

Optimal Excitation Controller Design for Wind Turbine Generator Using H^∞ Control Technique

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Abstract

An optimal excitation controller design based on multirate-output controllers (MROCs) having a multirate sampling mechanism with different sampling period in each measured output of the system is presented. The proposed H^∞ -control technique is applied to the discrete linear open-loop system model which represents a wind turbine generator supplying an infinite bus through a transmission line.

Keywords: Multirate – output controllers, H^∞ -control, wind turbine generators, disturbance.

1. Introduction

In the recent years H^∞ -control problem has great interest [1-5, 7, 8, 12-17]. The H^∞ -control problem for discrete-time and sampled-data single rate and multirate systems has been treated successfully [2-6]. In both the continuous and discrete-time cases, when the state vector is not available for feedback, the H^∞ -control problem is usually solved using dynamic measurement feedback approach.

For the solution of the H^∞ -disturbance attenuation problem a new technique [6] is presented. In order to solve the sampled-data, based on multirate-output controllers (MROCs), H^∞ -disturbance attenuation problem relies mainly on the reduction, under appropriate conditions, of the original H^∞ -disturbance attenuation problem, to an associated discrete H^∞ -control problem for which a fictitious static state feedback controller is to be designed, even though same state variables are not available for feedback.

In this paper discrete linear open-loop power system model was obtained through a systematic procedure, using a linearized continuous, with impulse disturbances, 8th-order SIMO open-loop model representing a practical power system.

The sought digital controller, which will lead to the associated designed discrete closed-loop power system model displaying enhanced dynamic stability characteristics, is accomplished by applying properly the presented MROCs technique.

2. H^∞ -Control Technique Using MROCS [4, 6]

Consider the controllable and observable continuous linear state-space system model of the general form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{D}\mathbf{q}(t), \mathbf{x}(0) = \mathbf{0} \quad (1a)$$

$$\mathbf{y}_m(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{J}_1\mathbf{u}(t)$$

$$\mathbf{y}_c(t) = \mathbf{E}\mathbf{x}(t) + \mathbf{J}_2\mathbf{u}(t) \quad (1b)$$

Where:

$$\mathbf{x}(t) \in \mathbf{R}^n, \mathbf{u}(t) \in \mathbf{R}^m, \mathbf{q}(t) \in \mathbf{L}_2^d,$$

$$\mathbf{y}_m(t) \in \mathbf{R}^{p_1}, \mathbf{y}_c(t) \in \mathbf{R}^{p_2}$$

are the state, input, external disturbance, measured output and controlled output vectors, respectively. In Eqn. 1 all matrices have real elements and appropriate dimensions.

The definition below is useful in what will follow.

Definition. For an observable matrix pair (\mathbf{A}, \mathbf{C}) , with $\mathbf{C}^T = [\mathbf{c}_1^T \quad \mathbf{c}_2^T \quad \dots \quad \mathbf{c}_{p_1}^T]$ and \mathbf{c}_i with $i=1, \dots, p_1$, the i th row of the matrix \mathbf{C} , a collection of p_1 integers $\{n_1, n_2, \dots, n_{p_1}\}$ is called an observability index vector of the pair (\mathbf{A}, \mathbf{C}) , if the fol-

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lowing relationships simultaneously hold

$$\sum_{i=1}^{p_1} n_i = n,$$

$$\text{rank} \left[\mathbf{c}_1^T \dots (\mathbf{A}^T)^{n_1-1} \mathbf{c}_1^T \dots \mathbf{c}_{p_1}^T \dots (\mathbf{A}^T)^{n_{p_1}-1} \mathbf{c}_{p_1}^T \right] = n$$

Next the multirate sampling mechanism depicted in Figure 1. is applied to Eqn. 1.

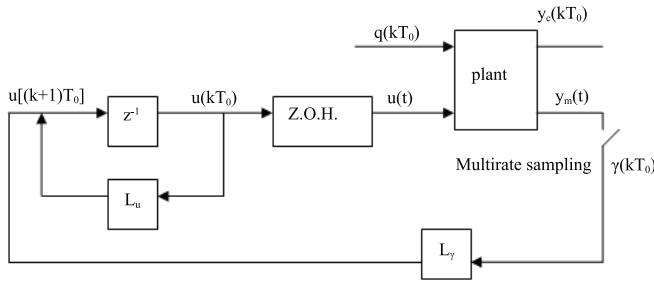


Figure 1. Control of linear systems using MROCs

Assuming that all samplers start simultaneously at $t=0$, a sampler and a zero-order hold with period T_0 is connected to each plant input $u_i(t)$, $i=1,2,\dots,m$, such that

