PSEUDO-BOND GRAPH MODEL FOR THE ANALYSIS OF THE THERMAL BEHAVIOUR OF BUILDINGS

by

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In this work, a simplified graphical modeling tool, which in some extent can be considered in halfway between detailed physical and data driven dynamic models, has been developed. This model is based on Bond Graphs approach. This approach has the potential to display explicitly the nature of power in a building system, such as a phenomenon of storage, processing, and dissipating energy such as heating, ventilation, and air-conditioning systems. This paper represents the developed models of two transient heat conduction problems corresponding to the most practical cases in building envelope, such as the heat transfer through vertical walls, roofs, and slabs. The validation procedure consists of comparing the results obtained with this model with analytical solution. It has shown very good agreement between measured data and Bond Graphs model simulation. The Bond Graphs technique is then used to model the building dynamic thermal behavior over a single zone building structure and compared with a set of experimental data. An evaluation of indoor temperature was carried out in order to check our Bond Graphs model.

Key words: *bond graph approach, heating, ventilation and air-conditioning systems, reaction hydrocarbon systems, thermal behavior, building*

Introduction

The rapidly growing energy use gave rise to exhaustion of energy resources and heavy environmental impact. The building sector alone stands for the most energy consuming sector in France with 43% of final energy consumption and 25% of greenhouse gases emission [1]. Thus, efforts should be taken in new buildings construction and existent buildings thermal renovation in order to satisfy low energy building criterions [2]. To meet this objective, it is necessary to use a representation for building analysis in order to model both qualitative and quantitative dynamic aspects. Nowadays, several software packages such as ESP, DOE2, PLEIADES, TAS [3] have been developed to carry out parametrical studies during the design of low energy efficient buildings. Some commercial software packages are based on a simplified steady-state models. These ones benefit in reliability and time, but are poor in accuracy, so they are often applied at the preliminary design stage. Detailed dynamic models benefit in accuracy, they are often applied in the analysis of annual energy consumption and performance of building heating, ventilation and air-conditioning (HVAC) systems. Regarding the building system as a whole where dynamic exchanges of heat and mass transfer occur, the models can be mainly categorized into physical models, data driven models and gray models.

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Physical modeling, also called forward modeling, begins with a description of the building system or component of interest and defines the building according to its physical description. Most simulation models are based on these principals, such as EnergyPlus [4], DOE-2 [5], TAS [6], ESP-r [7], and TRNSYS [8]. These models require a large number of parameters as inputs for the simulation and the process of collecting a physical description is time consuming. Data driven dynamic models are often described by regression techniques [9, 10] where a standard parametric model is adapted to measured data obtained from an experiment on the building process. In this kind of models, it is generally necessary to acquire data over a long period of time with widely varying conditions in order to train the models for accurate predictions under all conditions. Also, these models do not reflect any specific physical structure. Gray models [11, 12] are a combination of model based on physical laws and with a parametric identification procedure using a limited number of parameters having a definite physical meaning. The thermal network model using electrical analogy parameters is an example of gray models. Gray models can represent the physical properties of building system and predict its thermal behavior and consumption and are suitable when we are facing a non-linear process such as solar radiation processes. They are considered as simplified physical models which can represent properly the physical properties of the building system. Therefore, much of such simplified models are commonly developed to overcome the heaviness of detailed physical models. However, there are few unified models whatsoever detailed physical or gray models which can be used for representing and analyzing different building sub-systems simultaneously [13].

The aim of this paper is to develop the dynamic model of the RHC system, contained within the test building, based on the bond graph approach. A bond graph is a graphical representation of a physical dynamic system. A final simulation tool is of modular technological level. This tool provides a comprehensive graphical user interface allowing future extensions.

Bond Graph methodology

The bond graph technique is based on a graphical formalism. It is well suited for modeling physical processes and multidisciplinary dynamic engineering systems including features and components involved in different energy domains. Its philosophy is founded on a systematic and common way representation of power flow between the model's components. Paynter [14] started the bond graph formalism and used it for modeling dynamic multiport systems. He suggested that energy and power are the fundamental dynamic variables which characterize all physical interactions.

