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MODIFICATION OF TOPSIS METHOD FOR SOLVING OF MULTICRITERIA TASKS

Zoran MARKOVIĆ
PE PTT Communications "Serbia"

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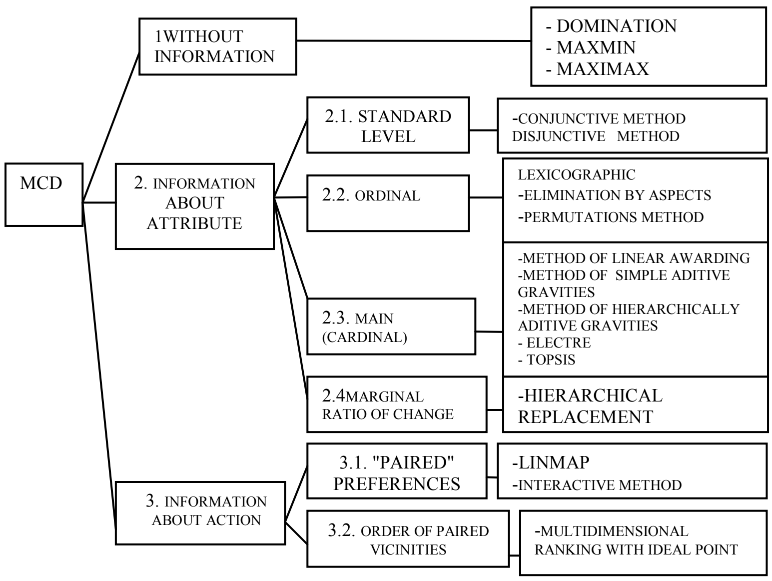
Abstract: This paper describes the possible modifications of one of the multi-criteria analysis methods that possess certain advantages in cases of solving the real business problems. We will discuss the TOPSYS method, whereas the modification reflects in change of the determination manner of the ideal and anti-ideal points in criteria environment, in standardization of quantification and fuzzycation of the attributes in cases of criteria expressed by linguistic variables.

Keywords: Decision-making, multi-criteria analyses, attributes, fuzzy attribute description.

1. INTRODUCTION

Modern operational methods in large hierarchy-structured business systems imply making numerous important business decisions in a short period of time, which means that managers are often forced to use specific tools in order to be able to make minimum risk in quality decisions. It could be said that the last quarter of the 20th century and the beginning of new millennium have flourished in various studies and researches aiming to develop the decision-making mechanisms and methods in situations in which relationships within the system and the environment are becoming ever more complicated and more dynamic and when the reaction time to actual or assumed dysfunctions becomes a considerable factor of success. The majority of business decisions are made in conflict or partially conflict criteria situations, in which cases the uni-criterion tasks' solving methods are almost inapplicable. Practice has imposed the development of new methods which have acknowledged the conflict quality of criteria or goals. This resulted in development of multi-criteria and multi-target methods of real problems' solving. Taxonomy of the multi-criteria tasks' solving method is shown bellow in the Picture 1. [5] [2].

This paper describes the TOPSYS method of solving the multi-criteria decision-choosing tasks that implies full and complete information on criteria, expressed in numerical form. The method is very useful for solving of real problems; it provides us with the optimal solution or the alternative's ranking. In addition to this, it is not so complicated for the managers as some other methods which demand additional knowledge. TOPSYS method would search among the given alternatives and find the one that would be closest to the ideal solution but farthest from the anti-ideal solution at the same time. The essence of it reflects in determination of the Euclidean distances from the alternatives (represented by points in n-dimensional criteria space) to the ideal and anti-ideal points. Modification of the method aims to set a different manner of determining the ideal and anti-ideal point – through standardization of linguistic attributes' quantification and introduction of fuzzy numbers in description of the attributes for the criteria expresses by linguistic variables.
INFORMATION TYPE
IMPORTANT
BASIC METHOD OF DECISION MAKER INFORMATION CLASSES CHARACTERISTICS



Picture 1 Taxometry of multi-criteria decision-making method (MCD)

2. SETTING OF PROBLEMS AND TOPSYS METHOD

In cases where real problems are to be solved, the managers often have to make a decision by choosing one out of many alternative solutions based on several decision-making criteria of opposite or partially opposite characteristics. Therefore, let us assume that we are given m – alternatives and that n-criteria is being assigned to each of them, meaning that we are choosing the most acceptable alternative a^* out of the final A alternative group, taking into account all criteria simultaneously.

$$A = [a_1, a_2, ..., a_m]$$

Each alternative $a_i; i = 1,2,...,m$ is described by attribute values $f_j; j = 1,2,...,n$ marked as follows: $x_{ij}; i = \overline{1,m}; j = \overline{1,n}$. Criteria f_j may be of profit (benefit) or expenditure (cost) type.[1] Profit type criteria means that greater value of attribute is preferred to lesser attribute value (herein represented by "**max"),** while cost type criteria means that lesser attribute value is preferred to greater value of attribute (herein represented by "**min**").

The above may be illustrated with the following matrix O:

$$\begin{array}{c|cccccc}
 & f_1 & f_2 & \cdots & f_j & \cdots & f_n \\
\hline
a_1 & x_{11} & x_{12} & \cdots & x_{1j} & \cdots & x_{1n} \\
a_2 & x_{21} & x_{22} & \cdots & x_{2j} & \cdots & x_{2n} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
a_i & x_{i1} & x_{i2} & \cdots & x_{ij} & \cdots & x_{in} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
a_m & x_{m1} & x_{m2} & \cdots & x_{mj} & \cdots & x_{mn} \\
 & \begin{pmatrix} \max \\ \min \end{pmatrix} & \begin{pmatrix} \max \\ \min \end{pmatrix} & & \begin{pmatrix} \max \\ \min \end{pmatrix} & & \begin{pmatrix} \max \\ \min \end{pmatrix}
\end{array}$$

The elements of the matrix O are real numbers (not negative) or linguistic expressions from the given group of expressions. Linguistic attributes have to be quantified within previously determined and agreed value scale. The most commonly used scales are as follows:

Ordinal scale
Interval scale
Relationship scale

Ordinal scale determines the ranking of actions, whereas the relative distances between the ranks are not taken into account, unlike the Interval scale where equal differences between the attribute values and defined benchmarks are determined. Ratio scale also ensures equal relations between the attribute values but the benchmarks are not defined beforehand. The author's opinion is that Interval scale represents the suitable tool to be used when performing quantification of qualitative attributes. The most commonly used scale is 1 to 9, since the extremes of the attributes for the criteria being analyzed are usually unknown. The table bellow shows one of the methods of translating the qualitative attributes into quantitative attributes.

Qualitative estimation	bad	good	avarage	very good	excellent	Type of criteria
Quantitative estimation	1	3	5	7	9	Max
	9	7	5	3	1	min

Quantification of qualitative criteria can be performed in many different ways. One of them is so called fuzzycation, which gives account of the ambiguities occurring at expression of linguistic variables. Therefore, the matrix O becomes quantified according to each criterion and as such, this matrix is called – quantified decision-making matrix O1.

