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EXERGO-ECOLOGICAL EVALUATION OF HEAT EXCHANGER

by

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Thermodynamic optimization of thermal devices requires information about the influence of operational and structural parameters on its behaviour. The interconnections among parameters can be estimated by tools such as CFD, experimental statistic of the deviceetc. Despite precise and comprehensive results obtained by CFD, the time of computations is relatively long. This disadvantage often cannot be accepted in case of optimization as well as online control of thermal devices. As opposed to CFD the neural network or regression is characterized by short computational time, but does not take into account any physical phenomena occurring in the considered process. The CFD model of heat exchanger was built using commercial package Fluent/Ansys. The empirical model of heat exchanger has been assessed by regression and neural networks based on the set of pseudo-measurements generated by the exact CFD model. In the paper, the usage of the developed empirical model of heat exchanger for the minimisation of TEC is presented. The optimisationconcerns operational parameters of heat exchanger. The TEC expresses the cumulative exergy consumption of non-renewable resources. The minimization of the TEC is based on the objective function formulated by Szargut. However, the authors extended the classical TEC by the introduction of the exergy bonus theory proposed by Valero. The TEC objective function fulfils the rules of life cycle analysis because it contains the investment expenditures (measured by the cumulative exergy consumption of non-renewable natural resources), the operation of devices and the final effects of decommissioning the installation.

Key words: *thermo-ecological cost, heat exchanger, CFD modelling,*
optimization, neural networks

Introduction

The proper selection of devices in many cases requires the application of optimization procedure, which should lead to the answer of the question which device ensure the optimal operational results. After the device is selected, its parameters should be also optimized. However, the result of optimization strongly depends on the optimization criterion. These optimization criteria can be formulated based on economic, thermodynamic or other approaches [1, 2]; in this work the thermo-ecological cost (TEC) as the desired criteria is used [3]. The TEC approach makes use of exergy,which isa thermodynamic measure of the quality of resources [4-10]. In general, TEC is defined as the cumulative consumption of non-renewable exergytaking into account the harmful effects of the waste products connected with producing a particular useful product [11].

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The TEC is calculated not only for operational phase, but also includes other phases of production system [7, 12]. This implies that, the objective function fulfil the rules of life cycleassessment (LCA).This an approach contains all phases of the device's life, among others the investment expenditures (measured by the cumulative exergy consumption of non-renewable natural resources) and the final effects of decommissioning the installation are taken into account. Moreover, the TEC minimisation, which is proposed, goes step forward beyond the classic LCA since it introduces the combined measure of different ecological impacts. For example, the application of the TEC in full life cycle objective function for optimization the solar collector is presented in [13]; other TEC applications are presented in [3, 13-14].

The mathematical description of an optimized device is one of the most important part of the optimization procedure. This description in other wordsthe device characteristic should simulate the operation of the device, namely, these characteristics should be able to generate the values of the output parameters (temperature, pressure of the working fluids, heat loads, *etc*.) based on the device input parameters. Nowadays,it is very popular to use the experimental characteristic based on the measurements of the real devices. This attempt has weaknesses suchas it is expensive during preparation (a number of devices should be precisely investigated), and this method cannot be used for the brand new constructions. Another attempt is to use the approximate characteristic, based on the measurements and approximate theoretical relationships. This solution is cheaper but its accuracy in many cases is unsatisfying.

Nowadays, the device characteristic can be insertedin the numerical model due to the rapid development of computer technology and advancement of numerical modelling. The numerical model behaves as the characteristic of the devices; it returns the output parameters based on the values of the input parameters supplemented by the boundary and initial conditions. One of the most precise and sophisticated solution is computational fluid dynamic (CFD), in which thermal devices are efficiency modelled [15]. Despite comprehensive results, which are obtained by CFD, the time for computations is relatively long; moreover, advanced computer equipment is demanded, which very often cannot be accepted for examplein the case of online control.

The neural modelling is an alternative for using models, which directly based on statistic data and does not take into account any physical phenomena that occur in the investigated device. In contrast to CFD the neural network is characterized by short computational time andthe precision of this tool is also very high.

