Discrete Dynamics in Nature and Society, Vol. 5, pp. i–iii
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Foreword

Two-Dimensional Maps and Chaotic Synchronization

An important problem that arises time and time again in the biological sciences is related to the collective behavior of a group of cells or functional units that individually displays complicated nonlinear phenomena. The human kidney, for instance, contains of the order of one million nephrons, each having at its disposal mechanisms to regulate the incoming blood flow. Investigations on rats have revealed that this flow regulation tends to produce self-sustained oscillations in the intratubular pressures with a period of 30–40 s, and for rats with elevated blood pressure chaotic oscillations in the flow regulation can be observed. The nephrons interact with one another *via* muscular contractions that propagate along the afferent blood vessels, and experiments have shown how this interaction can lead to synchronization of the pressure and flow oscillations in neighboring nephrons. It is clearly of interest to understand to which extent this type of entrainment phenomena influence the overall function of the kidney. Will there be circumstances, for instance, where the coupling produces global synchronization or leads to the formation of local or global patterns, and will transitions between different modes of behavior be related to the development of particular diseases?

Similarly, each of the insulin producing β-cells of the pancreas exhibits a complicated pattern of bursts and spikes in its membrane potential and, depending on the operational conditions, this dynamics can be either regular or chaotic. Presumably *via* the effect that the variations in membrane potential have on the exchange of Ca^{2+}-ions between the cell and its surroundings, the fraction of time that the cell spends in the bursting state controls the release of insulin. At the same time, the β-cells interact with one another *via* a variety of different mechanisms, and again it is of interest to understand how the collective behavior of a group of cells is related to the function of the individual cell. Can synchronization between the cells, for instance, stimulate a cluster of cells to produce more insulin than they would otherwise do?

In the economic realm, each individual production sector with its characteristic capital life time and inventory coverage parameters may exhibit an oscillatory response to changes in the external conditions. Overreaction, time delays, or reinforcing positive feedback mechanisms may cause this dynamics to become destabilized and to produce self-sustained oscillations or other more complicated forms of nonlinear dynamic behavior. The sectors of the economy interact *via* the exchange of goods and services and *via* their competition for labor and materials. A basic problem in the development of a dynamic, macroeconomic theory is therefore to describe how interaction between the various sectors leads to a certain synchronization and, hence, to the formation of overall business cycles of different periodicities.

The same type of synchronization and anti-synchronization phenomena may arise in the competition between a pair of duopolists or in the trade relations between national economies. These are precisely the questions addressed by Bischi and Gardini in their paper on *global properties of symmetric competition models with riddling and blowout phenomena*, the first contribution to the present issue, and analyzed by Yousefi, Maistrenko and Popovych in their subsequent contribution on *complex dynamics in a simple model of interdependent open economies*.

In order to understand the complicated phenomena that can arise through the interaction of two (or more) chaotic oscillators, a variety of simplifying conditions must be considered. The most direct approach is clearly to represent each of the chaotic oscillators by a one-dimensional, noninvertible map. This is the approach that forms the basic theme of the present issue. If the chaotic oscillators are also assumed to be identical, a state of full synchronization may be achieved in which the oscillators perform precisely the same motions. Interesting questions that arise in this relation pertain to the stability of the synchronized state to noise or to a small mismatch in the parameters of the interacting oscillators. Other questions relate to the behavior of the coupled system, once the synchronization breaks down, and to the distribution in phase space of the initial points from which entrainment can be obtained.

Investigations of these and related problems have recently led to the discovery of a number of new phenomena, including riddled basins of attraction, attractor bubbling, and on-off intermittency. Riddled basins of attraction may be observed in regions of parameter space where the synchronized state is attracting on the average (the largest transverse Lyapunov exponent is negative), but particular orbits embedded in this state are transversely unstable (the corresponding transverse eigenvalues are numerically larger than one). From each point of such orbits as well as from their dense set of preimages, tongues will open up in which the trajectories are locally repelled from the synchronized state. However, whether these trajectories will diverge to infinity or approach some other limiting state depends on the character of the transverse instability (sub- or supercritical) as well as on the presence of nonlinear restraining mechanisms.

Laugesen *et al.*, also consider the interaction between two identical, noninvertible one-dimensional maps in their discussion of *type-II intermittency in a class of two coupled one-dimensional maps*. By introducing an antisymmetric coupling they can obtain that the sub-critical period-doubling bifurcation in the individual map is transformed into a sub-critical Hopf bifurcation in the coupled map system. Hence, one can observe the corresponding transformation from type-III intermittency in the individual map into type-II intermittency in the coupled maps system. Moreover, by a proper choice of the parameters of the individual map they can assure a fairly uniform reinjection of the trajectories near the unstable focus point. In this way the transition to type-I intermittency provides a universal route for the development of hyperchaos.

In their paper on the *scaling dynamics for a period-doubling map with external periodic driving*, Ivan'kov and Kuznetsov apply a renormalization group approach to discuss several types of scaling that can be observed in the neighborhood of the Feigenbaum accumulation point in the presence of a small amplitude periodic forcing. The type of scaling behavior is found to depend on the structure of the binary representation of the frequency parameter for the driving term. Normal Feigenbaum scaling is found for finite binary fractions, periodic or quasiperiodic scaling for periodic fractions, and statistical scaling for non-periodic binary fractions. If an economic system is found to undergo a period-doubling transition to chaos, the annual forcing of this economy through variations in domestic food production or oil consumption may be assumed to give rise to these various types of scalings.

Besides as models of coupled, highly dissipative chaotic oscillators, two-dimensional maps are also used as Poincaré models of three-dimensional flows with relatively weak dissipation. Such maps are clearly invertible. Originally developed in connection with a problem in astrophysics, over the years the much celebrated Hénon map has been put to a variety of different applications, including as a model of radiophysical generators with inertial nonlinearly and of technical control systems with pulse width modulation. The bifurcation structure of the Hénon map has also been studied by a significant number of investigators. In their paper on *bifurcation analysis of the Hénon map*, Zhusubaliyev *et al.*, apply two-dimensional continuation techniques to shed new light on this structure and particularly to discuss the shrimp (or swallow tail) structures observed in two-parameter transitions to chaos. These are the same structures as discussed by Yousefi *et al.*, and by Ivan'kov and Kuznetsov in their contributions.

In their paper on *simulated evolution in a linguistic model*, Knudsen and Cameron apply a system of coupled logistic maps to discuss evolutionary dynamics in children's language development. The basic assumption in this approach is that children learn from each other through their mutual interactions, with learning manifesting itself as an increase in the set of utterances that a child can produce through reproduction and combination of utterances spoken by other children.

Finally, in their paper on *chaos and complexity in a simple model of production dynamics*, Katzorke and Pikovski consider the complex dynamical behaviors that can arise in a simple funnel model of production dynamics. With continuous order flow, a model of three parallel funnels reduces to a one-dimensional Bernoulli-type map, which is known to display strongly chaotic dynamics. Optimization of the production costs is possible, however, through classic methods of chaos control. This brings us back to the economic applications of nonlinear dynamics and chaos theory, a field which has already attracted significant interest in the literature. However, by contrast to much other work, the purpose here is not to prove neither that economic development is chaotic or to show how nonlinear dynamics can be used to predict this dynamics. Rather, the interest is in using methods from nonlinear dynamics to optimize the behavior of a complex production system.

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March 5, 2000.

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