In order for the task to be solved it is necessary to normalize the attribute values, i.e. to perform the "unification" or "make the attributes non-dimensional", which means that the attribute values would be set within 0 – 1 interval. Normalization of the quantified matrix O1 can be performed in two ways, as follows:

1. Vectorial normalization, and
2. Linear normalization.

In vectorial normalization procedure each element of quantified decision-making matrix is divided by its own norm. The **norm** represents the square root of the addition of element value squares, according to each criterion. The procedure is as follows: [6]

The **norm** is calculated for each *j*-column of decision-making matrix:

$$norma_j = \sqrt{\sum_{i=1}^{m} x_{ij}^2}\ ; (j = 1,\dots, n)$$

Whereas x_{ij} - represents the value of *j*-attribute for *i*-alternative.

r_{ij} represents the elements of new, normalized decision-making matrix R, and are calculated in the following manner:

For the **max** type criteria,

$$r_{ij} = \frac{x_{ij}}{norma_j} ; (i = 1,2,\dots, m)(j = 1,2,\dots, n)$$

For the **min** type criteria,

$$r_{ij} = 1 - \frac{x_{ij}}{norma_j} ; (i = 1,2,\dots, m)(j = 1,2,\dots, n)$$

Depending on the criteria type, linear normalization of attributes is performed in a way in which attribute value is divided by maximum attribute value for given **max** type criteria, i.e. by supplementing - up to1 - for given **min** type criteria. This results in linear decision-making matrix R with the following elements:

For the **max** type criteria:

$$r_{ij} = \frac{x_{ij}}{x_j^*};\ x_j^* = \left\{ x_j \middle| \begin{matrix} \max \\ i \end{matrix} x_{ij} \right\};\ i = \overline{1,m};\ j = \overline{1,n}$$

For **min** type criteria:

$$r_{ij} = 1 - \frac{x_{ij}}{x_j^*};\ x_j^* = \left\{ x_j \middle| \begin{matrix} \max \\ i \end{matrix} x_{ij} \right\};\ i = \overline{1,m};\ j = \overline{1,n}$$

Nevertheless, in order to preserve the maximum initial information in the course or further action in relation to initial attribute values and attribute values of other criteria, for the **min** type criteria, it is necessary to perform more precise copying of attribute values into the 0 -1 interval. Namely, normalized attribute values for **max** type criteria would be in the interval *p*-1, and $0 \prec p \prec 1$, while in case of **min** type criteria that value belongs in the interval from 0-p, and $0 \prec p \prec 1$. From these grounds we suggest the linear normalization with copying, as in **max** type criteria, meaning that:

$$r_{ij} = 1 - \frac{x_{ij} - x_j^-}{x_j^*};\ x_j^* = \left\{ x_j \middle| \begin{matrix} \max \\ i \end{matrix} x_{ij} \right\};\ x_j^- = \left\{ x_j \middle| \begin{matrix} \min \\ i \end{matrix} x_{ij} \right\};\ i = \overline{1,m};\ j = \overline{1,n}$$

After the normalized decision-making matrix is made, it is necessary to determine the coefficients of relative criteria importance $w_j;\ j = 1,2,...,n$ - which are also being normalized, which results in the following: $\sum_{j=1}^{n} w_j = 1$

Relative importance of criteria represents a significant part of multi-criteria task set-up, since it ensures the relation between criteria which, by the rule, are not of the same value. Relative importance of criteria depends on subjective estimation of the DM (Decision Maker) and has a significant influence on the final result. Multiplication of each normalized matrix's element r_{ij} with the assigned weight coefficient w_j results in decision-making matrix V where one of the multi-criteria tasks' solving methods is applied. The elements of decision-making matrix are as follows: $v_{ij} = w_j r_{ij};\ i = 1,2,...,m;\ j = 1,2,...,n$

Selection of multi-criteria tasks' solving method depends on complexity of the task as well as on the preferred result (rank alternative, the best alternative, group of satisfactory alternatives, etc.)

In the text which follows we shall discuss the TOPSYS method resulting in rank alternative, being the best alternative at the same time, taking into consideration all criteria simultaneously.

TOPSYS – (**T**echnique for **O**rder **P**reference by **S**imilarity to **I**deal **S**olution) [5] method, determines the similarity to ideal solution. Therefore, it introduces the criteria space in which every alternative A_i is represented by a point in the n-dimensional criteria space and coordinates of those points are attribute values of decision-making matrix V. Next step is determining of ideal and anti-ideal points and finding the alternative with the closest Euclidean distance from the ideal point, but at the same time, the farthest Euclidean distance from the anti-ideal point. Picture 2 represents the example of two-dimensional criteria space in which every alternative A_i possesses the coordinates which are equal to normalized values of the assigned attributes multiplied by normalized weight coefficients, coordinates of ideal A^* and anti-ideal point A^-, as well as the Euclidean alternative distances from the ideal and anti-ideal point.

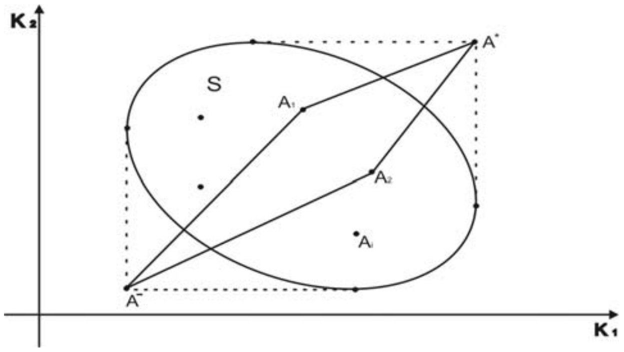


Figure 2 Euclidean alternative distances from the ideal and anti-ideal point.

TOPSYS method builds on the assumption that *mxn* decision-making matrix O includes m-alternatives and n-criteria:

O=

	f_1	f_2	$\cdots$	f_j	$\cdots$	f_n
a_1	x_{11}	x_{12}	$\cdots$	x_{1j}	$\cdots$	x_{1n}
a_2	x_{21}	x_{22}	$\cdots$	x_{2j}	$\cdots$	x_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
a_i	x_{i1}	x_{i2}	$\cdots$	x_{ij}	$\cdots$	x_{in}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
a_m	x_{m1}	x_{m2}	$\cdots$	x_{mj}	$\cdots$	x_{mn}
	$\begin{pmatrix}\max\\\min\end{pmatrix}$	$\begin{pmatrix}\max\\\min\end{pmatrix}$		$\begin{pmatrix}\max\\\min\end{pmatrix}$		$\begin{pmatrix}\max\\\min\end{pmatrix}$

a_i - i_ alternative ; x_{ij} - attribute value i_alternative for j_ criteria

It is also assumed that attributes expressed by linguistic terms have been quantified, as well as that benefits of each individual criterion have been determined and that relative criteria weights w_j have also been defined. Further procedure can be described in 6 steps, as follows:

1. First step – calculating the normalized matrix using the vector normalization, whereas the matrix elements for the **max** type criteria are:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x^2_{ij}}} \left(j = 1,\dots n \right)$$

and for the **min** type criteria:

$$r_{ij} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x^2_{ij}}} \left(j = 1,\dots n \right)$$

This results in normalized decision-making matrix as shown bellow:

	f_1	f_2	$\cdots$	f_j	$\cdots$	f_n
a_1	r_{11}	r_{12}	$\cdots$	r_{1j}	$\cdots$	r_{1n}
a_2	r_{21}	r_{22}	$\cdots$	r_{2j}	$\cdots$	r_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
a_i	r_{i1}	r_{i2}	$\cdots$	r_{ij}	$\cdots$	r_{in}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
a_m	r_{m1}	r_{m2}	$\cdots$	r_{mj}	$\cdots$	r_{mn}
	$\begin{pmatrix}\max\\ \min\end{pmatrix}$	$\begin{pmatrix}\max\\ \min\end{pmatrix}$		$\begin{pmatrix}\max\\ \min\end{pmatrix}$		$\begin{pmatrix}\max\\ \min\end{pmatrix}$

2. Second step – multiplication of normalized matrix elements with normalized weight coefficients $w_j; j = 1,2,\dots,n$ such as that: $\sum_{j=1}^{n} w_j = 1$ whereas the elements of the modified decision-making matrix are: $v_{ij} = w_j r_{ij}$

3. Third step – determining the ideal and anti-ideal points in n-dimensional criteria space, so that ideal point is as follows:

$$A^* = (\max_i v_{ij} \quad |j \in J), (\min_i v_{ij} \quad |j \in J^{'}) | i = 1,2,...,m$$

$A^* = (v_1^*, v_2^*,..., v_j^*,..., v_n^*)$ - Ideal alternative coordinates;

$$A^- = (\min_i v_{ij} \quad |j \in J), (\max_i v_{ij} \quad |j \in J^{'}) | i = 1,2,...,m$$

$A^- = (v_1^-, v_2^-,..., v_j^-,..., v_n^-)$ - Anti-ideal alternative coordinates;

Whereas $J \subset \{1,2,...,n) | j - \max\}$ applies for the **max** type criteria,

while $J^{'} \subset \{1,2,...,n) | j - \min\}$ applies for the **min** type criteria.