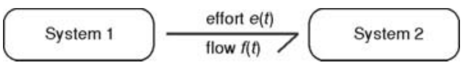

Figure 1. Bond graph link

In bond graph modeling, the interaction between two components is modeled by a bond with a semi-arrow in the end. The power is represented as product of two physical quantities, one extensive, the other intensive. These two power conjugated variables are called effort and flow and are denoted by the letters e and f (fig. 1).

The selection of these two physical quantities is specific for each physical domain. For instance, in electrical domain, we use the voltage u as effort variable and the current i as flow variable. In thermal domain, the effort variable is represented by the temperature T and the flow variable by the heat flow $\dot{Q}$. However, a product of the temperature and the heat flow is not the power transferred between ports. This has enabled researchers to introduce the pseudo-bond graphs (PBG) method in the modeling of thermal systems. The advantage of PBG is the fact that it facilitates the modeling of the thermal systems overcoming the non-linear thermal problems.

A classification of bond graph elements can be made up by the number of ports. Ports are placed where interactions with other processes take place. There are one port elements symbolizing inertial element (I), capacitive element (C), resistive element (R), effort source (Se) and flow source (Sf), and two ports elements representing transformer element (TF) and gyrator element (GY). The elements I, C, and R are passive elements because they convert the supplied energy into stored or dissipated energy. The sources Se and Sf are active elements because they supply power to the system. The bonds are inter-linked by two type junction elements 0-junction and 1-junction which serve to connect I, C, R, and source elements. At the 0-junction, the flow adds up to zero while all efforts are equal, and at the 1-junction all effort variables add up to zero while all flows are equal. The concept of causality is an important concept embedded in bond graph theory. This refers to cause and effect relationship. Thus, as part of the bond graph modeling process, a causality assignment is implicitly introduced.

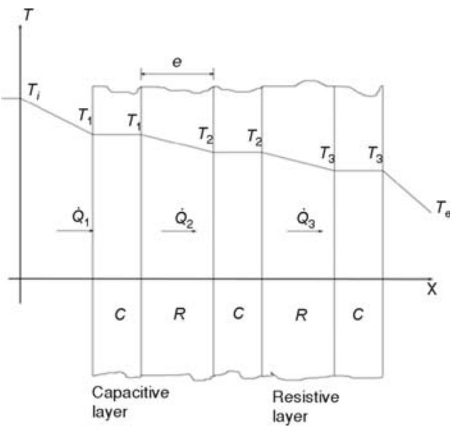

Figure 2. Representation of the spitted wall

In order to explain the PBG modeling approach, we propose to model the heat loss through a homogeneous wall constituting the building envelope. The absolute temperature T may be chosen as an effort variable and the heat flow as a flow variable $\dot{Q}$.

Lumped parameter assumption is usually adopted for this kind of cases. This is realized by splitting the wall into a number of layers, where, temperature and the thermo-physical properties are assumed homogeneous (fig. 2). Each layer stores and conducts heat simultaneously. The external ones are subject to convection heat exchange with inside and outside surrounding.

The PBG model representation of the heat conduction into the wall constituted of four layers is shown in fig. 3.

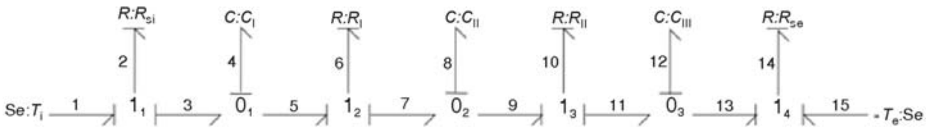

Figure 3. Numerated PBG model of the wall

Power bonds may join at one of two kinds of junctions: a "0" junction and a "1" junction. In a 0-junction, the flows and the efforts satisfy the eqs. (1) and (2):

$$\sum_{i=1}^{n} f_i = 0 \tag{1}$$

$$e_1 = e_2 = ... = e_n \tag{2}$$

In a 1-junction, the flows and the efforts satisfy the eqs. (3) and (4):

$$f_1 = f_2 = ... = f_n \tag{3}$$

$$\sum_{i=1}^{n} e_i = 0 \tag{4}$$

The R and C elements represent, respectively, the thermal resistance and the thermal capacity, given by:

$$R = \begin{cases} \dfrac{e_1}{\lambda A} \text{(conduction)} \\ \dfrac{1}{hA} \text{(conduction)} \end{cases} \tag{5}$$

$$C = \rho e_l Ac \tag{6}$$

where e_1 is the thickness of each layer, λ – the thermal conductivity of the material, h – the convection coefficient, A – the area of the layer, and c – the specific heat. The coefficient of heat convection (with or without radiation) can be estimated using correlations available in the literature [15-17]