$$\mathbf{u}(t) = \mathbf{u}(kT_0), \quad t \in [kT_0, (k+1)T_0) \quad (2)$$

while the i th disturbance $q_i(t)$, $i=1,\dots,d$, and the i th controlled output $y_{c,i}(t)$, $i=1,\dots,p_2$, are detected at time kT_0 , such that for

$$t \in [kT_0, (k+1)T_0)$$

$$\mathbf{q}(t) = \mathbf{q}(kT_0),$$

$$\mathbf{y}_c(kT_0) = \mathbf{E}\mathbf{x}(kT_0) + \mathbf{J}_2(kT_0)$$

The i th measured output $y_{m,i}(t)$, $i=1,\dots,p_1$, is detected at every T_i period, such that for

$$\mu = 0, \dots, N_i - 1$$

$$y_{m,i}(kT_0 + \mu T_i) = \mathbf{c}_i \mathbf{x}(kT_0 + \mu T_i) + (\mathbf{J}_1)_i \mathbf{u}(kT_0) \quad (3)$$

where, $(\mathbf{J}_2)_i$ is the i th row of the matrix \mathbf{J}_2 . Here $N_i \in \mathbf{Z}^+$ are the output multiplicities of the sampling and $T_i \in \mathbf{R}^+$ are the output sampling periods having rational ratio, i.e. $T_i = T_0 / N_i$ with $i=1,\dots,p_1$.

The sampled values of the plant measured outputs obtained over $[kT_0, (k+1)T_0)$, are stored in the N^* -dimensional column vector given by

$$\hat{\boldsymbol{\gamma}}(kT_0) = \left[y_{m,1}(kT_0) \dots y_{m,1}(kT_0 + (N_1 - 1)T_1) \dots \dots y_{m,p_1}(kT_0) \dots y_{m,p_1}(kT_0 + (N_{p_1} - 1)T_{p_1}) \right]^T \quad (4)$$

(where $N^* = \sum_{i=1}^{p_1} N_i$), that is used in the MROC of the form

$$\mathbf{u}[(k+1)T_0] = \mathbf{L}_u \mathbf{u}(kT_0) - \mathbf{L}_\gamma \hat{\boldsymbol{\gamma}}(kT_0) \quad (5)$$

where $\mathbf{L}_u \in \mathbf{R}^{m \times m}$, $\mathbf{L}_\gamma \in \mathbf{R}^{m \times N^*}$.

Assumptions:

a) The matrix triplets $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ and $(\mathbf{A}, \mathbf{D}, \mathbf{E})$ are stabilizable and detectable.

b)

$$\text{rank} \begin{bmatrix} \mathbf{A} & \mathbf{D} \\ \mathbf{C} & \mathbf{0}_{p_1 \times d} \end{bmatrix} = n + d, \quad \text{rank} \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{D} \\ \mathbf{C} & \mathbf{0}_{p_1 \times m} & \mathbf{0}_{p_1 \times d} \end{bmatrix} = n + m + d$$

c)

$$\mathbf{J}_2^T \begin{bmatrix} \mathbf{E} & \mathbf{J}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{m \times n} & \mathbf{I}_{m \times m} \end{bmatrix}$$

d) There is a sampling period T_0 , such that the open-loop discrete-time system model in general form becomes

$$\mathbf{x}[(k+1)T_0] = \Phi \mathbf{x}(kT_0) + \hat{\mathbf{B}} \mathbf{u}(kT_0) + \hat{\mathbf{D}} \mathbf{q}(kT_0) \quad (6)$$

$$\mathbf{y}_c(kT_0) = \mathbf{E} \mathbf{x}(kT_0) + \mathbf{J}_2 \mathbf{u}(kT_0)$$

where,

$$\Phi = \exp(\mathbf{A}T_0), \quad (\hat{\mathbf{B}}, \hat{\mathbf{D}}) = \int_0^{T_0} \exp(\mathbf{A}\lambda) (\mathbf{B}, \mathbf{D}) d\lambda \quad (7)$$

is stabilizable and observable and does not have invariant zeros on the unit circle.