In this way, the coordinates of the ideal A^* and anti-ideal point A^- in the n-dimensional criteria space have been determined.

4. Fourth step – calculating of Euclidean distance S_i^* of each alternative a_i, from the ideal point and S_i^- of each alternative a_i from the anti-ideal point A^-.

$S_i^* = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^*)^2}, i = 1,..., m$ - Euclidean distance of the i^n alternative from the ideal point;

$S_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}, i = 1,..., m$ - Euclidean distance of the i^n alternative from the anti-ideal point.

5. Fifth step – calculating the relative similarity of the alternatives from the ideal and anti-ideal points which is done in the following manner:

$$C_i = \frac{S_i^-}{S_i^* + S_i^-}; 0 \prec C_i \le 1; i = 1,..., n$$

If C_i=1 then $a_i = A^*$ and if C_i=0, then $a_i = A^-$. Therefore, the conclusion is that a_i is closer to A^* if the C_i is closer to value 1.

6. Sixth step – setting up the rank according to C_i , meaning that the bigger C_i is - the better the alternative would be.

3. MODIFICATION OF TOPSYS METHOD

The author is familiar with two modifications of TOPSYS method, whereas the first one aims to simplify the procedure of best action selection, while the other one deals with fuzzycation of attributes. First modification was performed by Yoon and Hwang [5] by using the simple additive weight method as the base. Modification reflects in the fact that relative closeness is not determined on the basis of the Euclidean distance but it is based on the city distance; therefore setting up the alternative rank according to the shortest city distance to the ideal point but, at the same time, the longest distance from the anti-ideal point. The basic TOPSYS method includes the exact numerical descriptions of attributes, whereas the authors of the above said modification translate linguistic descriptions into numerical forms within the determined value scale. In case the manager is doubtful about the available subjective estimations, the method provides the option of calculating the replacement margin by using the indifference curve. More detailed description of this modification can be found under reference [5].

Another modification in relation to the attribute fuzzycation (as described in detail under [3]), means that each attribute is described by a discrete fuzzy number. This being done, we determine the relations of order between discrete fuzzy groups, as well as the probabilities of belonging to a group and also the measures of inferiority of the alternatives according to a certain criterion. The rank is established based on belief that alternative is worse then ideal solution but better then anti-ideal solution. Modification is in deed interesting, but the author is of the opinion that it is not necessary to carry out fuzzycation of all criteria but only those which are being expressed by linguistic terms. In addition to this, the proposed modification makes its practical application more difficult.

The author will try to solve the problem of noticed deficiencies of TOPSYS method when applied in practice, through modification of basic method, as described in the text which follows.

3. 1. Implementation of ideal and anti-ideal alternative

The author's opinion is that determining of ideal and anti-ideal points also represents a deficiency of the original TOPSYS method, because in the original method, maximum and minimum values of attributes according to all criteria represent the coordinates of ideal and anti-ideal points. Nevertheless, the attribute values in specific tasks are not always ideal for the given criterion. When solving the real problems managers tend to define ideal and anti-ideal values for each criterion and compare the attributes with the extremes defined in that manner. Potential solutions in most cases deviate from the ideal, and therefore the task is to find the solution that would be closest to the ideal, taking into account all criteria simultaneously. Qualitative criteria are especially interesting when used to express evaluations of managers within some value scale. If we consider the 1 to 10 value scale, the attribute values are often to be found somewhere in between the extreme values and that is why in the original method, maximum and minimum attribute values (rather then extreme scale values) are taken as coordinates of the ideal and anti-ideal points. Therefore, the manager assumes that the ideal value is equal to 10 and then assigns other attribute values in accordance to that value, so it is logical to assign the value 10 for the attribute value of ideal alternative, i.e. to assign the value 1 for the anti-ideal. When dealing with the criteria whose attributes could be expressed in numerical terms, it is always questionable whether the maximum and minimum attribute values are truly ideal and anti-ideal or it is up to the manager himself to estimate if those values could be more extreme. This only adds to manager's subjectivity during the task solving process, but on the other hand, it contributes to more precise and clear definitions of the ideal and anti-ideal solutions which are later used as benchmarks for all other alternatives. Attribute values for the ideal and anti-ideal alternative must comply with the following requirement:

$$x+ \geq \max \quad x_{ij} \left(j = 1,.... n\right), (i = 1,... m)$$
$$x_j^- \leq \min \quad x_{ij} \left(j = 1,.... n\right), (i = 1,... m)$$

This paper suggests modification of the basic method through introduction of two new alternatives. One of the alternatives would possess the attributes of maximum theoretical value (i.e. ideal) as opposed to the other alternative that would possess the attributes of minimum theoretical value (i.e. anti-ideal). It goes without saying that when determining the ideal and anti-ideal values we have to bear in mind the criteria benefits, maintaining the possibility to translate the cost criteria into profit criteria by inversion of attribute values. Thus the attributes of the said alternatives would serve as ideal and anti-ideal points' coordinates.

This can be demonstrated by a simple example involving only two criteria. Let us assume that both criteria are of linguistic nature and that estimations are expressed in the interval from 1 to 10. Let us also assume that we have four alternatives and that the table bellow shows the decision-making matrix after the quantification process:

	Criteria 1	**Criteria 2**
Alternative 1	**9**	**2**
Alternative 2	**3**	**6**
Alternative 3	**4**	**6**
Alternative 4	**9**	**4**
Weight coefficients	**0,4**	**0,6**

After we perform all calculations, we would come to alternatives' coordinates, ideal and anti-ideal points, provided that calculation manner is a standard one and that ideal and anti-ideal alternatives have been introduced. As both criteria are of linguistic nature, let us assume they are of profit character coordinates of the ideal point are the attribute values of the alternative 1 for the first criterion and alternative 3 for second criterion, when standard manner is in question. Therefore, in case of standard calculation, it means that the ideal characteristic of criterion 1 is of value 9, which is not logical if we consider that evaluations are made within the value scale from 1 to 10. Also, the values of anti-ideal point coordinates are being changed in the identical manner. Introduction of additional alternatives resulted in change of criteria space as well as in alternatives' coordinates. Consequently, the change also occurred in Euclidean distances from the ideal and anti-ideal points, which may not necessarily influence the alternative ranking.

Standard way of calculation			**Modified way of**	
Ideal point	**0,26326**	**0,37533**	**0,2357**	**0,43189**
Alternative 1	**0,26326**	**0,12511**	**0,21213**	**0,08638**
Alternative 2	**0,08775**	**0,37533**	**0,07071**	**0,25913**
Alternative 3	**0,117**	**0,37533**	**0,09428**	**0,25913**
Alternative 4	**0,26326**	**0,25022**	**0,21213**	**0,17276**
Anti ideal point	**0,08775**	**0,12511**	**0,02357**	**0,04319**

Our example clearly shows that points within the criteria space have moved towards the coordinate beginning, as shown in the Picture 3.

