach this goal. The SNB did not indicate any specific time frame for this policy, but hinted at possible further measures if necessitated by economic prospects and deflationary risks.²

Until the time of writing (May 2014), this policy was successful: Since its inception, the exchange rate has always been at or above 1.20 (disregarding very short-term and minor violations of this intervention level). This, however, comes at the risk of domestic inflation. Especially in the first few months after announcing this policy, the SNB's euro reserves grew markedly, whereas in late 2012, the SNB managed to get rid of a sizable portion of its euro position without causing a slump in the exchange rate. The SNB has been called upon its promise repeatedly since September 2011 and has proven its commitment on these occasions.

From an economic point of view, this raises the question where the EUR/CHF exchange rate would be without the SNB's policy. This question is at the core of the present paper. Out of a number of possible approaches, we opt for an option-pricing setting, modeling traded options on the exchange rate as compound options on the latent exchange rate process that would prevail in the absence of the SNB's policy. As a by-product, we derive market estimates of the credibility of the SNB's commitment to the intervention level of 1.20 in the form of probabilities for a policy change occurring before the traded options' maturities. Part of the desired effect on the exchange rate level might come from decreased speculation following the SNB's announcement. One limitation of our approach is that we

¹Liechtenstein also uses the Swiss franc as its official currency.

²See http://www.snb.ch/en/mmr/reference/pre_20110906/source/pre_20110906.en.pdf for the original SNB press release.

do not attempt to measure the amount of speculation in the market or its effects on the observed exchange rate.

We are unaware of any other paper that tries to infer the latent exchange rate process from observed exchange rates in the presence of central bank interventions. The related paper by Hertrich and Zimmermann (2014) also uses an option-based analysis of the EUR/CHF exchange rate, but has a different focus: Its main goal is to assess the credibility of the SNB's commitment to the intervention level of 1.20. As an earlier contribution, Malz (1996) uses an option pricing approach based on a jump-diffusion process for the exchange rate to study the probability of an exchange rate leaving a specified target zone. Both models will be discussed and compared to the approach used in the present paper in Section 2.1.

The paper is structured as follows: Section 2 describes modeling of the exchange rate in more detail, relating our approach to the literature. Section 3 discusses the option pricing model. Section 4 describes our data and details the estimation procedure. Section 5 presents the empirical results, and Section 6 concludes.

2 Exchange rate modeling

In this section, we describe our approach to modeling the EUR/CHF exchange rate before and after the SNB's policy announcement on Sept. 6, 2011. This will be the basis for the option pricing model laid out in Section 3. To point out the differences of our model to related papers, we start by a description of the models used by Malz (1996) and Hertrich and Zimmermann (2014). Fig. 1 shows the development of the EUR/CHF exchange rate from Jan. 2011 to Oct. 2013. The political and banking crises in the eurozone in the second and third quarters of 2012 caused the EUR/CHF exchange rate to decrease towards the intervention level of 1.20, which was tested several times, e.g., in April 2012.

2.1 Related literature

Malz (1996) studies the behavior of the GBP/DEM exchange rate in the years 1990-1992. The British pound became part of the so-called Exchange Rate Mechanism (ERM), which marked an important step towards the euro. A target rate was fixed towards the European Currency Unit (ECU), together with a $\pm 6\%$ fluctuation band. Due to political and economic shocks during the observation period, there was speculation about a possible realignment of this target rate (Malz, 1996, p. 721). Malz uses a jump-diffusion process to

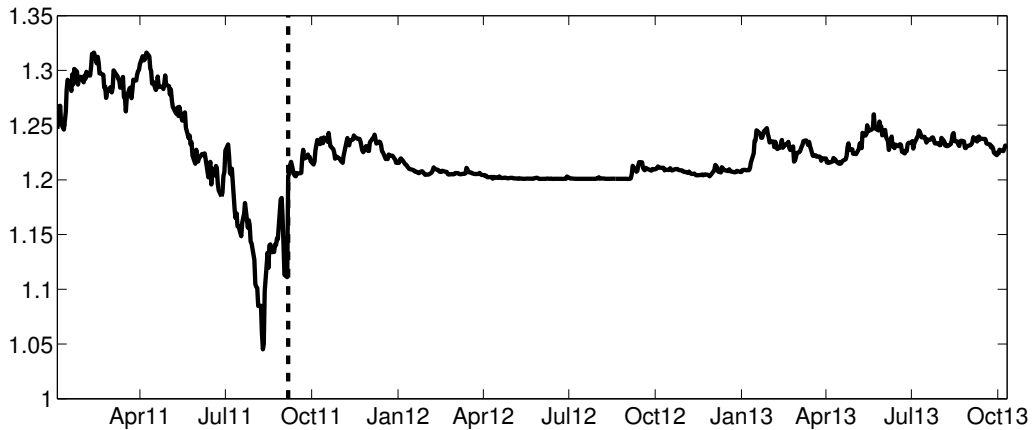


Figure 1: EUR/CHF exchange rate from Jan. 2011 to Oct. 2013. On Sept. 6, 2011 (vertical dashed line), the SNB announced its commitment to enforce a minimum exchange rate of 1.20 Swiss francs per euro.