In this paper, the CFD model is used as a numerical experiment in order to generate the grid of operational states of considered devices. The specified number of results of CFD is transferred to the empirical models,so the set of pseudo-measurements are identifiedas an inputsinthe empirical model [16]. In this paper, the heat exchanger is presented as an example of thermal devices. The heat exchanger (HEX), which is chosen for the analysis, is treated as a black box to which the following parameters are provided (1) diameters of tubes, (2) number of tubes, (3) lengths of tubes, (4) thermal energy of heat exchanger, (5) the pressure drop, and (6) Reynolds number. Based on these parameters of considered heat exchanger, the neural network and regression are assessed. Moreover, the results of training and verification of applied models are discussed. Additionally, the possibility of application of developed models for optimization purposes is described.

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Thermo-ecological cost

The index of operational thermo-ecological cost can be determined by solving the thermo-ecological cost balancesthe idea of which is described in details in previous works [11-14]. The equation for the operational TEC balance, where ρ is the thermo-ecological cost index, a – the inputs and outputs consist on the devices, b – the exergy of non-renewable natural resources, and ψ – TEC of emissions, is [11]:

$$\rho_j = \sum_i (f_{ij} - a_{ij})\rho_i - \sum_r a_{rj}\rho_j = \sum_s b_{sj} + \psi_{j0} \tag{1}$$

TEC expres the cumulative consumption of non-renewable natural resource, which means the impact of non-renewable fossil fuels and minerals is considered. Previous study of TEC [17] shows that the exergy of mineral resources has low impact on the TEC results.Even though, the analysis should consider both fuel and mineral exergy since not only the fuel deposits are in danger due to excessive exploitation of deposits but also the mineral deposits with really rich ores. It should be noted, that minerals are partially recycled and the non-renewable natural resources such as fuels could be partially replaced by renewable ones.The rapid usage of both deposits is taken into account in TEC minimization. Moreover, the TEC is supplemented with the exergy replacement cost concept proposed by Valero [18-21]. In this approach the chemical exergy of natural resource b_{sj} in eq. (1) is calculated as:

$$b_{sj} = \frac{1}{M_s}\{(Mb)_{ch,s} + [(Mb)_{c,s} - (Mb)_{m,s}]\} \tag{2}$$

where
– specific molar concentration exergy of substance in crustal earth:

$$(Mb)_{c,s} = -(MR)T_0 \left[\ln(z_{c,s}) + \frac{(1-z_{c,s})}{z_{c,s}}\ln(1-z_{c,s}) \right] \tag{3}$$

– specific molar concentration exergy of substance in ore:

$$(Mb)_{m,s} = -(MR)T_0 \left[\ln(z_{m,s}) + \frac{(1-z_{m,s})}{z_{m,s}}\ln(1-z_{m,s}) \right] \tag{4}$$

where z_c and z_m denote concentration of useful substance in the crust and in mineral.

To express the total natural resource expenditures, the thermo-ecological cost method includes the total life of installation [4]. A Thermo-Ecological Life Cycle Assessment (TEC-LCA) consists of three main parts [13]:
– Construction phase encompasses project, extraction of raw materials, semi-finished product fabrication, and transport expenditures in the construction phase. All of these expenses influence the thermo-ecological cost burdening the final useful consumptive product. This phase can have a significant contribution to thermo-ecological cost in the case of processes based on renewable energy. For instance, the thermo-ecological cost of a wind power plant results mainly from expenses in the construction phase.
– Operation phase is defined as the period of time between the end of the construction phase and the beginning of the decommissioning phase. In processes utilising non-renewable

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resources, this phase is the predominant consumer of natural resources, mainly energy carriers.
– Decommissioning phase concerns the period at the end of installation. Thermo-ecological cost in this phase results from expenditures for developing the remains of the system and for reclamation of terrain.

Thermo-ecological cost optimisation of heat exchanger

The general form of the objective function, based on TEC-LCA minimization idea takes into account the whole lifetime of the product [7, 8, 13] is expressed by the following formula:

$$(TEC) = \tau_n (\sum_j \dot{G}_j + \sum_k \dot{P}_k \zeta_k - \sum_u \dot{G}_u s_{iu}) \; + \frac{1}{\tau} [\sum_m G_m \rho_m (1 - u_m) + \sum_r G_r] \tag{5}$$

Equation (5) represents the yearly thermo-ecological cost of a given product with emphasion on its complete lifetime. This equation can also be used for optimisation of the construction and operational parameters of different resource intensive systems. In this case, the function should be minimised TEC–min. TEC optimisation based on eq. (5) requires amathematical model of the process or device.