Now, if we add the relative closeness of the alternatives and ideal and anti-ideal point we will come to the modified order of the alternatives as shown in the table bellow.

Standard method		Modified method	
Alternative 4	**0,632726**	**Alternative 3**	**0,504404**
Alternative 3	**0,632689**	**Alternative 2**	**0,480587**
Alternative 2	**0,587747**	**Alternative 4**	**0,467875**
Alternative 1	**0,412253**	**Alternative1**	**0,358391**

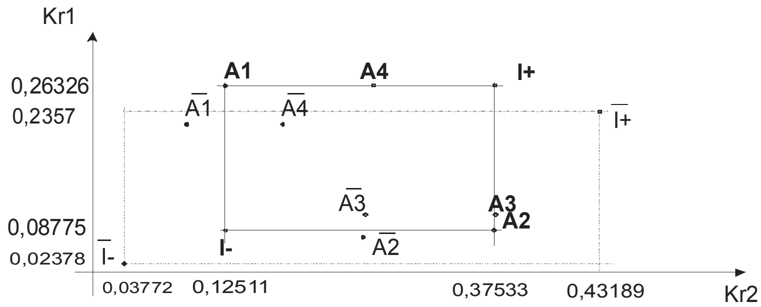


Figure 3 Points in criteria spaces for standard and modified calculation manner

When dealing with more complex tasks and when ideal and anti-ideal alternatives are introduced, the ideal point is more distant from coordinate beginning in comparison to the ideal point in standard method. Also, it is clearly shown that coordinates of the alternatives are quite different when those two calculation manners are applied, because the introduction of two additional alternatives results in change of attributes in the process of data matrix normalization. If greater number of criteria and alternatives are involved, that difference would diminish.

Same would happen in case of normalization performed through linear attributes' normalization, whereas the differences between normalized attribute values would be greater in modified manner of calculation then in standard manner of calculation. Bellow table and picture shows the change of criteria space in case of normalization done by linear attributes' normalization, in the same example.

Standard way of calculation			Modified way of	
Ideal point	**0,4**	**0,45**	**0,4**	**0,6**
Alternative 1	**0,4**	**0,15**	**0,36**	**0,12**
Alternative 2	**0,13333**	**0,45**	**0,12**	**0,36**
Alternative 3	**0,17778**	**0,45**	**0,16**	**0,36**
Alternative 4	**0,4**	**0,3**	**0,36**	**0,24**
Anti ideal point	**0,13333**	**0,15**	**0,04**	**0,06**

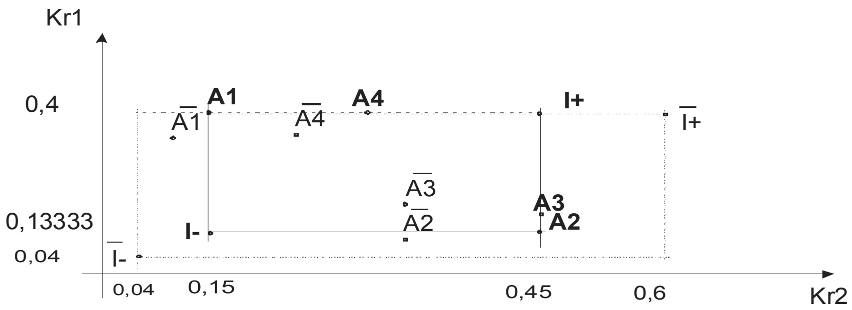


Figure 4 Points in criteria spaces at linear normalization.

It is shown that original method criteria space at linear attributes' normalization represents the criteria space sub-group when ideal and anti-ideal alternatives are introduced. If now we calculate the relative closeness of alternatives to ideal and anti-ideal point, we will get the unchanged order of alternatives as shown in the table bellow:

Standard method		**Modified method**	
Alternative 4	**0,671023**	**Alternative 3**	**0,503384**
Alternative 3	**0,57712**	**Alternative 2**	**0,487697**
Alternative 2	**0,529412**	**Alternative 4**	**0,457087**
Alternative 1	**0,470588**	**Alternative1**	**0,40332**

Therefore, in case that normalization is done by linear attributes' normalization, the rank would differ from the one obtained by vectorial normalization. Nevertheless, we can also see that the alternatives are closer to one another in modified calculation manner then in the standard one, as opposed to the case of vectorial normalization. If attribute values change, the change of rank would be likely to happen even in case of linear normalization. The author's opinion is that linear normalization is more suitable if ideal and anti-ideal alternatives are introduced, because the relative ratio between attribute values and the extremes would remain unchanged.

In any case, the end result may reflect in different rank of alternatives, leading us to conclusion that introduction of ideal and anti-ideal alternative is useful. Namely, if the basic idea of TOPSYS method is finding an alternative which would be closest to the ideal and farthest to anti-ideal, it leads us to the question of how we can decide which alternative is ideal/anti-ideal. To be more precise, would it be correct if we take the values from the group of values of given alternatives to represent ideal/anti-ideal alternative? The author is of the opinion that it would be more correct to define ideal and anti-ideal solution, and then compare the potential solution to the previously defined extremes. Even more, managers find it easier to define the attributes for qualitative criteria if the ideal and anti-ideal alternative values are familiar to them, because it implies comparison between the attributes as well as with respect to the extremes.

3.2. Quantification of attributes of quality

In most cases of solving the real problems, the ranking of the alternatives is being performed based on the qualitative criteria, as well. Each multi-criteria task solving method implies quantification of the attributes expressed by linguistic terms. We have already discussed the types of attribute quantification scales, but the author noticed a weak point of TOPSYS method in the fact that it does not include a unique scale for quantification of qualitative attributes which would be strictly applied in all cases. It could prove that alternative ranks may differ if different scales for quantification of two independent qualitative criteria are used [6]. Quantification of qualitative attributes usually includes translation of standard linguistic terms group into numeric values within previously agreed value scale. The standard linguistic terms group may be as follows:

$$x_{ij} \in \{lttle, middle, big\} \Rightarrow x_{ij} \in \{1,3,5\}$$

$$x_{ij} \in \{bad, goot, excellent\} \Rightarrow x_{ij} \in \{1,5,9\}$$

$$x_{ij} \in \{bad, enough, goot, riping, excellent\}$$
$$\Rightarrow x_{ij} \in \{1,2,3,4,5\}$$

$$x_{ij} \in \{bad, enough, goot, riping, excellent\}$$
$$\Rightarrow x_{ij} \in \{1,3,5,7,9\}$$

Therefore, if we use one standard group of terms for one qualitative criterion as well as the corresponding quantification scale and if for the other qualitative criterion we use other group which differs with respect to the number of group elements but also with respect to the range of scale, then we risk of failing to set the relative inter-connection between those two criteria in a correct and adequate manner. For this reason, it is essential that we determine a unique way of quantifying the qualitative attributes.

Nevertheless, when managers express their qualitative evaluations, they usually determine those evaluations by comparisons to some reference values. When a professor evaluates the knowledge of his student, he bears in mind the highest mark as the benchmark and then he compares the knowledge of his student to the knowledge required for the highest mark, or to the knowledge threshold necessary for passing the exam. It is often the case that student's knowledge deserves the mark which belongs somewhere in between the possible values. Example: When a professor says:" You have showed the knowledge which can be graded higher then 7 but not sufficient for 8" he creates the problem since it is just not allowed to express marks with decimal numbers.