From the above it follows that the procedure for H^∞ -disturbance attenuation using MROCs, essentially consists in finding for the control law a fictitious state matrix \mathbf{F} , which equivalently solves the problem and then, either determining the MROC pair $(\mathbf{L}_\gamma, \mathbf{L}_u)$ or choosing a desired \mathbf{L}_u and determining the \mathbf{L}_γ . As it has been shown in [3], matrix \mathbf{F} takes the form

$$\mathbf{F} = (\mathbf{I} + \hat{\mathbf{B}}^T \mathbf{P} \hat{\mathbf{B}})^{-1} \hat{\mathbf{B}}^T \mathbf{P} \Phi \quad (8)$$

where \mathbf{P} is an appropriate solution of the Riccati equation

$$\begin{aligned} \mathbf{P} = & \mathbf{E}^T \mathbf{E} + \Phi^T \mathbf{P} \Phi - \Phi^T \mathbf{P} \hat{\mathbf{B}} (\mathbf{I} + \hat{\mathbf{B}}^T \mathbf{P} \hat{\mathbf{B}})^{-1} \hat{\mathbf{B}} \mathbf{P} \Phi \dots \\ & \dots + \mathbf{P} \hat{\mathbf{D}}_\gamma (\mathbf{I} + \hat{\mathbf{D}}_\gamma^T \mathbf{P} \hat{\mathbf{D}}_\gamma) \hat{\mathbf{D}}_\gamma^T \mathbf{P} \\ \hat{\mathbf{D}}_\gamma = & \gamma^{-1} \hat{\mathbf{D}} \end{aligned} \quad (9)$$

It is to be noted that $\gamma \in \mathbf{R}^+$, such that $\| \mathbf{T}_{\mathbf{q}_c}(z) \| \geq \gamma$ where $\| \mathbf{T}_{\mathbf{q}_c}(z) \|_\infty$ is the H^∞ -norm of the proper stable discrete transfer function $\mathbf{T}_{\mathbf{q}_c}(z)$, from sampled-data external disturbances $\mathbf{q}(kT_0) \in \ell_2^d$ to sampled-data controlled output $\mathbf{y}_c(kT_0)$.

Once matrix \mathbf{F} is obtained the MROC matrices \mathbf{L}_γ and \mathbf{L}_u (in the case where \mathbf{L}_u is free), can be computed according to the

following relations

$$\mathbf{L}_\gamma = [\mathbf{F} \quad \mathbf{0}_{m \times d}] \tilde{\mathbf{H}} + \Lambda \left(\mathbf{I}_{N^* \times N^*} - [\mathbf{H} \quad \Theta_q] \tilde{\mathbf{H}} \right) \quad (10)$$

$$\mathbf{L}_u = \left\{ [\mathbf{F} \quad \mathbf{0}_{m \times d}] \tilde{\mathbf{H}} + \Lambda \left(\mathbf{I}_{N^* \times N^*} - [\mathbf{H} \quad \Theta_q] \tilde{\mathbf{H}} \right) \right\} \Theta_u \quad (11)$$

Where,

$\tilde{\mathbf{H}}[\mathbf{H} \quad \Theta_q] = \mathbf{I}$ and $\Lambda \in \mathbf{R}^{m \times N^*}$ is an arbitrary specified matrix. In the case where $\mathbf{L}_u = \mathbf{L}_{u,sp}$, we have

$$\mathbf{L}_\gamma = [\mathbf{F} \quad \mathbf{L}_{u,sp} \quad \mathbf{0}_{m \times d}] \hat{\mathbf{H}} + \Sigma \left(\mathbf{I}_{N^* \times N^*} - [\mathbf{H} \quad \Theta_u \quad \Theta_q] \hat{\mathbf{H}} \right) \quad (12)$$

where $\hat{\mathbf{H}}[\mathbf{H} \quad \Theta_u \quad \Theta_q] = \mathbf{I}$ and $\Sigma \in \mathbf{R}^{m \times N^*}$ is arbitrary. The explicit expressions for \mathbf{H} , $\tilde{\mathbf{H}}$, $\hat{\mathbf{H}}$, Λ , Θ_u and Θ_q are given in [4, 6].

The resulting closed-loop system matrix ($\mathbf{A}_{cl/d}$) takes the following general form

$$\mathbf{A}_{cl/d} = \mathbf{A}_{ol/d} - \mathbf{B}_{ol/d} \mathbf{F} \quad (13)$$

where **cl** = closed-loop, **ol** = open-loop and **d** = discrete.

3. Open and Closed Loop Model of Power System Under Study

In the present work the power system under study (shown in Figure 2) consists of a 1 MVA wind turbine generator [9-10], which was extrapolated from that of the 100kW unit for the NASA-Lewis Research Center [11], supplying an infinite bus through a transmission line (i.e. it refers to the plant of Figure 1).