capture what he considers to have been the prevailing market view of the likely consequence of such a realignment: A sudden jump in the value of the observed exchange rate, but no change in exchange rate volatility. While the assumption of unchanged volatility may be justified for the type of realignment studied by Malz, it is quite different from the likely consequences of a revocation of the SNB's minimum exchange rate policy. Fig. 1 shows that the EUR/CHF exchange rate volatility declined sharply following the SNB's announcement on Sept. 6, 2011. Moreover, in the second and third quarters of 2012, the EUR/CHF rate approached the intervention level of 1.20. At the same time, exchange rate volatility reached historical lows, which clearly indicates level-dependence in volatility. Since the jump-diffusion process used by Malz (1996) assumes volatility to be level-independent, it is not suitable for the economic situation analyzed here. Moreover, this model assumes that a realignment event is triggered externally, which means that realignment probabilities are independent of the current level of the process. In the case of the SNB's minimum rate policy, we can reasonably assume that the probability of an end to the SNB's commitment is higher when the exchange rate is close to the intervention level. Hui and Lo (2009) analyze the same situation and data as in Malz (1996), but use a first-passage time approach to estimate realignment probabilities. This overcomes an important limitation of Malz (1996) by modeling these probabilities as path-dependent, i.e., they increase when the exchange rate approaches the boundaries of the target-rate band. However, the economic situation

of a target-rate band differs from that of the SNB’s minimum rate policy, which is why Hui and Lo model the exchange rate before the realignment event as mean-reverting. Since a pronounced increase in the EUR/CHF rate, which would move the exchange rate way above 1.20 (say, to 1.35) would not be a reason for an intervention on the part of the SNB, a mean-reverting process does not seem to be an adequate model for its minimum rate policy. Neither approach, whether treating realignment probabilities as external (Malz, 1996) or internal (Hui and Lo, 2009), is designed to model the exchange rate before and after the realignment event in an internally consistent manner: Both use descriptive processes with different parameters before and after realignment, whereas the minimum rate policy of the SNB allows us to put more structure on the link between these two phases. This will be an important aspect for the modeling approach taken in the present paper, which will be discussed in Section 2.2.

Hertrich and Zimmermann (2014) model the EUR/CHF exchange rate in the presence of the SNB’s commitment as a geometric Brownian motion (GBM). Observing non-zero prices for put options with strikes below an exchange rate of 1.20, they conclude that this minimum exchange rate is not perceived as fully credible by the market. To rationalize non-zero prices for puts with strikes below 1.20 while maintaining the GBM assumption for the observed exchange rate, they invoke the additional assumption of a market-anticipated decrease of the intervention level of 1.20 to a – fully credible – lower level $b < 1.20$. They use this lower bound b as a reflection barrier on the geometric Brownian motion. Option pricing in such a setting has first been studied by Veestraeten (2008), with an extension for continuous dividend yields provided by Hertrich and Veestraeten (2013). We do not use the Hertrich and Zimmermann (2014) approach in the present paper for the following reasons. First, the assumption of an anticipated reset of the intervention level to a level $b < 1.20$ seems difficult to justify: If, at some stage, the SNB should be unable or unwilling to defend an exchange rate of 1.20, markets simply would not perceive resetting the intervention level to, e.g., 1.15 as credible. Moreover, the Hertrich and Zimmermann (2014) model implicitly assumes the barrier b to exist forever, but allows the *level* of the barrier to change daily, with sizable fluctuations (Fig. 6 in Hertrich and Zimmermann, 2014, p. 35). Second, their model treats the observed exchange rate process as external and does not address consistency between observed exchange rates and their process assumptions. Observing the exchange rate path in the second and third quarters of 2012 (cf. Fig. 1) is extremely unlikely given the assumption of GBM. The put option implied by the SNB’s policy, which will be discussed in Section 2.2, affects the observed exchange rate process,

e.g., by decreasing volatility when exchange rates approach the intervention level. These effects are ignored when assuming a GBM for the observed exchange rates in the presence of the SNB’s policy. Third, the model requires historical volatilities to estimate the reflection barrier b . Thus, it assumes a constant exchange rate volatility, which is clearly at odds with the empirically observed exchange rate process (see Fig. 1).

2.2 Modeling observed and latent exchange rates

In contrast to Hertrich and Zimmermann (2014), our model for the EUR/CHF exchange rate is designed to apply both in the presence and in the absence of the SNB’s policy, and it links the two regimes in an internally consistent manner. It explicitly accounts for a possible future removal of the intervention level of 1.20, which allows us to estimate the corresponding probabilities of such a removal before the expiration of traded options. Furthermore, we endogenize the observed exchange rate process by explicitly considering the value of the SNB’s policy to the holders of euros. For each euro in the market, the SNB’s promise to defend a level of 1.20 can be interpreted as an American put option with uncertain lifetime. Once a euro is bought by the SNB, the put option becomes meaningless, given that the writer of the put now holds the underlying. If the SNB sells the euro again, it comes with a newly written put. Since the SNB’s announcement, the observed exchange rate reflects the value of this put option. Denoting the EUR/CHF exchange rate before Sept. 6, 2011, by V_t , this leads to two processes from this date onwards: The observed exchange rate S_t is the sum of the latent exchange rate V_t (“freely floating”, no longer observable) and the value of the American put option. Once the SNB abandons its minimum exchange rate policy, S_t will be equal to V_t again.

Instead of trying to explain the exchange rate by fundamental factors, we follow the literature discussed in Section 2.1 and choose a descriptive model, specifying a stochastic process for V_t . Using option pricing theory, we compute the corresponding process for the observed exchange rate S_t in the presence of the SNB’s policy. Market data are then used to calibrate the model and back out the latent exchange rate process V_t from the observed exchange rate and traded exchange rate options.

We use the following notation:

- V_t . . . value of the EUR/CHF spot exchange rate at time t without the SNB’s minimum exchange rate policy (observable until Sept. 5, 2011, unobservable since, and again observable upon revocation of the policy),

- σ_V ... the volatility of V_t ,
- r_t and i_t ... domestic (CHF) and foreign (EUR) spot rates,
- K ... intervention level imposed by the SNB, equal to 1.20,
- S_t ... observed EUR/CHF spot exchange rate; in the absence of the SNB's policy, $S_t=V_t$, whereas in the presence of the policy, S_t is given by

$$S_t(V_t, \cdot) = V_t + P^A(V_t, \cdot), \quad (1)$$

where $P^A(V_t, \cdot)$ is the value of the American put option implied by the SNB's policy,

- τ_t ... maturity of this American put option.