The constitutive equations between the efforts and flows corresponding to R and C-elements are, respectively, given by:

$$e(t) = Rf(t) \tag{7}$$

$$e(t) = \frac{1}{C}\int f\mathrm{d}t \tag{8}$$

For each layer inside the wall, the heat quantity is decomposed into two parts: the first part is dissipated by conduction; modeled by a 1-junction related to R-element representing the conductive resistance, whereas the second part is stored by the layer; modeled by a 0-junction related to C-element representing the thermal capacity. Once written, the equations are given in the following matrix form:

$$\dot{X} = \begin{bmatrix} -\left(\dfrac{1}{R_{si}} + \dfrac{1}{R_1 C_I}\right) & \dfrac{1}{R_1 C_{\mathrm{II}}} & 0 \\ \dfrac{1}{R_1 C_{\mathrm{II}}} & -\left(\dfrac{1}{R_1 C_{\mathrm{II}}} + \dfrac{1}{R_2 C_{\mathrm{II}}}\right) & \dfrac{1}{R_2 C_{\mathrm{III}}} \\ 0 & \dfrac{1}{R_2 C_{\mathrm{III}}} & -\left(\dfrac{1}{R_2 C_{\mathrm{III}}} + \dfrac{1}{R_{se} C_{\mathrm{III}}}\right) \end{bmatrix} X + \begin{bmatrix} \dfrac{1}{R_{si}} & 0 \\ 0 & 0 \\ 0 & \dfrac{1}{R_{se}} \end{bmatrix} U \tag{9}$$

with

$$U = \begin{pmatrix} T_1 \\ T_{15} \end{pmatrix} \equiv \begin{pmatrix} T_i \\ -T_e \end{pmatrix} \tag{10}$$

$$X = \begin{pmatrix} q_4 \\ q_8 \\ q_{12} \end{pmatrix} \equiv \begin{pmatrix} q_{\mathrm{I}} \\ q_{\mathrm{II}} \\ q_{\mathrm{III}} \end{pmatrix} \tag{11}$$

where q_{I}, q_{II}, and q_{III} are the heat stored by three capacitive layers, and T_i and T_e represent the indoor and outdoor temperatures.

The system of differential eq. (9) gives the heat flow in each layer, this allows to obtain the temperature profile inside the wall.

Simulation results

Validation

In this section, the performed simulations of transient heat conduction through walls will be illustrated by calculating the surface temperatures, heat fluxes, and energy stored quantities.

The thermo-physical and geometric characteristics of the chosen wall are presented in tab. 1. This table includes also the boundary conditions values.

Table 1. Characteristics data of the studied heat conduction problems

Geometric parameters	Wall thickness L [m]	0.2	Thermo-physical properties	Wall material	Concrete
				Thermal conductivity λ [Wm^{-1}K^{-1}]	0.963
	Wall heat transfer section A [m^2]	1		Specific heat c [Jkg^{-1}K^{-1}]	650
				Density ρ [kgm^{-3}]	1300
Inside and outside conditions	Inside thermal resistance R_{si} [m^{2}KW^{-1}]				0.13
	Outside thermal resistance R_{se} [m^{2}KW^{-1}]				0.04
	Inside temperature T_i [°C]				21
	Outside temperature T_e [°C]				0
	Initial temperature T_0 [°C]				12

The following dimensionless variables are introduced to compare the simulation results of PBG model with the analytical model in the case of the wall: $x^* = x/L$; $T^* = T(x, t) - T_e/\Delta T_{wall}$; $\varphi^* = \varphi e/\lambda \Delta T_{wall}$, $Q^* = Q/\rho Lc\Delta T_{wall}$, $\Delta T_{wall} = T_0 - T_e$, where x^*, T^*, φ^*, and Q^* are the dimensionless variables corresponding, respectively, to the position, temperature, heat flow, and the heat stored (subscripts e and l are, respectively, for entering and leaving).