Similarly, the managers evaluate some qualitative values, so the author thinks that it is good to introduce the standard scale of values from 1 – 10 in multi-criteria problem analysis, expressing the evaluations with respect to the given extremes, whereas the attribute may take any of the values within the given interval. It is undoubtedly possible to form the standard group of linguistic terms which could be quantified within the given scale, as in the example given bellow:

Very bad	**1**
Bad	**2**
Sufficient	**3**
Satisfactory	**4**
Good	**5**
Very good	**6**
Very good indeed	**7**
Excellent	**8**
Extraordinarily	**9**
Perfectly	**10**

If we allow the attribute to take decimal value, i.e. if we allow a professor to use maximum precision in expressing his evaluations, as for example by expression “almost excellent”, then we will create the possibility for the attribute to take any of the values from 1 – 10 interval, so quantifying the manager’s expression with 7, 8. Even more, the manager can quantify the attribute himself without linguistic terms as a measure of correlation to whole number values and/or to the scale extremes. In this way, the manager would quantify the expressions such as “almost”, “nearly”, “scarcely less”, “slightly over”, “just above” etc, as his subjective estimations of "reaching the measure". If the manager rules over techniques of multi-criteria analysis, which is often the case lately, then quantification of qualitative attributes represents direct allocation of numeric value to the attribute within the defined scale.

3.3. Fuzzification of attribute

Translation of attributes into numeric form represents the deficiency of the original method, for the criteria expressed by linguistic measures within a determined value scale, as accounted for in the previous sections of this paper. When dealing with such criteria, the subjective manager’s estimation is crucial, so that the evaluation itself may vary. This is the reason way, in addition to standard translation scale described in this paper the author proposes the allocation of **group** of numbers to each qualitative attribute, i.e. determining the intervals within which evaluations could move with certain degree of manager’s certainty.

In this way, alternative coordinates (for criteria expressed by linguistic terms) may take any of the values from the defined interval of values. Thus the alternative does not represent a point in n-dimensional criteria space, but k-dimensional criteria space in n-dimensional criteria space. Ideal and anti-ideal alternative possess fixed attribute values so that they represent the points in above mentioned n-dimensional criteria space.

In this situation, the question is posed of how to determine the closeness from the alternative to the ideal point. The possible approach would involve determining the center of alternative space, distribution of space density and its mass, determining the force of gravity on ideal and anti-ideal point, as function of mass and Euclidean distance of centre. Alternative mass would be a function of volume and density, while force of gravity to ideal and anti-ideal point would be proportional to mass and counterproportional to square of Euclidean centre distance. The best alternative would be the one with highest force of gravity to ideal point and lowest force of gravity to anti-ideal point at the same time.

This approach would complicate the calculations because it would arouse number of issues which could hardly be given answers to. How to find the points of center? How to calculate the alternative space mass if the distribution functions inside the groups are not familiar? One of the solutions might be the fuzzycation of the qualitative attributes where the attributes are described with different forms of FUZZY numbers, this resulting in changing the manner of calculation of gravity force depending on the form of integration function, for each individual problem. It is possible to facilitate the calculating process if we take the so called “triangular” FUZZY number each time, which implies linear descending and ascending integration functions. The author’s opinion is that it is possible to set up the alternative rank or group of “good alternatives”, taking into consideration the points of alternative spaces which are closest to both ideal and anti-ideal points. It is also possible to elect the best alternative as well as those **close** to it, based on those points. When all other alternatives are eliminated, the manager decides on the manner in which he would elect the alternatives (by repeating the procedure with additional criteria, by changing alternative space through change of degree of certainty for qualitative criteria, by changing relative weights, by direct comparison or otherwise).

Above all, it is necessary to define the procedure of determining the FUZZY numbers. Based on his experience, the author claims that managers quantify the qualitative attributes by comparison to the extremes and usually by expressions as: “around x”, “not less then x and not more then y”, “between x and y” and likewise, which basically represent linguistic expressions and can be represented with “triangular” type FUZZY number. Sometimes we have the expressions as “between x and y but not less then p and not more then q”, which represents the FUZZY number of trapezoid type which can be approximated by FUZZY number of “triangular” type where mean value of the x- y interval is taken for $\mu(x)=1$. Therefore, if the manager expresses his evaluation of qualitative attribute in ambiguous manner, then such evaluation can be expressed by a FUZZY number.

If we adopt the triangular FUZZY number as a form of FUZZY number used to describe linguistic manager’s expressions, then the mentioned interval could be described with three discrete values as shown in the Picture 5.

$$p = x_0, \forall \mu(x_0) = 1$$

$$p^- = x_1; \forall \mu(x_1) = 0 \wedge x_1 \leq x_0$$

$$p^+ = x_2; \forall \mu(x_2) = 0 \wedge x_2 \geq x_0$$

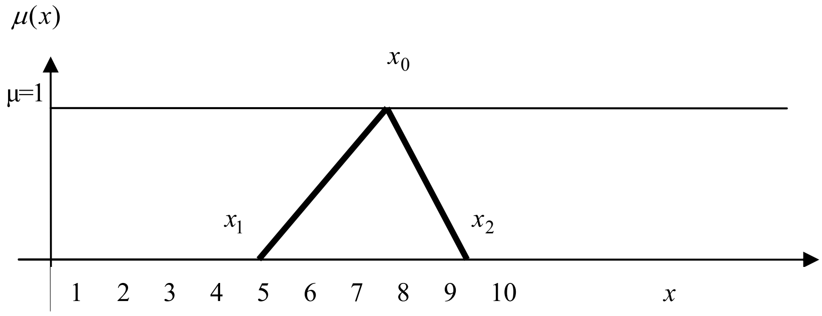


Figure 5 The example of FUZZY number allocated to the attribute.

It goes without saying that there is no such x which could be applied in the bellow formula: $\mu(x) \succ 0 \wedge (0 \leq x \prec x_1 \vee x_2 \prec x \leq 10); 0 \leq x \leq 10$

The presented FUZZY number which, of course, has to be normalized and convex, represents subjective manager's estimation and evaluation of the matter which is not defined in an exact manner but expressed with linguistic terms or is quantified within an adopted value scale instead. Linguistic terms are quantified within the 1 – 10 value scale interval so that the end values of the scale correspond to terms such as "unacceptable" = 1 or "perfect" = 10.

In our example, the manager claims with high certainty degree that the attribute possesses the value which corresponds to the term "just above 6". When asked to determine the lowest and the highest value he would assign to the attribute, the manager's answer was "just above 8" and "not bellow 4" which if translated into numerical form corresponds to x_1 =4 and x_2 =8,2. Therefore, the manager believes that evaluation for the attribute analyzed can range from 4 to 8,2 with the highest certainty degree x_0 =6,3. It is understood that expression "around 6" implies that x_0 =6 and that x_1 and x_2 have been determined using the attribute values taken from the scope of the lowest and highest possible limits previously set by the manager. It is clear that evaluation may take rational value which practically means that the number of values that could be assigned to attributes within defined value scale is limitless.

On the other hand, when giving the subjective evaluations, managers often tend to express them in vague, i.e. not clearly defined terms, such as: almost 8, more then 6, approximately 7 or in some other terms based on which it is very difficult to determine the interval limits. It is necessary to insist on more precise expressions in order to be able to define values $x_i; i \in \{0,1,2\}$ for each attribute which is expressed linguistically and determine the triangular fuzzy number uniformly.

Therefore, the FUZZY number can have various forms but still, we can say that in most cases linguistic terms and expressions provided by managers can be approximated with triangular FUZZY number, where values for μ(x)=0, as well as for μ(x)=1 are analyzed and distribution within intervals is linearized.

Managers' subjectivity is also present at the process of determining the weight coefficients. Nevertheless, when setting the weights one must first consider the fixed values because the condition of $\sum_{j=1}^{k} w_j = 1$ must be met, since the change of value of only one coefficient influences all other weight coefficients. It is possible to set more tasks with different weight coefficients and to analyze alternative rank in accordance with the introduced changes.