We model V_t using a geometric Brownian motion, mainly for reasons of simplicity and robustness. We note that for a zero risk-free rate and positive dividend yield, an American put should never be exercised early, making its price equal to that of a European put. This result is less well-known compared to the “no-early-exercise” result for American call options on non-dividend-paying underlyings. Using the well-known no-arbitrage relation $P(V_t, K, \tau_t, \cdot) > K \exp(-r_t \tau_t) - V_t \exp(-i_t \tau_t)$ for the European put option and noting that the intrinsic value $K - V_t < P(V_t, K, \tau_t, \cdot)$ for $r_t = 0$ and $i_t > 0$ proves non-optimality of early exercise.

Short-term interest rates on the Swiss franc between the inception of the minimum exchange rate policy and the time of writing were very close to zero, with interest rates on the euro (the “dividend yield” of the underlying) consistently above Swiss franc interest rates. This justifies treating the American put representing the SNB's policy as European, which allows us to use the Garman and Kohlhagen (1983) model for its valuation:

$$P^A(\cdot) \simeq P(V_t, K, r_t, i_t, \tau_t) = K e^{-r_t \tau_t} N(-d_2) - V_t e^{-i_t \tau_t} N(-d_1) \quad (2)$$

$$d_1 = \frac{\ln(V_t/K) + (r_t - i_t + \sigma_V^2/2)\tau_t}{\sigma_V \sqrt{\tau_t}}, \quad d_2 = d_1 - \sigma_V \sqrt{\tau_t}. \quad (3)$$

For European currency options, the following put-call parity relation holds:

$$V_t e^{-i_t \tau_t} + P(V_t, \cdot) = K e^{-r_t \tau_t} + C(V_t, \cdot). \quad (4)$$

Our option pricing model in Section 3 will be based on rewriting equation (1) to approxi-

mate S_t as

$$S_t(V_t, \cdot) \simeq Ke^{(i_t - r_t)\tau_t} + C(V_t, \cdot), \quad (5)$$

which is inspired by multiplying both sides in equation (4) by $\exp(i_t\tau_t)$ and comparing the resulting left-hand side to equation (1). To assess the error associated with this approximation, note that $K=1.20$, while V_t is expected to be roughly in the range of 1-1.15. Observable exchange rates in the range 1.20-1.25 imply values for the put option $P(V_t, \cdot)$ in equation (4) of 0.1-0.2, and for the call option in the range 0-0.05. Replacing the right-hand side in equation (1) by $\exp(i_t\tau_t)$ times the right-hand side of equation (4) would increase S_t by the interest (at rate i_t) on the put option. Part of this error has been reduced in equation (5) by multiplying only K by $\exp(i_t\tau_t)$, but not $C(V_t, \cdot)$. Then, the error is approximately

$$(e^{i_t\tau_t} - 1)[P(V_t, \cdot) - C(V_t, \cdot)]. \quad (6)$$

During our sample period, the average value of i_t was 0.505%. Preliminary calculations indicate values for τ_t (which can be interpreted as the market's consensus estimate of the expected end of the SNB's policy) of 0.5-1 years. Using a value for τ_t of 0.75 and (from the arguments provided above) a difference between put and call values of 0.1, equation (6) gives an error of $3.8 \cdot 10^{-4}$, which is much smaller than the accuracy achievable by our method given the noise present in option prices/implied volatilities. For this reason, we feel comfortable with replacing equation (1) by equation (5) as the basis for our option pricing model in Section 3. The unobservable input parameters V_t , σ_V and τ_t will then be estimated using market data. Data and estimation procedures will be described in more detail in Section 4.

3 The compound option pricing model

Call and put options on the EUR/CHF exchange rate are standard options on V_t until Sept. 6, 2011. Afterwards, they can be viewed either as standard options on S_t or as compound options (as analyzed in Geske, 1979) on the latent exchange rate V_t . Once the SNB changes its policy back to a freely floating exchange rate, these options will again be standard options on V_t . Therefore, values of currency options depend on the market's estimate of the probability of such a policy change. As noted by Hertrich and Zimmermann (2014), put options with strikes $X_k \leq K$ can *only* have a positive payoff if the policy change happens before the option's maturity. Positive prices for these options therefore imply that

the market attaches a positive probability to a policy change before the options' maturities.

Denoting the maturity of traded option k by τ_k ,³ two cases have to be distinguished: With risk-neutral probability $\pi_{\tau_k,t}$, the SNB's policy will still be in place when the option expires, i.e., $\tau_t \geq \tau_k$. This implies that with risk-neutral probability $1 - \pi_{\tau_k,t}$, the option payoff will not depend on S_t given by equation (1) (including the "SNB put"), but on V_t . Since risk premia in currency markets are very small, risk-neutral probabilities will be close to real-world probabilities, see, e.g., De Santis and Gérard (1998). Using γ_k for either call or put options k on the exchange rate, we can write their values as

$$\gamma_{k,t}(V_t, \cdot) = \pi_{\tau_k,t} \gamma_t(S_t, X_k, \cdot) + (1 - \pi_{\tau_k,t}) \gamma_t(V_t, X_k, \cdot) \quad (7)$$

$$= \pi_{\tau_k,t} \gamma_t(K e^{(i_t - r_t) \tau_t} + C_t(V_t, K, \tau_t, \sigma_V), X_k, \cdot) + (1 - \pi_{\tau_k,t}) \gamma_t(V_t, X_k, \cdot), \quad (8)$$