In order to validate our PBG model, we have compared all simulation results with those of analytical method. Figure 4(a) exhibits the results of temperature variations with time for three positions in the wall (x = 0 m; 0.1 m and 0.2 m). It shows that there is a good agreement between bond graph and both analytical calculations. Figure 4(b) indicates the magnitude of absolute deviations on the calculated temperatures between bond graph model and analytical method. It can be observed from fig. 4(b) that the maximum of deviations occurs at the first hours then tend towards zero, for each position in the wall, after about 20 hours of simulation.

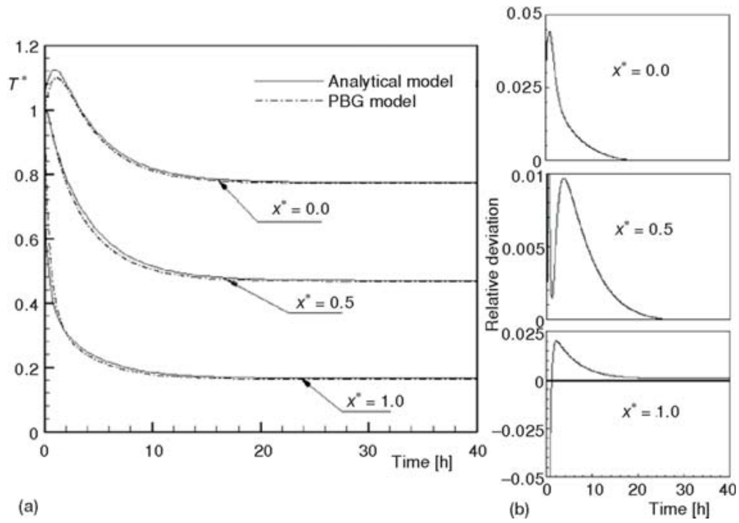

Figure 4. Temperature profiles in the wall (a) and the corresponding absolute deviations between bond graph model and analytical method (b) (case 1: two convective boundary conditions)

The PBG determined profiles of heat stored and heat flow leaving and entering the plane wall are compared in figs. 5(a) and 5(b) with their analytical counterparts. Agreement between the two simulation results is shown to be

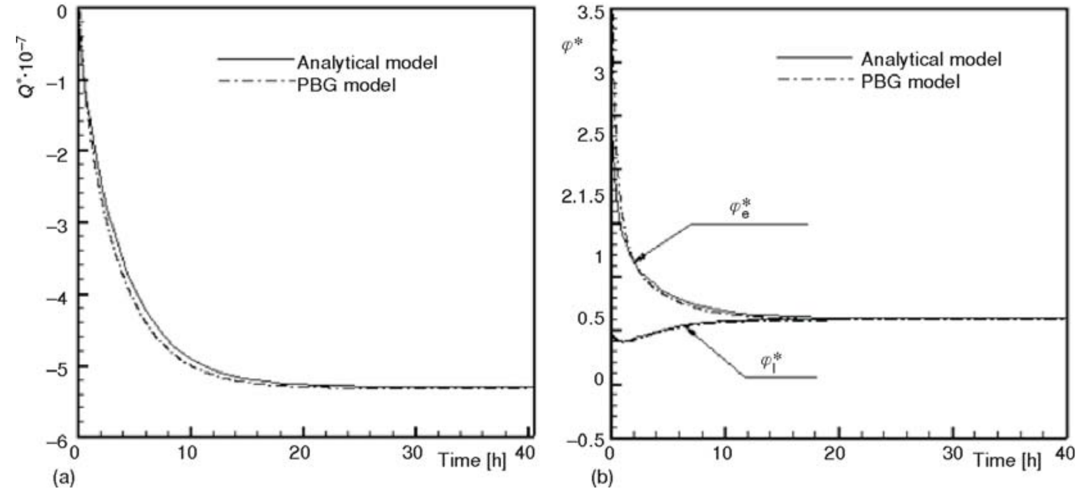

Figure 5. Heat stored by the wall (a) and heat flows leaving and entering the wall (b) (case 1: two convective boundary conditions)

satisfactory. The steady state is reached after 20 hours of simulation. Furthermore, these graphs clearly illustrate the fact that the heat flow leaving is more important than the heat flow entering the wall. This can be explained by the important temperature gradient between the outside and the wall ($[T_e - T(L, t)] > [T_i - T(0, t)]$). Moreover, the heat stored by the wall (or, in this case, evacuated) decreases to reach a value of -1.08 MJ/m^2 at the steady-state.