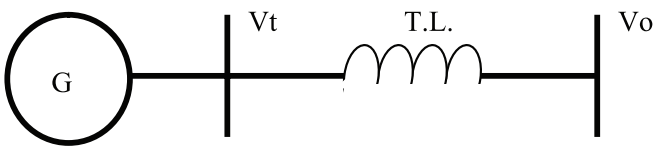


Figure 2. Schematic diagram of 1 MVA wind turbine generator

The detailed equations characterizing the mechanical dynamics, generator and excitor systems and blade pitch controls, leading to the state space formulation, are taken from [9]. Its continuous description in the form of Eqn. 1 is as follows

$$\dot{x} = [\delta \quad \omega \quad E'_q \quad E_{fd} \quad v_R \quad v_3 \quad \theta_1 \quad \theta_2]^T,$$

$$u = [v_{ref.}]^T, \quad q = u, \quad y_m = [\delta \quad \omega]^T,$$

$$y_c = x, \quad \mathbf{E} = \mathbf{I}_{8 \times 8}, \quad \mathbf{J}_1 = \mathbf{0}_{2 \times 2}, \quad \mathbf{J}_2 = \mathbf{0}_{8 \times 8}$$

The computed discrete linear open-loop power system model, based on the associated linearized continuous open-loop system model described in Appendix 2 of [9], is given below in terms

of its matrices with sampling period $T_0 = 1.0$ sec.

$$\mathbf{A}_{ol/d} = \begin{bmatrix} 0.0010 & -0.1100 & 0.0905 & -0.2314 & -0.1084 & 3.5993 & -0.0019 & 0.0010 \\ -0.0004 & -0.0010 & 0.0069 & 0.0009 & -0.0004 & 0.0138 & -0.0001 & 0.0 \\ 0.1332 & 2.2387 & 0.0737 & 0.1748 & 0.0870 & -2.8903 & -0.0088 & -0.0020 \\ -0.0250 & 107.0600 & -4.1567 & -0.5589 & 0.1601 & -5.3517 & 0.1462 & 0.0143 \\ -1.2283 & 121.8777 & -6.2567 & -2.7826 & -0.2696 & 8.8617 & 0.2745 & 0.0380 \\ -0.0379 & -0.9717 & -0.0076 & -0.0481 & -0.0266 & 0.8825 & 0.0020 & 0.0005 \\ -0.0589 & 4.5445 & -0.2025 & 0.0044 & 0.0105 & -0.3505 & 0.0004 & -0.0011 \\ 0.0330 & -21.9751 & -0.1481 & -0.1098 & 0.0023 & -0.0800 & 0.0763 & 0.0106 \end{bmatrix}$$

$$\mathbf{B}_{ol/d} = \begin{bmatrix} -4.7483 \\ -0.0234 \\ 3.9848 \\ 11.1723 \\ 7.6413 \\ -0.2659 \\ 0.4806 \\ 0.4304 \end{bmatrix}$$

$$\mathbf{C}_{ol/d} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{D}^T = 10^{-4} [4501 \quad -1 \quad -2600 \quad 9660 \quad 37687 \quad 729 \quad -3457 \quad 967]$$

Based on Figure 1, the H^∞ -control using MROCs (given in section 2), and the computed discrete linear open-loop model of the power system under study, and the discrete closed-loop power system model were designed considering with $\gamma = 0.08$ the feedback gain computed as:

$$f = 10^{-4} [2184 \quad -843087 \quad -477 \quad -1475 \quad 340 \quad -11369 \quad 2418 \quad 336]$$

The numerical values of the matrices referring to the discrete closed-loop power system model of the above case are not included here due to space limitations.

The magnitude of the eigenvalues of the discrete original open-loop and of the designed closed-loop power system model are shown in Table 1. By comparing the eigenvalues of the designed closed-loop power system model to those of the original open-loop power system model the resulting enhancement in dynamic system stability is judged as being remarkable.

Table 1. Magnitude of eigenvalues of discrete original open-loop and designed closed-loop power system models.

Original open-loop power system model	$ \lambda $	1.1294	1.1294	0.3450	0.1820
		0.1820	0.0087	0.0087	0.0000
Designed closed-loop power system model	$ \lambda $	0.8314	0.5634	0.5634	0.3365
		0.2069	0.0137	0.0119	0.0000

The responses of the output variables (δ , ω , E_{fd} , E'_q , θ_1 , θ_2 , v_3 , and V_R) of the original open-loop and designed closed-loop power system model for zero initial conditions and unit step input disturbance are shown in Figure 3 and Figure 10, respectively.