where equation (8) follows from equation (5). The second part of equation (8) is just a standard call or put option on V_t . The first part is a compound option (call on a call or put on a call), because in this case $\tau_t > \tau_k$. If $\gamma_{k,t}$ is a traded call, then on the first expiration date τ_k its value is

$$C_{k,\tau_k}(V_t, \cdot) = \begin{cases} \max[C_{\tau_k}(V_{\tau_k}, K, \tau_t - \tau_k, \sigma_V) - (X_k - K e^{(i_t - r_t) \tau_t}), 0] & \text{with prob. } \pi_{\tau_k,t} \\ \max[V_{\tau_k} - X_k, 0] & \text{with prob. } 1 - \pi_{\tau_k,t} \end{cases}, \quad (9)$$

and in case of a put

$$P_{k,\tau_k}(V_t, \cdot) = \begin{cases} \max[(X_k - K e^{(i_t - r_t) \tau_t}) - C_{\tau_k}(V_{\tau_k}, K, \tau_t - \tau_k, \sigma_V), 0] & \text{with prob. } \pi_{\tau_k,t} \\ \max[X_k - V_{\tau_k}, 0] & \text{with prob. } 1 - \pi_{\tau_k,t} \end{cases}. \quad (10)$$

Equations (9) and (10) hold if $X_k - K e^{(i_t - r_t) \tau_t} > 0$.⁴ In this case, the critical exchange rate $V_{\tau_k}^*$ for the compound option part is (in both cases) given by

$$\gamma_t(V_{\tau_k}^*, K, \tau_t - \tau_k, \sigma_V) = X_k - K e^{(i_t - r_t) \tau_t}. \quad (11)$$

³In light of the definition of τ_t , using τ_k instead of, e.g., τ_t^k or $\tau_{k,t}$ is a slight abuse of notation. However, since the options used in our empirical analysis have constant maturities of one and three months (without any time dependence) and the meaning should be clear from the context, we prefer this notation for reasons of brevity and simplicity. Furthermore, the distinction between the conditional/unconditional time to maturity of the guarantee (τ_t) should be clear from the context.

⁴The case $X_k - K e^{(i_t - r_t) \tau_t} < 0$ will be treated at the end of this section.

This allows us to derive the time t price of C_k in equation (9) as

$$C_{k,t} = \pi_{\tau_k,t} \left[V_t e^{-i_t \tau_t} N_2 \left(a_+, b_+; \sqrt{\tau_k/\tau_t} \right) - K e^{-r_t \tau_t} N_2 \left(a_-, b_-; \sqrt{\tau_k/\tau_t} \right) - e^{-r_t \tau_k} (X_k - K e^{(i_t - r_t) \tau_t}) N(a_-) \right] + (1 - \pi_{\tau_k,t}) \left[V_t e^{-i_t \tau_k} N(c_+) - X_k e^{-r_t \tau_k} N(c_-) \right], \quad (12)$$

and the time t price of P_k in equation (10) as

$$P_{k,t} = \pi_{\tau_k,t} \left[K e^{-r_t \tau_t} N_2 \left(-a_-, b_-; -\sqrt{\tau_k/\tau_t} \right) - V_t e^{-i_t \tau_t} N_2 \left(-a_+, b_+; -\sqrt{\tau_k/\tau_t} \right) + e^{-r_t \tau_k} (X_k - K e^{(i_t - r_t) \tau_t}) N(-a_-) \right] + (1 - \pi_{\tau_k,t}) \left[X_k e^{-r_t \tau_k} N(-c_-) - V_t e^{-i_t \tau_k} N(-c_+) \right], \quad (13)$$

with:

$$\begin{aligned} a_+ &= \frac{\log(V_t/V_{\tau_k}^*) + (r_t - i_t + \sigma_V^2/2)\tau_k}{\sigma_V \sqrt{\tau_k}}, & a_- &= a_+ - \sigma_V \sqrt{\tau_k}, \\ b_+ &= \frac{\log(V_t/K) + (r_t - i_t + \sigma_V^2/2)\tau_t}{\sigma_V \sqrt{\tau_t}}, & b_- &= b_+ - \sigma_V \sqrt{\tau_t}, \\ c_+ &= \frac{\log(V_t/X_k) + (r_t - i_t + \sigma_V^2/2)\tau_k}{\sigma_V \sqrt{\tau_k}}, & c_- &= c_+ - \sigma_V \sqrt{\tau_k}, \end{aligned}$$

and $N_2(x, y; \rho)$ is the two-dimensional normal distribution function with correlation coefficient ρ . If $X_k - K e^{(i_t - r_t) \tau_t} < 0$, the compound option component disappears, and pricing of traded options reduces to the Garman-Kohlhagen model. For the out-of-the-money calls used in our numerical analysis (see Section 4), this case never occurs. For the out-of-the-money puts in our sample, the first summand in equation (13) is zero in this case.

4 Data and estimation process

We retrieve daily exchange rates from Thomson Reuters Datastream. Bloomberg provides implied volatilities (annualized) for EUR/CHF calls and puts for various maturities and deltas. For each trading day t , we use 8 options: Calls and puts with maturities of 1 and 3 months and deltas of 10 and 25 percentage points (“10D” and “25D”). For each of these options, we convert the implied volatilities to prices using the Garman and Kohlhagen

(1983) model. To this end, we start by determining the respective strikes corresponding to the 10D and 25D levels for all options. As proxies for the risk-free interest rates, we use the 3-month LIBOR rates in both currencies for all maturities. An alternative would be to use repo rates. However, while the EUREPO is available for a range of maturities, a repo rate for the Swiss franc is only available for a maturity of one week. We are well aware of potential distortions in LIBOR rates resulting from varying credit risk and liquidity premia. However, when it comes to option pricing, it is primarily the *difference* between the interest rates in the two currencies that is relevant. This difference should be rather similar regardless whether we compute it from LIBOR or from repo rates. Short-term interest rates in both currencies were very low and close to flat throughout the sample period (Sept. 7, 2011 until Oct. 11, 2013). Moreover, noise in implied volatilities from other sources (cf. Table 1) dominates possible distortions arising from the maturity-dependence of rates. For these reasons, we are comfortable with the assumption of a flat term structure in each currency for each trading day at the level of the respective 3-month rate.