Results related to the heat conduction in the wall subject to a fixed temperature condition in one side and one convective condition on the other are depicted hereafter. Temperature profiles plots, for three different positions in the wall ($x = 0$ m; $x = 0.1$ m; $x = 0.2$ m), are shown in fig. 6(a).

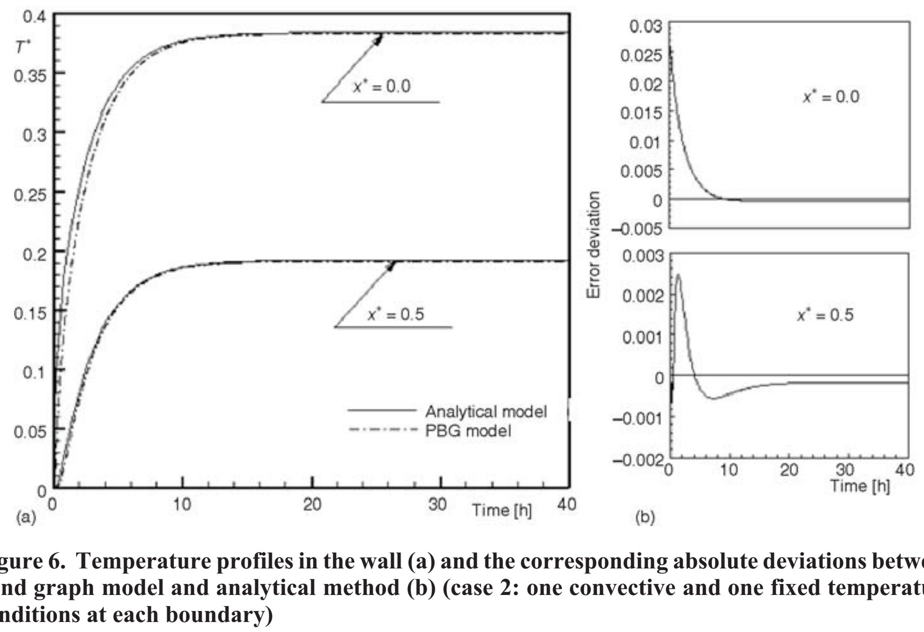

Figure 6. Temperature profiles in the wall (a) and the corresponding absolute deviations between bond graph model and analytical method (b) (case 2: one convective and one fixed temperature conditions at each boundary)

The corresponding deviations plots, between analytical and bond graph methods, on the calculated temperatures are presented in fig. 6(b). Obviously, good agreement can be observed between the analytical predictions and those obtained by the pseudo bond graph model. From fig. 6(a), we can analysis the thermal behaviour of the wall. Indeed, the achieved steady state temperature decreases more and more when going away from inside surface boundary to the outside one. Upon observing fig. 6(b), it can be noticed that the deviations on the calculated temperature does not exceed ±0.4 °C for the boundary surface $x = 0$ m and ±0.025 °C for $x = 0.1$ m.

Thereafter, the heat stored into the wall, as well as the heat flows entering and leaving the wall is illustrated in fig. 7. From fig. 7(b), the leaving flows (heat losses) decrease rapidly at the first times then increase gradually with time before reaching the steady-state. Alternatively, the entering flows (heat gains) decrease continually until reaching the same steady-state. It is also indicated, from fig. 7(a), that the heat stored by the wall decreases firstly then increases with time because the inside gradient of temperature $[T_i - T(0, t)]$ becomes more important than the outside one. However, for the first moments, the calculated bond graph heats and flux shows some disparities with the analytical ones. Upon comparing these results it can be stated that the PBG model agrees accurately with analytical method.