Certain decisions require multi-disciplinary knowledge, due to which it is necessary to include more managers in the decision-making process as they could give their independent evaluations. Discrepancies between subjective evaluations can be considerable, especially when dealing with criteria of aesthetic nature, meaning that it is necessary that Decision Maker sets the values for $x_i; i \in \{0,1,2\}$ and weight coefficients by using statistic methods, depending on a case. In this way, group decision-making would make sense and the decisions made in this way are of higher quality.

Fazzycation of qualitative attributes introduces the vagueness of managers' subjective evaluations into the task but it is impossible to set the alternative rank without discrete values. For this reason it is necessary to set more tasks with different values for qualitative attributes described by FUZZY numbers. In addition to characteristic values for $\mu(x)=0$; $x \in \{x_1, x_2\}$ and $\mu(x)=1$ describing the FUZZY number, other values would be considered as well. For example, the values of x such that $\mu(x)=0{,}8$, $\mu(x)=0{,}6$, $\mu(x)=0{,}4$ and $\mu(x)=0{,}2$. Then we would consider the change of rank with respect to the changes of attribute values.

Discrete attribute values defined in this manner, after being normalized and multiplied by normalized weights, then represent coordinates of the alternatives in n-dimensional criteria space. If we assume that all qualitative criteria are of max type, which results from the quantification manner, and if we perform linear attribute normalization, then it could be asserted that lower attribute value results in higher value of Euclidean distance from the ideal point and lower value of Euclidean distance from the anti-ideal point. The consequence of this would reflect in lower value of relative closeness coefficient, i.e. the alternative would be correspondingly worse.

Proof:

If

$$a \prec b \Rightarrow \frac{a}{c} \prec \frac{b}{c}; c = \max_j x_{ij}; c \geq b \Rightarrow aw_j \prec bw_j \Rightarrow (aw_j - cw_j)^2 \succ (bw_j - cw_j)^2$$

Then $x_{ij} \downarrow \Rightarrow S_i^+ \uparrow \wedge S_i^- \downarrow \Rightarrow C_i \downarrow; x_{ij} \uparrow \Rightarrow S_i^+ \downarrow \wedge S_i^- \uparrow \Rightarrow C_i \uparrow$

Based on the above assertion, we can also assert the following:

1. The change in rank alternatives would not happen only in case the FUZZY numbers are identical $\frac{x_2 - x_0}{j} = const \wedge \frac{x_0 - x_1}{j} = const$. Otherwise, the change of rank alternative may occur.

2. If we take $\frac{x_1}{j}$ for values of qualitative attributes, then we would have $\min C_i(x); x_1 \le x \le x_2$, i.e. if we take $\frac{x_2}{j}$ then we would have $\max C_i(x); x_1 \le x \le x_2$, regardless of whether the FUZZY numbers are identical according to criteria.

Let us assume that the alternative rank changed after the values which include the manager's certainty degree - less then 1 - had been taken for values of qualitative attributes. In that case, a dilemma would be: which alternative rank should we adopt? Logically, the rank possessing the parameters of highest manager's certainty degree should be adopted. But then again, how can we be sure that the manager's evaluation was precise enough or that his opinion would remain unchanged in other moment in time. That leads us to conclusion that FUZZY groups represent the qualitative attribute value and that there are number of combinations determined by alternative coordinates. The author's opinion is that we have to consider the ambiguities present in the process of quantification of qualitative attributes and that an attribute can be assigned with any of the values from the chosen group. In this way, each alternative represents k-dimensional criteria space in n-dimensional criteria space (whereas "k" represents the number of qualitative criteria).

Forming of FUZZY groups for each qualitative attribute would be performed based on the corresponding FUZZY number and chosen certainty degree. Namely, if we decide for a certainty degree $\mu(x)=0,8$„ then we would define the FUZZY group where all values x in which $\mu(x)\ge 0,8$ can be taken for attribute values. After this being done, next step would be to calculate the relative closeness to ideal and anti-ideal point for the following: $\mu(x^-) = 0,8 \wedge x^- \le x_0$

$$\mu(x^-) = 0,8 \wedge x^- \le x_0 \ ; \ \mu(x^+) = 0,8 \wedge x^+ \ge x_0$$

Finally, we compare the values of relative closeness coefficients and search for close alternatives. First, $C_p = \max C_i(x^-); i = \overline{1,n}; p \in \{1,2,...,n\}$ is found, and then each $C_i(x^+) \ge C_p(x^-)$. All alternatives A_i that meet this condition are considered to be close to p-alternative. If there is not one alternative that meets the above condition, then p-alternative would be considered the best alternative.

Group of close alternatives can be determined in the same way also in cases where some other values for qualitative attributes are taken in which manager's certainty degrees are $\mu(x)=0,6$, $\mu(x)=0,4$ and $\mu(x)=0,2$ or otherwise chosen by the decision-making manager. Normally, lower certainty degree would increase the possibility of having the greater number of alternatives closer to the best alternative. It can be graphically shown as in the picture 6 bellow:

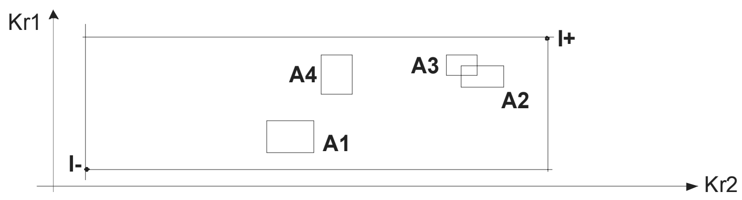


Figure 6 Two-dimensional criteria alternative space.

It is clearly shown that A_2 and A_3 alternatives are close because the average of possible alternative coordinates value groups is not Ø.

If we consider all of the above, the modified TOPSYS method contains the following steps:

1. **First step** – determining the criteria and alternatives, their attributes and weight coefficients, ideal and anti-ideal alternatives, as well as FUZZY numbers for each qualitative attribute. Then we determine the manager's certainty degree for which further calculations are performed (for example $\mu(x)\geq 0,8$) based on which we would get two decision-making matrixes: with attribute values for highest $x_{ij}^{+} = \max x_{ij} \wedge \mu(x_{ij}) = 0,8$ and lowest group limits. When exact attributes are in question then $x_{ij}^{+} = x_{ij}^{-}$.

$$
\begin{array}{c|cccccc}
 & f_1 & f_2 & \cdots & f_j & \cdots & f_n \\
\hline
a_1 & x_{11}^{+} & x_{12}^{+} & \cdots & x_{1j}^{+} & \cdots & x_{1n}^{+} \\
a_2 & x_{21}^{+} & x_{22}^{+} & \cdots & x_{2j}^{+} & \cdots & x_{2n}^{+} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
a_i & x_{1j}^{+} & x_{2j}^{+} & \cdots & x_{ij}^{+} & \cdots & x_{in}^{+} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
a_m & x_{m1}^{+} & x_{m2}^{+} & \cdots & x_{mj}^{+} & \cdots & x_{mn}^{+} \\
 & \begin{pmatrix}\max\\ \min\end{pmatrix} & \begin{pmatrix}\max\\ \min\end{pmatrix} & & \begin{pmatrix}\max\\ \min\end{pmatrix} & & \begin{pmatrix}\max\\ \min\end{pmatrix}
\end{array}
$$

$$
\begin{array}{c|cccccc}
 & f_1 & f_2 & \cdots & f_j & \cdots & f_n \\
\hline
a_1 & x_{11}^{-} & x_{12}^{-} & \cdots & x_{1j}^{-} & \cdots & x_{1n}^{-} \\
a_2 & x_{21}^{-} & x_{22}^{-} & \cdots & x_{2j}^{-} & \cdots & x_{2n}^{-} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
a_i & x_{i1}^{-} & x_{i2}^{-} & \cdots & x_{ij}^{-} & \cdots & x_{in}^{-} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
a_m & x_{m1}^{-} & x_{m2}^{-} & \cdots & x_{mj}^{-} & \cdots & x_{mn}^{-} \\
 & \binom{\max}{\min} & \binom{\max}{\min} & & \binom{\max}{\min} & & \binom{\max}{\min}
\end{array}
$$

m – a number of alternatives including ideal and anti ideal
n – a number of criteria
We adopt: a1 - ideal alternative and am- anti ideal alternative.