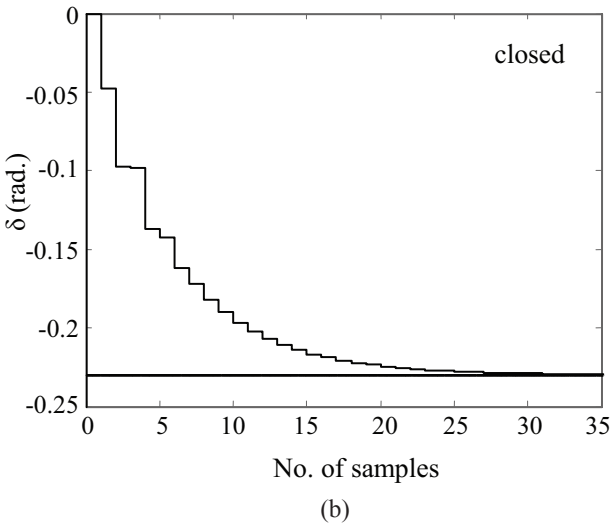
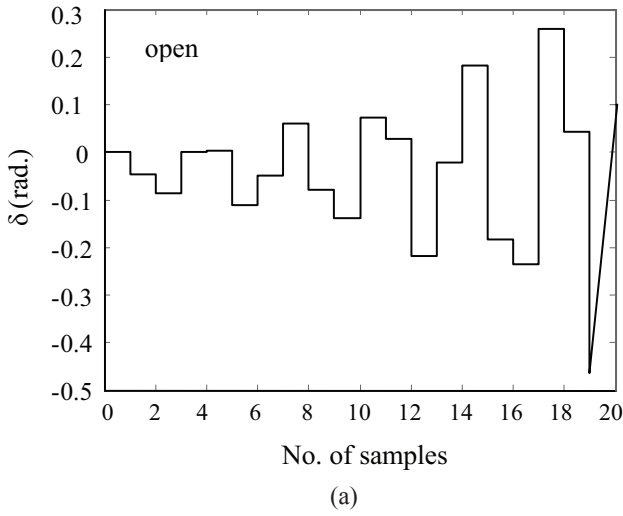


Figure 3. (a) (b): δ responses of original discrete open and close-loop power system model subject to unit step input disturbance

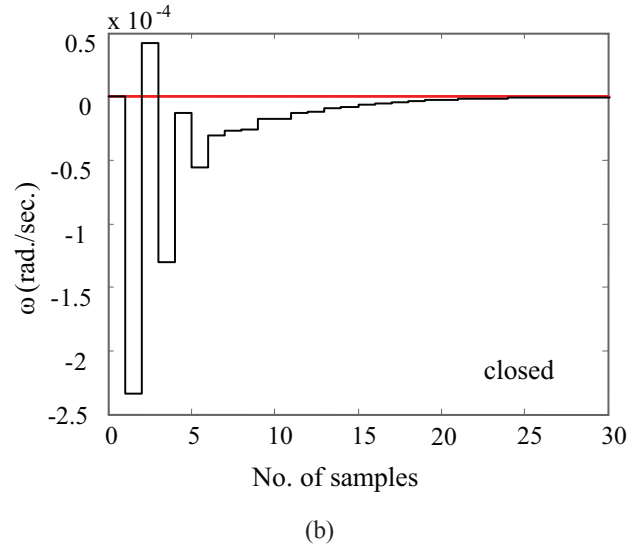
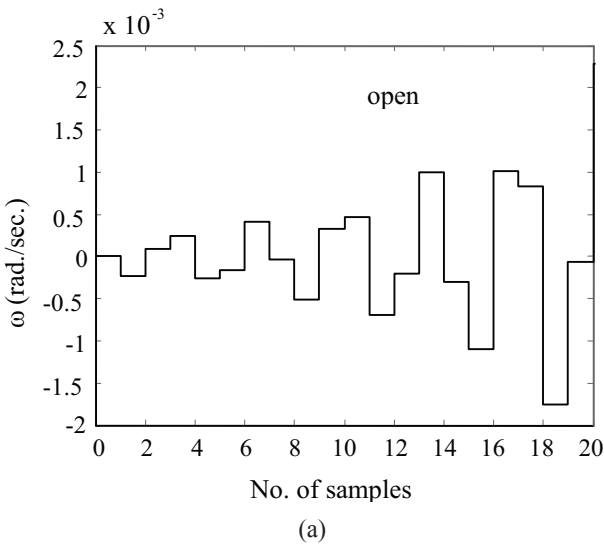


Figure 4. (a) (b): ω responses of original discrete open and close-loop power system model subject to unit step input disturbance