The scheme of the modelled heat exchanger is shown in fig. 1. The amount of exchanged heat strongly depends on the size of this area and on the medium temperature difference. It is governed by the Peclet equation. Due to evaluate the heat load one should
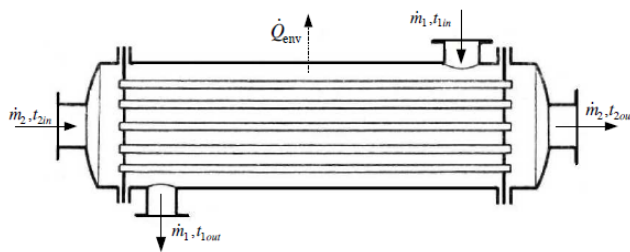

Figure 1. Scheme of the considered heat exchanger

know the values of the heat transfer area A and the heat transfer coefficient k. Both quantities result, among the others, from the geometry of the exchanger. The size of inner tubes directly influences the heat transfer area and additionally changes the flow inside and outside the tubes, what is strongly connected with overall heat transfer coefficient.

The yearly thermo-ecological cost of the heat-exchanger operation can be expressed as follows [7, 11, 13]:

$$TEC = \frac{\rho_{el}}{\eta_p} \int_0^{\tau_p} (\dot{V}_1 \delta p_1 + \dot{V}_2 \delta p_2) \mathrm{d}\tau + \frac{\rho_s (1 - u_s)}{\tau_z} (G_p + G_s') + E_{el} \rho_{el} \rightarrow \min \tag{6}$$

Equation (6) contains two main components:
– the operational components expressing the consumption of electricity (the integral part), and
– the investment component containing the TEC of the materials used for the construction of the exchanger.

The investment component takes into account the lifetime τ_Z of the installation and the possibility to reuse steel after the lifecycle of the installation. The operational components take into account the yearly operation time τ_P of the pump.

Numerical simulation of heat exchanger HEX using CFD modelling

The numerical model of a device can be developed from the physical laws appropriate for the considered device. In thermal technology, the numerical description of devices is based on conservation equations.In general, the mass balance, the energy conservation equation and the momentum equation are calculated, andthey cover the phenomena to be found in the solid parts of a device (*e. g.* heat transfer if the solid wall) as well as in working mediums (fluid flow in engines, turbines *etc.*). The branch of fluid mechanics which utilizes the numerical methods for the problem's solution is called CFD [22, 23] and can be successfully employed for the creation of the numerical model of thermal devices [15].

In this work, each configuration of geometry, the velocity, pressure and temperature fields are computed inAnsys Fluent software [24]. The procedure of CFD model creation can be divided into several steps:
– geometry creation,
– meshing,
– computations, and
– results processing.

In the computations, the following parameters were assumed as a constant:
– number of tubes,
– outer diameter of tubes,
– hot fluid inlet temperature and mass flow rate, and
– cold fluid inlet temperature and mass flow rate.

Detailed information on CFD modelling of considered HEX are presented in [24, 16].

Neural model of HEX

While the computation time of CFD procedures is relatively high the results of the exact CFD model are used for training the neural empirical model. Such model is useful for multi-variant simulation or for TEC optimisation of HEX that is the aim of this paper.

Figure 2 presents the scheme of a single artificial neuron. Several signals (several inputs of the model) x_k are supplied to the neural inputs. These signals come either from the inputs of the neural network or from the outputs from other neurons. Each signal reaching the neuron is multiplied by its weight w_k and then summed up. The sum of the input signals φ multiplied by their weights represents the argument of the neuron activation function. The output (answer) of the neuron is the result of the neuron activation function $y = f(\varphi)$. The possibilities of application of a single neuron as a model are considerably low due to its constrained
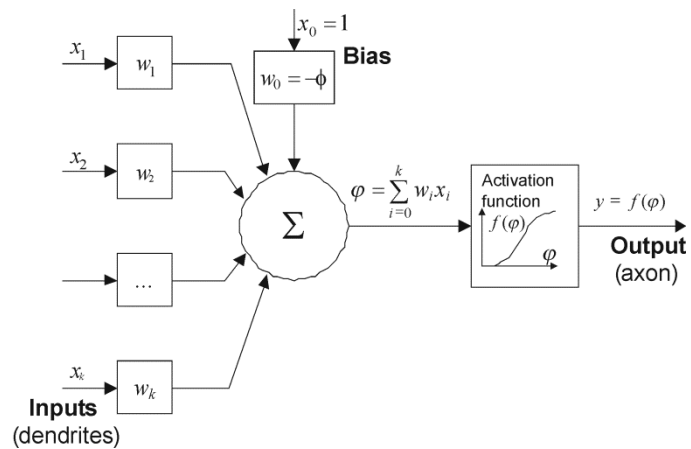