2. **Second step** – calculating the normalized matrixes by setting the attributes to (0,1), which means we should make the attributes non-dimensional through linear attribute normalization, so that the elements of matrixes would be:

For **max** type criteria

$$r_{ij}^{+} = \frac{x_{ij}^{+}}{x_{1j}};\ r_{ij}^{-} = \frac{x_{ij}^{-}}{x_{1j}};\ i = \overline{1, m},\ j = \overline{1, n}$$

For **min** type criteria

$$r_{ij}^{+} = \frac{x_{ij}^{+}}{x_{1j}};\ r_{ij}^{-} = \frac{x_{ij}^{-}}{x_{1j}};\ i = \overline{1, m},\ j = \overline{1, n}$$

3. **Third step** – multiplication of normalized matrixes elements by normalized weight coefficients so that:

$$v_{ij}^{l} = r_{ij}^{l} w_j \text{ , where } \sum_{j=1}^{n} w_j = 1; (i = 1,...,m), (j = 1,...,n), l \in \{+,-\}$$

4. **Fourth step** – calculating the Euclidean distance measure

$$S_{i+}^{+} = \sqrt{\sum_{k=1}^{n} (v^{+}{}_{ik} - v_{1k})^2}\ , i = 2,..., m-1$$

$$S_{i+}^{-} = \sqrt{\sum_{k=1}^{n} (v^{-}{}_{ik} - v_{1k})^2}\ , i = 2,..., m-1$$

$$S_{i-}^{+} = \sqrt{\sum_{k=1}^{n} (v^{+}{}_{ik} - v_{mk})^{2}}\,, i = 2,...,m-1$$

$$S_{i-}^{-} = \sqrt{\sum_{k=1}^{n} (v^{-}{}_{ik} - v_{mk})^{2}}\,, i = 2,...,m-1$$

5. **Fifth step** – calculating the relative closeness:

$$C_i^{+} = \frac{S_{i-}^{+}}{S_{i+}^{+} + S_{i-}^{+}}, i = 2,...,m-1$$

$$C_i^{-} = \frac{S_{i-}^{-}}{S_{i+}^{-} + S_{i-}^{-}}, i = 2,...,m-1$$

6. **Sixth step** – defining the best alternative and group of close alternatives according to C_i^{+} and C_i^{-}

To find $C_p^{-} = \max C_i^{-}, i = 2,...,m-1; 2 \le p \le m-1$

And each A_i for which $C_i^{+} \ge C_p^{-}, i = 2,...,m-1; 2 \le p \le m-1$

If there is not one A_i which meets the above condition, then the alternative A_p would be considered the best. If, nevertheless, there are alternatives which meet the given conditions, then we consider those alternatives to be close to the alternative A_p and eliminate the rest of the alternatives.

Decision Maker can decide on which alternative to elect by comparison - if 2 or 3 alternatives are close, by introduction of additional criteria for evaluation or simply by accepting the alternative A_p as the best alternative. It is possible to repeat multi-criteria task with close alternatives, change the weight coefficients or choose the best alternative in some other manner. Decision Maker compares the groups of close alternatives for different manager's certainty degrees described by FUZZY numbers, and decides which close alternative group to submit to further analysis.

If we repeat the multi-criteria task by basic TOPSYS method, it is quite possible that we would get different alternative ranks, because the attribute values, as well as ideal and anti-ideal point coordinates would change due to vector normalization. Nevertheless, upon introduction of ideal and anti-ideal alternatives and performed linear normalization, change of rank would not occur, so that is why it is necessary to introduce additional criteria or change some other parameters as for example, the weight coefficients. The author's opinion is that Decision Maker must find the way to elect three alternatives (at the most) from the group of close alternatives, and to choose the best alternative based on his subjective estimation, by himself alone or by using the group decision-making method.

If the multi-criteria task does not possess qualitative criteria then the TOPSYS method modification relates only to introduction of ideal and anti-ideal alternatives and linear attribute normalization, which results in uniform alternative rank. Decision Maker's subjectivity is present only at determining of weight coefficients.

4. EXAMPLE OF MULTI-CRITERIA TASK SOLVING BY USING THE MODIFIED METHOD

Typical example of multi-criteria task is the election of products in the procurement procedure. Let us take the example of procurement of delivery vehicles for transportation of postal items.

The first step would be to define the problem and to describe it. Analysis has shown that available company's fleet could not support all business activities planned, from number of reasons, as follows:

Age-structure of the fleet is high which then requires high maintenance costs,

There are several different types of vehicles, which additionally increases the maintenance and exploitation costs,

New business deals have been made, which requires greater number of vehicles in order to perform business activities in a satisfactory manner,

Vehicles with standardized loading space, according to Euro-box palette standards are required,

Liquid fuel consumption of the existing vehicles is high and vehicles do not comply with ecology standards,

Security of postal items and people would be endangered with further exploitation of old vehicles.

The analysis has showed that it is necessary to procure 50 new vehicles for the Company, from one supplier in order to gradually standardize the fleet and decrease the maintenance costs. It has also been determined that vehicles must be equipped with diesel motors, due to the reasons of rationalization of fuel costs and longer exploitation time. It was found that market offered quite enough suppliers that would be able to fulfill the defined requirements and that it was necessary to issue the tender in order to elect the most favorable supplier. The following criteria are determined for evaluation of the most favorable supplier:

1. Procurement price
2. Guarantee Period Validity
3. Other Requirements within the Guarantee
4. Fuel Consumption (per 100 km)
5. Loading Space Size
6. Design
7. Cabin Commodity
8. Motor Power
9. Ecology Parameters
10. Payment Conditions

Weight coefficients were determined for the above criteria within 1 – 10 scale, as shown in the table bellow:

Crit.	1	2	3	4	5	6	7	8	9	10
T.k.	9	7	5	8	7	4	6	5	7	9
N.t.k.	0,134	0,104	0,075	0,119	0,104	0,060	0,090	0,075	0,104	0,134

1., 2., 4., 5., and 8 represent criteria described with exact data while other criteria are described with linguistic variables. 1st and 4th criteria are of cost type (min), while others are of benefit type (max).

Upon collection of offers, alternative solutions were determined based on fulfillment of all tender criteria upon which the attributes were assigned, in addition to assigning the attributes to ideal and anti-ideal alternatives. Let us assume that we have the following alternative matrix with attributes assigned for quantitative criteria and with FUZZY numbers $F_{ij};\, i \in \{3,6,7,9,10\};\, j = \overline{2,7}$ for qualitative attributes according to methodology described in this paper.