Figure 2 plots the implied (annualized) volatilities reported by Bloomberg against the respective strikes, standardized by the observed exchange rate S_t on the corresponding trading day. For each maturity (1M and 3M) and trading day, we have four options available, two calls and two puts. Diamonds (1M) and squares (3M) indicate averages of implied volatilities for 10D and 25D calls and puts for the respective maturities, which are plotted against averages of the standardized strikes $X_{k,t}/S_t$. All averages are taken across the entire dataset. Volatilities show the well-known symmetric smile pattern, which is typical for currency options. Table 1 provides descriptive statistics of our options data, both for the implied volatilities retrieved from Bloomberg and the option prices computed from these implied volatilities using the Garman-Kohlhagen model. Option prices in our sample fluctuate considerably, note in particular the high option price volatilities of one-month put options (P1M10 and P1M25) in column 4, which are caused by high relative price changes for very small option prices. Prices of this option category exhibit particularly high levels of noise. Fluctuations of option prices decrease considerably at later stages of our dataset, as shown in the final column for a subsample containing the last 50 observations from our dataset.

The parameters to be estimated using exchange rate and option price data are V_t , σ_V , τ_t , $\pi_{1/12,t}$ and $\pi_{3/12,t}$. We assume a fixed volatility σ_V across all eight options on each trading day, i.e., a flat volatility across strikes and maturities for all eight options. An alternative would be to explicitly account for volatility smiles and term structures (see

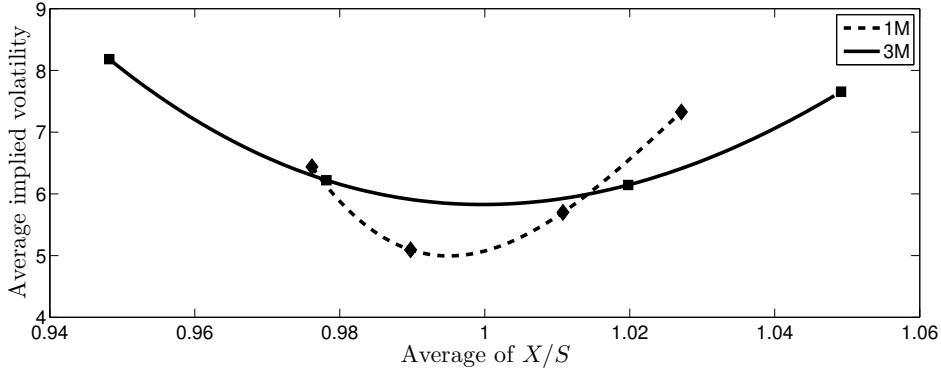


Figure 2: Average volatility smile for 1M and 3M EUR/CHF calls and puts over the entire sample period (Sept. 7, 2011, to Oct. 11, 2013). Diamonds (1M) and squares (3M) indicate averages of implied volatilities (annualized) quoted by Bloomberg for 10D and 25D calls and puts for the respective maturities, which are plotted against averages of their standardized strikes $X_{k,t}/S_t$. Lines shown interpolate between these points using splines.

Option	Entire dataset			Last 50 obs.
	Impl. Vol. Average	Averages of relative option prices ($\times 10^3$)	Volatility of option prices (in %)	Volatility of option prices (in %)
C1M25	5.70	2.47	25.8	2.68
C3M25	6.14	4.63	12.6	0.90
C1M10	7.33	1.02	19.4	2.54
C3M10	7.66	1.86	17.2	0.74
P1M25	5.09	2.18	33.3	3.51
P3M25	6.22	4.57	13.7	2.19
P1M10	6.44	0.87	26.3	4.54
P3M10	8.18	1.87	10.8	3.08

Table 1: Descriptive statistics of the options used to estimate the parameters of the compound option model. Sample period: Sept. 7, 2011 until Oct. 11, 2013 (total number of observations: 547). Abbreviations in first column: First letter indicates call (C) or put (P), next two letters indicate maturity (one month or three months), final two digits indicate moneyness (10D or 25D). The next three columns provide statistics for the entire dataset: The second column provides the averages of implied volatilities (annualized) quoted by Bloomberg, which are also shown in Figure 2. The third column contains averages of relative prices $C_{k,t}/S_t$ and $P_{k,t}/S_t$, computed from these implied volatilities using the Garman-Kohlhagen model (multiplied by 10^3). The fourth column shows volatilities of option prices (i.e., the annualized standard deviations of daily log returns computed from option prices). The final column provides the volatilities of option prices (using the same definition) computed for a subsample containing only the last 50 observations.

Figure 2 for average smiles). However, smiles and term structures also vary over time, and our main goal is to robustly estimate the latent exchange rate as opposed to pricing each individual option as precisely as possible.⁵ We allow σ_V to vary over time when re-calibrating our model on each trading day, which corresponds to the standard market practice of assuming time-varying volatility when using the Black/Scholes-model and its variants. To make this time-dependence explicit, we will write $\sigma_{V,t}$ instead of σ_V from now on. Using traded options with different maturities τ_k will result in different values for $\pi_{\tau_k,t}$. The longer the maturity τ_k of the traded option, the higher the probability that the SNB's policy will end before the option's maturity. Assuming a specific functional form for the dependence of $\pi_{\tau_k,t}$ on τ_k will reduce the number of parameters and stabilize the estimation process. For simplicity and robustness, we postulate linear dependence between $\pi_{\tau_k,t}$ and τ_k to this end:

$$\pi_{\tau_k,t} = 1 - g_t \tau_k. \quad (14)$$

Hence, instead of a separate estimation of $\pi_{1/12,t}$ and $\pi_{3/12,t}$, we compute only the common parameter g_t determining both probabilities via equation (14).