Figure 2. Model of an artificial neuron

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computation capacity. In order to obtain advanced computation possibilities and a higher calculation accuracy the single neurons are interconnected into a net (fig. 3) [25, 26]. The calculation of neural network is called learning process. The Lavenberg Marquardt algorithm has been applied to train the developed empirical model [27].
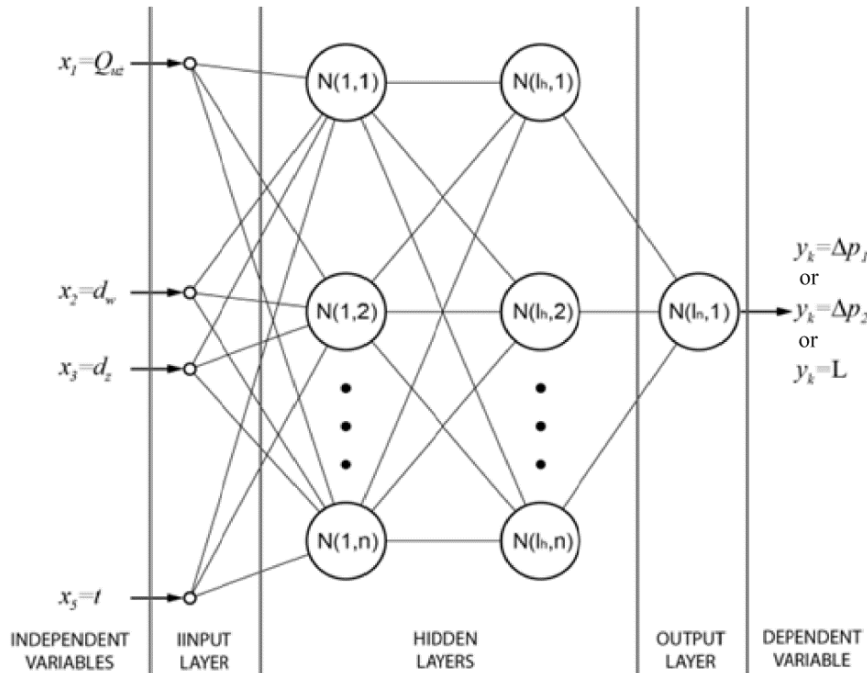

Figure 3. Structure describing change in pressures in inner tubes and outer tube

Table 1. Correlation coefficient for neural network and regression

	δp_1	δp_1	Length
Layer	2	1	2
Neurons in layer	15	15	15
Correlation coefficient			
Neural network	0.9136	0.9812	0.8568
Regression	0.9993	0.9314	0.9727
Full regression	0.8630	0.8681	0.7898