A/K	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10
I+	10300	36	10	5,5	2,0	10	10	85	10	10
A1	16300	24	F_{32}	7,2	1,4	F_{62}	F_{72}	55	F_{92}	F_{102}
A2	13200	36	F_{33}	6,1	1,2	F_{63}	F_{73}	62	F_{93}	F_{103}
A3	11900	24	F_{34}	6,3	1,2	F_{64}	F_{74}	62	F_{94}	F_{104}
A4	14100	18	F_{35}	6,5	1,4	F_{65}	F_{75}	62	F_{95}	F_{105}
A5	15600	12	F_{36}	6,9	1,8	F_{66}	F_{76}	75	F_{96}	F_{106}
A6	16800	18	F_{37}	7,0	2,0	F_{67}	F_{77}	80	F_{97}	F_{107}
I-	18000	12	1	8,0	1,0	1	1	45	1	1
N.t.k.	0,134	0,104	0,075	0,119	0,104	0,060	0,090	0,075	0,104	0,134
type	min	max	max	min	max	max	max	max	max	max

A1- WF
A2- PEUGEOT
A3- CITROEN

A4- RENAULT
A5- OPEL
A6- FIAT

Let us assume that Decision Maker has determined value groups for qualitative attributes by certainty degree $\mu(x) \geq 0,6$, due to which we get two decision-making matrixes with the elements $x_{ij}^{l};\, (i = 1,\dots, m);\, (j = 1,\dots. n),\, l \in \{+,-\}$, resulting in the bellow matrixes:

x_{ij}^{-}	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10
I+	10300	36	10	5,5	2,0	10	10	85	10	10
A1	16300	24	7,3	7,2	1,4	4,8	5,8	55	7,8	0,6
A2	13200	36	8,4	6,1	1,2	7,2	8,7	62	8,4	4,6
A3	11900	24	6,5	6,3	1,2	9,3	8,4	62	9,2	6,5
A4	14100	18	8,2	6,5	1,4	8,6	6,6	62	6,3	8,4
A5	15600	12	4,7	6,9	1,8	4,2	4,2	75	2,6	0,7
A6	16800	18	3,6	7,0	2,0	5,5	7,4	80	3,8	3,6
I-	18000	12	0	8,0	1,0	0	0	45	0	0
N.t.k.	0,134	0,104	0,075	0,119	0,104	0,060	0,090	0,075	0,104	0,134
type	min	max	max	min	max	max	max	max	max	max

x_{ij}^{+}	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10
I+	10300	36	10	5,5	2,0	10	10	85	10	10
A1	16300	24	8,7	7,2	1,4	5,2	6,2	55	8,2	1,4
A2	13200	36	9,6	6,1	1,2	8,8	9,3	62	9,6	5,4
A3	11900	24	7,5	6,3	1,2	10	9,6	62	10	7,5
A4	14100	18	9,8	6,5	1,4	9,4	7,4	62	7,7	9,6
A5	15600	12	5,3	6,9	1,8	5,8	5,8	75	3,4	1,3
A6	16800	18	4,4	7,0	2,0	6,5	8,6	80	4,2	4,4
I-	18000	12	0	8,0	1,0	0	0	45	0	0
N.t.k.	0,134	0,104	0,075	0,119	0,104	0,060	0,090	0,075	0,104	0,134
type	min	max	max	min	max	max	max	max	max	max

$C_1^- = 0,407225$

$C_2^- = 0,626849$

$C_3^- = 0,659539$

$C_4^- = 0,624051$

$C_5^- = 0,282808$

$C_6^- = 0,419059$

$C_1^+ = 0,446424$

$C_2^+ = 0,682853$

$C_3^+ = 0,713972$

$C_4^+ = 0,688485$

$C_5^+ = 0,337089$

$C_6^+ = 0,47307$

Alternatives A2 and A4 can be considered to be close to the A3 alternative because the coefficients are $C_2^+, C_4^+ \geq C_3^-$. The rest of the alternatives (A1, A5 and A6) are eliminated. If Decision Maker should decide to repeat the procedure but this time with the manager's certainty $\mu(x) = 0,8$, only A3 and A4 alternatives would be considered to be close. In order for the Decision Maker to choose between close alternatives it is possible to repeat the election procedure by considering the additional criteria or by direct comparison of alternative pairs. It is possible to assign new weight coefficients and so perform the alternative ranking once more. The author's advice is to repeat the procedure with additional criteria or by repeating the linguistic attribute evaluation, i.e. by assigning the new weight coefficients in case of more then three close alternatives. If two or three close alternatives are got as the result, direct comparison would be the most realistic option. Let us assume that service network is the criterion which was not considered and that alternative A4 is better according to that criterion, so the manager decides to add one more criterion in consideration and after the calculation is done, he eliminates A2 alternative. A3 and A4 represent the alternatives which are absolutely close and it could be said that they are both equally good, so the manager finally makes the decision based on the general impression.

The example shows that coordinates of ideal and anti-ideal alternatives are different for the exact criteria as well, because Decision Maker's opinion is that offered vehicles' price is not ideal and he takes upon him self to define ideal and anti-ideal price. It is the same in case of "motor power" criterion, in which the manager assigns new values to the ideal and anti-ideal alternative, choosing from those contained in the offers.

We can also see that manager's subjectivity is almost always present at real problems, when giving evaluations on qualitative criteria and that by assigning the FUZZY number to each qualitative attribute the possibility of mistake is decreased. The above example clearly shows that when dealing with criteria where manager's subjectivity degree is rather high, it is not always easy to find the best alternative. Instead, in most cases the groups of alternatives are presented as those that are "better then the rest", whereas the election is made through additional ranking or in some other manner chosen by the Decision Maker.

It is also clear that if there are no qualitative criteria in the task and each alternative represents the point in n-criteria space, then the attribute quantification would not be necessary, i.e. there would be no attributes which could be described by FUZZY numbers and there is only one decision-making matrix. Ideal and anti-ideal point is being determined based on the manager's estimations and it could happen they are identical as in the original TOPSYS method. Then we would have the alternative ranking where in cases when it could be asserted that the alternative with the maximum coefficient C_i is the best alternative according to all criteria simultaneously and with defined weight coefficients.

5. CONCLUSION

We can conclude that there are a number of real business problems, the nature of which is such that their solving requires the methods of multi-criteria analysis, due to the opposite or partly opposite criteria or targets. Many different methods are available to managers who can use them in solving the problems, more or less successfully. The author considers the TOPSYS method to be one of such methods from the reason of its efficiency in practical application, especially with modifications proposed in this paper. Criteria described by linguistic terms are present at most of the real problems so that it is essential to bear in mind the variations of attribute values at subjective evaluations or group decision-making. In this way managers are provided with the practical tools for instant decision-making in case the problem is familiar, i.e. they are provided with tools for determining and evaluation of criteria for estimation of possible solutions.

Modification of TOPSYS method as presented in this paper is the result of long-term utilization of this method by the author, in cases of solving the real business problems. Certain deficiencies of conventional TOPSYS method have been discovered, from which reason the author had decided to correct such deficiencies by modification of basic method. Additional two alternatives have been introduced into the task, as follows:

- Ideal alternative, assigned with the ideal attribute values according to manager's subjective estimation and maximum value in value scale which is used for evaluation of criteria described by linguistic variables, and
- Anti-ideal alternatives, as the alternative possessing the least advantageous attribute values according to manager's subjective estimation and the lowest value in the scale used to quantify the qualitative attributes.

The above alternatives represent the ideal and anti-ideal point, compared to which we are able to set the alternative rank, i.e. we are able to determine the closeness to ideal solution.

Since the manager's subjectivity is present in cases of multi-criteria tasks, at qualitative criteria, it is necessary to determine the standard methods of quantification and assign the FUZZY numbers to attributes. The author's opinion is that decision-making requires the introduction of group decision-making principles, and therefore it would be very useful to introduce the FUZZY attribute descriptions for the criteria which are described by linguistic terms. The author also thinks that attribute normalization must be performed by linear normalization instead of vectorial, in order to perform the introduction of ideal and anti-ideal alternative in the proper manner, from the aspect of keeping the initial information.

Finally, this paper opens the door to other possibilities of further improvement of the method presented and analyzed in this paper.

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