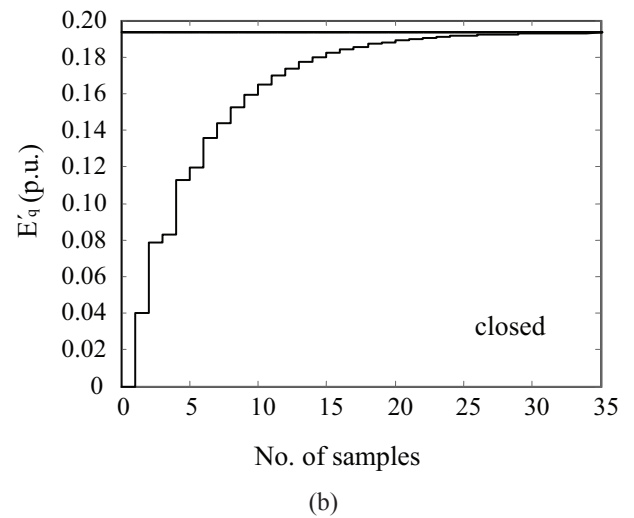
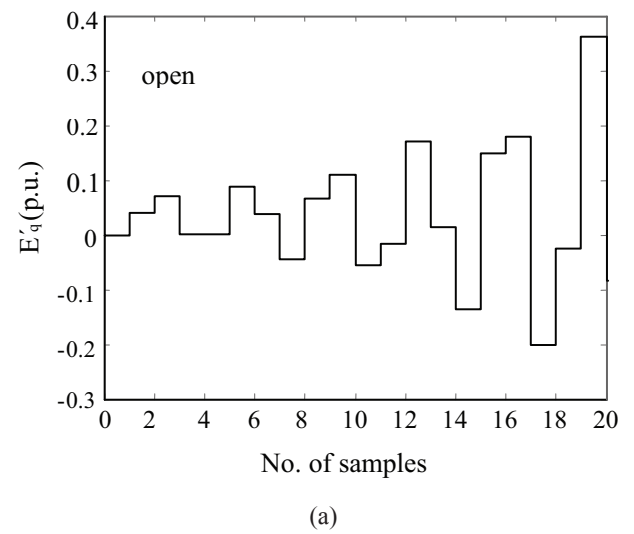
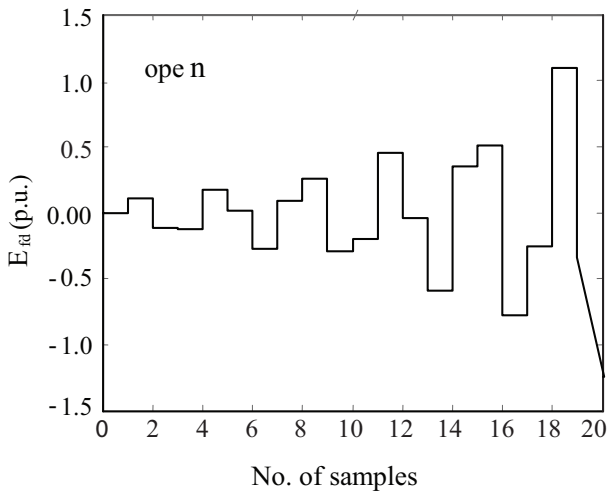
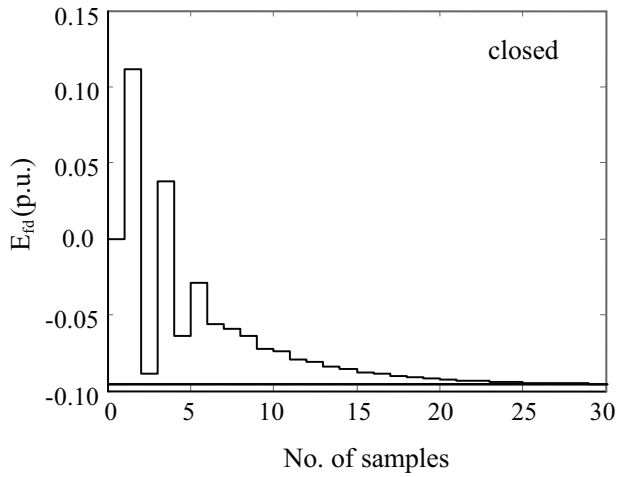


Figure 5. (a) (b): E'_q responses of original discrete open and close-loop power system model subject to unit step input disturbance

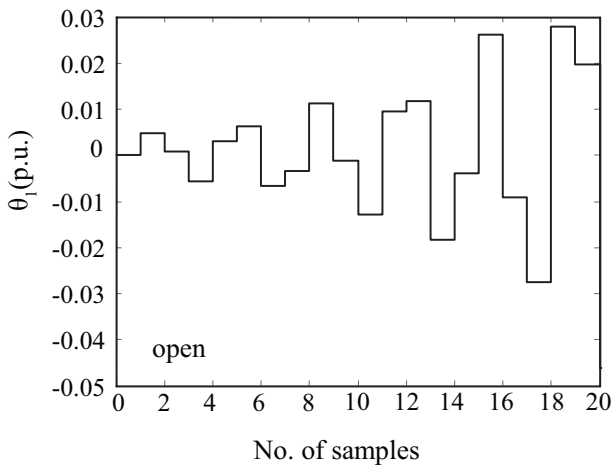


(a)