We estimate the parameters V_t , $\sigma_{V,t}$, τ_t and g_t by minimizing the sum of squared differences between market prices $\gamma_{k,t}^m$ (computed from implied volatilities as described above) and model prices $\gamma_{k,t}$ (computed from equations (12) and (13)) for each trading day in our sample.⁶ As an additional component, we add the squared difference between the observed EUR/CHF exchange rate and the exchange rate V_t implied by equation (5) to impose internal consistency of our model as an additional requirement.⁷ More precisely, we compute

$$(V_t, \sigma_{V,t}, \tau_t, g_t) = \arg \min \left[(S_t^m - S_t(V_t, \cdot))^2 + \sum_k (\gamma_{k,t}^m - \gamma_{k,t}(V_t, \sigma_{V,t}, \tau_t, g_t))^2 \right] \quad \forall t, \quad (15)$$

where the superscript m denotes observed market values for exchange rates and option

⁵Additional information regarding the shape of the risk-neutral pricing density might be inferred from risk reversals, i.e., differences in implied volatilities of calls and puts with identical absolute deltas and maturities, see e.g. Campa et al. (1998) and Hertrich and Zimmermann (2014, pp. 14 ff.). We leave a possible extension to our model based on such information to future research.

⁶We use the MATLAB function *fmincon* for this purpose.

⁷Alternatively, the estimation could be formulated as a constrained optimization problem, using in the objective function only the squared differences between $\gamma_{k,t}^m$ and $\gamma_{k,t}$, and adding a constraint in the form of an upper bound on the absolute deviation between the observed exchange rate S_t^m and the model exchange rate $S_t(V_t, \cdot)$. The results of this approach (using an upper bound of 0.01) are very similar to those shown in Section 5.

prices. Minimizing the equally weighted sum of squares across these terms (in our base case, these are eight options and the exchange rate) per trading day is the simplest approach to be used. We refrain from using weights on the components to avoid any impression of tweaking parameters, which – in the absence of sound theoretical arguments in favor of any particular weighting scheme – could easily lead to data snooping. Note that implicitly, however, weighting does take place in some way because of different price levels among the options and the exchange rate. We will address this issue using robustness checks with different weighting schemes.

5 Results

The estimates resulting from equation (15) are quite noisy on some trading days, to some extent because of the complicated non-linear interrelations between the parameters, which leads to substitutive effects when estimating four parameters simultaneously. Therefore, we show smoothed values using splines (dashed black lines) in Fig. 3 in addition to the daily estimates (solid gray lines).

The top left panel in Fig. 3 compares the observed EUR/CHF exchange rate to the latent exchange rate V_t , estimated using equation (15). The latent exchange rate starts around 1.03, close to the August 2011 all-time low of the observed EUR/CHF exchange rate. It increases in the weeks after the SNB’s announcement, but drops again below 1.10 at the beginning of 2012. For most of 2012, with a short exception in September, it remains between 1.01 and 1.10 (also in Q2 and Q3, when the observed exchange rate almost flattens out closely above 1.20). November 2012 marks the beginning of an upward trend for V_t , which continues until February 2013. Afterwards, the latent exchange rate stabilizes between 1.12 and 1.18.

The estimated volatility of the latent exchange rate, $\sigma_{V,t}$, shows a decreasing trend over the entire dataset, albeit with sizeable fluctuations. Starting around 24% in Sept. 2011, it decreases to 13% at the end of the sample (top right panel). The values seem high in a historical context, which can be attributed to ongoing concerns of the markets about future developments in the eurozone. Moreover, the estimates are driven by implied volatilities of out-of-the-money options, which tend to be higher than realized volatilities.

The market’s estimate of the remaining life of the SNB’s policy, τ_t (bottom left panel), increases from 0.285 years in Sept. 2011 to around 0.9 years in the second quarter of 2013. Until the end of 2012, we consider the main driver of this development to be the increasing