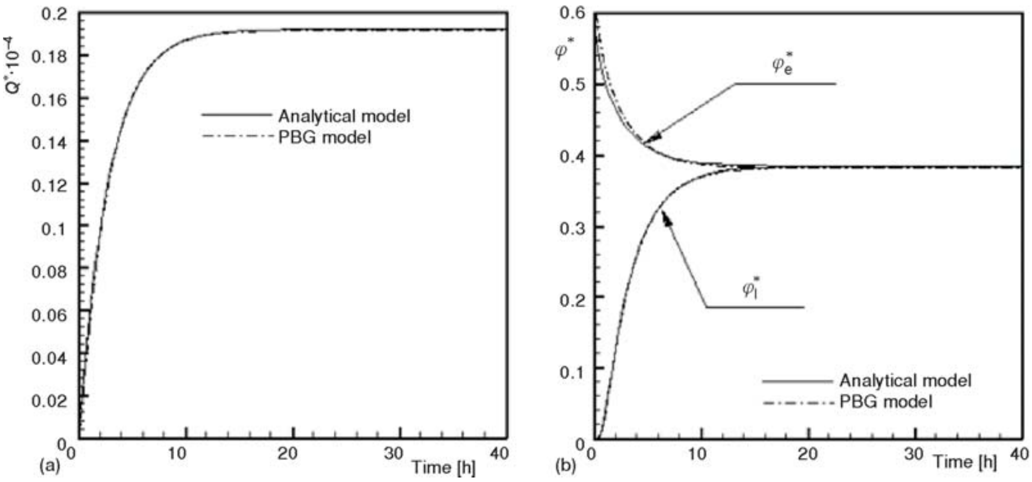

Figure 7. Heat stored by a wall (a) and heat flows leaving and entering the wall (b) (case 2: one convective and one fixed temperature conditions at each boundary)

Test case

The experimental building is a workshop space of 2050 m^2 assimilated to a single-zone building located in Nancy, France. It is a heated space with the following dimensions: 9 × 25 × 82 m (height × width × length). Table 2 presents the building envelope material properties.

In tab. 2, the *U*-value coefficients are calculated including the internal and external heat transfer coefficients, which have respectively the values of 3 and 17 W/m^{2}K.

The heating system consists of a natural gas boiler allowing up to 200 kW of heating rate. The heat supply is insured by 8 air heaters. The temperature is controlled by means of a thermostatic valve system and a temperature sensor located at each blowing orifice of the air

Table 2. Material specifications of the building

Elements	Material (inside → outside)	Thermal conductivity λ [Wm^{-1}K^{-1}]	Thickness [m]	U-value [Wm^{-2}K^{-1}]
Exterior wall	Concrete block Rock wool Steel	1.05 0.04 46	0.2 0.08 0.001	0.423
Roof	Asbestos-cement Glass wool Bituminous roofing	0.95 0.04 0.5	0.075 0.075 0.01	0.473
Floor	Concrete	0.963	0.2	2.394

heaters. In order to validate our model, accurate temperature measurements have been performed during three successive days through a platinum resistance sensor which provides results with an uncertainty not higher than ±0.01 K. During the tests, the set-point temperature is fixed to 17 °C for day time (from 8 a. m. to 8 p. m.) and 15 °C for night time. The local is unoccupied and we consider that solar gain is negligible because of closing windows during this period. The exterior temperature variation file has been generated by means of a local weather station providing maximum and minimum temperatures of the day.

The model of the present building invokes smaller models (sub-models) that are related to the constructive elements of the building: external walls, roof and slab.

First, the sub-models have been created to describe the physics of heat transfer considering conduction and convection. Heat flow through these elements is affected by convection and conduction phenomena as well as by outdoor/indoor temperature difference.

In the next step, all elements at the boundaries of building are connected to the thermal capacity C_{zone} which takes the form:

$$C_{\text{zone}} = r_{\text{air}} \times V_{\text{building}} \times c_{\text{air}} \tag{12}$$

The PID controller drives the heating process in order to minimize the difference between the indoor set and measured temperatures. The PBG of the building can be represented in fig. 8.

PBG model was run using measured weather data for three successive days of the winter period (February 5$^{\text{th}}$ to 7$^{\text{th}}$, 2009). Heating load are calculated and compared between two models and a comparison against measured indoor temperature is carried out. Figure 9 represents the distribution of the outdoor temperature, the set point temperature and the measured indoor temperature. The set point temperature is fixed to 15 °C for night time and 17 °C for day time, whereas, the outdoor temperature fluctuates between the minimum and the maximum of the considered day. We can observe the important fluctuations of the measured internal temperature around the set point which can be estimated to about ± 0.5 °C, this is due to the control system quality.