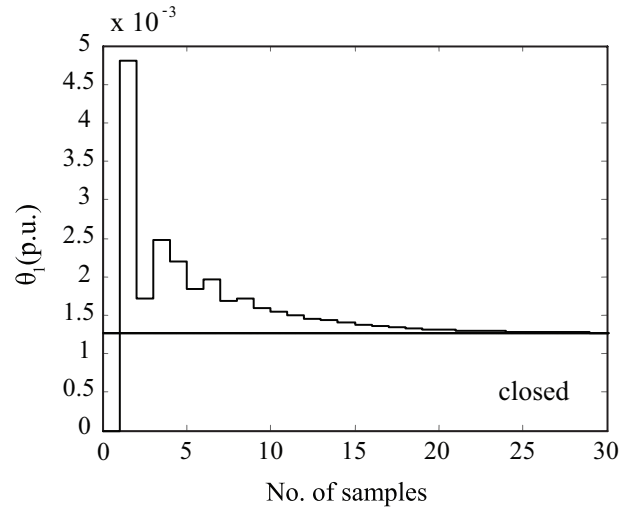


(b)

Figure 6. (a) (b): E_{ref} responses of original discrete open and close-loop power system model subject to unit step input disturbance

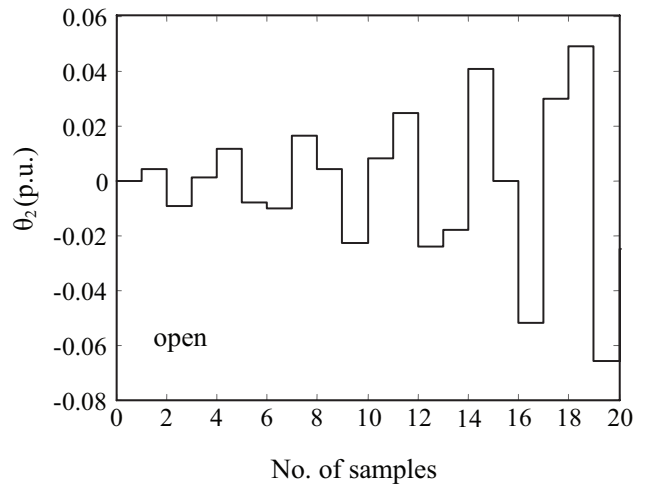


(a)

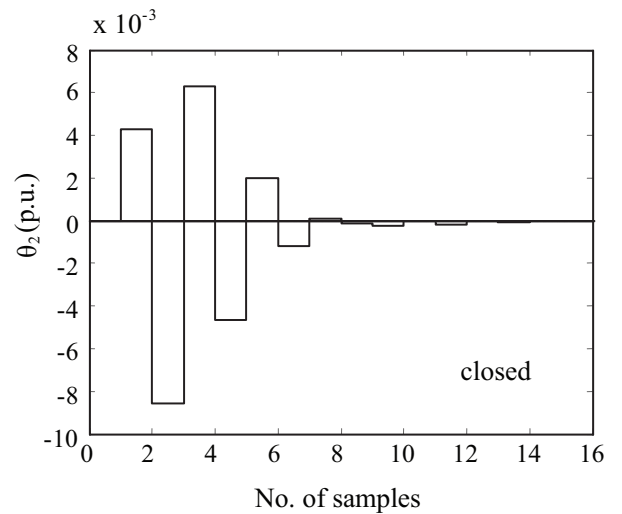


(b)

Figure 7. (a) (b): θ_1 responses of original discrete open and close-loop power system model subject to unit step input disturbance



(a)



(b)

Figure 8. (a) (b): θ_2 responses of original discrete open and close-loop power system model subject to unit step input disturbance