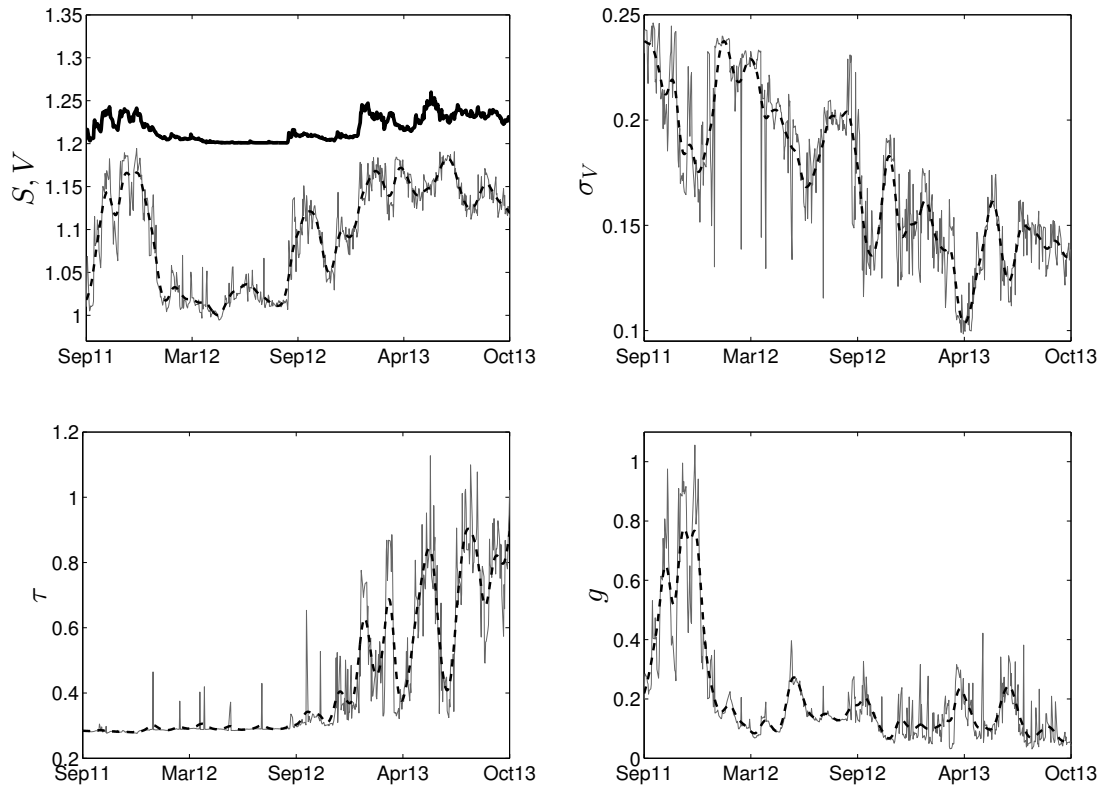


Figure 3: Daily estimates for V_t , $\sigma_{V,t}$, τ_t and g_t from equation (15), smoothed using splines (dashed lines). Top left: latent exchange rate process V_t compared to observed exchange rates S_t (solid black line), top right: volatility $\sigma_{V,t}$ of the latent exchange rate, bottom left: market's unconditional estimate τ_t of the remaining life of the SNB's minimum exchange rate policy, bottom right: parameter g_t from equation (14).

confidence of the market in the SNB’s commitment to its policy. Increased levels of both observed and latent exchange rate during the year 2013 may lead to an “implicit” increase in the perceived remaining life of the SNB’s policy in the sense that a higher exchange rate implies a higher probability that it will remain above 1.20 even without any intervention on the part of the SNB.

The parameter g_t , which determines the probability for a policy change before the traded options’ maturity, is above 0.5 at the end of 2011, but decreases afterwards. Values of $g_t \approx 1$ at the beginning imply a three-month probability for policy revocation of 25%. During the second and third quarters of 2013, g_t declines further and reaches 0.04 on the final day in our sample. This value implies a break probability of 1% for three months and 0.3% for one month, which means a high degree of confidence by the markets in the SNB’s commitment. Although our estimates of break probabilities show some variation, they are considerably lower than the values reported by Hertrich and Zimmermann (2014), who use the same options data, in the majority of cases: E.g., for Oct. 31, 2012, they report implied probabilities for breaking the 1.20 barrier of 37% (25D 3-month puts, their Table 2) and 41% (10D 3-month puts, their Table 3). For this day, we estimate a g_t of 0.11, which, using equation (14), implies a corresponding break probability of 2.8%. We attribute these high deviations to two sources: First, the strong process assumptions in Hertrich and Zimmermann (2014, see the discussion in Section 2.1) combined with their use of a constant (historical) volatility in their estimation. Second, their “break probabilities” are driven by market participants constantly revising their estimates of the barrier value b which will never be broken,⁸ while our probabilities are directly associated with an (implicit or explicit) end to the SNB’s minimum rate policy, since this is the only way in which a break of the intervention level can occur in our model.

As noted in Section 4, we minimize the equally weighted sum of squares across nine observations per trading day. One way to assess the quality of the model and the estimation is by analyzing the deviations between observed and fitted values. The average absolute deviation between observed exchange rate S_t^m and the model-implied exchange rate S_t (computed from equation (5)) is 0.0012, which is very small compared to range of observed exchange rates (between 1.20 and 1.25). Fig. 4 shows the mean absolute error $\bar{\varepsilon}$ between the estimated values for V_t and $\gamma_{k,t}$ and the corresponding market prices, see equation (15). Comparatively higher values for $\bar{\varepsilon}_t$ at the beginning indicate that the market seemed to be somewhat “unsettled” about pricing options in the presence of the SNB’s minimum

⁸For a precise description, see Hertrich and Zimmermann (2014, p. 11).

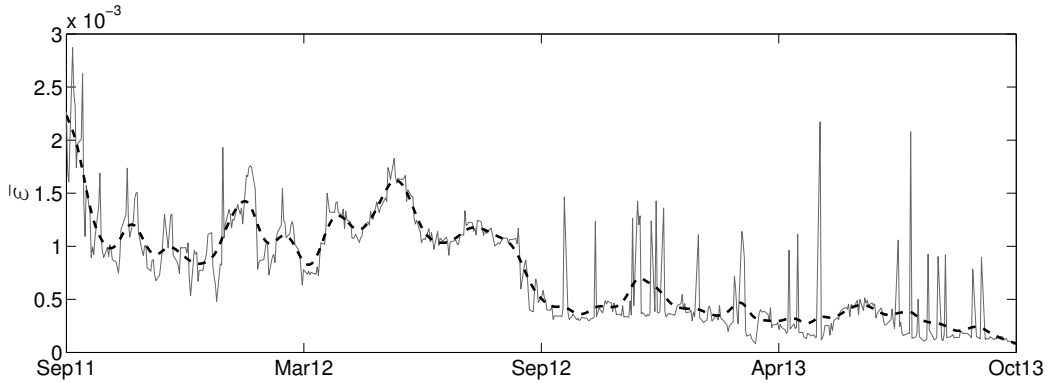


Figure 4: Mean absolute error $\bar{\varepsilon}_t$ from estimating our model according to equation (15) (grey line, average taken across the 9 components in equation (15), smoothed using splines, black dashed line). Following initially high values during the “unsettled” period immediately after the SNB’s announcement on Sept. 6, 2011, $\bar{\varepsilon}_t$ declines considerably as time passes, indicating an improving model fit.