Neural simulations were carried out ten times for $j = 5, 10,.., 50$ neurons for each layer with $k = 1, 2, 3$ layers. The best R^2 coefficient for hot stream obtained in case of one layer and 15 neurons in it; however, for cool stream in case of two layers and also 15 neurons in each layer. In case of length the best correlation obtained also for one layer and 15 neurons in each layer. The results of neural modelling were compared with a regression model. The correlation coefficient for regressions and a neural network is presented in tab. 1, moreover, the adjustment is shown in fig. 4.
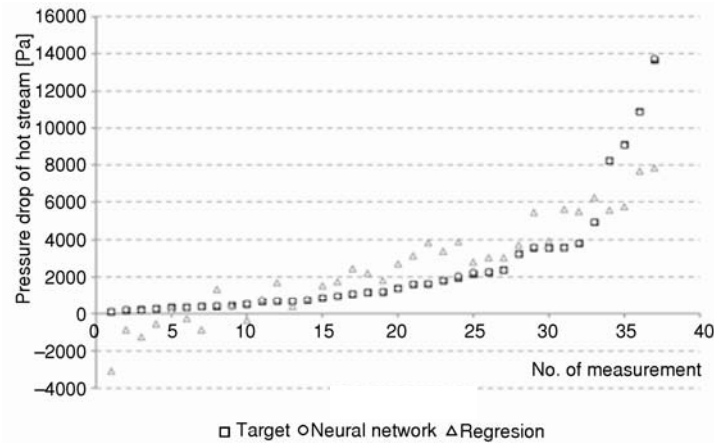
Figure 4. Adjustment of neural network and regression to results of CFD simulation in case of hotstream
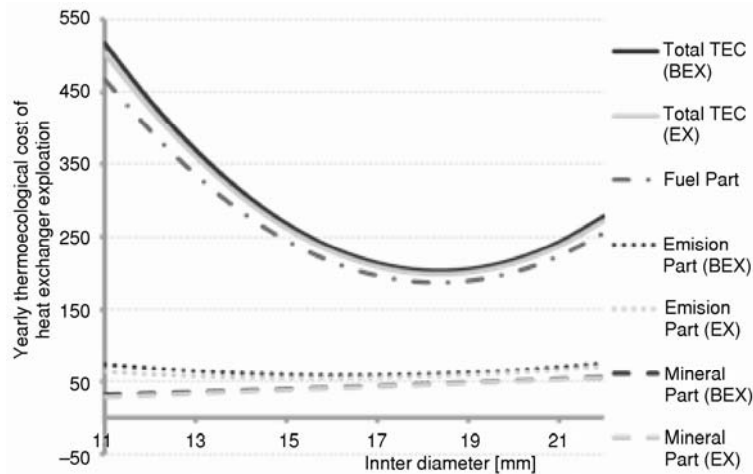
Results of thermo-ecological cost optimisation of heat exchanger

The results of the minimisation of heat exchanger TEC are shown in figs. 5 and 6.

The presented results show the total TEC and its decomposition as a function of HEX tube inner diameter and as function of Re number. The thermo-ecological optimum is located at Re = 50000 and inner diameter d = 18 mm. The decomposition included: BEX = total TEC with the exergy bonus application, EX = total TEC in classic approach, Fuel Part = the fuelpart of cumulative exergy in total TEC, emission part of TEC and mineral part – both with and without bonus). It can be noticed that there is not too much difference in the results of BEX, EX and fuel part. It can be concluded that from the point of view of optimisation of HEX the it is enough to include only the cumulative exergy consumption of energy carriers. The mineral part and emission part of total TEC is negligible.
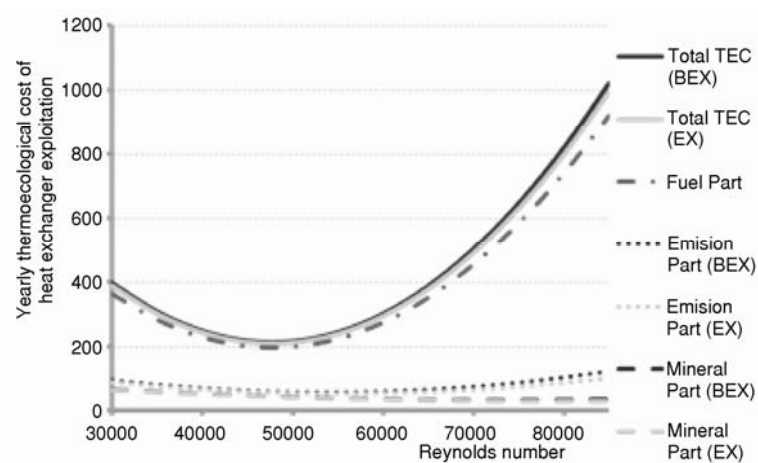
Figure 5. Yearly thermo-ecological cost of heat exchanger exploitation as a function of Reynolds number

Figure 6. Yearly thermo-ecological cost of heat exchanger exploitation as a function of inner diameter

Summary and conclusions

The results of preliminary attempts of utilization of CFD models in thermo-ecological optimization procedure is quite encouraging. The replacing experimental characteristics of devices with numerical models decrease the costs of obtaining the characteristics and extend the possible application of the method to the brand new devices. The main problem lies in the computational time, which significantly enlarges the time of optimization. The rapid development in computer technology gives hope for the machines which will be, in not far future, able to solve so complicated problems in reasonable time. Nowadays, this problem can be overcome by the utilization of the neural network, which operates on the data cloud generated by the multi-variant numerical model.