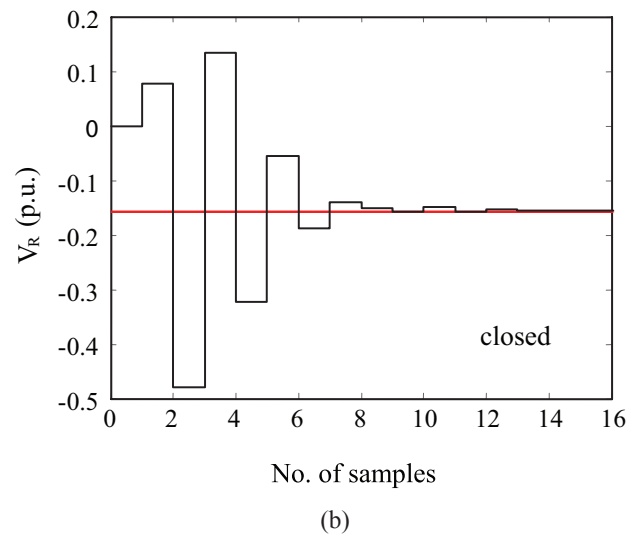
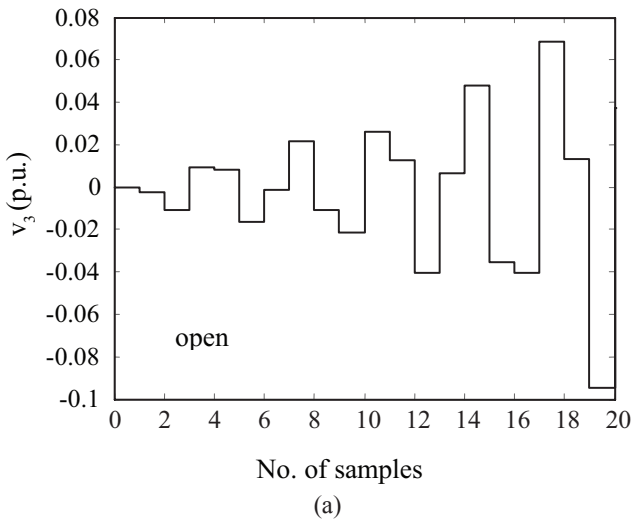
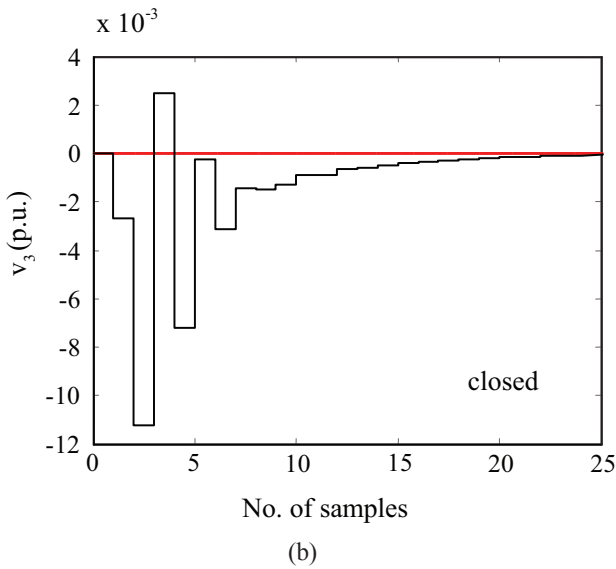


Figure 10. (a) (b): V_R responses of original discrete open and close-loop power system model subject to unit step input disturbance



From Figure 3 to Figure 10 it is clear that the dynamic stability characteristics of the designed discrete closed-loop system-model are far more superior than the corresponding ones of the original open-loop model, which attests in favour of the proposed H^∞ -control technique.

4. Conclusions

An efficient H^∞ -control technique based on multirate-output controllers has been presented in concise form for the purpose of attenuating in an effective manner system disturbances which otherwise degrade the performance of the synchronous generator. The method was applied successfully to a discrete open-loop power system model, which was computed from an original continuous linearized open-loop model of the practical power system, resulting in the design of an associated discrete closed-loop power system model. The results of the simulations performed on the discrete open- and closed- loop power system models demonstrated clearly the significant enhancement of the dynamic stability characteristics achieved by the designed closed-loop power system model. Thus the H^∞ -control technique used is an appropriate/reliable tool for the design of implementable multirate-output controllers.

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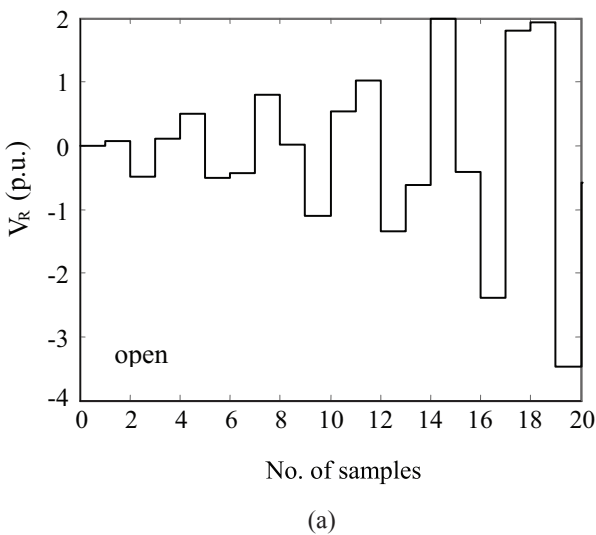


Figure 9. (a) (b): v_3 responses of original discrete open and close-loop power system model subject to unit step input disturbance

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