exchange rate policy. Reassuringly, this mean absolute error decreases quite steadily over time: The more time the market has had to learn how to price options in the presence of the SNB’s policy, the better is the fit of our model. Hence, in a way, market prices seem to converge towards the model proposed in this paper over time. The precision of estimated option prices follows a similar pattern (not shown): In the period after Sept. 2011, implied option volatilities fluctuate considerably. The noise in option prices also leads to larger deviations between market prices and model prices of options compared to later periods in our sample, when pricing accuracy is very good (e.g., average relative pricing errors of 7.5% for calls over the last 50 days of our sample period).

To check the robustness of our results, we repeat our calculations for various variations in our setting: In case (i), we omit the 10D options, i.e., we use only the 25D options for estimating the parameters. In cases (ii) and (iii), we use all options, but we apply different weighting schemes by putting more weight on the options deviations relative to the exchange rate deviation:

$$(V_t, \sigma_{V,t}, \tau_t, g_t) = \arg \min \left[w(S_t^m - S_t(V_t, \cdot))^2 + (1 - w) \sum_k (\gamma_{k,t}^m - \gamma_{k,t}(V_t, \sigma_{V,t}, \tau_t, \tau_k, g_t))^2 \right] \quad \forall t. \quad (16)$$

The original estimation in equation (15) implies $w=50\%$. Since this leads to very high accuracy levels for the estimates of V_t , we use as robustness checks values of (ii) $w=40\%$

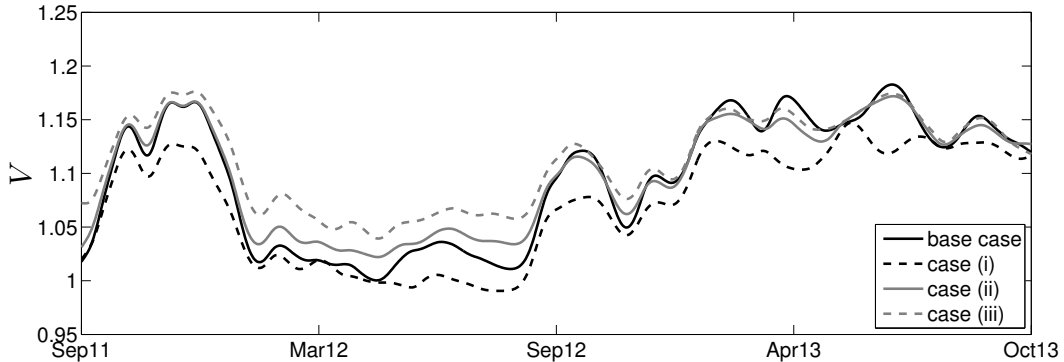


Figure 5: Robustness checks, comparing the estimated latent exchange rate process (smoothed using splines) for various variations of the base case setting (solid black line). Dashed black line: case (i), omitting the 10D options and using only the 25D options in equation (15). Alternative weighting schemes are indicated by grey lines. Solid grey line: case (ii), $w=40\%$ in equation (16), dashed grey line: case (iii), $w=20\%$ in equation (16).

and (iii) $w=20\%$, which puts higher weight on estimation errors from option prices. Fig. 5 compares the estimated latent exchange rate processes (smoothed using splines similar to Figure 3) from these robustness checks. The solid black line represents our base case and is identical to the top left panel in Figure 3. Excluding the 10D options leads to the latent exchange rate process depicted by the dashed black line, see case (i). Most of the time, the two estimated processes differ by not more than 2 or 3 basis points. Larger deviations (up to 5 bps) occur at the beginning and in the middle of 2013, but are rather short-lived. Alternative weighting schemes are shown in grey. Case (ii) with $w=0.4$ differs only marginally from the base case, showing deviations below 2 bps for most of the time. Case (iii) with $w=0.2$, which puts even more weight on accuracy in option prices rather than the latent exchange rate process, leads to slightly larger deviations during certain periods (particularly in the third quarter of 2012). However, even in these periods the deviations from the base case are never larger than 5 bps. Overall, we consider the results of these robustness checks to confirm the soundness of our modeling approach in general.

One limitation of our model is that it ignores the impact of speculative activity on the EUR/CHF exchange rate. We neither try to measure this impact, nor the effect on speculative activity caused by the SNB's policy. If the SNB's policy did indeed reduce downward speculative pressure on the Swiss franc, our results can be viewed as an upper bound on exchange rate levels that would have prevailed in the presence of unchanged

speculative behavior.

6 Conclusions

Using observed exchange rate data and implied volatilities of exchange rate options, we showed how to back out the latent process of the EUR/CHF exchange rate that would prevail in the absence of the SNB's minimum exchange rate policy. We found latent exchange rate levels between 1.01 and 1.18, which are markedly lower than the intervention level of 1.20 set by the SNB. As by-products, we estimated two statistics describing implied market's expectations regarding the SNB's policy: (i) the expected remaining life of the policy and (ii) implied probabilities for an end to the policy within the next one/three month(s). Our results indicate that the market's confidence in the SNB's commitment increased considerably over time, leading to both increased expected lifetime and decreased probability for an end to the policy in the near future. Robustness checks confirmed relative insensitivity of our results to variations in data and weighting schemes used in the estimation. The absolute estimation error in our model declines markedly over time, which indicates convergence of market prices towards the model proposed in this paper.

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