Figure 10 shows a comparison between the calculated indoor temperatures with the bond graphs approach against the experimental data.

Obviously, a good agreement is observed between the simulated data and the measured ones. The models reproduce properly the thermal behavior highlighted by the experimental data. We can, especially, denote a good harmony between the two simulation models, which demonstrates the potential of the new bond graphs approach and its ability to model the building thermal behavior systems.

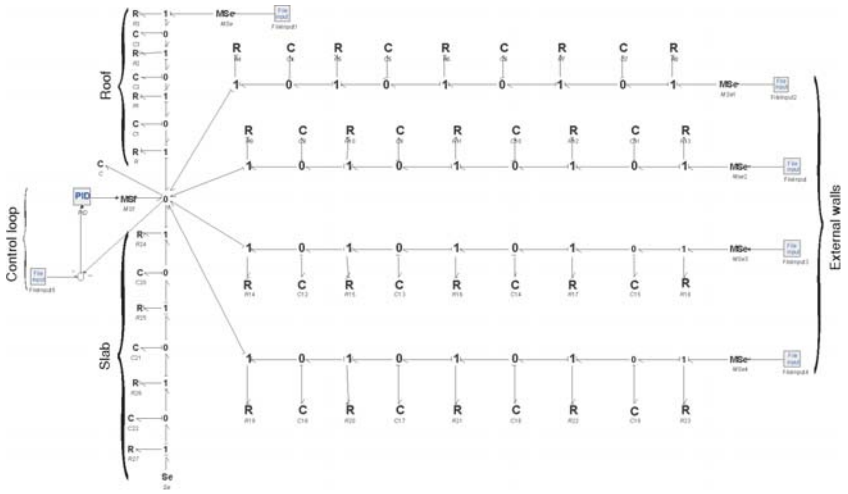

Figure 8. Pseudo-Bond Graph model of the building

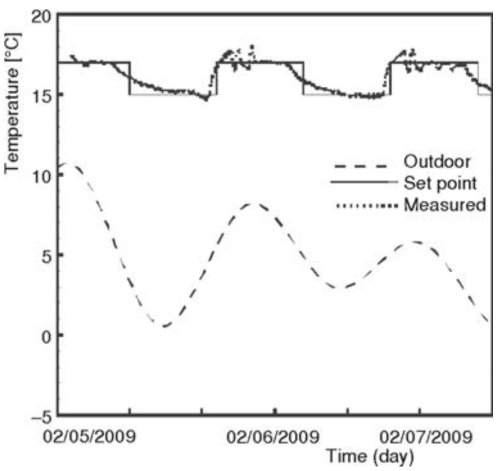

Figure 9. Outdoor, set point and measured indoor temperatures

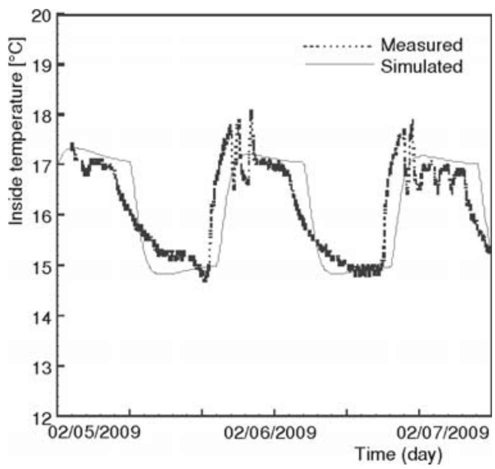

Figure 10. Comparison between measured and simulated indoor temperature data

Conclusions

In this paper the PBG model of one-dimensional transient heat conduction through plane walls has been developed. Two kinds of problems, regarding boundary conditions, were selected: The first heat conduction problem involves two different convective conditions at each boundary surface of the wall which symbolizes a roof or a vertical wall in a building. The second problem deals with heat conduction in a wall which is subject to one convective condition in one boundary surface and a fixed temperature condition in the other side; this symbolizes heat transfer in a slab. The bond graph method can be extended to multidimensional heat conduction problems applied to the thermal constriction phenomenon [18-24].

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