Neural modelling is much faster and for once trained, validated and tested network with a high correlation coefficient for the given range of parameters is an easy and fast way to get the desired results. This article shows the adjustment of points with neural models and multi-regression to the results of CFD modelling. In this case, for the pressure drops and the length of the tubes was obtained a high correlation coefficient between the input and output parameters. Both regression and neural network tools are very useful to determine the correlation between variables and the results are obtained very fast.

The TEC decomposition presented in the paper include: BEX = total TEC with the exergyreplacement cost application, EX = total TEC in classic approach, Fuel Part = the fuel part of cumulative exergy in total TEC, emission part of TEC and mineral part – both with and without bonus. It can be noticed that there is not too much difference in the results of BEX, EX and fuel part. It can be concluded that from the point of view of optimisation of HEX the it is enough to include only the cumulative exergy consumption of energy carriers. The mineral part and emission part of total TEC is negligible.

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Nomenclature

A – heat transfer area, [m^2]

a_{ij} – coefficient of the consumption of the i-th product per unit of the j-th major product,*e.g.* [MJMJ^{-1}]

a_{rj} – coefficient of the consumption of the r-th imported product per unit of the j-th major product, [MJMJ^{-1}]

b_{sj} – exergy of the s-th non-renewable natural resource immediately consumed in the process under consideration per unit of the$j$-th product, [MJunit^{-1}]

D – internal diameter of tubes, [m]

d – inside dimension, [m]

E_{el} – electricity consumption during exchanger construction, [MJunit^{-1}s^{-1})]

f_{ij} – coefficient of by-production of the i-th product per unit of the j-th major product, [MJMJ^{-1}]

$\dot{G}_j$ – nominal flow rate of the j-th major product, [kgs^{-1}]

G_m – consumption of m-th material or energy carrier used for construction of installation, [kg unit^{-1}]

G_p, G'_s – mass of steel tubes and exchanger jacket, [kg]

G_u – nominal flow rate of the useful u-th by-product, [kgs^{-1}]

k – overall heat transfer coefficient, [kW·m^{-2}K^{-1}]

L – length of the tube, [m]

$m_0,..m_k$ – regression coefficients, determined during the calibration of the model, [–]

n – number of tubes, [–]

n-p – degrees of freedom, [–]

P_k – nominal flow rate of the k-th deleterious waste product rejected to the environment, [kgs^{-1}]

p_{kj} – amount of the k-th aggressive component of waste products rejected to the environment per unit of the j-th product, [kgunit^{-1}]

δp – difference of inlet and outlet total pressure, [Pa]

$\dot{Q}$ – heat load of the apparatus,

Re   – Reynolds number, [–]

R_R – regression sum of squares, [–]

R_T – regression total sum of squares, [–]

t – thickness of pipe wall, [mm]

Δt_m – logarithmic temperature difference in the exchanger, [K]

u_m – expected recovery factor of the m-th material, [kg kg^{-1}]

$\dot{V}$ – volumetric flow rate, [m^{3}s^{-1}]

y_i – actual observation from measurements, [–]

z_{lj} – amount of the l-th aggressive component of waste products entering the cleaning installation, [kg]

$)_1$ – hot medium

$)_2$ – cold medium

Greek symbols

γ_s – density of steel, [kgm^{-3}]

ε – error, uncorrelated random variable, [–]

ζ_k – cumulativeexergy consumption of non-renewable resources due to the emission of unit of the k-th waste product, [MJkg^{-1}]

η_p – electric efficiency of pump and electric engine, [–]

κ, σ – coefficients resulting from construction of exchanger, [–]

μv – proportionality coefficients (as determined by producer of exchanger, for example), [–]

ρ_i, ρ_j – thermo-ecological cost of the i-the product, [MJkg^{-1}]

ρ_{el} – unit thermo-ecological cost of electricity, [MJkg^{-1}]

ρ_s – thermo-ecological cost of steel, [MJkg^{-1}]

ρ_m – thermo-ecological cost of m-th material or energy carrier used for construction of installation, [MJkg^{-1}]

ρ_r – specific thermo-ecological cost of the r-th imported good, [MJkg^{-1}]

τ_n – annual operation time with nominal capacity, [years]

τ_z – nominal lifetime of installation, [years]

σ_l – cumulativeexergy consumption of non-renewable resources due to the removing of k-th aggressive product from wastes, [MJkg^{-1}]

τ – lifetime of installation, [years]

ψ_{j0} – requirement for natural resources exergy to compensate or to avoid the environmental losses resulting from operation of j-th production process, [MJMJ^{-1}]

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